

# Adaptive Lowpass Filters for Zero-Crossing Detectors

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**Abstract** – Computationally efficient adaptive digital filters are introduced for 50/60 Hz line frequency signal processing in zero-crossing detectors in power converters. The purpose of the filter is to extract the fundamental sinusoidal signal from noise and impulsive disturbances so that the output is accurately in phase with the primary input signal. The proposed filters have fixed coefficients which are multiplied by adaptive parameters in order to adapt to the instantaneous line frequency. Only two adaptive parameters are needed in this method. Two linear-phase approaches are considered for the basis filters, one designed for good stopband attenuation and the other for computational simplicity.

## 1. INTRODUCTION

When designing signal processing and control for power electronics and power delivery systems, it is often desirable to attenuate superimposed harmonics, commutation notches, noise, and other disturbances without phase shifting the primary sinusoid. Such a filtering task is encountered, for example, in zero-crossing detectors, as in triac and thyristor power converters, where satisfactory line synchronization requires reliable zero-crossing information [1-3]. For a fixed mains frequency the filter design task would be straightforward. However, fixed filters are not satisfactory if considerable frequency variation is possible, because the phase shift depends on the frequency.

A fixed digital filter-based signal smoothing method was proposed in [4]. It is a cascade of a median filter and a predictive Finite Impulse Response (FIR) filter, and suppresses notch-type disturbances efficiently. For such a fixed filter, the main limiting factor is phase delay variation when the line frequency deviates from the nominal value. Therefore, adaptive filters for the task have been considered by the present authors [5-6]. A truly adaptive filtering solution based on the Widrow-Hoff LMS algorithm was described in [6] and supports even large frequency variations which are encountered, for instance, with local stand-alone generators. However, a problem with full-rank adaptive filters is the tendency to adapt to the harmonic frequencies, in addition to the principal signal, which is undesirable in the present application. Also the computational complexity in standard adaptive schemes is high, especially if long filters are needed for high selectivity.

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Previously, adaptive methods with prefiltering have been developed for line-frequency signal processing by Kamwa and Grondin [7], using an FIR prefilter. An analog adaptive approach was developed by Luo and Hou [8]. Neural network-based estimation has been also investigated for phasor detection and adaptive identification [9].

In this paper, a novel adaptive filtering approach is considered, consisting of a fixed basis filter and an adaptation scheme, called the Multiplicative General Parameters (MGP) algorithm. Linear-phase filters are used as the basis filter, having better passband and stopband characteristics than forward predicting filters [4],[5]. First, a lowpass filter-based design is presented, with design freedom in the stopband properties, targeting primarily  $\pm 2\%$  tuning range. Second, a computationally efficient algorithm using averaging filters is described. Two adaptive parameters are used in both cases for tuning. Combined with a median filter, the adaptive filters are evaluated for a test signal with considerable notching.

## 2. LINEAR-PHASE FILTERS AS SINE PREDICTORS

The prediction requirement for sine wave predictors is specified as

$$x(n+p) = \sum_{k=0}^N h(k)x(n-k), \quad (1)$$

where  $p$  is the prediction step and the  $h(k)$ 's are the filter coefficients. The signal  $x(n)$  is assumed to be a sinusoidal signal of the given nominal angular frequency  $\omega_0$ , i.e.,

$$x(n) = A \sin(\omega_0 n + \phi), \quad (2)$$

where  $A$  and  $\phi$  are arbitrary. Such predictors are most often designed [4] to minimize the white Noise Gain, given by

$$NG = \sum_{k=0}^N [h(k)]^2, \quad (3)$$

while satisfying the prediction constraint (1). At the nominal frequency, the phase shift of the resulting predictor is equal to  $M2\pi + p\omega_0$ , where  $M$  is an integer. With arbitrary design parameters, the predictor has nonlinear phase, i.e., the coefficients are not symmetrical with respect to the center of the

window. The predictor has linear phase only when  $p = -N/2$ . This value of  $p$  gives the lowest NG.

Linear-phase FIR filters have symmetric or antisymmetric impulse responses [10]. Type I and Type II linear-phase filters have a frequency response of the form

$$H(e^{j\omega}) = e^{-j\omega N/2} A(\omega), \quad (4)$$

where  $A(\omega)$  is real. Such a filter is therefore characterized by a constant phase delay equal to  $N/2$ . In order to achieve the desired prediction property, the following condition must be met:

$$\omega_0 N/2 = M2\pi - p\omega_0. \quad (5)$$

Notice that the filter coefficients are not involved in this equation. In practice, there are limitations concerning the sampling rate, which preferably should be a multiple of the line frequency. When  $p$  is given, a quick search over the practical values of the other parameters reveals if the condition can be met.

Since an adaptive technique is considered in this paper, it is not strictly necessary for the fixed part to satisfy (5) exactly. However, even in an adaptive scheme, a good match makes parameter initialization easier and maximizes the feasible tuning range. In our system, there is the precondition  $p = 2$  in order to compensate for the delays (phase shift) in the other parts of the system [4]. From several possible candidate solutions, we have chosen the parameter combination  $N = 46$ ,  $\omega_0 = 0.08\pi$ , and  $M = 1$  as the most practical. The filter coefficients are designed using standard least squares optimization such that the passband is from zero to  $0.08\pi$  and the stopband starts at  $0.24\pi$ . Thus, the third harmonic of the nominal line frequency coincides with the edge of the stopband. Here, a relatively wide transition band allows high attenuation at the stopband. Decreasing  $\omega_0$ , i.e., increasing the sampling rate would make the transition band narrower, since it is required that the third harmonic should be well attenuated.

The amplitude response of the resulting filter is shown in Fig. 1 (solid line).

### 3. THE MGP ADAPTIVE ALGORITHM

The proposed adaptive algorithm is called the multiplicative general parameter method. Only two adaptive parameters are used, providing sufficient freedom for adapting to a sine wave within the desired tuning range. The filter is divided into two blocks of lengths  $N_1 + 1$  and  $N - N_1$ , respectively, and the output is computed as

$$y(n) = g_1(n) \sum_{k=0}^{N_1} h(k)x(n-k) + g_2(n) \sum_{k=N_1+1}^N h(k)x(n-k), \quad (6)$$

where  $g_1(n)$  and  $g_2(n)$  are the adaptive MGP parameters, and the  $h(k)$ 's are the coefficients of the fixed FIR basis filter. The

coefficients of the composite filter are  $\theta(k) = g_1(n)h(k)$  for  $k = 0, 1, \dots, N_1$ , and  $\theta(k) = g_2(n)h(k)$  for  $k = N_1 + 1, \dots, N$ . The parameters  $g_1(n)$  and  $g_2(n)$  are updated according to the following equations:

$$g_1(n+1) = g_1(n) + \gamma e(n) \sum_{k=0}^{N_1} h(k)x(n-k) \quad (7)$$

$$g_2(n+1) = g_2(n) + \gamma e(n) \sum_{k=N_1+1}^N h(k)x(n-k), \quad (8)$$

where  $\gamma$  is the gain factor and  $e(n)$  is the prediction error between the reference signal and the filter output. With the nominal input signal,  $g_1(n) = g_2(n) = 1$ , and  $\theta(k) = h(k)$ ,  $k = 0, 1, \dots, N$ . When the frequency deviates from the nominal value, the parameters adapt in order to correct the prediction for the instantaneous line frequency.

The amplitude responses for  $\pm 2\%$  frequency deviation are shown in Fig. 1. Frequency deviation deteriorates stopband attenuation but preserves the lowpass nature of the filter without gain peaking on the passband. This is an advantage compared to forward predicting filters, where the gain typically peaks above 0 dB for some frequencies [4]. The noise gain is also affected, but it should be noticed that here the basis filter is not optimized with respect to NG. As the length of the basis filter of Fig. 1 is odd ( $N+1 = 47$ ), we have set  $N_1+1 = 23$ . As the lengths of the two filter blocks differ by one, the adaptive filter is not strictly linear-phase, except at the nominal frequency. This does not have any practical significance, because the primary signal is a single-frequency sinusoid, and the phase response for the other frequencies is irrelevant.

The overall filter system for the zero crossing detector is shown in Fig. 2. The signal is first median filtered, and then filtered by the MGP adaptive filter. The double delay, denoted by  $z^{-2}$ , is included in the adaptive loop in order to achieve two step-ahead prediction in the MGP filter.

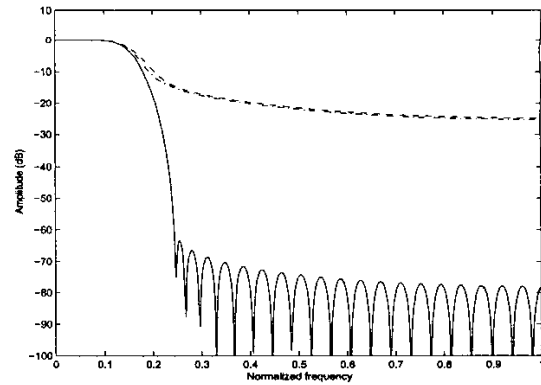


Fig. 1. Amplitude response of the adaptive filter for the nominal line frequency (solid line), 2% increased (dash-dotted), and 2% decreased frequency (dashed). The respective noise gains are 0.1452, 0.1568, and 0.1623.

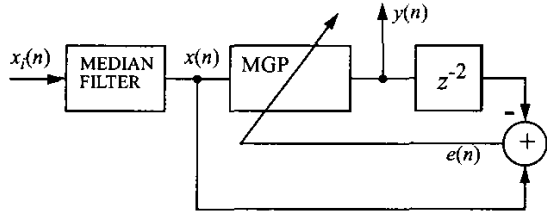


Fig. 2. Filter system for the zero crossing detector.

As a test signal, we use a voltage waveform resembling that observed by Weidenbrüg *et al.* [2] at the input terminal of a thyristor power converter, shown in Fig. 3(a). Contaminated by strong superimposed notching, the signal as such is not suitable for synchronization. The output signal is shown in Fig. 3(b). The zero crossings of the filtered waveform are seen to be clean of any notches or disturbances. With  $\gamma = 0.02$ , the total distortion of the output sine wave is -57.4 dB, while that of the input is about -16.7 dB. When the signal frequency is increased 2% above the nominal, the corresponding output distortion is -45.2 dB.

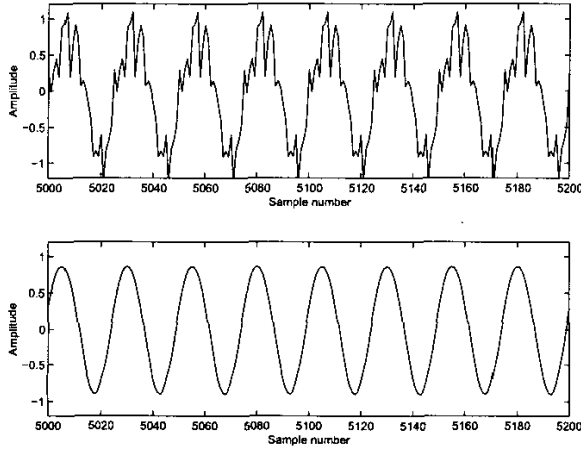


Fig. 3. Test signal input (upper plot) and output (lower).

#### 4. SIMPLIFIED BASIS FILTERS

With long filters, the phase curve is rather steep, and the basis filter should be designed to match the desired prediction characteristics. For short filters, however, we can rely more on the adaptive parameters to shape the predictor response, even if the basis filter is a less accurate approximation. Thus, computationally efficient structures can be used as basis filters. Probably the simplest structure is an averaging filter, i.e., an FIR filter with all unity-valued coefficients. Such a filter is implemented efficiently using the recursive running sum structure [11], where the arithmetic complexity does not depend on the filter length. The adaptive filter is then similar to the scheme presented in [12].

The frequency response of the averager of length  $N + 1$  is given by

$$H(e^{j\omega}) = e^{-j\omega N/2} \frac{\sin[(N+1)\omega/2]}{(N+1)\sin(\omega/2)}. \quad (9)$$

The main design consideration is that  $\omega_0$  should be located on the 'passband' of the sinc function, i.e.,  $\omega_0 < 2\pi/N$ .

As in [4], the sampling rate is set at 1.67 kHz, and therefore the 50 Hz line frequency corresponds to  $\omega_0 = 0.06\pi$ . A suitable filter length is 18, which supports  $\pm 5\%$  frequency variations without significant gain peaking. The corresponding amplitude responses are shown in Fig. 4. For the test signal, the output after median and MGP filtering has -47.0 dB distortion. Using an NG-optimized predictor of the same length as the basis filter results in -50.8 dB output distortion.

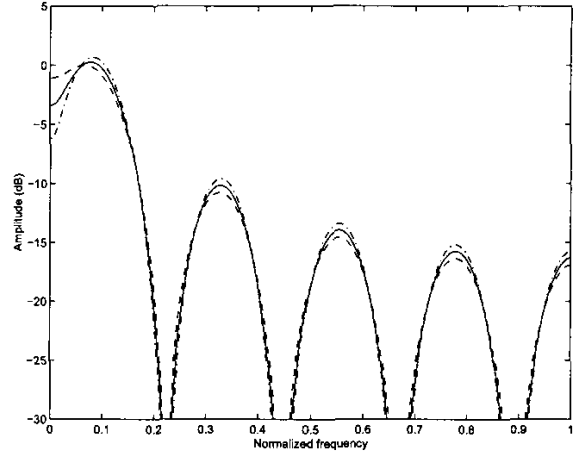


Fig. 4. Amplitude response of the adaptive filter for the nominal line frequency (solid line), 5% increased (dashed), and 5% decreased frequency (dash-dotted). The respective noise gains are 0.1316, 0.1352, and 0.1339.

#### 5. STABILITY OF THE MGP ALGORITHM

The stability condition for the MGP algorithm can be derived following a similar approach as in [12] and [13]. A necessary condition for stability is that the gain factor  $\gamma$  is limited by

$$0 < \gamma < \frac{2}{E[\mathbf{h}\mathbf{S}(n)\mathbf{x}(n)]}, \quad (10)$$

where  $E[\cdot]$  denotes the expectation value,

$$\mathbf{h} = [h(0) \ h(1) \ \dots \ h(N)],$$

$$\mathbf{x}(n) = [x(n) \ x(n-1) \ \dots \ x(n-N)]^T, \text{ and}$$

$$\mathbf{S}(n) = \begin{pmatrix} S_1(n) & 0 & \dots & 0 & 0 \\ 0 & S_1(n) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & S_2(n) & 0 \\ 0 & 0 & \dots & 0 & S_2(n) \end{pmatrix},$$

where

$$S_1(n) = \sum_{j=0}^{N_1} h(j)x(n-j),$$

and

$$S_2(n) = \sum_{j=N_1+1}^N h(j)x(n-j).$$

The matrix  $\mathbf{S}(n)$  is an  $(N+1) \times (N+1)$  diagonal matrix where the first  $N_1+1$  rows have the entry  $S_1(n)$  and the rest of the rows include  $S_2(n)$ .

## 6. CONCLUSIONS

Linear-phase and nearly linear-phase adaptive filters were proposed for line-frequency signal processing in zero crossing detectors. The MGP adaptive algorithm is a computationally efficient method for tuning the filter within the band of frequency tolerance. Guidelines for designing highly selective basis filters and, on the other hand, very simple structures were provided.

## 7. REFERENCES

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