Noise Reduction in Zero Crossing Detection by Predictive Digital Filtering

Olli Vainio, *Senior Member, IEEE,* and Seppo J. Ovaska, *Senior Member, IEEE*

***Abstract*—A simple combination of nonlinear and linear digital signal processing methods is proposed for efficient noise reduction in zero crossing detectors. The method is very robust against strong impulsive noise, typically encountered in thyristor power converters, where reliable zero crossing detection is required for firing synchronization. A systematic design procedure is described for the proposed filter-based synchronization method, taking into account the specified line frequency tolerance. The fully digital signal processing approach allows compact implementations, and supports flexible interfacing to digital motor control systems.**

I. Introduction

ACCURATE detection of true zero crossings is important in many fields of industrial electronics, particularly in thyristor power converters, where satisfactory line synchronization requires reliable zero crossing information. In practice, the 50/60 Hz sinusoid is seldom distortionless, and heavy noise that typically consists of deep and wide notches, can easily lead to harmful faulty zero crossings. This makes zero crossing detection an important topic of research and development.

A concise comparative summary of existing solutions to zero crossing noise reduction was recently published by Weidenbrüg *et al.* [1]. They classified totally six major approaches to the present problem. All those approaches have at least one of the following undesired characteristics:

- specialized instrumentation is needed;
- a phase shift is added to the primary signal;
- the method is sensitive to component parameter variations;
- the dynamic response to a line disturbance is slow;
- the calculation time represents a delay that must be compensated; or
- a large application-specific look-up table is needed.

An additional consideration is that the zero crossing signal is phase shifted with respect to the infinite bus and varies as a function of the converter firing angle and load. The adaptive approach proposed in [1] produces a reliable zero crossing signal whose phase shift is not influenced by the value of the commutation overlap angle, load changes, or frequency changes. Its implementation is an analog-digital system with a moderate complexity. Kumar *et al.* [2] reported a heuristic zero crossing detection method that is also tailored with regard to the implementation hardware. On the other hand, a practicing engineer would prefer a digital hardware-independent zero crossing detection method.

The motivation of our paper is to introduce a straightforward, efficient, and flexible digital signal processing approach to zero crossing noise reduction. Only the noise filtering of the incoming voltage signal is performed. Our method is straightforward because it contains only three cascaded filtering blocks, and needs no heuristic reasoning or decision making operations. The proposed multistage filter suppresses notch-type disturbances very efficiently, and due to its bandpass nature, it attenuates wide-band noise. We use a predictive bandpass filter because it also reduces the possible dc offset in the digitized voltage waveform. Computational efficiency is a natural consequence of the straightforward structure, and thus the proposed algorithms can be easily implemented in a low-cost microcontroller that is a standard component in modern motor drives. The functional flexibility is a result of the multistage, multi-rate, predictive filtering structure. Each individual filtering stage contributes to a single primary task, and the filter type and its design parameters are selected accordingly. A median filter is used for notch suppression, a linear bandpass predictor for overall delay compensation, and upsampling with interpolation for final resolution enhancement. We also provide the necessary parameters to construct a zero crossing detector which is compatible with the Western European line frequency tolerance, and evaluate its performance.

This paper is organized as follows. In Section II, we present the analytical design procedure of predictive FIR (Finite Impulse Response) digital filters for sinusoidal signals, and discuss the effects of frequency variations. 50 Hz nominal line frequency is assumed throughout this paper, but the method is directly applicable for 60 Hz as well, if the sampling rate is increased correspondingly. In Section III, we describe the motivation and structure of the multistage signal processing method. Section IV shows some highly encouraging results of processing a noisy test signal. Section V concludes the paper.

II. Design of Predictive FIR Filters for Sinusoidal Signals

Let us assume that the input signal, in the absence of noise, can be modeled by a sinusoid:

$$x(n) = \sin(\omega_0 n + \phi) \tag{1}$$

where ω_0 is the nominal frequency and ϕ is an arbitrary phase shift. The signals observed in industrial environments typically

Manuscript received January 21, 1994; revised May 7, 1994.

O. Vainio is with the Signal Processing Laboratory, Tampere University of Technology, FIN-33101 Tampere, Finland.

S. J. Ovaska is with the Department of Information Technology, Lappeenranta University of Technology, FIN-53851 Lappeenranta, Finland.

IEEE Log Number 9407574.

$$\lambda_0 = \frac{\frac{1}{2}\sin[\omega_0(1-p)]\sum_{k=1}^{N}[\cos[(1-k)\omega_0]\sin[(1-k)\omega_0]] - \frac{1}{2}\cos[\omega_0(1-p)]\sum_{k=1}^{N}[\sin[(1-k)\omega_0]]^2}{-\frac{1}{4}\sum_{k=1}^{N}[\cos[(1-k)\omega_0]\sin[(1-k)\omega_0]]^2 + \frac{1}{4}\sum_{k=1}^{N}[\cos[(1-k)\omega_0]]^2\sum_{k=1}^{N}[\sin[(1-k)\omega_0]]^2} \tag{8a}$$

and

$$\lambda_1 = \frac{-\frac{1}{2}\sin[\omega_0(1-p)]\sum_{k=1}^{N}[\cos[(1-k)\omega_0]]^2 + \frac{1}{2}\cos[\omega_0(1-p)]\sum_{k=1}^{N}[\cos[(1-k)\omega_0]\sin[(1-k)\omega_0]]}{-\frac{1}{4}\sum_{k=1}^{N}[\cos[(1-k)\omega_0]\sin[(1-k)\omega_0]]^2 + \frac{1}{4}\sum_{k=1}^{N}[\cos[(1-k)\omega_0]]^2\sum_{k=1}^{N}[\sin[(1-k)\omega_0]]^2}. \tag{8b}$$

consist of the primary component $x(n)$, and some additive noise that must be reduced by signal processing. We want to estimate $x(n-p)$ by $\hat{x}(n-p)$ which is obtained by filtering the input samples $x(n-k), k = 1, \ldots, N$, with a predictive FIR filter of length N, having the coefficients $h(k), k = 1, \ldots, N$. Thus the parameter p determines the length of the prediction step, for example, $p = 0$ for one-step-ahead prediction. The values of p are not limited to integers; fractional values can be used to accurately compensate for various delays in the physical implementation. The delay caused by a low-cost A/D converter is typically 10... 20 μs, which can be compensated by decreasing p by 0.017... 0.033, assuming a 600-μs sampling period.

There are two preconditions for the task of determining the coefficients $h(k)$. First, exact prediction of sinusoids of the nominal frequency is required, i.e.,

$$\sin[\omega_0(n-p)+\phi] = \sum_{k=1}^{N} h(k)\sin[\omega_0(n-k)+\phi]. \tag{2}$$

Second, the noise attenuation is optimized with respect to white noise, as this approach leads to favorable bandpass characteristics. Because the noise components in each sample are assumed to be independent, the noise power gain is given by $\sum_{k=1}^{N}[h(k)]^2$ [3]. The optimization can be carried out analytically using the method of Lagrange multipliers, which has been previously applied for the design of polynomial predictors [3], [4]. The procedure is easy to program, for example, using the Mathematica software [7].

In order to achieve correct sinusoid prediction regardless of signal phase, we require that:

$$\cos[(n-p)\omega_0] = \sum_{k=1}^{N} h(k)\cos[(n-k)\omega_0] \tag{3a}$$

$$\sin[(n-p)\omega_0] = \sum_{k=1}^{N} h(k)\sin[(n-k)\omega_0]. \tag{3b}$$

Without loss of generality, we set $n = 1$ and get the following constraints for optimization:

$$g_0 = \sum_{k=1}^{N} h(k)[\cos\omega_0\cos k\omega_0 + \sin\omega_0\sin k\omega_0] - \cos[(1-p)\omega_0] = 0 \tag{4a}$$

$$g_1 = \sum_{k=1}^{N} h(k)[\sin\omega_0\cos k\omega_0 - \cos\omega_0\sin k\omega_0] - \sin[(1-p)\omega_0] = 0. \tag{4b}$$

The Lagrange function is

$$L(h(1),\ldots,h(N),\lambda_0,\lambda_1) = \sum_{k=1}^{N}[h(k)]^2 + g_0\lambda_0 + g_1\lambda_1. \tag{5}$$

The optimal coefficients $h(k), k = 1, \ldots, N$, are found by setting the partial derivatives of $L(h(1),\ldots,h(N),\lambda_0,\lambda_1)$ with respect to all of the arguments equal to zero. As the first step, we get:

$$\frac{\partial L}{\partial h(k)} = 2h(k) + \lambda_0[\cos\omega_0\cos k\omega_0 + \sin\omega_0\sin k\omega_0] + \lambda_1[\sin\omega_0\cos k\omega_0 - \cos\omega_0\sin k\omega_0] = 0 \tag{6}$$

which is solved for the $h(k), k = 1, \ldots, N$:

$$h(k) = -\frac{1}{2}\lambda_0[\cos\omega_0\cos k\omega_0 + \sin\omega_0\sin k\omega_0] - \frac{1}{2}\lambda_1[\sin\omega_0\cos k\omega_0 - \cos\omega_0\sin k\omega_0]. \tag{7}$$

We then back substitute $h(k)$ into (4a) and (4b), and solve the pair of equations for the unknowns λ_0 and λ_1, which results in the following [see (8a) and (8b), shown at the top of the page].

The coefficients $h(k), k = 1, \ldots, N$, are then obtained from (7). The predictor is implemented so that $h(1)$ multiplies the latest input sample, etc.

Generally, the noise power gain of a predictive FIR filter depends on p and N. A long filter has narrow-band characteristics and high noise attenuation. For line frequency signals, however, we must take into account the possible frequency variation, which in Western European countries is typically less than ±2%. If the predictor is designed to have too narrow a prediction band (N is too large), then the prediction results for band edges, 50 ± 1 Hz, are erroneous although prediction at the nominal frequency of 50 Hz is accurate. Thus the design of a predictor is always a compromise between noise attenuation and bandwidth.

For the zero crossing detector, our objective is to obtain ± 100 μs timing resolution, which corresponds to 200 equally spaced samples for each full period of the 50 Hz sinusoid. Noticing that $\sin x \approx x$ in the neighborhood of zero crossings,

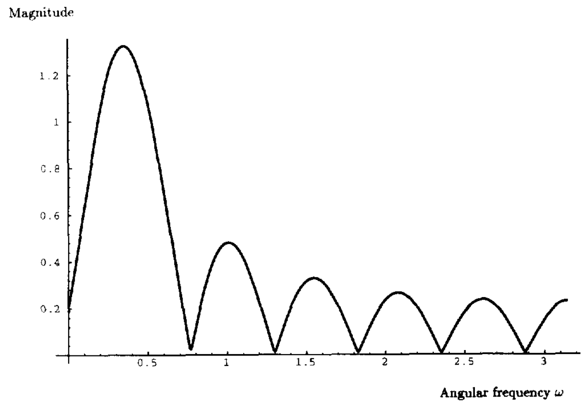


Fig. 1. Amplitude response $|H(e^{j\omega})|$ of the predictive FIR filter.

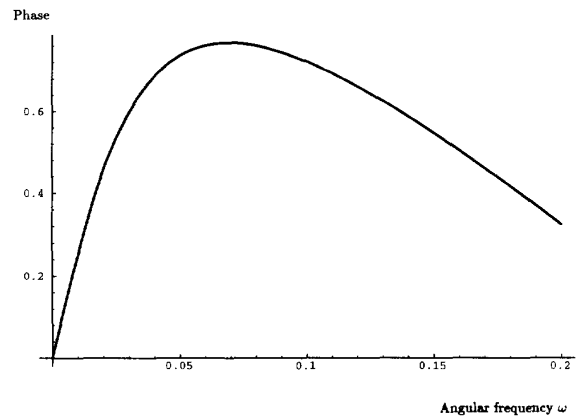


Fig. 2. Zoomed phase response $Arg[H(e^{j\omega})]$ for ω range [0, 0.2].

the timing error is approximately measured by subtracting the ideal sinusoid from the predicted waveform. Thus we specify the following limit for the prediction error of the system: *the error must be smaller than one sampling interval or $\pi/100$ rad.* For the zero crossing detector, it would be sufficient to apply this limit only at the zero crossings, but in this work we apply the specification strictly for the entire signal. This is because we expect to find applications also in signal restoration filtering.

As an example, we present a two-step-ahead predicting filter ($p = -1$) with $N = 12$, to be used as a part of the complete zero crossing detector. The input sampling rate is 1.67 kHz, and the 50 Hz line frequency corresponds to $\omega_0 = 0.06\pi$ in the discrete-time domain. The resulting transfer function is:

$$\begin{aligned} H(z) = {} & 0.225439 + 0.200691z^{-1} + 0.168833z^{-2} \\ & + 0.130994z^{-3} + 0.0885146z^{-4} + 0.0428996z^{-5} \\ & - 0.00423519z^{-6} - 0.0512199z^{-7} - 0.0963902z^{-8} \\ & - 0.138146z^{-9} - 0.175007z^{-10} - 0.205669z^{-11}. \end{aligned} \tag{9}$$

The amplitude response of the predictor is shown in Fig. 1, and the zoomed phase response is shown in Fig. 2. On the nominal frequency of 0.06π, the amplitude response is equal to unity, and the phase delay is $-0.12\pi/0.06\pi = -2$ time units, corresponding to a *forward* phase shift of two sampling periods. The noise power gain, $\sum_{k=1}^{N}[h(k)]^2 = 0.25$.

The predictive FIR filter alone would be sufficient in applications where white noise is dominant. This is not the case in industrial electronics, however, where impulsive disturbances are usually the main concern. Therefore, we need an additional nonlinear preprocessing stage to eliminate the impulses. Cascaded nonlinear and linear smoothing filters have been previously proposed for speech processing by Rabiner *et al.* [8].

III. The Multistage Filter

Although we aim at 100-μs resolution, the input sampling rate is lower than 10 kHz, for two reasons. First, we want to keep the computational load low. Second, the worst case duration of commutation disturbances in thyristor converters is about 600 μs [2]. By choosing the sample rate of 1.67 kHz (= 1/600 μs), we typically get only one severely corrupted sample from each commutation notch, among the "better" samples (which may also be noisy). The final desired resolution is obtained by interpolation, after first filtering for sufficient noise attenuation.

A block diagram of the proposed multistage filter is shown in Fig. 3. The first block is a three-point median filter which removes the disturbing impulses. The median filter is a nonlinear filter which operates by sorting the samples inside the moving filter window by magnitude, and choosing the middle value, i.e., the median as the output. The median filter is very suitable for this application, as it completely removes isolated impulses, regardless of their magnitude [5]. However, the median filter has some drawbacks: It causes a one sample delay, and it does not fully restore a sinusoidal signal after removing impulses.[1] Both of these problems are compensated by a predictive FIR filter, which is the second block of the system. With a long FIR filter we could restore the sinusoid almost completely. However, the length is limited by the requirement of tolerance to frequency variations, and we choose the filter of (9) with 12 coefficients. The filter predicts *two* steps ahead: one to compensate for the delay of the median filter, and the other to allow linear interpolation rather than extrapolation in the final processing stage. This is done after raising the sampling rate to 10 kHz. Thus the interpolator produces five new samples between every two nonzero samples [6]. The impulse response of the minimum-length linear interpolator is shown in Fig. 4(a) and the interpolation process is illustrated in Fig. 4(b)-(d). Very little error is caused by the linear approximation because the sine wave behaves almost linearly near zero crossings.

The signal timing relationships in the filter system are illustrated in Fig. 5 for 50-Hz line frequency signals. The system detects zero crossings with the specified 100-μs resolution.

[1] Generally, a median filter with window size $M = 2k + 1$ causes a delay of k samples and removes impulses up to length k.

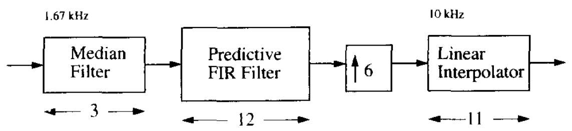


Fig. 3. Block diagram of the proposed multistage filter for zero crossing detectors. The input sampling rate is 1.67 kHz and the output sampling rate is 10 kHz.

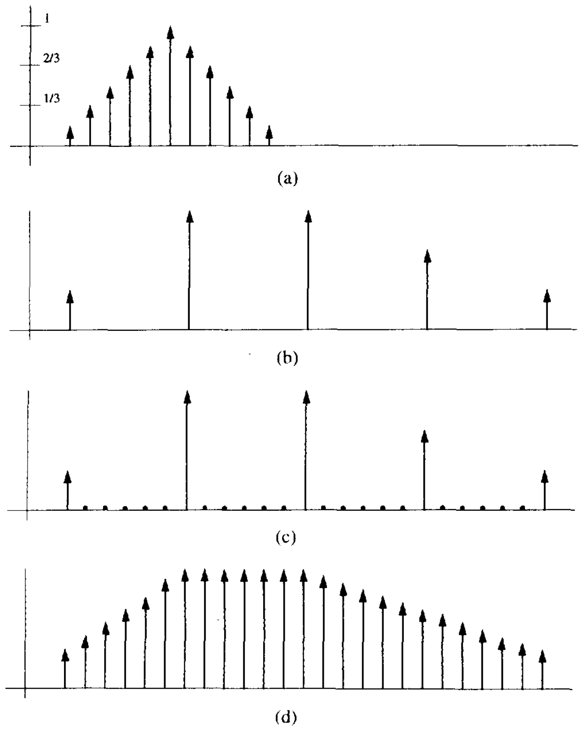


Fig. 4. (a) Impulse response of the linear interpolator for sampling rate increase by a factor of six. (b) An example of a sample sequence. (c) The sequence after increasing the sampling rate by a factor of six. (d) The result of interpolation, which is obtained as the convolution of sequences (c) and (a).

IV. Evaluation

As the median filter is nonlinear, only approximate information can be given about the noise attenuation characteristics of the system. With uniformly distributed white noise, the three-point median filter has a noise power gain of 3/5 [3]. The performance of the FIR predictor is affected by the fact that the noise spectrum has been shaped by the median filter, and thus full white noise attenuation (0.25) is not achieved. The interpolator has very little effect on the average noise power. Simulations show that the noise power gain of the entire system is about 0.33 for uniform white noise.

If the noise is impulsive, the median filter contributes to considerably higher attenuation. Such noise is often modeled by the Laplacian distribution, for which the noise power gain of the system is approximately 0.14. By construction, the system completely removes isolated impulses of length less than 600 μs, such as those observed by Kumar *et al.* [2].

For evaluation of more practical interest, we have used a replica of the test signal used recently by Weidenbrüg *et al.* in [1]. The input signal is shown in Fig. 6 and the output of the proposed signal processing system is shown in Fig. 7. Although the sinusoid is not completely restored—because of the limited FIR predictor length—the zero crossing points are free of any disturbances. This is despite the fact that the input signal is severely corrupted by strong disturbances very close to true zero crossings.

The prediction errors for 49 Hz and 51 Hz sinusoids (corresponding to $\pm 2\%$ frequency variation) are shown in

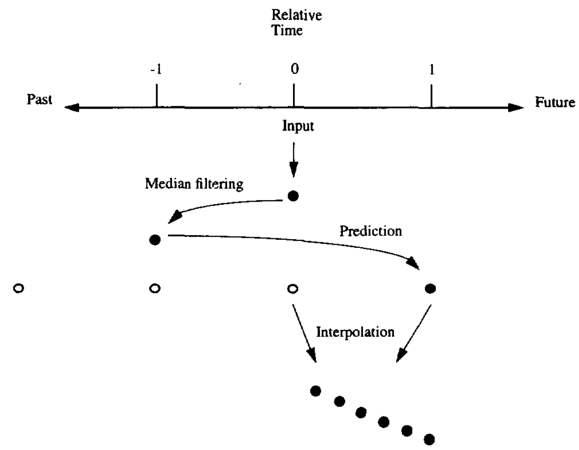


Fig. 5. Signal timing relationships in the multistage filter.

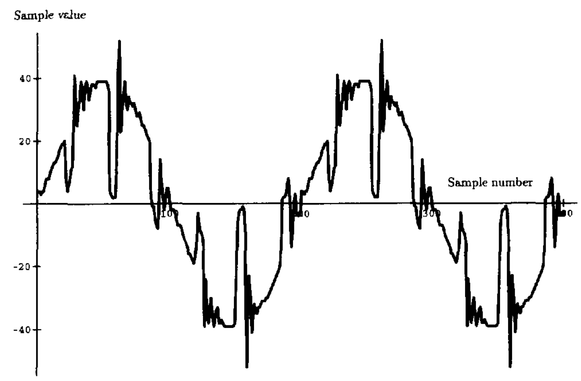


Fig. 6. Test signal input.

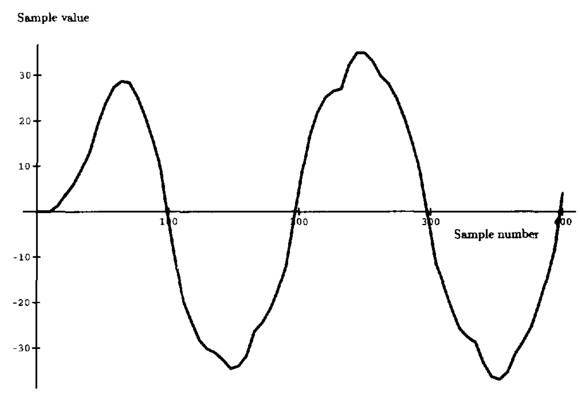


Fig. 7. System response to the test signal.

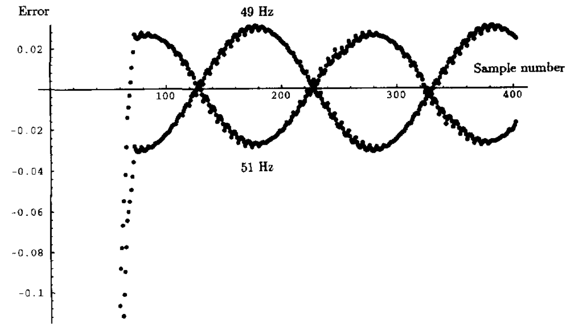


Fig. 8. Prediction error curves for 49 Hz and 51 Hz pure sinusoids.

Fig. 8. The steady-state errors are below the specified limit of ±0.0314 rad. On the nominal 50-Hz frequency, the worst case prediction error is ±0.0065 rad.

The proposed approach cannot tolerate the large frequency variations that may happen with stand-alone generator sets. When the frequency range is made wider by introducing several sinusoids to the design procedure, the wide-band noise attenuation is also reduced. However, the effective frequency range could be enhanced by replacing the sinusoidal predictor by an appropriate lowpass predictor. This is an interesting topic of future research. An analytical design procedure for least-squares optimal lowpass predictors was recently introduced by Honkanen *et al.* in [9].

V. Conclusions

A systematic and straightforward design method for digital filter based zero crossing detectors was presented. The proposed method is robust against strong impulsive disturbances, and tolerates also other kinds of noise. Satisfactory performance is achieved for the entire band of frequencies which is specified for ac power distribution systems. The proposed design can easily be adapted for different system specifications and noise characteristics.

Acknowledgment

The authors are grateful to the referees for their comments and valuable suggestions aimed at improving the paper.

References

[1] R. Weidenbrüg, F. P. Dawson, and R. Bonert, "New synchronization method for thyristor power converters to weak ac-systems," *IEEE Trans. Ind. Electron.*, vol. 40, pp. 505–511, Oct. 1993.

[2] P. P. Kumar, R. Parimelalagan, and B. Ramaswami, "A microprocessor-based dc drive control scheme using predictive synchronization," *IEEE Trans. Ind. Electron.*, vol. 40, pp. 445–452, Aug. 1993.

[3] P. Heinonen and Y. Neuvo, "FIR-median hybrid filters with predictive FIR substructures," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 36, pp. 892–899, June 1988.

[4] T. G. Campbell, "Design and implementation of image filters," Doctoral dissertation, Tampere University of Technology, Tampere, Finland, 1992.

[5] J. W. Tukey, *Exploratory Data Analysis.* Menlo Park, CA: Addison-Wesley, 1971.

[6] R. E. Crochiere and L. R. Rabiner, *Multirate Digital Signal Processing.* Englewood Cliffs, NJ: Prentice-Hall, 1983.

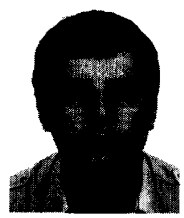
[7] S. Wolfram, *Mathematica: A System for Doing Mathematics by Computer.* Reading, MA: Addison-Wesley, 1988.

[8] L. R. Rabiner, M. R. Sambur, and C. E. Schmidt, "Applications of a nonlinear smoothing algorithm to speech processing," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 23, pp. 552–557, Dec. 1975.

[9] J. A. Honkanen, T. I. Laakso, S. J. Ovaska, and I. O. Hartimo, "Design of optimal predictors with lowpass characteristics," in *Proc. 11th European Conf. on Circuit Theory and Design,* Davos, Switzerland, Aug. 1993, pp. 515–520.

Olli Vainio (S'84–M'88–SM'94) received the Diploma Engineer and Doctor of Technology degrees in electrical engineering from the Tampere University of Technology, Tampere, Finland, in 1984 and 1988, respectively.

He has held research and teaching positions with the Tampere University of Technology and the Academy of Finland. In 1986–1987, he was a Visiting Scholar at the University of California, Santa Barbara. He now holds the position of Senior Research Fellow of the Academy of Finland. He is also Docent of Microelectronics in the Lappeenranta University of Technology. His research interests are in VLSI signal processing and microelectronics.



Seppo J. Ovaska (M'90–SM'91) received the Diploma Engineer degree in electrical engineering from the Tampere University of Technology, Tampere, Finland, in 1980, the Licentiate of Technology degree in information technology from Helsinki University of Technology, Finland, in 1987, and the Doctor of Technology degree in electrical engineering from Tampere University of Technology, in 1989.

He is presently an Associate Professor of Electronics at Lappeenranta University of Technology, Finland. He is also a Docent in Signal Processing Methods and Industrial Applications at the Helsinki University of Technology. His research interests are in digital signal processing and industrial electronics. He holds eight patents in the area of elevator instrumentation.

Dr. Ovaska is a member of the editorial review committee of the IEEE Transactions on Instrumentation and Measurement.