NEW APPARENT POWER AND POWER FACTOR WITH NON-SINUSOIDAL WAVEFORMS

F. Ghassemi, MIEEE, MIEE.
99A Fortess Road, London NW5 1AG, England
Email: fznghassemi@compuserve.com

ABSTRACT
The new concept in ac power theory is investigated for three phase systems. It will be shown that there is discrepancy in the definition of reactive and phasor (complex) power in the time and frequency domains when the conventional power theory is considered. It will also be shown that the new power theory consistently defines the power quantities in the time and frequency domains. A comparison between the results given by the new method and the other two methods, known as RMS and System or equivalent apparent power will be presented. It is shown that the new method does not posses the shortcomings of the conventional methods.

I- INTRODUCTION

The importance of phase angle difference between ac voltages and currents was first debated in 1888 [1,2]. The subsequent works by many researchers led to the introduction of power factor. This was used as a convenient figure of merit to reflect the quality of loads and utilisation of the supplying systems. Power factor (pf) was defined as the ratio of active or true power to apparent power (AP) [3], or alternatively, it can be calculated using reactive power.

All these parameters have specific definitions that are presented in standards and textbooks. System design procedures, systems control strategies and billing policies are, in one way or other, based on these quantities. These definitions have unique, clear meaning in single phase and balanced 3-phase systems. However, with increasing number of non-linear loads operating in power systems, the issues of determining the quality of such loads and their effects on supplying systems have become a major concern for industry and academia. Only active power is known to have an accepted definition for all situations [4]. The ambiguity in defining reactive power, AP and hence pf remains to be solved [3, 4, 5].

The author believes that persistence of the problem is due to the fact that nearly all new attempts for defining and measuring the basic power quantities in non-ideal situations have been based on the same concept that was originally proposed in 1888. This seemed adequate for single phase and 3-phase balanced systems (can effectively be regarded as single phase) with sinusoidal waveforms. In other words, the conventional power theory has been taken as a basis for deriving a modified set of definitions that can be used in abnormal conditions encountered increasingly in practical systems. These attempts have so far led to inconsistent and largely different definitions for power quantities when unbalanced and/or non-sinusoidal waveforms are considered.

A new concept for ac power theory was briefly introduced in [6]. This is reviewed in the following section for 3-phase systems.

Objectives of any new approach should be focused to provide a set of definitions for the measurement of power which·

a) is consistent for all system topologies and conditions.
b) covers single and polyphase systems.
c) is valid for balanced and unbalanced polyphase systems.
d) is valid for pure or distorted waveforms.
e) does not violate any principle of electrical engineering.
f) can be readily implemented in practical equipment.

It has been shown that the new concept meets the above main requirements.

A comparison between the results of the new concept and two other well-known definitions, known as RMS AP and System AP will be presented.

II- THE CONVENTIONAL POWER THEORY REVISITED

(1) and (2) give the voltage across and current through a load respectively. The instantaneous power (IP) is defined as (3). These imply a simple single phase system with sinusoidal waveform.

$$v(t) = \hat{V}\cos(\omega_1 t + \alpha) \qquad (1)$$

$$i(t) = \hat{I}\cos(\omega_1 t + \beta) \qquad (2)$$

$$p_p(t) = v(t)i(t) = \frac{\hat{V}\hat{I}}{2}\cos\varphi + \frac{\hat{V}\hat{I}}{2}\cos(2\omega_1 t + \lambda) \qquad (3)$$

where. $\varphi=\alpha-\beta$ and $\lambda=\alpha+\beta$ (4)

Considering voltage as reference and resolving the current onto the voltage as in-phase and quadrature phase components, (3) can be rearranged as (5) and its average is given by (6).

$$p_p(t) = v(t)i(t) = P + P\cos(2\omega_1 t + 2\alpha) + Q\sin(2\omega_1 t + 2\alpha) \qquad (5)$$

$$P = \frac{1}{T}\int_0^T v(t)i(t)dt = \frac{\hat{V}\hat{I}}{2}\cos\varphi \qquad (6)$$

where T is the period of the signals. The standard active and reactive powers are defined as given by (7.a) and (7.b).

$$P \overset{\Delta}{=} \frac{\hat{V}\hat{I}}{2}\cos\varphi = VI\cos\varphi \qquad (7.a)$$

$$Q \overset{\Delta}{=} \frac{\hat{V}\hat{I}}{2}\sin\varphi = VI\sin\varphi \qquad (7.b)$$

V and I are the rms values of voltage and current respectively.

Note that reactive power is defined, in the time domain, as the magnitude of one of the oscillatory components. AP is defined as the product of the rms values of voltage and current as shown below by (8). Equation (8) is the standard relationship between AP and active and reactive powers.

$$S = VI = \sqrt{(VI\cos\varphi)^2 + (VI\sin\varphi)^2} = \sqrt{P^2 + Q^2} \qquad (8)$$

In normal electrical engineering practices, such as load flow studies, phasor or complex power defined by (9) is considered.

$$\vec{S} = \vec{V}\vec{I}^* = P + jQ = |\vec{S}|\angle\varphi \qquad (9)$$

where $|\vec{S}|$ is considered to be AP in single and balanced 3-phase systems and also $\vec{V}$ and $\vec{I}$ are the voltage and current phasor respectively. (9) is a frequency domain expression as it is written in terms of phasor quantities. Considering complex power, it is assumed that P and Q have the same nature, i.e. average values, as only quantities with the same frequency can be shown as phasor in the form illustrated in (9). This contradicts the basic definition of reactive power in the time domain that is defined as the amplitude of an ac term. Note that although reactive power is defined as the magnitude of one of the oscillatory terms of instantaneous power but conventionally, it is measured as an average value. This is usually achieved by phase shifting the voltage signal by -90° and

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multiplying it by the current signal. Integration of the product over one period gives the mean value, which is Q as given by (7.b) [7].

In two ways the frequency spectrum of power relationship, Equation (3), can be obtained. The first is to identify the frequency components of voltage and current and to perform convolution in the frequency domain. This is equivalent to the multiplication of voltage and current in the time domain. The second is to calculate IP as given by (3), in the time domain and to perform the Fourier transform. Both methods must give the same result.

The Fourier transform of any signal is defined as in (10).

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad (10)$$

By performing the Fourier transform on (1) and (2), the frequency spectrum of voltage and current are determined.

$$V(f) = \frac{1}{2}\left[\hat{V}e^{+j\alpha}\delta(f-f_1) + \hat{V}e^{-j\alpha}\delta(f+f_1)\right] \qquad (11.a)$$

$$I(f) = \frac{1}{2}\left[\hat{I}e^{+j\beta}\delta(f-f_1) + \hat{I}e^{-j\beta}\delta(f+f_1)\right] \qquad (11.b)$$

where δ is the Dirac function.

The following relationship exists between time and frequency domain quantities. $(\cdot)$ and $(*)$ denote multiplication and convolution respectively

$$v(t) \cdot i(t) \leftrightarrow V(f) * I(f) \qquad (12)$$

Convoluting the voltage and current spectrum reveals that the power spectrum will have a value at dc, which is active power and ac components at $2f_1$ and $-2f_1$. Active power is calculated from (13.a) or (13.b).

$$P = V(-f_1)I(f_1) + V(f_1)I(-f_1) = V^*(f_1)I(f_1) + V(f_1)I^*(f_1) \qquad (13.a)$$

$$P = \sum_{f=-\infty}^{\infty} V(f)I^*(f) = \sum_{-f_1}^{f_1} V(f)I^*(f) \qquad (13.b)$$

where superscript • denotes the phasor conjugate. The power spectrum is shown in Fig 1.

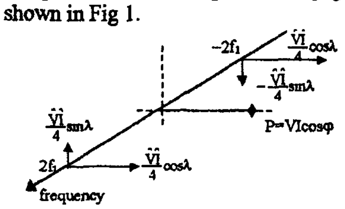

Fig 1- Frequency Spectrum of $P_p(f)$

The power frequency spectrum does not provide any information about reactive power, Q. Note that the ac terms are functions of λ only. If Fourier transform is applied on (3), the same result as shown in Fig 1 is obtained. This is again in contrast to the time domain definition of reactive power.

The power in any two signals, $v(t)$ and $i(t)$ may be found in terms of either a time or frequency description based on Plancherel's relation. Plancherel's relation is defined as in (14).

$$\frac{1}{T}\int_0^T v(t)i^*(t)dt = \sum_{-\infty}^{\infty} V(f)I^*(f) \qquad (14)$$

L.H.S of (14) is the average power that is given by (6). R.H.S of (14) is equal to that calculated by the convolution theorem in frequency domain that is described by (13.b). Thus Plancherel's relation can be considered as a basis for defining the power in ac circuits. The definition of phasor or complex power in the frequency domain clearly violates Plancherel's relationship. This can be deduced by comparing (9) and (14). In other words, (9) is not analytically valid.

Thus it can be said that the artificial decomposition of the instantaneous power into dc and ac terms does not reflect the actual physical phenomena of the circuit.

The main disadvantage with all proposed methods for calculating AP is the fact that all depends on the voltage of reference point at which measurement is made. In single phase circuits the rms values of the load voltage and current are clearly defined. In 3-phase systems the current, active and reactive powers flow do not depend on the reference voltage [4]. But such a voltage will affect the value of the conventional apparent power.

The emphasis, in recent years, has been on two main definitions for AP. These are 'rms' or 'arithmetic' and 'system' or 'equivalent' AP. These are defined in Appendix A. Appendix A shows that RMS and System APs give similar results if the load is balanced irrespective of waveform distortion.

It is clearly evident that both apparent powers are dependent on the reference voltage [3, 4, 6]. Thus, apparent power for a load may not be unique if the reference point of voltage measurement changes. This is critical when a load that is a combination of several 3-phase loads with different kind of connections are considered. One requirement for the conventional AP measurement is the definition of reference point [8].

The use of RMS AP would indicate a pf of unity for pure resistive load weather or not the load is balanced. This is now viewed as unacceptable result as it is well known that unbalanced loads cause extra losses in the supplying systems and this must be reflected in pf [3].

The use of System (Sys) AP in a system with a broken line indicates two different APs that depend on the position of the voltage measuring devices with respect to the broken point. If the voltage-measuring device is on the source side of the broken line the voltage of that phase will be determined by the source voltage and if it is on the load side then it will be influenced by the load impedance and the current in other two lines. Hence, Sys AP gives different values for the same load.

III- REVIEW OF THE NEW CONCEPT

III.1- Complex Instantaneous Power

Assume that (15) and (16) give the voltages and currents for an ideal 3-phase source respectively.

$$e_k(t) = \hat{E}\cos(\omega_1 t + \alpha_k)\Big|_{k=a,b,c} \qquad (15)$$

$$i_k(t) = \sum_{h=1}^{\infty} \hat{I}_{kh}\cos(\omega_h t + \beta_{kh})\Big|_{k=a,b,c} \qquad (16)$$

where: $\alpha_a=0°$, $\alpha_b=-120°$, $\alpha_c=120°$ and k and h denote phase number and harmonic order respectively. (15) can be regarded as to give the back emf inside a generator. Note that the source can only generate the positive phase sequence (pps), fundamental frequency component (ffc) voltage. The current has been assumed distorted, implying that the source is supplying a non-linear load.

The frequency spectrum of each phase voltage is given by (11.a) where 'E' substitutes 'V'. The current spectrum is given below;

$$I_k(f) = \frac{1}{2}\left[\hat{I}_{kh}e^{j\beta_{kh}}\delta(f-f_h) + \hat{I}_{kh}e^{-j\beta_{kh}}\delta(f+f_h)\right]\Big|_{k=a,b,c} \qquad (17)$$

According to the new technique, the voltages must be phase shifted by $-90°$ to produce the quadrature voltages. Thus;

$$e_{qk}(t) = \hat{E}\cos(\omega_1 t + \alpha_k - \tfrac{\pi}{2})\Big|_{k=a,b,c} = \hat{E}\sin(\omega_1 t + \alpha_k)\Big|_{k=a,b,c} \qquad (18)$$

The complex voltage signals are defined as follows;

$$\vec{e}_{mk}(t) = e_{pk}(t) + je_{qk}(t)\Big|_{k=a,b,c} = \hat{E}e^{j(\omega_1 t + \alpha_{k1})}\Big|_{k=a,b,c} \qquad (19)$$

where e_{pk} is given by (15). Subscript 'p' has been used to denote the real voltage. By the new definition, the complex instantaneous power (CIP) is defined from the product of the voltage given by (19) and the current signals on the per phase basis.

$$\vec{p}(t) = \sum_{k=a,b,c} \vec{p}_k(t) = \sum_{k=a,b,c}\left(\vec{e}_{mk}(t)\sum_{h=1}^{\infty} i_{kh}(t)\right) \qquad (20)$$

$$\vec{p}(t) = \sum_{k=a,b,c} EI_{k1} e^{j\varphi_{k1}} + \sum_{k=a,b,c} EI_{k1} e^{j(2\omega_1 t+\lambda_{k1})} + \sum_{k=a,b,c} \sum_{h=2}^{\infty} \vec{e}_{mk} i_{kh}^{*}$$

$$\vec{p}(t) = \sum_{k=a,b,c} EI_{k1} e^{j\varphi_{k1}} + \sum_{k=a,b,c} EI_{k1} e^{j(2\omega_1 t+\lambda_{k1})} + \sum_{k=a,b,c} \sum_{h=2}^{\infty} EI_{kh} e^{j(\omega_{1h} t+\varphi_{k1h})} + \sum_{k=a,b,c} \sum_{h=2}^{\infty} EI_{kh} e^{j(\omega^{1h} t+\lambda_{k1h})} \tag{21}$$

where: E and I with subscripts denote rms values, and

φ_{k1}=phase angle between ffc voltage and current $=\alpha_{k1}-\beta_{k1}$

$\lambda_{k1}=\alpha_{k1}+\beta_{k1}$

$\omega_{1h}=\omega_1-\omega_h$, $\omega^{1h}=\omega_1+\omega_h$ for $h\neq 1$

$\varphi_{k1h}=\alpha_{k1}-\beta_{kh}$, $\lambda_{k1h}=\alpha_{k1}+\beta_{kh}$ for $h\neq 1$

Note that $\vec{e}_{mk}(t)$ and $\vec{p}(t)$ are the time domain complex signals.

Note also that CIP has distinct positive (+ve) and negative (−ve) frequency components. This means that the product of different frequencies in the current signal with the ffc voltage will produce ω_{1h} and ω^{1h} that are unique.

It can be seen that active and reactive powers are specified as the real and imaginary mean values of CIP. These are given below,

$$P = \mathrm{Real}\left(\frac{1}{T_1}\int_0^{T_1} \vec{p}(t)dt\right) = \frac{1}{T_1}\int_0^{T_1}\left[\sum_{k=a,b,c}\left(e_{pk}(t)\sum_{h=1}^{\infty} i_{kh}(t)\right)\right]$$
$$= \sum_{k=a,b,c} EI_{k1}\cos\varphi_{k1} = \sum_{k=a,b,c} P_{k1} \tag{22.a}$$

$$Q = \mathrm{Imag}\left(\frac{1}{T_1}\int_0^{T_1} \vec{p}(t)dt\right) = \frac{1}{T_1}\int_0^{T_1}\left[\sum_{k=a,b,c}\left(e_{qk}(t)\sum_{h=1}^{\infty} i_{kh}(t)\right)\right]$$
$$= \sum_{k=a,b,c} E_k I_{k1}\sin\varphi_{k1} = \sum_{k=a,b,c} Q_{k1} \tag{22.b}$$

Note that (22.a) and (22.b) are active and reactive powers of the ideal source that are of the pps, ffc nature only.

The power spectrum can be obtained by convoluting the voltage and current spectra or by performing Fourier transform on (21). The power spectrum is shown in Fig 2, where the frequency components are the sum of three phases and S_{hf1} are the magnitude of each component given as follows;

$$S_0 = \left|\sum_{k=a,b,c} EI_{k1} e^{j\varphi_{k1}}\right| = \sqrt{P^2+Q^2} \tag{23.a}$$

$$S_{2f1} = \left|\sum_{k=a,b,c} EI_{k1} e^{j\lambda_{k1}}\right| \tag{23.b}$$

$$\underset{h=-1\to-\infty}{S_{-hf1}} = \left|\sum_{k=a,b,c} EI_{kh} e^{j\varphi_{k1h}}\right| \tag{24.a}$$

$$\underset{h=3\to\infty}{S_{hf1}} = \left|\sum_{k=a,b,c} EI_{kh} e^{j\lambda_{k1h}}\right| \tag{24.b}$$

The significance of the component at dc, S_0, has already been mentioned. Note that the conventional quantity known as phasor or complex power can be easily identified on the spectrum as the component at zero frequency as shown below,

$$S(0) = P + jQ = \sum_{k=a,b,c} P_{k1} + j\sum_{k=a,b,c} Q_{k1} = S_0\angle\varphi_0 \tag{25}$$

where: S_0 is given by (23.a) and $\varphi_0=\cos^{-1}(P/S_0)=\tan^{-1}(Q/p)$.

This will satisfy Plancherel's relationship, as the average value in the time domain is equivalent to the same quantity in the frequency domain with both active and reactive powers present.

The component at $2f_1$ is present even if the current signals are pure sinusoid. This term is zero only if the system is balanced, since;

$I_{a1}=I_{b1}=I_{c1}=I_1$ and $\beta_{a1}=\beta_1$, $\beta_{b1}=-120+\beta_1$, $\beta_{c1}=120+\beta_1$ thus;

$\lambda_{a1}=\beta_1$, $\lambda_{b1}=-120-120+\beta_1=120+\beta_1$, $\lambda_{c1}=+120+120+\beta_1=-120+\beta_1$

(24.a) and (24.b) give the magnitude of other frequency components. These are present when the currents are distorted.

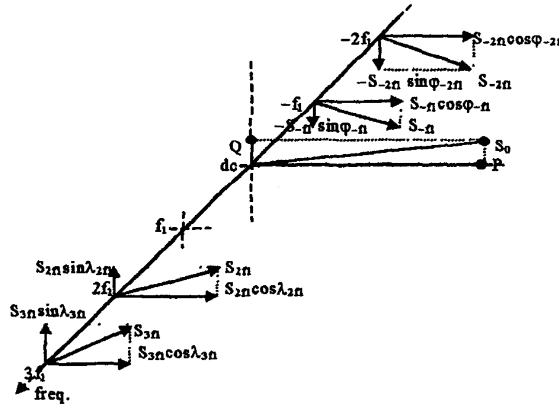

Fig 2- Power Spectrum for the Ideal Source Using the New Concept

III.2- The New Apparent Power

2-norm or rms of a signal give the size of that signal The 2-norm or rms of any periodic signal is defined as in (26).

$$X = \sqrt{\frac{1}{T}\int_0^T |x(t)|^2 dt} \tag{26}$$

Note that for generalisation of the relationship the magnitude of signals is considered to include complex signals [9]. The new AP is defined as the 2-norm or rms value of the CIP [6].

$$S = \sqrt{\frac{1}{T}\int_0^T |\vec{p}(t)|^2 dt} \quad \text{in the time domain} \tag{27.a}$$

$$S = \sqrt{\sum_{w=-\infty}^{\infty} S_w^2} \quad \text{in the frequency domain} \tag{27.b}$$

where S_w are obtained from (23.a) to (24.b). For the calculation of S in the time domain see Appendix B. S can be decomposed into different components according to their nature as follows;

$$S = \sqrt{S_s^2 + S_u^2 + D^2} \tag{28}$$

where:

$$S_s = S_0 = \sqrt{P^2+Q^2} = E\left[I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + 2I_{1a}I_{b1}\cos(\varphi_{a1}-\varphi_{b1}) + 2I_{b1}I_{c1}\cos(\varphi_{b1}-\varphi_{c1}) + 2I_{c1}I_{a1}\cos(\varphi_{c1}-\varphi_{a1})\right]^{\frac{1}{2}} \tag{29.a}$$

$$S_u = S_{2f1} = \left|\sum_{k=a,b,c}\left(\frac{\hat{E}\hat{I}_{k1}}{2}\cos\lambda_{k1} + j\frac{\hat{E}\hat{I}_{k1}}{2}\sin\lambda_{k1}\right)\right|$$
$$= E\left[I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + 2I_{a1}I_{b1}\cos(\lambda_{a1}-\lambda_{b1}) + 2I_{b1}I_{c1}\cos(\lambda_{b1}-\lambda_{c1}) + 2I_{c1}I_{a1}\cos(\lambda_{c1}-\lambda_{a1})\right]^{\frac{1}{2}} \tag{29 b}$$

$$D = E\sqrt{\sum_{h=2}^{\infty}\left|\sum_{k=a,b,c} I_{kh}e^{j\varphi_{k1h}}\right|^2 + \sum_{h=2}^{\infty}\left|\sum_{k=a,b,c} I_{kh}e^{j\lambda_{k1h}}\right|^2} \tag{29.c}$$

S_s , S_u and D are called symmetrical, unsymmetrical and distortion powers respectively. S_u is present whenever there is unbalanced in the circuit and D is present when the waveforms are distorted. Note that for balanced 3-phase systems without waveform distortion phasor or complex power and AP are given as below,

$$\vec{S} = 3P_{1\,phase} + j3Q_{1\,phase} = 3S_{0\text{-}1\,phase}\angle\varphi_0 \tag{30}$$

$$S = 3EI_1 \tag{31}$$

(31) implies that the New AP for balanced 3-phase systems with pure waveforms is the same as the conventional definition. In this case all frequency components other than dc are zero.

Power factor is defined as the ratio of active power to AP. Thus,

$$pf = \frac{P}{S} = \frac{\sum_{k=a,b,c} P_{k1}}{\sqrt{S_s^2+S_u^2+D^2}} \tag{32}$$

Note that pf can only be unity if all following conditions are met;

a) The system is balanced.
b) The waveforms are not distorted.
c) The value on the imaginary dc axis is zero.

III.3- Power Parameters at the Load Terminal

The voltage drop across the system impedance, assuming a symmetrical impedance matrix at all frequencies is given by (33).

$$\begin{bmatrix}\Delta v_a(t)\\ \Delta v_b(t)\\ \Delta v_c(t)\end{bmatrix}=\begin{bmatrix}z_{sh} & z_{mh} & z_{mh}\\ z_{mh} & z_{sh} & z_{mh}\\ z_{mh} & z_{mh} & z_{sh}\end{bmatrix}\begin{bmatrix}\sum_{h=1}^{\infty} i_{ah}(t)\\ \sum_{h=1}^{\infty} i_{bh}(t)\\ \sum_{h=1}^{\infty} i_{ch}(t)\end{bmatrix} \tag{33}$$

where:

$$z_{sh}=R_{sh}+L_{sh}\frac{d}{dt}+\frac{1}{C_{sh}}\int dt \text{ and } z_{mh}=R_{mh}+L_{mh}\frac{d}{dt}+\frac{1}{C_{mh}}\int dt$$

R, L and C are the line resistance, inductance and capacitance respectively. Subscript 's' and 'm' denote system self and mutual impedances and the differentiation and integration operators operate on currents.

The voltage at the load terminal is given by (34);

$$v_k(t)=e_k(t)-\sum_{h=1}^{\infty}\Delta v_{kh}(t)\Big|_{k=a,b,c} \tag{34}$$

Appendix C shows that the voltages are given by (35);

$$v_k(t)=\hat{V}_{k1}\cos(\omega_1 t+\alpha_{k1})-\sum_{h=2}^{\infty}\Delta v_{kh}(t)\Big|_{k=a,b,c} \tag{35}$$

For amplitudes and angles see Appendix C. (36.a) and (36.b) give the complex voltages for the system impedance and load terminal respectively.

$$\Delta\vec{v}_{mk}(t)=\sum_{h=1}^{\infty}\Delta\hat{V}_{kh}e^{j(\omega_h t+\alpha_{\Delta kh})} \tag{36.a}$$

$$\vec{v}_{mk}(t)=\sum_{h=1}^{\infty}\hat{V}_{kh}e^{j(\omega_h t+\alpha_{kh})} \tag{36.b}$$

The corresponding CIP are given below;

$$\begin{aligned}\Delta\vec{p}(t)&=\sum_{k=a,b,c}\Delta\vec{v}_{mk}(t)i_k(t)\\ &=\sum_{k=a,b,c}[\Delta\hat{V}_{k1}\hat{I}_{k1}e^{j(\omega_1 t+\alpha_{\Delta k1})}\cos(\omega_1 t+\beta_{k1})\\ &\quad+\sum_{h=2}^{\infty}\Delta\hat{V}_{k1}\hat{I}_{kh}e^{j(\omega_1 t+\alpha_{\Delta k1})}\cos(\omega_h t+\beta_{kh})\\ &\quad+\sum_{h=2}^{\infty}\Delta\hat{V}_{kh}\hat{I}_{kh}e^{j(\omega_h t+\alpha_{\Delta kh})}\cos(\omega_h t+\beta_{kh})\\ &\quad+\sum_{n=2}^{\infty}\sum_{\substack{m=1\\ m\neq n}}^{\infty}\Delta\hat{V}_{kn}\hat{I}_{km}e^{j(\omega_n t+\alpha_{\Delta kn})}\cos(\omega_m t+\beta_{kn})]\end{aligned} \tag{37.a}$$

$$\begin{aligned}\vec{p}(t)&=\sum_{k=a,b,c}\vec{v}_{mk}(t)i_k(t)=\sum_{k=a,b,c}[\hat{V}_{k1}\hat{I}_{k1}e^{j(\omega_1 t+\alpha_{k1})}\cos(\omega_1 t+\beta_{k1})\\ &\quad+\sum_{h=2}^{\infty}\hat{V}_{k1}\hat{I}_{kh}e^{j(\omega_1 t+\alpha_{k1})}\cos(\omega_h t+\beta_{kh})\\ &\quad-\sum_{h=2}^{\infty}\Delta\hat{V}_{kh}\hat{I}_{kh}e^{j(\omega_h t+\alpha_{\Delta kh})}\cos(\omega_h t+\beta_{kh})\\ &\quad-\sum_{n=2}^{\infty}\sum_{\substack{m=1\\ m\neq n}}^{\infty}\Delta\hat{V}_{kn}\hat{I}_{km}e^{j(\omega_n t+\alpha_{\Delta kn})}\cos(\omega_m t+\beta_{kn})]\end{aligned} \tag{37.b}$$

It can be seen that those terms of the CIPs that do not involve the ffc voltages have equal magnitudes and opposite signs. This implies that the non-linear load actually supplies these components of the CIP to the system impedance. The sources of these components of CIP are those components that source can produce. As was explained, the source can only produce pps, ffc voltage. (21) clearly shows that all power components in the source CIP are produced by this voltage. Thus, any other circuit requirement must be met by the source of the pollution, which produces distortion. The analysis proving the effect of the load pps voltage is lengthy and not possible to be presented here but will be reported in another paper. Hence, there exists a difference between the power components that are delivered by the supplying system, characterised by the power components at the ideal source given by (21), and those that non-linear loads use. The difference is injected back to the system to meet the system requirement [6].

The CIP in terms of phase co-ordinate quantities and distorted waveforms, depicting the load consumption, excluding the power components that are injected back to the system is given below;

$$\begin{aligned}\vec{p}(t)=\sum_{k=a,b,c}[&\sum_{h=1}^{\infty}V_{kh}I_{kh}e^{j\varphi_{kh}}+\sum_{h=1}^{\infty}V_{kh}I_{kh}e^{j(2\omega_{kh}t+\lambda_{kh})}+\\ &\sum_{n=1}^{\infty}\sum_{\substack{m=1\\ m\neq n}}^{\infty}V_{kn}I_{km}e^{j(\omega_{nm}t+\varphi_{knm})}+\sum_{n=1}^{\infty}\sum_{\substack{m=1\\ m\neq n}}^{\infty}V_{kn}I_{km}e^{j(\omega^{nm}t+\lambda_{knm})}]\end{aligned} \tag{38}$$

where: $\varphi_{kh}=\alpha_{kh}-\beta_{kh}$, $\lambda_{kh}=\alpha_{kh}+\beta_{kh}$, $\varphi_{knm}=\alpha_{kn}-\beta_{km}$, $\lambda_{knm}=\alpha_{kn}+\beta_{km}$

$\omega_{nm}=\omega_n-\omega_m$, $\omega^{nm}=\omega_n+\omega_m$

Note that the average imaginary of (38) is the Budeanu's reactive power [4]. In that definition the total reactive power was defined as the sum of the magnitude of ac components with different frequencies. That was not acceptable to many researchers [5]. However, with the new concept, reactive power is deduced as the sum of dc terms. This does not present any analytical problem. The real average value of (38) is the active power consumed by the load. Thus;

$$P+jQ=\sum_{k=a,b,c}\sum_{h=1}^{\infty}V_hI_h\cos\varphi_h+j\sum_{k=a,b,c}\sum_{h=1}^{\infty}V_hI_h\sin\varphi_h \tag{39}$$

The frequency of the ac terms in (38), depends on the frequency spectra of the voltages and currents. It is clear that the product of different frequency components of voltage and current in time domain (or convolution in frequency domain) may result in the formation of ac components with equal frequencies. Thus in general, combining the ac components of CIP with equal frequencies, CIP at the load terminal can be written as follows;

$$\vec{p}(t)=\sum_{k=a,b,c}[\sum_{h=1}^{\infty}V_hI_he^{j\varphi_h}+\sum_{w=-\infty}^{\infty}S_we^{j(\Omega_w t+\theta_w)}] \tag{40}$$

where: S_w= magnitude of the ac terms for different frequencies

θ_w= angle of the ac terms

AP is thus given by (41).

$$S=\sqrt{P^2+Q^2+\sum_{\substack{w=-\infty\\ w\neq 0}}^{\infty}S_w^2} \tag{41}$$

where P and Q are given by (39). pf is equal to P/S. Thus using the load itself active power given by (39) and AP described by (41), pf can be calculated.

If impedance matrix in (33) can be assumes to be approximately symmetrical for all frequencies, then it is possible to calculate power quantities related to the power components that the system delivers to the load. This is achieved by considering pps, ffc voltage at the load. Thus, the load pps, ffc voltage is given by (15) where $e(t)$ and $\hat{E}$ are replaced by $v^+(t)$ and $\hat{V}_1^+$ respectively. The angles also change;

$$\alpha_a^+=\alpha^+,\alpha_b^+=-120+\alpha^+,\alpha_c^+=+120+\alpha^+$$

pps angle α^+ is measured with respect to phase 'a' of the source CIP is then defined by (21) where E is replaced by V_1^+, which is the rms value of pps, ffc voltage at the load terminal. The angles are also appropriately substituted.

The delivered power quantities have the same form as those obtained for the ideal source Since the delivered power parameters reflect the quality of the power that is taken from the source, it is more desirable from the system point of view that these quantities are used for system studies and billing purposes. The following remarks can be made;

1. For non-linear loads, delivered active and reactive powers are larger than the load consumed active and reactive powers.
2. For linear loads in parallel with the non-linear loads, delivered active and reactive powers are smaller than the load consumed active and reactive powers. This is because, the harmonic non-pps active and reactive powers are added to the delivered powers.
3. Any other non-pps, non-ffc CIP components are supplied by the delivered CIP components with the same frequency.
4. If by the virtue of filtering the spectrum of the current at the load terminal changes, some of the CIP components are supplied by the filter(s). Only pps, ffc active power will be unchanged and is still supplied by the source, if the ffc load current is unchanged.

IV- COMPARISON BETWEEN DIFFERENT DEFINITIONS

Comparison between RMS, System and the new concept AP and pf is presented in this section. A circuit containing phase-controlled thyristors was simulated using ATP (EMTP) package. The system data is shown in table 1.

Source Symmetrical SCC 8.94 pu	Source $\frac{X_1^+}{R_1^+}=2,\ \frac{Z_1^o}{Z_1^+}=2$

Table 1- Simulated System Data

superscript 'o' denotes zero phase sequence (zps) impedance.

Table 2 gives the description of the circuit and the measuring quantities in each test. The reference point for voltage measurement in tests A and B is the grounded star point of the load. Artificial neutral was built in the circuit, for tests X, D and F, by using three high resistance (10^5 pu) star connected resistors at the load terminal. The star point of the unit was used as the artificial reference point.

Table 3, 4 and 5 illustrate the test results. All quantities are in pu. Firing angle zero denotes full conduction and an angle of larger than zero reflects progressively less conduction. Pps, ffc active and reactive power are measured according to (22.a) and (22.b) where the pps, ffc voltage at the load terminal is used. P and Q in Table 3 are those used by the load itself.

The following observations can be made from the test results;

I. All three methods give the same results for balanced system without distortion, test X. pf for all three is unity. As the load is balanced resistive, no reactive power is injected back to the system. P and P_1^+ are equal.

II. Test A, E and F indicate that for balanced load RMS and Sys AP give the same result even if distortion is present. See Appendix A.

III. The effect of the reference point voltage on the RMS and Sys AP measurement is evident from tests C and D. The AP by the new method is unaffected as the reference point changes.

IV. It can be seen, in pps, ffc voltage pf test that the New and Sys methods give similar results if the system is a 3-wire system. It can be shown that the new AP is calculated from (42).

$$S = 3V_1^+\sqrt{\frac{1}{3}\left(\sum_{h=1}^{\infty}I_{ah}^2+\sum_{h=1}^{\infty}I_{bh}^2+\sum_{h=1}^{\infty}I_{ch}^2\right)-\sum_{h=1}^{\infty}I_h^{o\,2}} \tag{42}$$

For 3-wire system the zps current is zero then (42) conforms to that of equivalent current used in Sys AP [3].

V. For pure resistive load without distortion, RMS method gives unity pf irrespective of system symmetry, tests B and C. The non-unity pf in test D for RMS method is due to the change in reference point.

VI. Since in unbalanced and distorted waveforms situation, the powers taken from the system is larger than the powers consumed by the load and also it can be said that non-linear and/or unbalanced loads have detrimental effects on the system, this effects must be reflected by AP and pf. The New method reflects this by calculating a lower pf for pps, ffc voltage pf. RMS and Sys pf do not have this characteristic. Sys pf actually give a higher pf when pps, ffc voltage pf is considered.

Test ID	System & Measurement Configuration
X	Delta conn., phase V's & I's, measu. wrt artificial neutral
A	Earthed, star conn., phase V's & I's
B	Earthed. star conn., phase V's & I's
C	Unearthed star conn, phase V's & I's, measu. wrt load neutral
D	Unearthed star conn., phase V's & I's, measu. wrt artificial neutral
E	Delta conn., line V's & phase I's
F	Delta conn., phase V's measu. wrt artificial neutral & Line I's,

Table 2- Test and Measurement Configuration

Test	Load Impedance (pu)			Firing	P(pu)	Q(pu)	P_1^+ (pu)	Q_1^+ (pu)
ID	Z_a	Z_b	Z_c	Angle				
X	1+j0	1+j0	1+j0	0.0	2.1229	.0	2.1229	.0
A	2+j0	2+j0	2+j0	72°	.4310	.0694	.4322	.0767
B	2+j0	∞	∞	0.0	.1554	.0	.1593	.0078
C	1+j0	2+j0	2.5+j0	0.0	.5431	.0	.5440	.0017
D	1+j0	2+j0	2.5+j0	0.0	.5431	.0	.5440	.0017
E	1+j0.5	1+j0.5	1+j0.5	144°	.0283	.2104	.0296	.2240
F	1+j0.5	1+j0.5	1+j0.5	144°	.0283	.2104	.0296	.2240

Table 3- Test Data and Active and Reactive Powers

Test	Apparent Power (pu)			PPS, ffc Voltage Apparent Power(pu)		
ID	New	RMS	Sys	New	RMS	Sys
X	2.1229	2.1229	2.1229	2.1229	2.1229	2.1229
A	.4426	.4523	.4523	.4444	.4501	.4501
B	.2198	.1554	.2765	.2256	.1595	.2763
C	.5570	.5431	.5823	.5587	.5520	.5587
D	.5570	.5516	.5587	.5587	.5520	.5587
E	.2507	.2723	.2723	.2708	.2708	.2708
F	.2507	.2723	.2723	.2708	.2708	.2708

Table 4- Measured Apparent Power

Test	Power Factor			PPS, ffc Voltage Power Factor		
ID	New	RMS	Sys	New	RMS	Sys
X	1.0	1.0	1.0	1.0	1.0	1.0
A	.974	.953	.953	.973	.960	.960
B	.707	1.0	.562	.706	.999	.577
C	.975	1.0	.933	.974	.986	.974
D	.975	.985	.972	.974	.986	.974
E	.113	.104	.104	.109	.109	.109
F	.113	.104	.104	.109	.109	.109

Table 5- Measured Power Factor

V- CONCLUSSIONS

It was shown that there is a contradiction between the definition for reactive powers and complex or phasor powers in the time and frequency domain, when the conventional power theory is considered. Reactive power as defined in the time domain cannot be determined in the frequency domain when the spectra of voltage and current are convoluted. Phasor or complex power defined in the frequency domain cannot be determined the time domain without violating Plancherel's theorem.

A technique was proposed and examined here, which does not violate any rule of electrical engineering. In this method, reactive power is determined as an average value on the imaginary axis both in the time and frequency domains. Thus it is in quadrature to active power. Phasor or complex power is determined as the mean value of the complex instantaneous power (CIP) in the time domain or the dc value on the power spectrum.

Apparent power is defined as the 2-norm or rms value of the CIP. This technique can be used without any reservation or assumption for any circuit topologies and condition from single to three phase. Here only 3-phase systems were considered. It was shown that the technique is independent of the reference point voltage at which measurement is made. A comparison between the new technique and other two well-known methods was presented.

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VIII- APPENDICES

VIII.1- Appendix A

RMS and System (Sys) AP are defined as follows;

$$S_{rms} = \sum_{k=a,b,c} V_k I_k \tag{A1}$$

where;

$$V_k = \left.\sqrt{\sum_{h=1}^{\infty} V_{kh}^2}\right|_{k=a,b,c}, \quad I_k = \left.\sqrt{\sum_{h=1}^{\infty} I_{kh}^2}\right|_{k=a,b,c} \tag{A2}$$

$$S_{sys} = 3 V_{sys} I_{sys} \tag{A3}$$

$$I_{sys} = \sqrt{\frac{1}{3} \sum_{k=a,b,c} \sum_{h=1}^{\infty} I_{kh}^2} \tag{A4}$$

Four wire system

$$V_{sys} = \sqrt{\frac{1}{3} \sum_{k=a,b,c} \sum_{h=1}^{\infty} V_{kh}^2} \tag{A5.a}$$

Three wire systems

$$V_{sys} = \sqrt{\frac{1}{9}\left[\sum_{h=1}^{\infty} V_{abh}^2 + \sum_{h=1}^{\infty} V_{bch}^2 + \sum_{h=1}^{\infty} V_{cah}^2\right]} \tag{A5.b}$$

where k and h denote phase number and harmonic order. V and I with subscript denote rms values of each harmonic of each phase.

Considering 4-wire systems, Sys AP can be written as follows;

$$S_{sys} = \sqrt{\sum_{h=1}^{\infty} V_{ah}^2 + \sum_{h=1}^{\infty} V_{bh}^2 + \sum_{h=1}^{\infty} V_{ch}^2}\sqrt{\sum_{h=1}^{\infty} I_{ah}^2 + \sum_{h=1}^{\infty} I_{bh}^2 + \sum_{h=1}^{\infty} I_{ch}^2} \tag{A6}$$

If the system is balanced then the rms value of each harmonic voltage and current in each phase are equal, thus (A6) can be written as follows;

$$S_{sys} = \sqrt{\sum_{h=1}^{\infty} V_h^2 + \sum_{h=1}^{\infty} V_h^2 + \sum_{h=1}^{\infty} V_h^2}\sqrt{\sum_{h=1}^{\infty} I_h^2 + \sum_{h=1}^{\infty} I_h^2 + \sum_{h=1}^{\infty} I_h^2} \tag{A7.a}$$

$$S_{sys} = \sqrt{3\sum_{h=1}^{\infty} V_h^2}\sqrt{3\sum_{h=1}^{\infty} I_h^2} = 3\sqrt{\sum_{h=1}^{\infty} V_h^2}\sqrt{\sum_{h=1}^{\infty} I_h^2} \tag{A7.b}$$

It is clear that RMS AP also conforms to the same relationship if the system is balanced. The same proof can be used for 3-wire systems.

VIII.2- Appendix B

CIP in terms of its real and imaginary components is given below;

$$\vec{p}(t) = p_p(t) + j\, p_q(t) \tag{B1}$$

where:

$$p_p(t) = \sum_{k=a,b,c}\left(e_{pk}(t)\sum_{h=1}^{\infty} i_{kh}(t)\right) \;\&\; p_q(t) = \sum_{k=a,b,c}\left(e_{qk}(t)\sum_{h=1}^{\infty} i_{kh}(t)\right)$$

AP is defined as 2-norm or rms as follows;

$$S = \sqrt{\frac{1}{T}\int_0^T |\vec{p}(t)|^2 dt} = \sqrt{\frac{1}{T}\int_0^T \vec{p}(t)\vec{p}^*(t) dt} \tag{B2}$$

$$S = \sqrt{\frac{1}{T}\int_0^T [p_p(t) + j\, p_q(t)][p_p(t) - j\, p_q(t)] dt} \tag{B3}$$

$$S = \sqrt{\frac{1}{T}\int_0^T [p_p^2(t) + p_q^2(t)] dt} = \sqrt{\frac{1}{T}\int_0^T p_p^2(t) dt + \frac{1}{T}\int_0^T p_q^2(t) dt} \tag{B4}$$

From (B4), it can be seen that S is obtained from the rms of the real and quadrature components as shown below;

$$S = \sqrt{\{\mathrm{rms}[p_p^2(t)]\}^2 + \{\mathrm{rms}[p_q^2(t)]\}^2} \tag{B5}$$

The implementation of (B5) is a standard rms calculation of two signals. Prior to the rms calculation, the voltage signal phase shifting and production of instantaneous power components must be carried out.

VIII.3- Appendix C

The phase voltages at the load terminal are given by (C1).

$$v_k(t) = e_k(t) - \Delta v_{k1}(t) - \left.\sum_{h=2}^{\infty} \Delta v_{kh}(t)\right|_{k=a,b,c} \tag{C1}$$

Since the source can only produce ffc voltage, the harmonic components of the load terminal voltages will be equal in magnitude to the system impedance voltage drop but opposite in sign. Thus;

$$\Delta v_{kh}(t) = \left.\left.\Delta\hat{V}_{kh}\cos(\omega_h t + \alpha_{\Delta kh})\right|_{h=1}^{\infty}\right|_{k=a,b,c} \tag{C2}$$

$$v_k(t) = \hat{V}_{k1}\cos(\omega_1 t + \alpha_{k1}) - \left.\sum_{h=2}^{\infty} \Delta v_{kh}(t)\right|_{k=a,b,c} \tag{C3.a}$$

$$v_k(t) = \hat{V}_{k1}\cos(\omega_1 t + \alpha_{k1}) + \sum_{h=2}^{\infty} \hat{V}_{kh}\cos(\omega_h t + \alpha_{kh}) \tag{C3.b}$$

where :

$$\hat{V}_{k1} = \left.\mathrm{Mag}\left[e_k(t) - \Delta v_{k1}(t)\right]\right|_{k=a,b,c} = \left.\left|e_k(t) - \Delta v_{k1}(t)\right|\right|_{k=a,b,c} \tag{C4.a}$$

$$\alpha_{k1} = \left.\mathrm{Angle}\left[e_k(t) - \Delta v_{k1}(t)\right]\right|_{k=a,b,c} \tag{C4.b}$$

$$\hat{V}_{kh} = \left.\Delta\hat{V}_{kh}\right|_{h=2}^{\infty} \tag{C5}$$

$$\alpha_{kh} = \left.\alpha_{k\Delta h} + \pi\right|_{h=2}^{\infty} \tag{C6}$$