Midterm-Project

in Dynamical Systems

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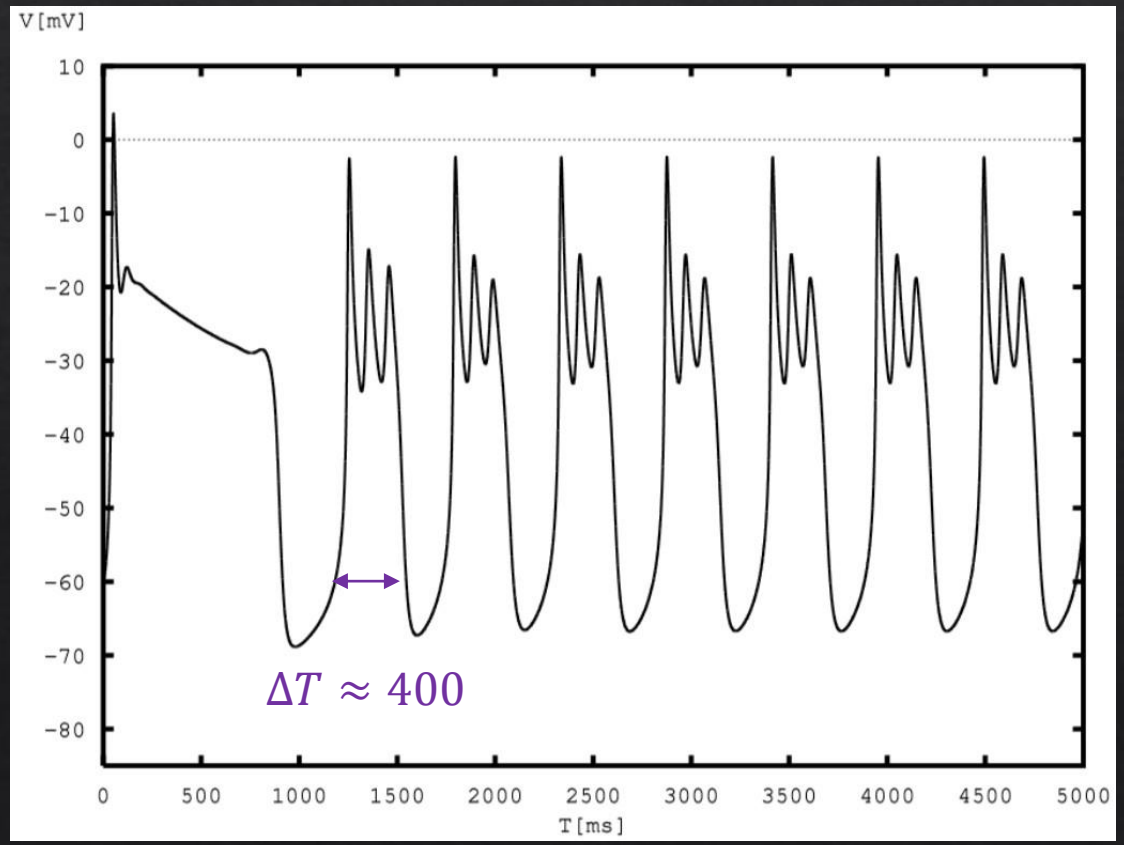
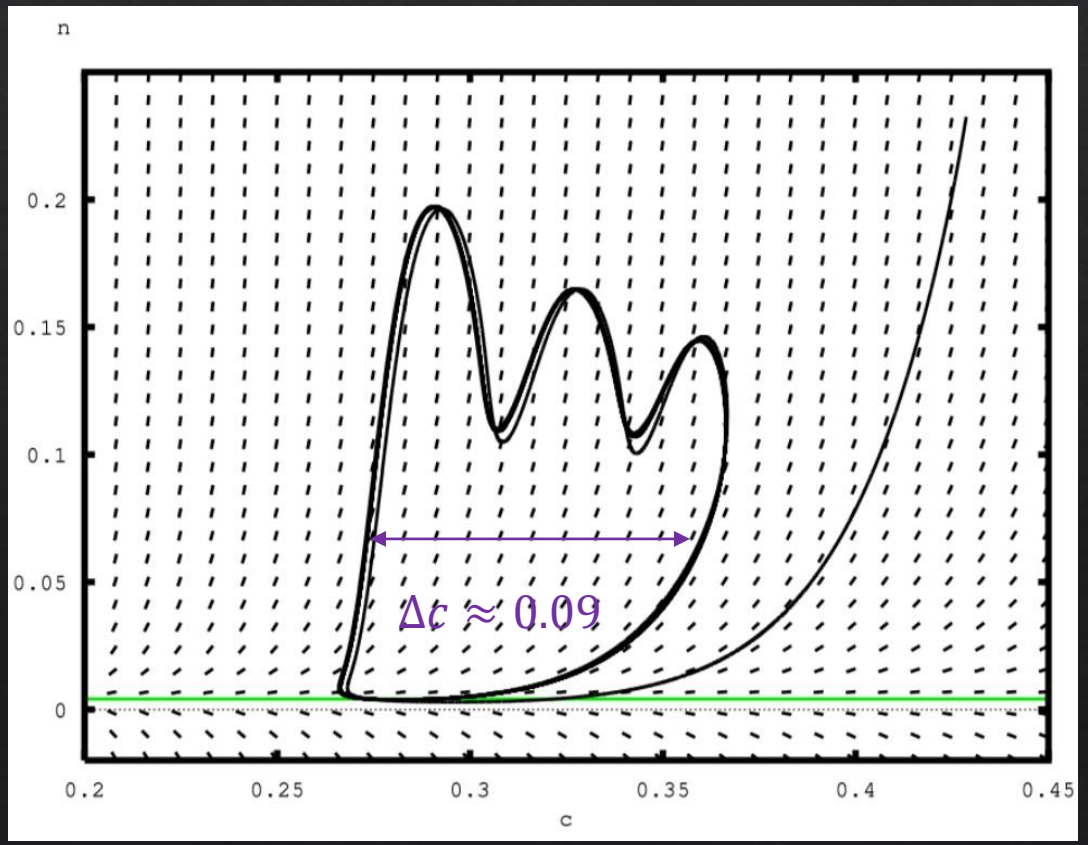
Direction

My general direction is analyzing the parameters that affect the plateau-firing:





Dynamic 1 – The plateau

In order to analyze the plateau-firing, first we need to find what causes the plateau. In order to do that, let us look at the dynamics of V:

$$V' = -\frac{I_{ca} + I_k + I_{kca} + I_{bk}}{C_m}$$

$$I_{ca} = G_{ca} m_\infty(V)(V - V_{ca}) \ , \ I_k = G_k n(V - V_k)$$

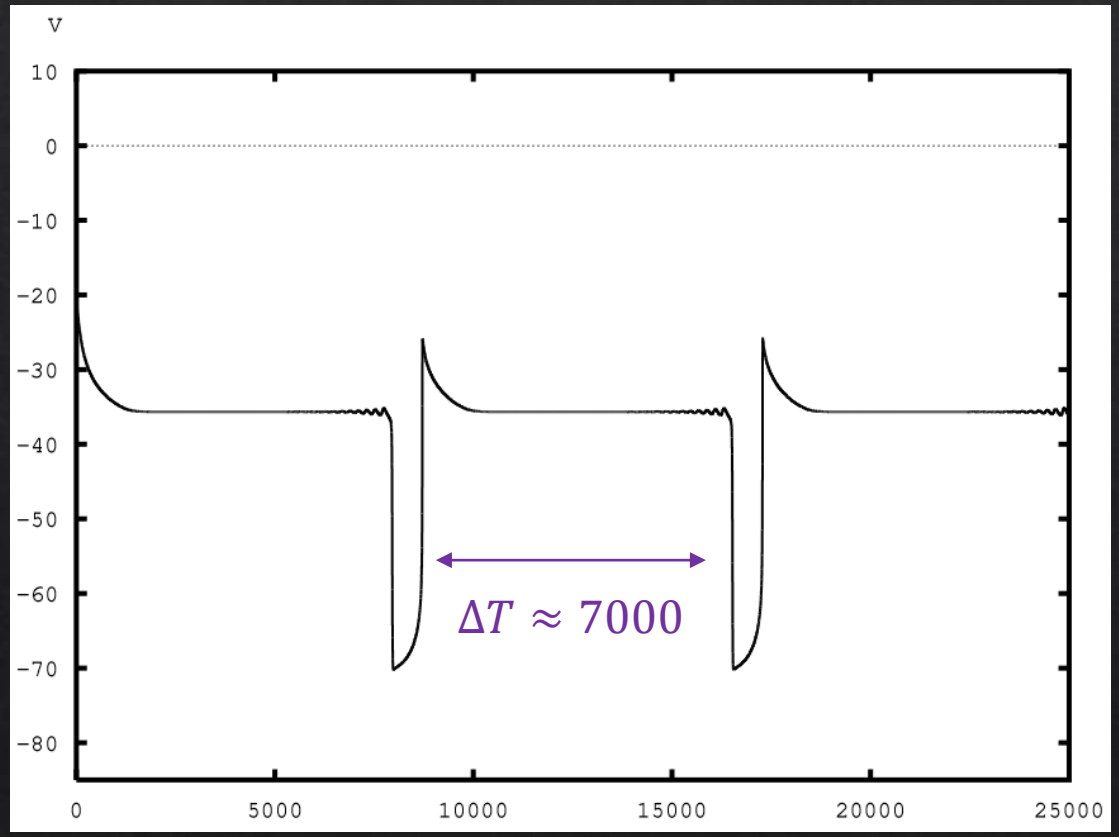
$$I_{kca} = G_{kca} s_\infty(c)(V - V_k), \ I_{bk} = G_{bk} b_\infty(V - V_k)$$

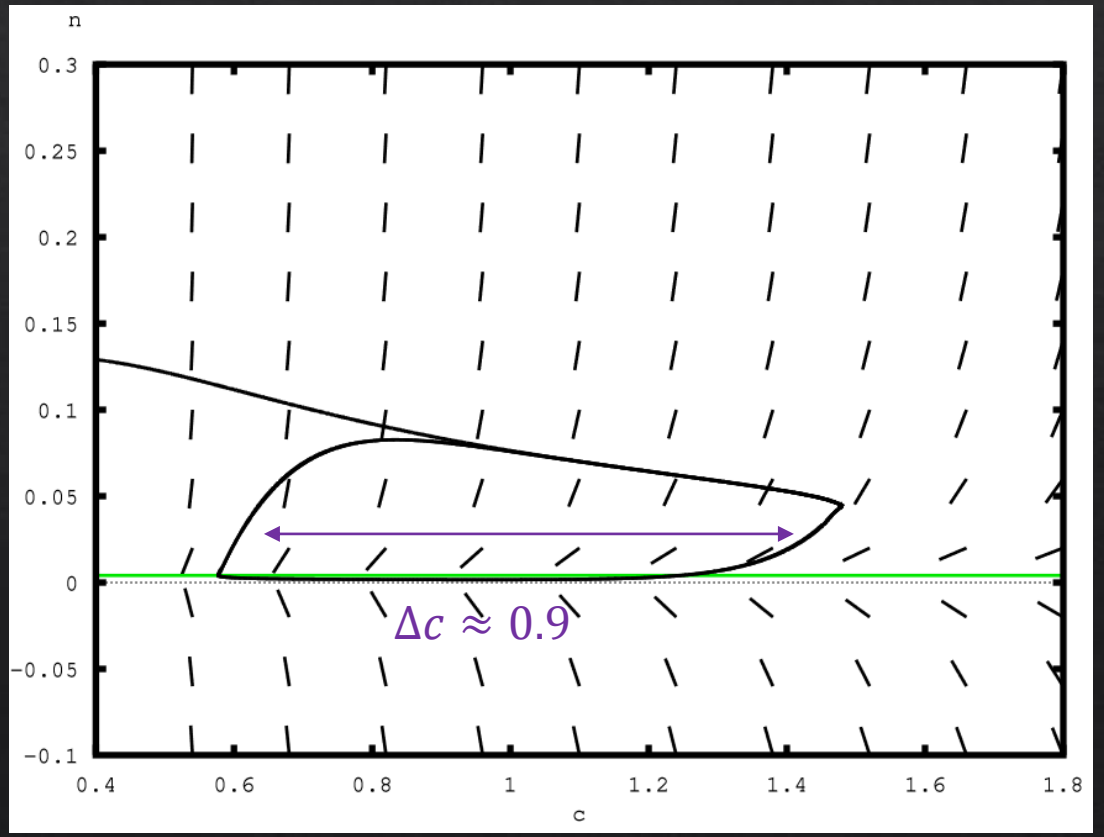
The plateau is a state which satisfays $\boldsymbol{V' \to 0}$, $V \approx -25$ *.
Considering appropriate $(V - V_i)$, $m_\infty, b_\infty \to 1$, and taking $n \approx 0.15, c \approx 0.32$ (from the $n - c$ PP) we get during the plateau:

$$G_{ca} = \frac{2}{3}(0.15G_k + 0.3524G_{kca} + G_{BK})$$

Dynamic 1 – The plateau

Using the last equation and adjusting the G_i's accordingly, we find:

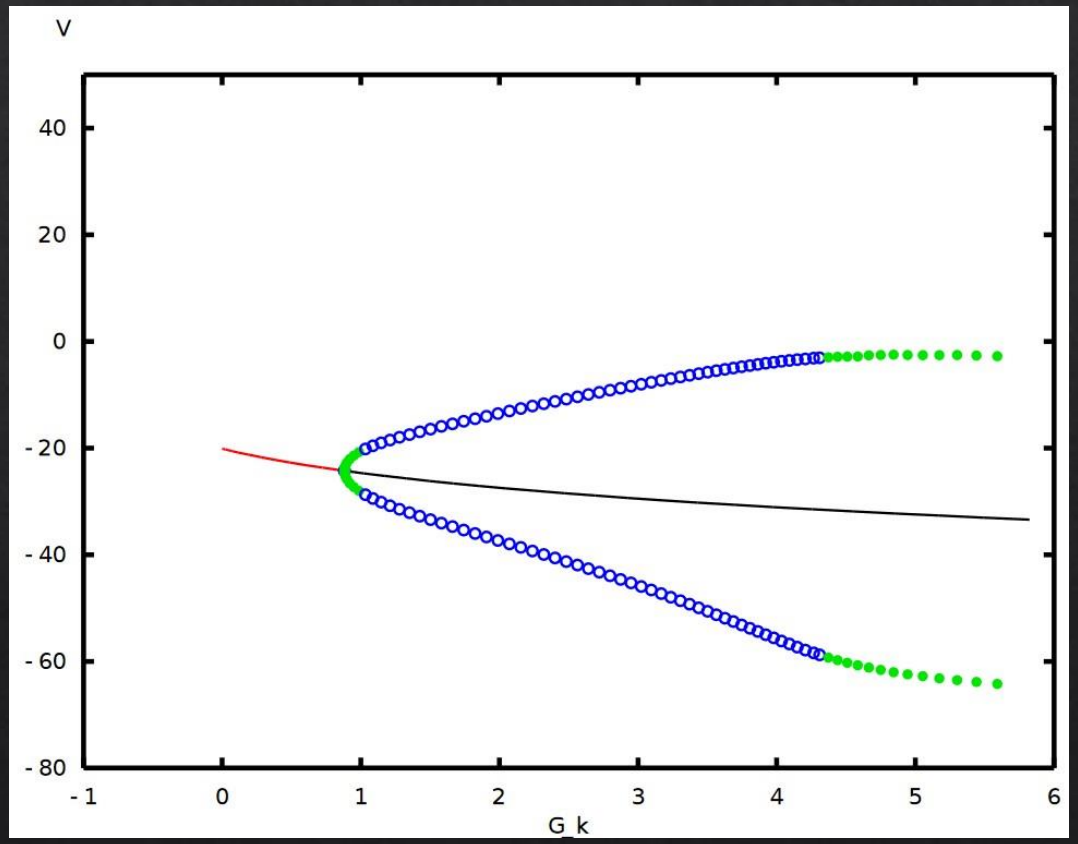




The Biggest Plateau!

Dynamic 2 – G_k

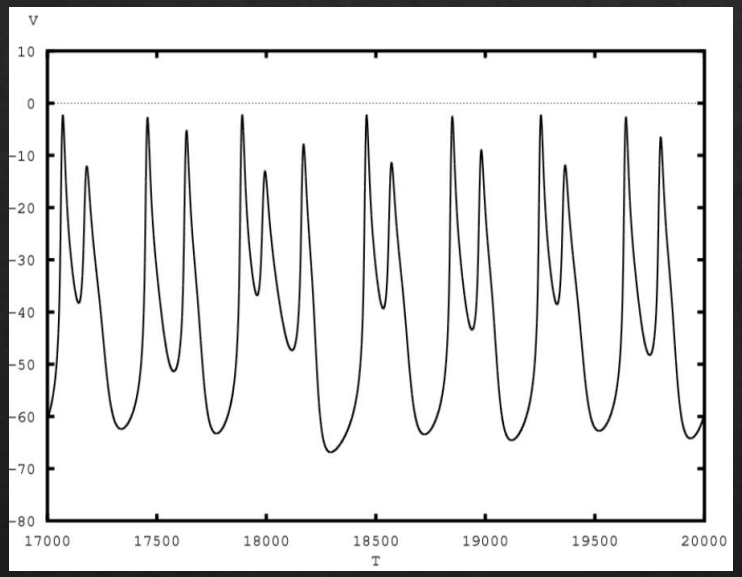
G_k has a big impact on the overall dynamics, we can see this from its bifurcation:



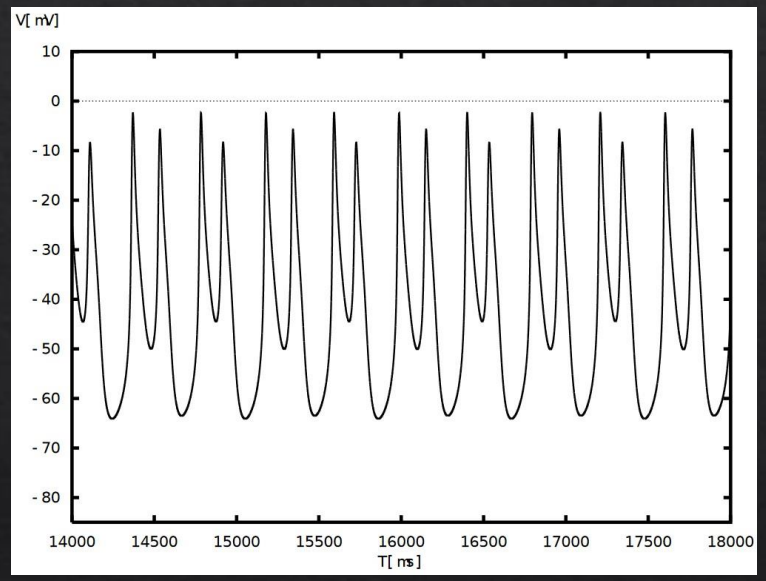
Notice there is a (backwards) supercritical Hopf bifurcation at $G_k \approx 0.9$

Dynamic 2 – G_k

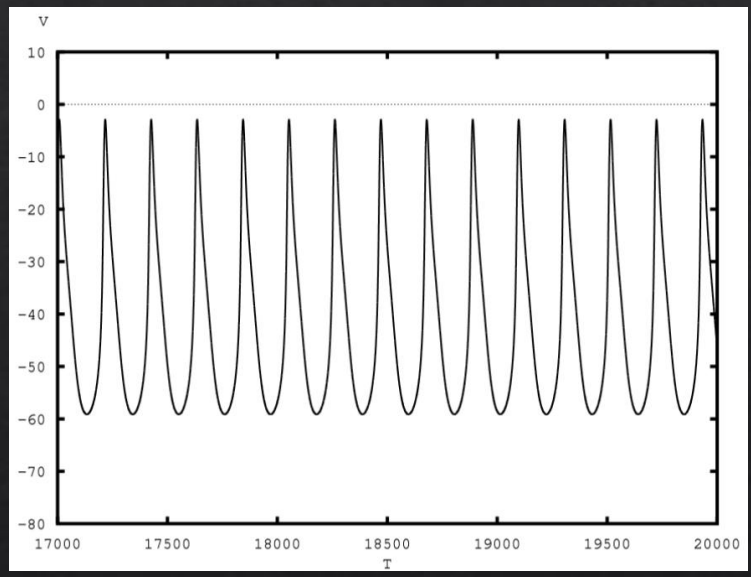
Additionally, we see in the diagram that the unstable limit cycle (that indicates plateau-firing) becoms stable (indicates stable firing) at $G_k \approx 4.3$:



$G_k = 4.1$



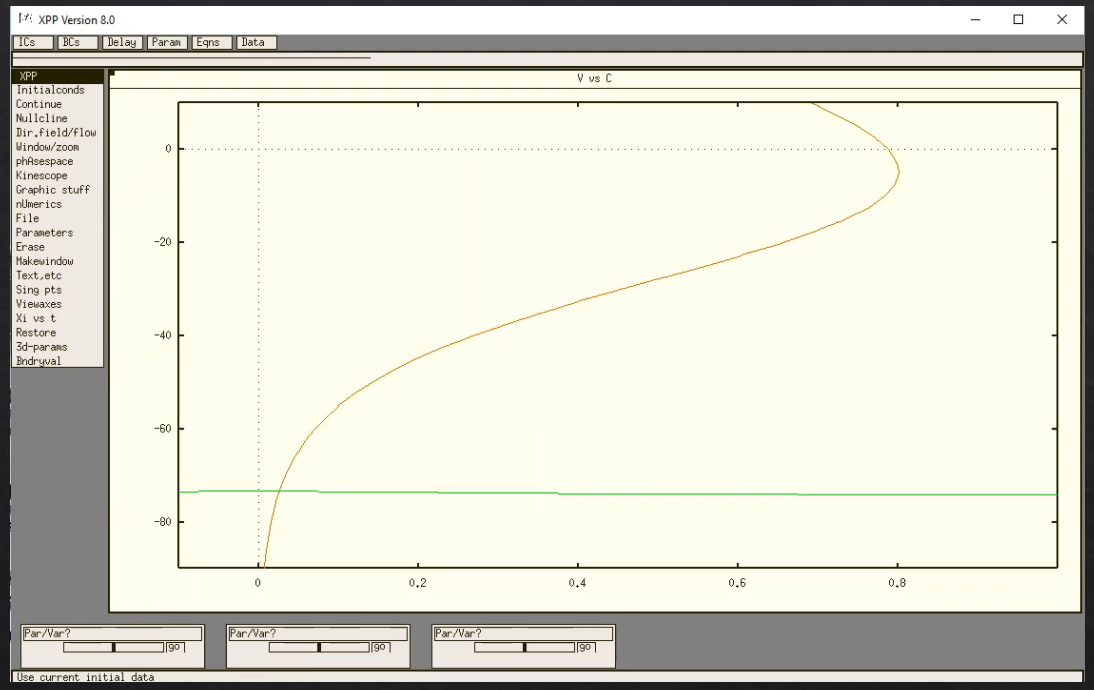
$G_k = 4.2$



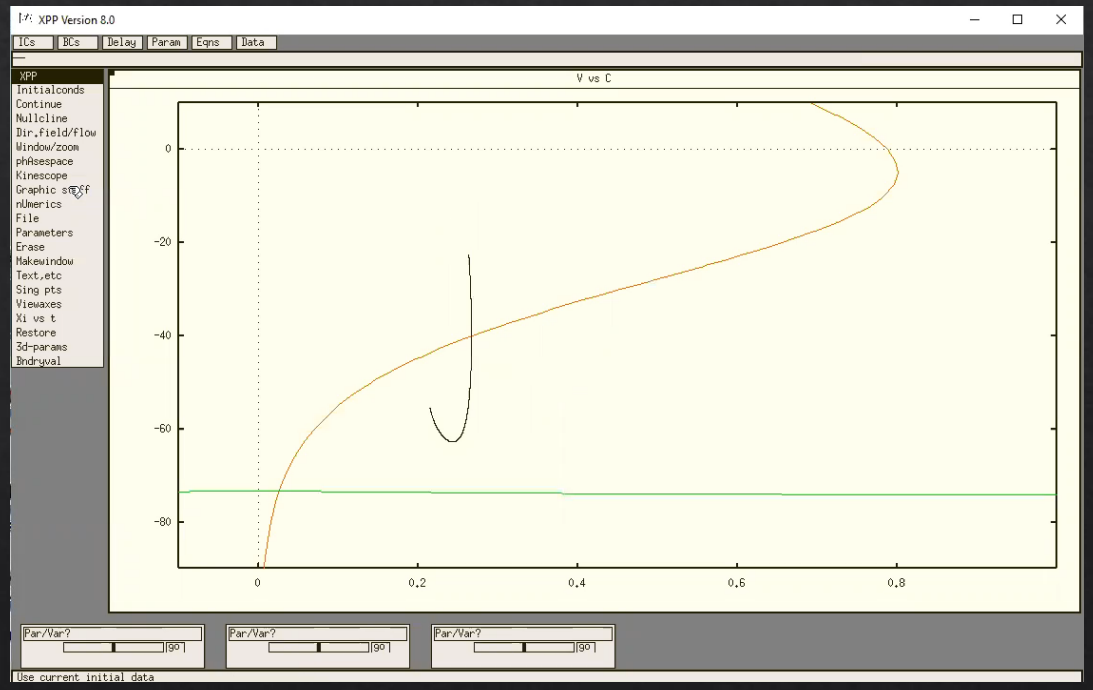
$G_k = 4.35$

Dynamic 3 – C_m and frequency

Different values of C_m give different qualitative behaviors even though it does not change the nullclines:



$C_m = 30$

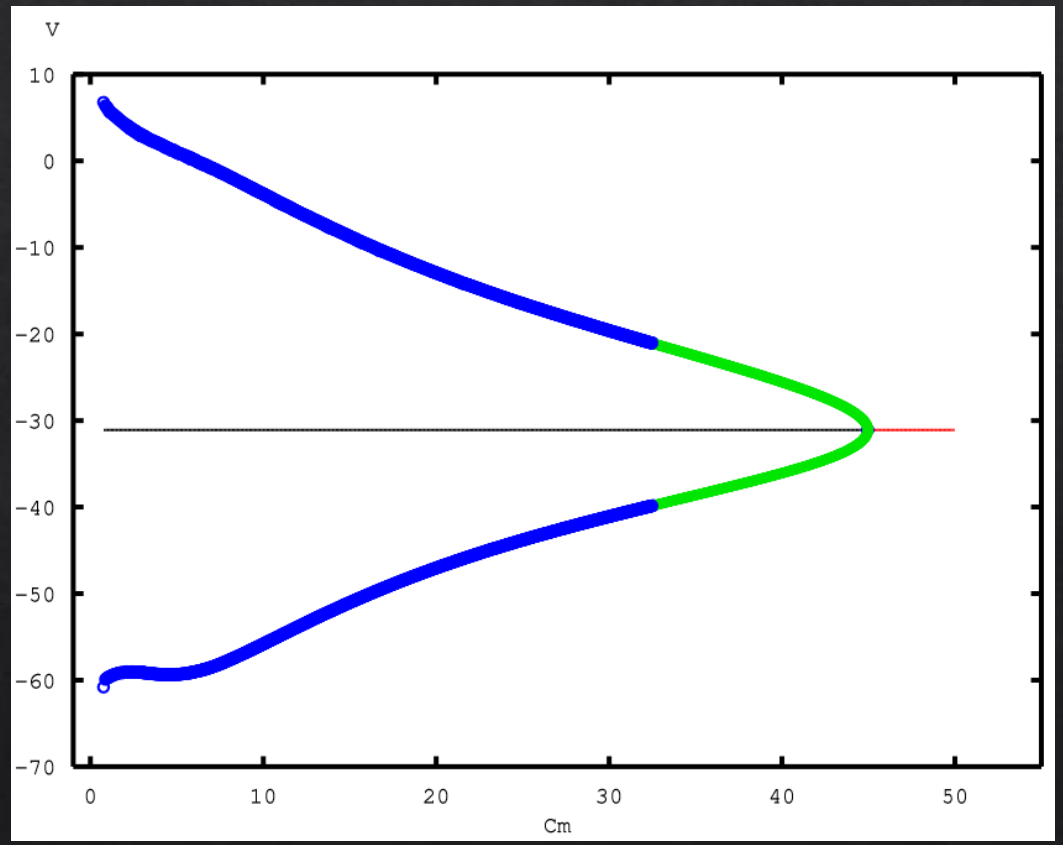


$C_m = 34$

It seems that there is a new stable limit cycle that is created at $C_m = 34$.

Dynamic 3 – C_m and frequency

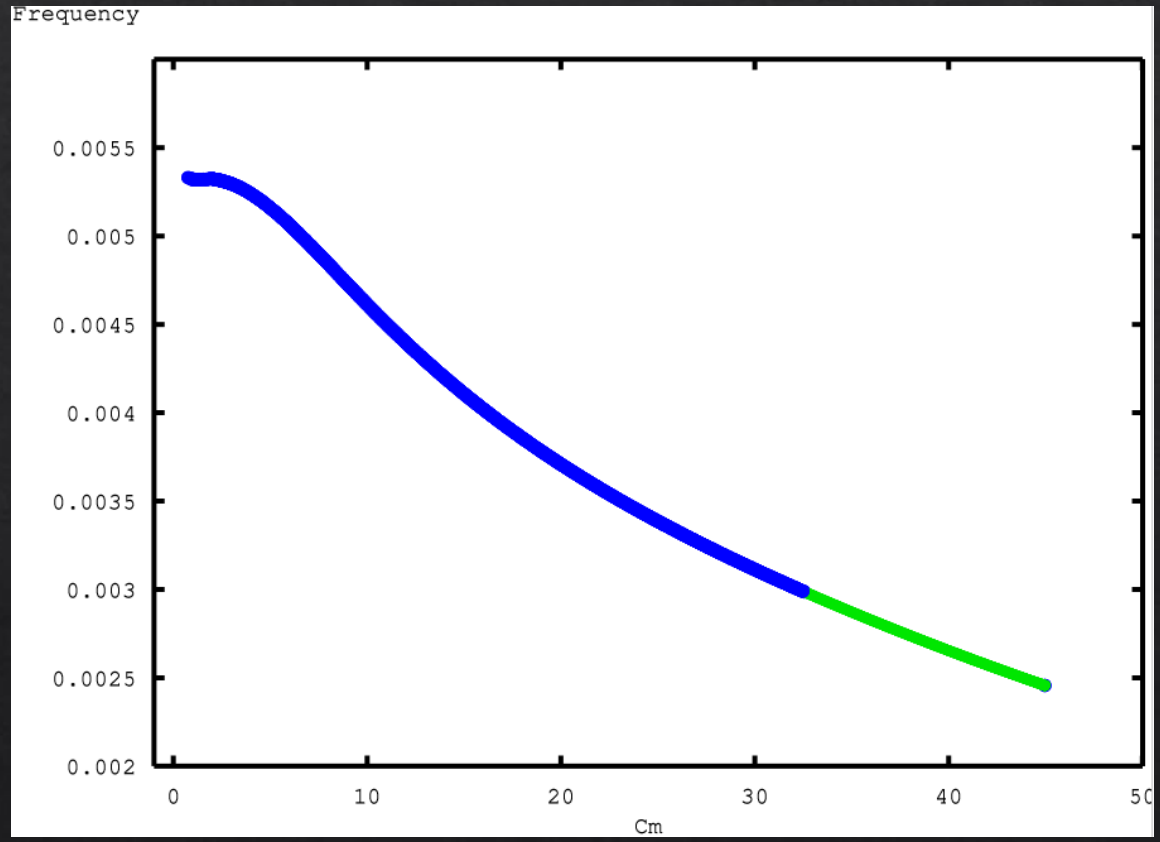
We can see exactly this limit cycle creation in the C_m bifurcation diagram:



That's another supercritical Hopf bifurcation!

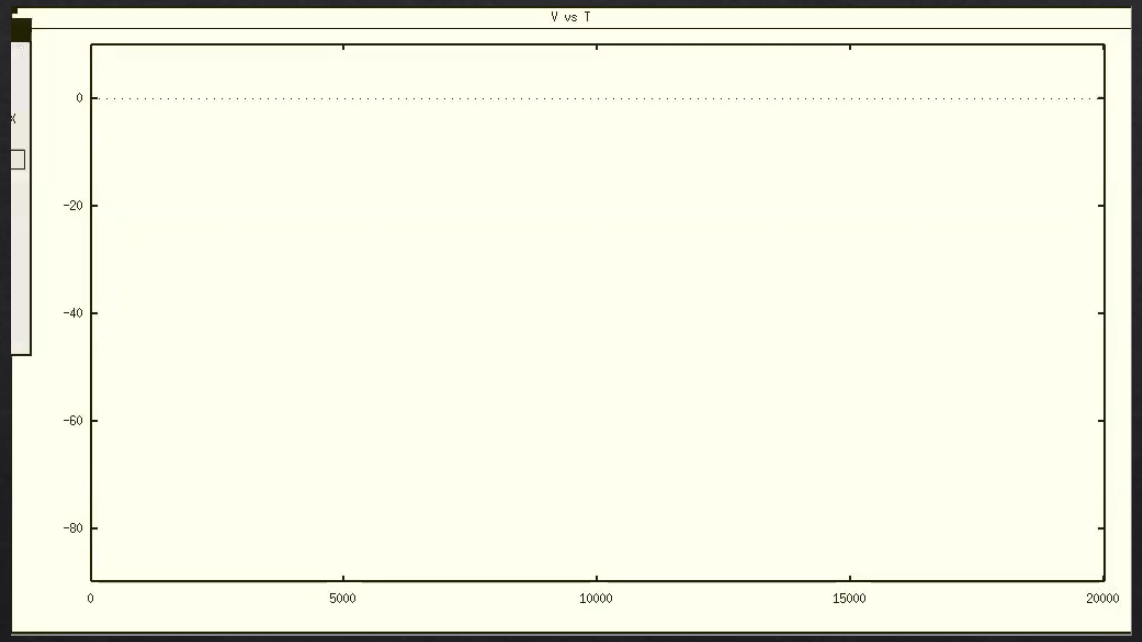
Dynamic 3 – C_m and frequency

It is interesting to look at the frequency plot that matches the previous bifurcation diagram:

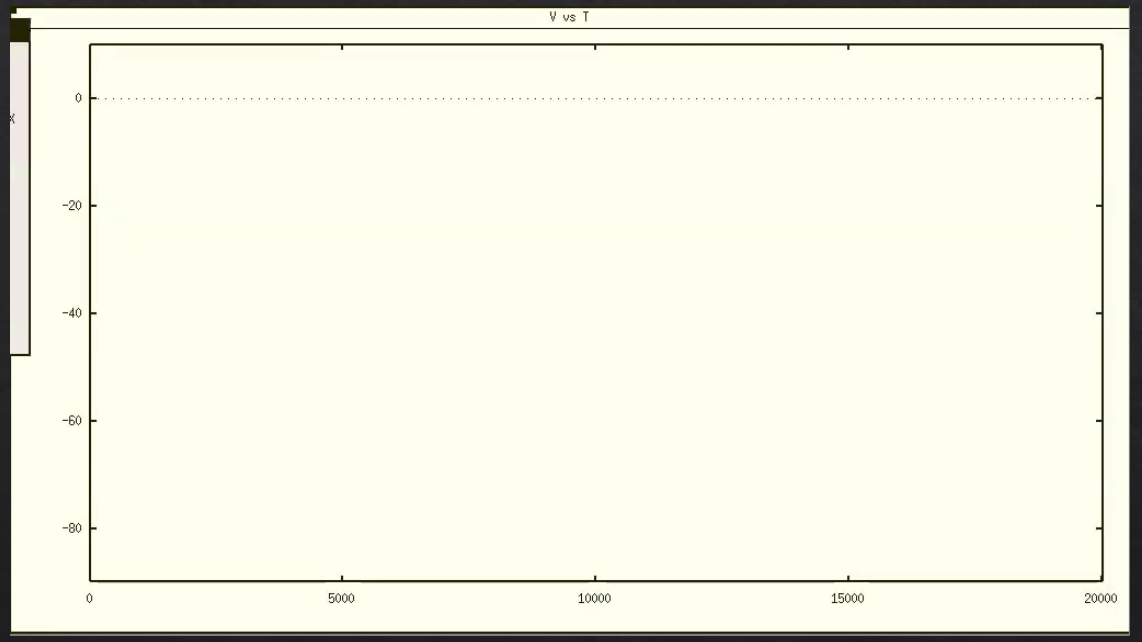


Dynamic 3 – C_m and frequency

And to confirm it:



$C_m = 32$



$C_m = 2$