Dept of Industrial Engineering & Management

Intelligent Robotic Systems

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My background

- study of physics at University of Ulm and Heidelberg, Germany
- Ph.D.: applied mathematics and computer science, WIS, Israel
- research in physics, computational neuroscience, robotics, AI in industry (GM) and academia
- joined IEM in June 2016, *Robotics and Machine Intelligence Lab*
- research focus: skill learning in robots
  - reinforcement learning
  - imitation learning
  - deep learning
  - computational human motor control, computational neuroscience

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office hours: Mo, 12-13 (schedule before!)
lab: -122,-124

Literature

Classical Robotics

- K.M. Lynch, F.C. Parker, *Modern Robotics*, Cambridge University Press, 2017.
- M.W Spong, S. Hutchinson, M. Vidyasagar, *Robot Modeling and Control*, J. Wiley & Sons, 2006.

Probabilistic Robotics

- S. Thrun, W. Burgard, D. Fox, *Probabilistic Robotics*, MIT Press, 2015

Artificial Intelligence

- S. Russell, P. Norvig, *Artificial Intelligence, a Modern Approach*, 3rd edition, Pearson 2010

Reinforcement learning

- R.S. Sutton, A. G. Barto, *Reinforcement Learning: An Introduction*, MIT Press, 1998.
  second, unpublished version is online available

Additional resources

- David Silver, Reinforcement Learning, lecture notes
- Marc Touissant, Introduction to Robotics, lecture notes
- Cyrill Stachniss, SLAM lecture notes

Prerequisites

Basic knowledge of probability theory, analysis and linear algebra.
Knowledge of a programming language (Matlab, Python, C/C++)

Where and when

Time and Location:

Wed 14-17
Building 90, Room 237

Course Outline

1. Introduction
2. Robot Estimation: Bayesian filters
3. Robot Localization: Particle filters
4. Robot Motion and Perception: Motion and sensor models
5. Robot Mapping: Occupancy grid mapping
6. Robot Localization and Mapping: SLAM
7. Robot Motion Planning I: Markov decision process (MDP)
8. Robot Motion Planning II: Dynamic programming (DP)
9. Robot Learning I: Reinforcement learning (RL)
10. Robot Learning II: Reinforcement learning (Monte-Carlo, TD/Q - learning)
11. Robot Programming I: ROS - Introduction (Shai Givati)
12. Robot Programming II: ROS - Gazebo simulator (Shai Givati)
13. Robot Programming III: ROS - Control of a real robot (Turtlebot 3) (Shai Givati)

Course grades

:

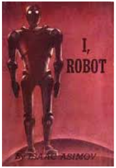
- 6 exercises (3 ROS exercises)
- final grade = average of all exercises

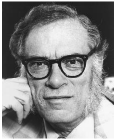
Lecture 1: Introduction

What is robotics?

- the term “robot” is derived from the Czech word “robota” (forced labor) and appeared first in 1921 in the play R.U.R. (Rossum’s Universal Robots) by Czech Writer Karel Čapek

- the term “robotics” was coined by russian-born american science-fiction writer Isaac Asimov in 1942 in his short story “Runaround”





- In this story Asimov also proposed three "Laws of Robotics", which define the ethics of human-robot interaction

Ethics of robot-human interaction

Asimov's three laws of robotics

1. A robot may not injure a human being or, through inaction, allow a human being to come to harm.
2. A robot must obey orders given it by human beings except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

Unfortunately, not all robots have obeyed these laws ...

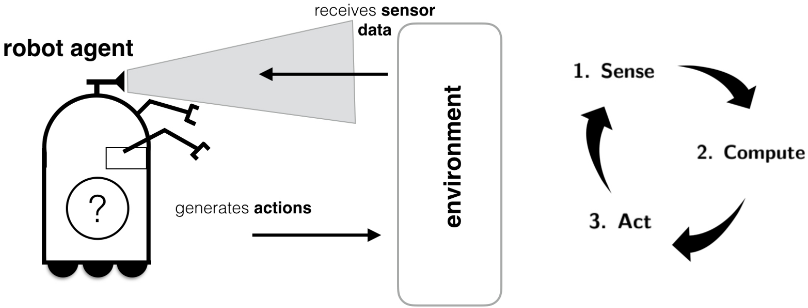
- First time a robot killed a human: 1979
  manufacturing robot at a Ford Motor Company factory, MI, USA
- Last time a robot killed a human: 2016
  self-driving car from Tesla, FL, USA

Definition of robotics

So what exactly is a robot and robotics? Many definitions exist ...

- "A reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of task." - *Robot Institute of America*, 1979
- "An automatic device that performs functions normally ascribed to humans or a machine in the form of a human." - *Webster's Dictionary*
- " Robots are physical agents that perform tasks by manipulating the physical world" - from the book *Artificial Intelligence - a modern approach*
- "Robotics is that field concerned with the intelligent connection of perception to action." - *Mike Brady*
- "a robot is a goal oriented machine that can sense, plan and act." - *Peter Corke*

What is robotics - our working definition for this course



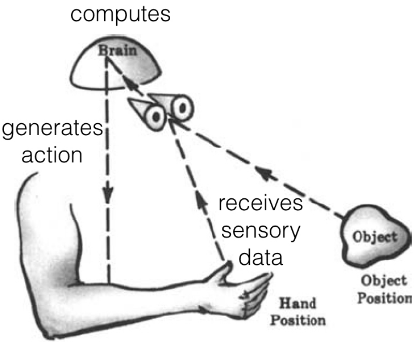
robotics: science of transferring sensor data into actions

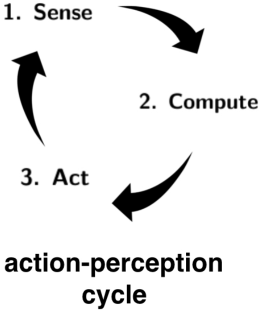
action-perception cycle

note: In robotics, the agent (robot) is **physical** or **embodied** in contrast to a software agent (or software robots or softbots).

Comparison: Action-perception cycle in humans

task: grasping an object

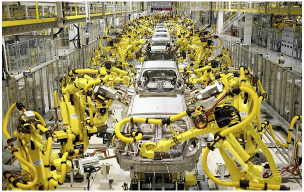


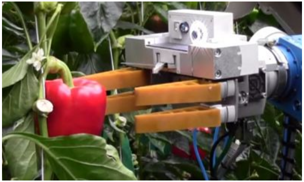


Q: Are we very intelligent robots? If not, what makes us different from robots?
A: To find out, watch "Westworld" :)

Why robotics?

- **Commercial**
  Industrial, military, health care, entertainment, agriculture, surgery, etc





Why robotics?

- **Robotics as intelligence research in the real world**
  AI: Machine Learning, probabilistic reasoning, optimization
  Real world: Interaction, manipulation, perception, navigation, etc

  Motion was the driving force to develop intelligence
  - motion needs control & decision making $\Leftrightarrow$ fast information processing
  - motion needs anticipation & planning
  - motion needs perception
  - motion needs spatial representations

  Manipulation requires to acknowledge the structure (geometry, physics, objects) of the real world. Classical AI does not

Tunicates digest their brain once they settled!

So don't get stuck …

Ultimate goal of robotics - whether you like it or not

Building of intelligent machines that can act, think and learn like humans in the real world

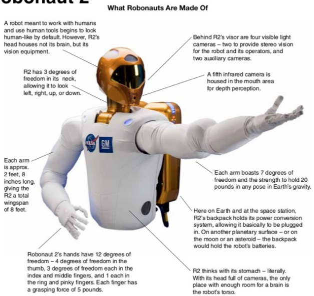
Can a machine think?

Some popular humanoid robots

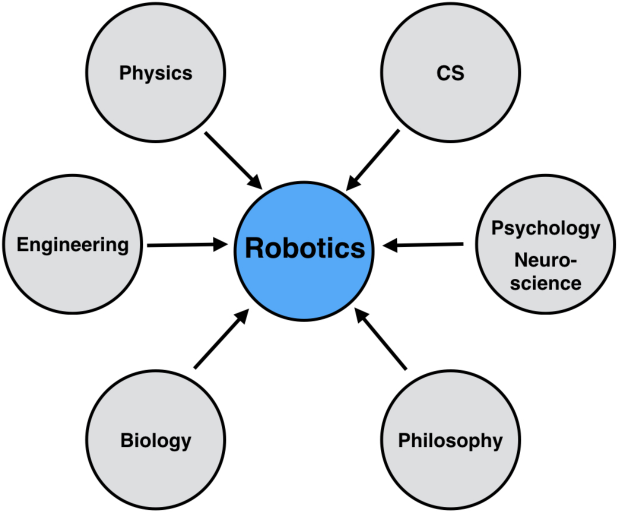
Nao

Justin

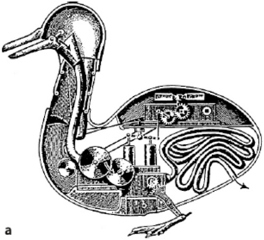
Robonaut 2



Fields that contribute to robotics



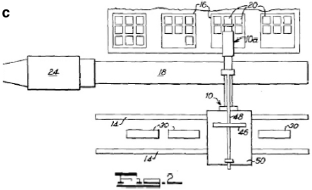
History of robotics

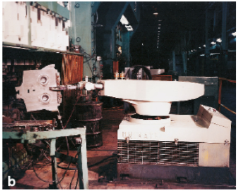


a

Early programmable machines. **a** Vaucanson's duck (1739) was an automaton that could flap its wings, eat grain and defecate. It was driven by a clockwork mechanism and executed a single program; **b** The Jacquard loom (1801) was a reprogrammable machine and the program was held on punched cards (photograph by George P. Landow from www.victorianweb.org)

c





first robotics company:
Unimation, 1956
(*Devol & Engelberger*)

Universal automation. **a** A plan view of the machine from Devol's patent; **b** the first Unimation robot working at a General Motors factory (photo courtesy of George C. Devol)

History of robotics: 1950-2000

What has been achieved until today?

- robot development between 1950-2000

movie

History of robotics: 2000-2015

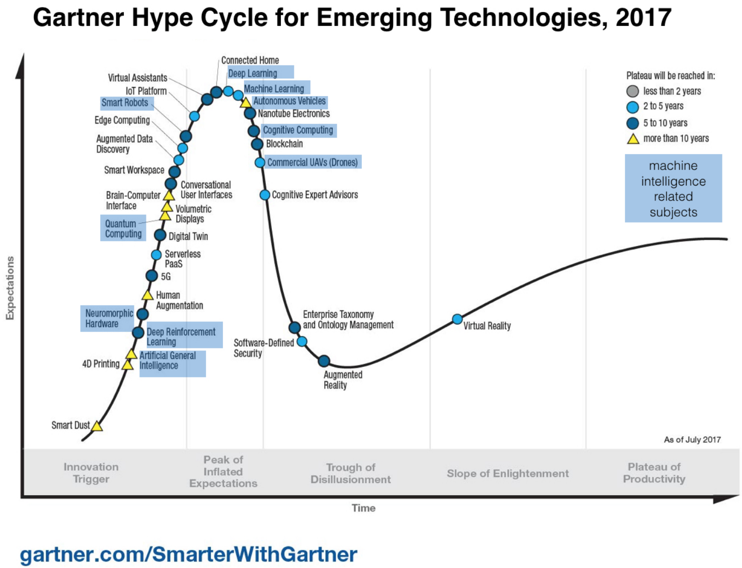
- robot development between 2000-2015

movie

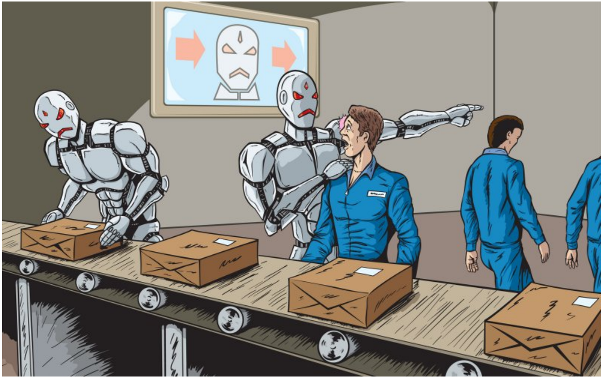
Atlas/Boston Dynamics - movie I

Boston Dynamics - movie II

The future of robotics - Emerging Technologies

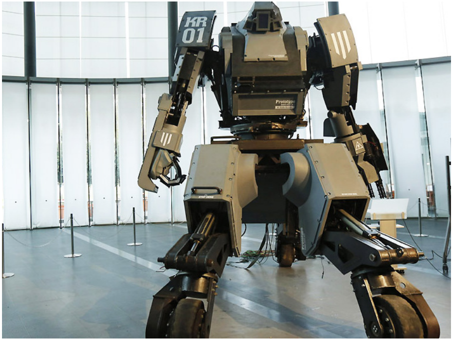


Danger of developing robots and AI - will there be superhuman intelligence and will robots take over?



Amazon is just beginning to use robots in its warehouses

Danger of developing robots and AI - Autonomous weapon systems

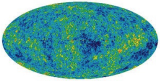


In July 2015, an *open letter from artificial-intelligence experts and roboticists* called for a ban on autonomous weapon systems (AWS), comparing their revolutionary potential to that of gun powder and nuclear weapons.

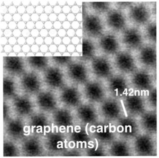
Danger of developing robots and AI - will superhuman intelligence lead to the end of human evolution?

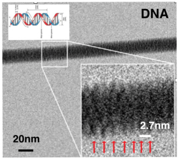
we went quite a long way ...

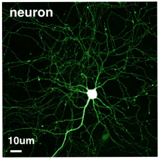
in a nutshell:
? $\rightarrow$ physics $\rightarrow$ chemistry $\rightarrow$ biology $\rightarrow$ neuroscience $\rightarrow$ human brain/hands $\rightarrow$ our final invention (the singularity)?



cosmic background radiation from the big bang



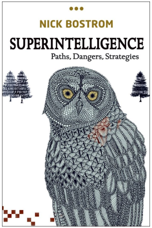


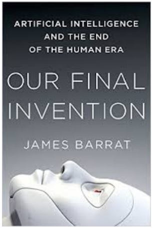


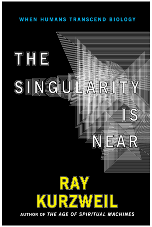
Quiz: Can you tell which pictures are real?

Danger of developing AI - Will superhuman intelligence lead to the end of human evolution?

some recent books on the topic that became bestsellers ...







Q: What do you think? Is this nonsense or a real problem?
A: One answer may be: "*Prediction is difficult, in particular, of the future.*" (Niels Bohr, 1885-1962)

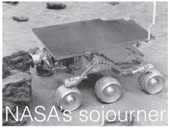
after these philosophical considerations concerning the future of robotics and AI let's go back to the present state-of-the-art of robotics ...

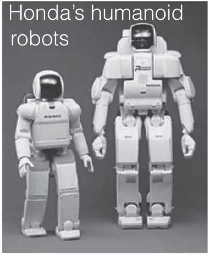
Classification of robots

a coarse taxonomy of robots: fixed manipulators vs mobile robots

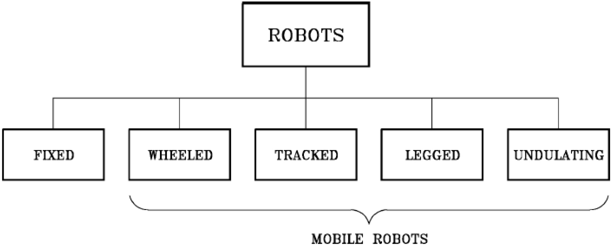
1. manipulators
2. mobile robots (UGVs, UAVs, AUVs, self-driving cars, micro-robots)
3. mobile manipulators, e.g., humanoid robots

others: prosthetic devices (artificial limbs, hands, ears and eyes)
multibody systems (swarms of small cooperating robots)



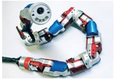


Classification of robots









manipulators: interaction with the world
mobile robots: mainly localization and mapping

Robotic system - Hardware

a robotic system consists of **several** components

example: **KUKA LWR**[1] **iiwa**[2]

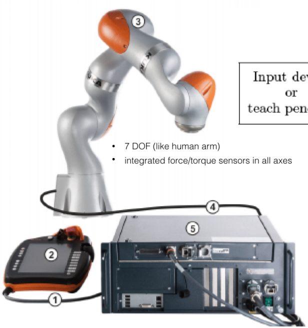
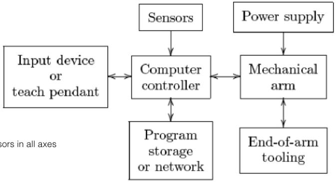


Fig. 3-1: Overview of robot system

1 Connecting cable to the smartPAD
2 KUKA smartPAD control panel
3 Manipulator
4 Connecting cable to KUKA Sunrise Cabinet robot controller
5 KUKA Sunrise Cabinet robot controller



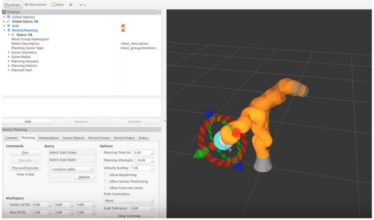
different pneumatic grippers

[1] leight weight robot
[2] intelligent industrial work assistant

Robotic system - Software

- ROS: **R**obot **O**perating **S**ystem
- URDF: **Universal Robot Description Format (URDF)** is an XML (eXtensible Markup Language) file format used by the Robot Operating System (ROS) todescribe the **kinematics**, **inertial properties**, **link geometry** and **visual appearance** of robots.
- Gazebo: robot simulator
- MoveIt: motion planning software

example: URDF of the KUKA is uploaded to ROS/Gazebo:
the robot can then be visualized and simulated



movie

Robot sensors - classification

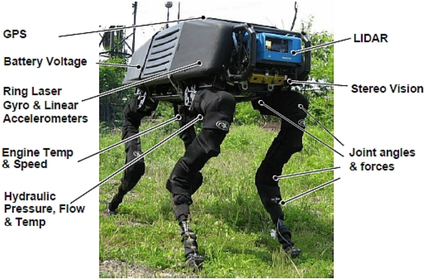
- **passive** vs **active** sensors (e.g., camera vs radar)
  active sensors emit energy to the environment, which is reflected back to the sensor
- active and passive sensors can be classified into three groups:

1. **sensors for the environment**
   sonar sensor, radar, infra-red, stereo vision, LIDARs (light detection and ranging)
   tactile sensors (whiskers, touch-sensitive)
2. **sensors for the robot's location**
   GPS (Global Positioning System), 31 satellites in orbit, accuracy $\sim$5m
   Differential GPS involves a second ground receiver with known location, accuracy $\sim$mm
3. **sensors for the robot's internal configuration**
   shaft decoders (odometry), inertial sensors (accelerometer, gyroscopes), force and torque sensors (e.g., for robot grasping)

Robot sensors - examples

self-driving car, Boss

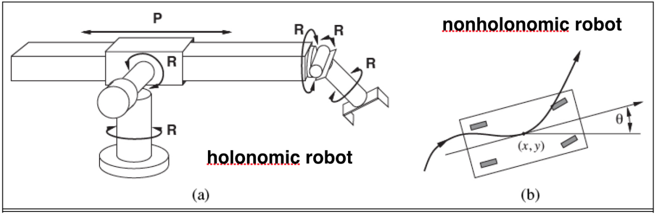
BigDog, Boston Dynamics



Robot effectors

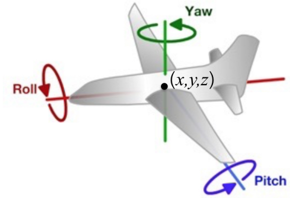
Effectors are the means by which robots move and change the shape of their bodies.

- **degrees of freedom** (DOFs)
  number of independent parameters that define the robot's configuration
- **prismatic** (P) vs **revolute** (R) joints
- **effective DOF** vs **controllable DOFs**
- **holonomic robot** vs **nonholonomic robot**
  effective DOF = controllable DOFs: holonomic (a)
  effective DOF > controllable DOFs: nonholonomic (b)
  example: car - DOF = (x, y, θ), control = (acceleration, steering)



Robot effectors

- **kinematic state** or **pose**
  independent parameters that define the robot's configuration
- **dynamic state**
  kinematic state + rate of change of independent parameters
  example: plane - kinematic state: $\boldsymbol{x} = (x, y, z, yaw, roll, pitch)$,
  DOFs: $n = 6$
  dynamic state: $(\boldsymbol{x}, \dot{\boldsymbol{x}})$, dimension: $2n$



- **differential drive**: robot posses two independently actuated wheels
- effectors are driven by **electric motors**, **pneumatic** or **hydraulic** actuation

Robot task environment types

- **fully observable vs partially observable**
  a task environment is fully observable if an agent's sensors give it access to the complete state of the env at each point in time (chess vs poker)
- **deterministic vs stochastic**
  a task environment is deterministic if the next state of env is completely determined by current state and action executed by agent (chess vs driving)
- **discrete vs continuous** task environments - in its state (including time), action and measurement spaces
  (chess vs driving)
- **static vs dynamic**
  if the environment does not change while an agent is deciding on an action, then the env is static (chess without a clock vs driving)

Note: real-world robotic problems have task environments, that are partially observable, stochastic, continuous and dynamic
(self-driving cars, Mars explorer mobile robot)

What is intelligence, i.e., artificial intelligence (AI)?

- the design of agents that can think and act like humans
- the design of rational agents, i.e., agents that maximize their expected utility given the information it has acquired from the environment.

Thinking Humanly	**Thinking Rationally**
"The exciting new effort to make computers think . . . *machines with minds*, in the full and literal sense." (Haugeland, 1985) "[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning . . ." (Bellman, 1978)	"The study of mental faculties through the use of computational models." (Charniak and McDermott, 1985) "The study of the computations that make it possible to perceive, reason, and act." (Winston, 1992)
Acting Humanly	**Acting Rationally**
"The art of creating machines that perform functions that require intelligence when performed by people." (Kurzweil, 1990) "The study of how to make computers do things at which, at the moment, people are better." (Rich and Knight, 1991)	"Computational Intelligence is the study of the design of intelligent agents." (Poole *et al.*, 1998) "AI . . . is concerned with intelligent behavior in artifacts." (Nilsson, 1998)

Figure 1.1 Some definitions of artificial intelligence, organized into four categories.

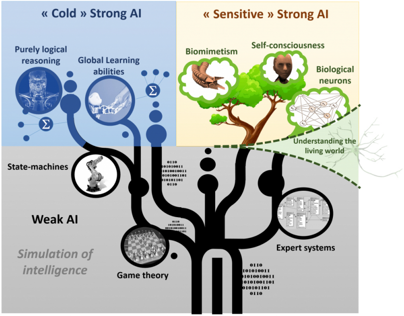
Weak AI vs Strong AI

Weak AI

- machines can act as if they were intelligent → simulated intelligence

Strong AI

- machines can actually think (and not only simulate thinking) → real intelligence

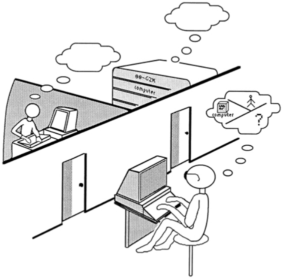


Most AI researchers do not care about strong AI, as long as their programs work.

Q: How do we know when a machine has intelligence?

Probing intelligence - Turing test (1950)

(Turing, A.M. (1950). Computing machinery and intelligence. Mind, 59, 433-460.)

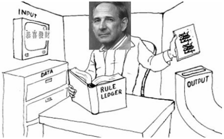


Turing test: hide human in a room and a machine in another and type them questions: if you cannot find out which one is which based on their answers, then the machine is intelligent

- a machine is said to be "intelligent" if it behaves exactly like a human being

Q: but does it really proof intelligence? ...

Probing intelligence - Searle's chinese room experiment (1980)



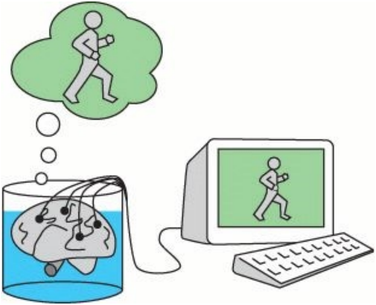
Chinese room: You are in a room with an opening through which Chinese sentences are passed. You have rule book that allows you to look up these sentences although you do not know Chinese. The book tells you how to reply to them in Chinese.

- shows that a Turing test cannot be used to test whether a machine thinks because a program cannot give machines a mind or understanding
- a program is formal (syntactic); minds have mental content (semantics)
- shows that conscious computer/machines using syntactical programs are impossible

Q: OK, so what is consciousness and where is it coming from?
If we can find out we just add it to the machine ...

Probing consciousness - brain-in-the-vat experiment

Q: Can we fool the brain or are you living in a computer simulation?



Q: Would a "disembodied" brain continue to have perfectly normal conscious experiences?

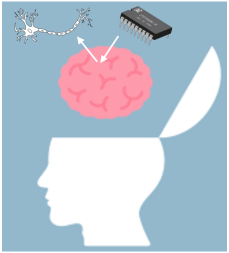
Does the same brain state cause the same mental state, e.g.,*I am jogging* or *I am eating a hamburger*

Further studies into the topic ;): watch the movie"Matrix".

Probing consciousness - brain replacement experiment

replace individual neurons by electronic devices without disturbing the whole operation of the brain - until all neurons are replaced

Q: Does consciousness remain unaffected?



Functionalism: a mental state is any intermediate causal condition between input and output.

If you claim that there is a difference then there must be some non-computational aspects in the brain that lead to consciousness

Aspects of intelligence

think about a self-driving car:

- perception
- attention
- awareness
- memory
- situation assessment
- decision making
- adaptation
- anticipation
- self-organization
- inference
- autonomy
- cooperation
- emotions

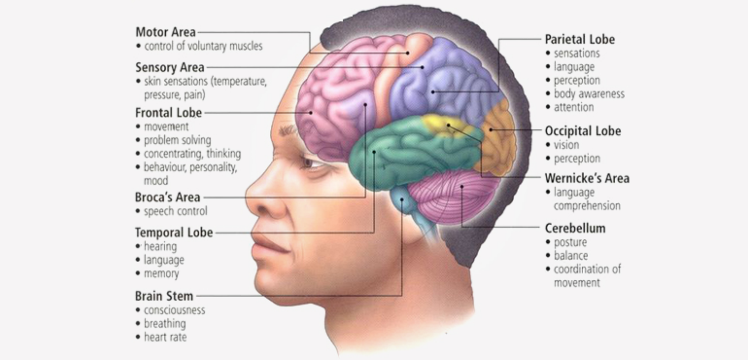
Q: Which important aspect of intelligence did I forget in the list?

Aspects of intelligence

Learning

How do we build intelligent machines?

- imagine you have to build an intelligent machine - where would you start?
- human brain
  input-output relations



Features of a single learning algorithm

Is there a **single** flexible algorithm or are there algorithms for each module, such as seeing, hearing, smelling, ...
what must this single algorithm do?

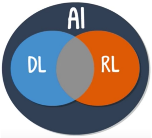
- interpret rich sensory input

- choose complex actions

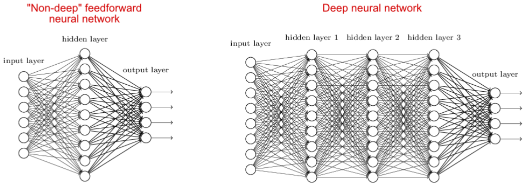
One approach: Deep Reinforcement Learning (DRL)

DRL = Deep Learning (DL) in neural networks + Reinforcement Learning (RL)

$\Rightarrow$ general-purpose framework for artificial intelligence



- **deep networks**
  can process complex sensory input (e.g., images)
  good in nonlinear function approximation
- **reinforcement learning**
  can choose optimal output actions



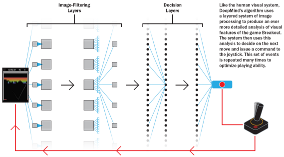
remember: a feedforward network defines a nonlinear parametrized mapping from an input $\boldsymbol{x}$ to an output $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{x}; \boldsymbol{w}, \mathcal{A})$, where $\boldsymbol{w}$ are the weights and $\mathcal{A}$ is the architecture.

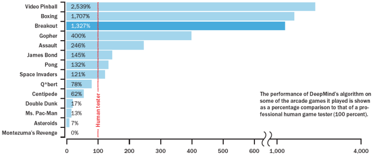
Deep Reinforcement Learning (DRL)

some success stories

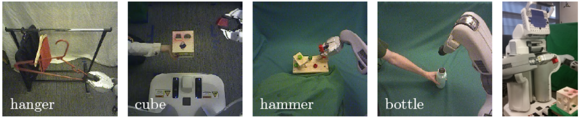
- superhuman "Go" player
  DeepMind develops a system which beats one of the best "Go" players

- superhuman "Atari" player
  DeepMind develops a system which outperforms professional human game tester





- Robotic manipulation



Learning Machines

"Instead of trying to produce a programme to simulate the adult mind, why not rather try to produce one which simulates the child's? If this were then subjected to an appropriate course of education one would obtain the adult brain."

Q: Who said this?

Intelligent robotic systems(?) - Servant robots

- **iRobot**, vacuum cleaner

movie

implements a SLAM = **S**imultaneous **L**ocalization **A**nd **M**apping algorithm

- **Moley**, cooking robot

movie

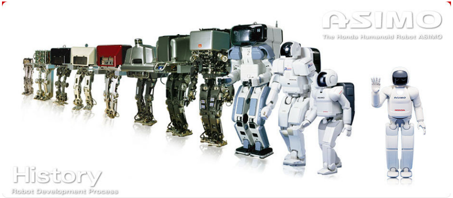
Q: Will it stand the Turing cooking test?

Intelligent robotic systems(?) - Humanoids and Animalnoids

- **Boston Dynamics**

movie - part 1

- **Honda - Asimo**



movie - part 2

Intelligent robotic systems ? - Self-driving cars

- **Stanley**, DARPA Grand Challenge, 2005

movie

- **Boss**, DARPA Urban Challenge, 2007

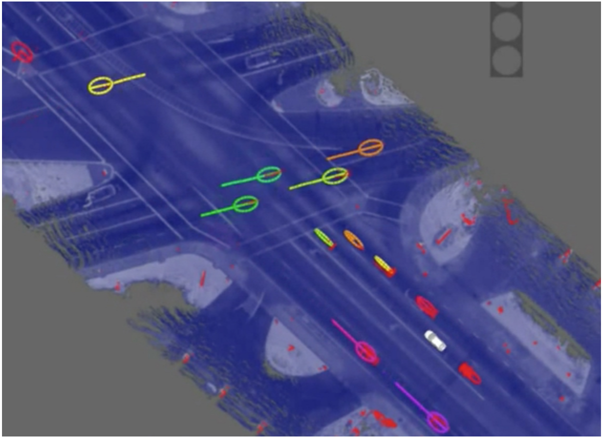
movie

Probabilistic Robotics

- many state-of-the-art algorithms in robotics are probabilistic
  $\Rightarrow$ **probabilistic robotics**
- probability comes into play due to the **uncertainty** in robot perception and action
- key idea:
  explicit representation of uncertainty using the calculus of probability theory
- in this framework:
  perception = state estimation
  action = utility optimization

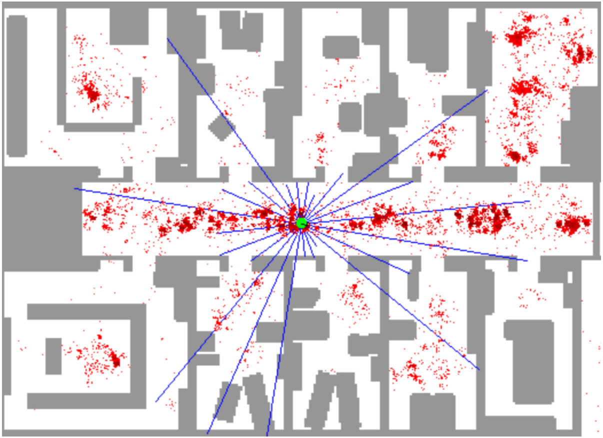
What will we cover in this course?

State estimation: Tracking the state (position and velocity) of cars, robots, etc ...



What will we cover in this course?

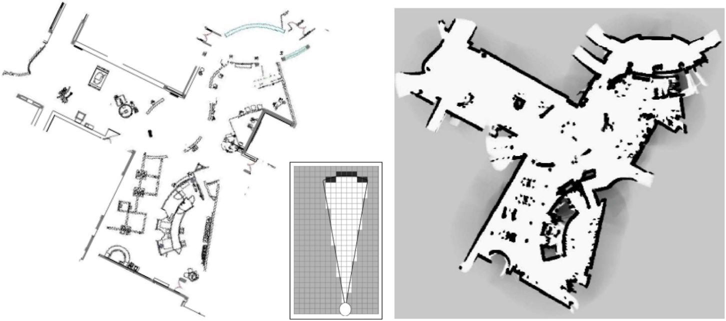
State estimation: Mobile robot localization



movie @ 1:45

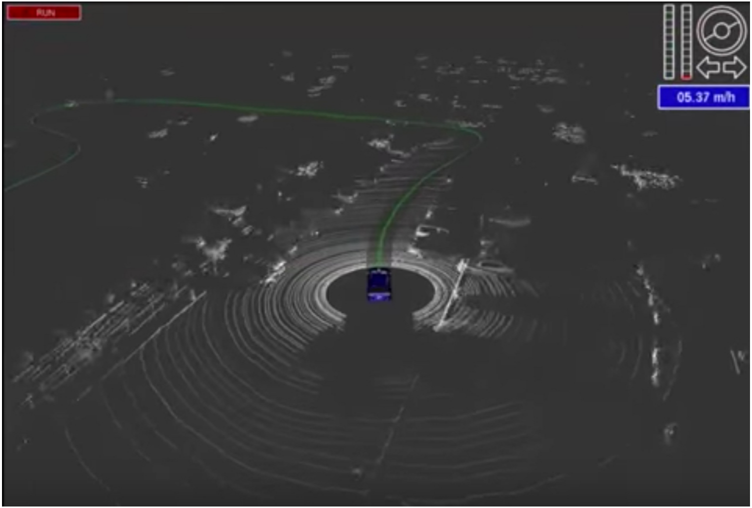
What will we cover in this course?

Mapping



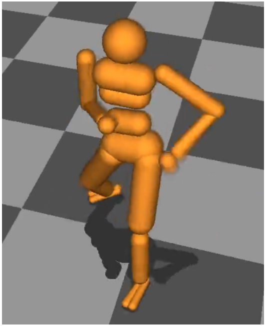
What will we cover in the course?

Planning of robot motion

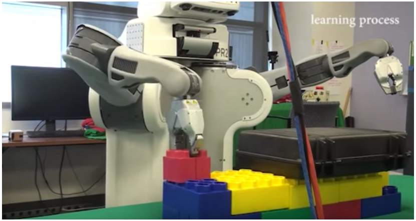


What will we cover in the course?

Learning



movie



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Basic Concepts of Probability

Axioms of probability theory

$\mathsf{Pr}(A)$ denotes the probability that proposition A is true

1. $0 \leq \mathsf{Pr}(A) \leq 1$
2. $\mathsf{Pr}(true) = 1 \quad \mathsf{Pr}(false) = 0$
3. $\mathsf{Pr}(\neg A) = 1 - \mathsf{Pr}(A)$

Discrete random variables

- X denotes a **random variable**
- X can take on a countable number of values in a **sample space** $S_X = \{x_1, x_2 \ldots, x_n\}$
- $P(X = x_i)$ or $P(x_i)$ or p_i is the **probability** that the random variable X takes on value x_i, i.e. there is a set $\mathcal{P}_X = (p_1, p_2, \ldots, p_n)$
- the triple $(X, S_X, \mathcal{P}_X)$ is called an **ensemble**

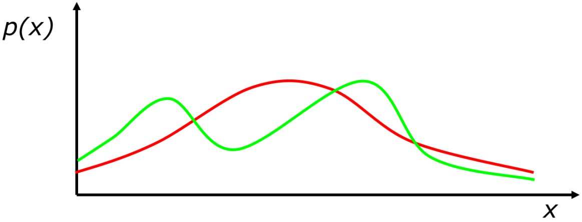
Example: rolling a dice

$X =$ value on the face of a die
$S_X = \{1, 2, 3, 4, 5, 6\}$
$P(X = i) = 1/6$, where $i \in S_X$, $\mathcal{P}_X = (\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$

Continuous random variables

- X takes on values in the continuum
- $p(X = x)$ or $p(x)$ is the **probability density function**

$$\Pr(x \in (a, b)) = \int_a^b p(x)dx$$



Example: Gaussian distribution $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

Joint, conditional and marginal probability

- $P(X = x \,\text{and}\, Y = y) = P(x, y)$ is the **joint probability** of X and Y if X and Y are **independent** then

$$P(x, y) = P(x) \cdot P(y)$$

- **marginal probability**

$$\begin{aligned} P(x) &= \sum_y P(x, y) \\ P(y) &= \sum_x P(x, y) \end{aligned}$$

- **conditional probabilities**

$$\begin{aligned} P(x|y) &= \frac{P(x, y)}{P(y)}, \quad (P(y) \neq 0) \\ P(y|x) &= \frac{P(x, y)}{P(x)}, \quad (P(x) \neq 0) \\ P(x, y) &= P(x|y) \cdot P(y) = P(y|x) \cdot P(x) \end{aligned}$$

read: "$P(x|y)$ is the (conditional) probability of x given y"
note if X and Y are **independent** then $P(x|y) = P(x)$

The rules of probability

	Discrete Case	Continuous Case
Law of total probability	$\sum_x P(x) = 1$	$\int p(x)dx = 1$
Sum rule	$P(x) = \sum_y P(x,y)$	$p(x) = \int p(x,y)dy$
Product rule	$P(x,y) = P(x\|y)P(y)$	$p(x,y) = p(x\|y)p(y)$

Example: Joint and marginal probabilities

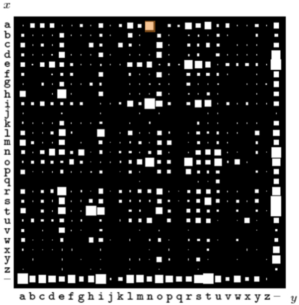
i	x_i	$P(x_i)$
1	a	0.0575
2	b	0.0128
3	c	0.0263
4	d	0.0285
5	e	0.0913
6	f	0.0173
7	g	0.0133
8	h	0.0313
9	i	0.0599
10	j	0.0006
11	k	0.0084
12	l	0.0335
13	m	0.0235
14	n	0.0596
15	o	0.0689
16	p	0.0192
17	q	0.0008
18	r	0.0508
19	s	0.0567
20	t	0.0706
21	u	0.0334
22	v	0.0069
23	w	0.0119
24	x	0.0073
25	y	0.0164
26	z	0.0007
27	–	0.1928

Figure 2.1. Probability distribution over the 27 outcomes for a randomly selected letter in an English language document (estimated from *The Frequently Asked Questions Manual for Linux*). The picture shows the probabilities by the areas of white squares.

marginal

joint ensemble:
XY = ordered pairs of successive letters in a document

XY = {aa,ab,ac, …, zz}



$P(x, y)$

Figure 2.2. The probability distribution over the 27×27 possible bigrams xy in an English language document, *The Frequently Asked Questions Manual for Linux*.

ex: $P(\text{first letter } x = \text{'a'}, \text{second letter } y = \text{'n'})$

Example: Conditional probabilities

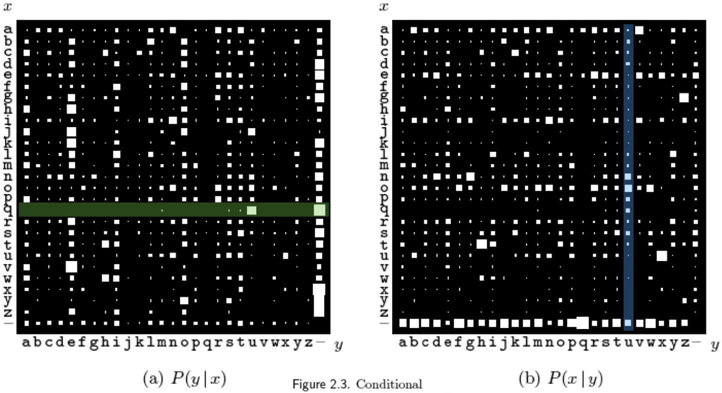


Figure 2.3. Conditional probability distributions. (a) $P(y\,|\,x)$: Each *row* shows the conditional distribution of the second letter, y, given the first letter, x, in a bigram xy. (b) $P(x\,|\,y)$: Each *column* shows the conditional distribution of the first letter, x, given the second letter, y.

ex: P(second letter y|first letter x='q')

ex: P(first letter x|second letter y='u')

Bayes Law

form the product rule:

$$P(x,y) = P(x|y) \cdot P(y) = P(y|x) \cdot P(x)$$

$$\Rightarrow \quad P(x|y) \quad = \quad \frac{P(y|x) \cdot P(x)}{P(y)}$$

$$\text{posterior} \quad = \quad \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

Since $P(y)$ does not depend on x it will be the same for all x. For this reason $P(y)^{-1}$ is often written as a normalizer η in Bayes law

$$P(x|y) \quad = \quad \eta P(y|x) \cdot P(x)$$

where $\eta = P(y)^{-1} = \frac{1}{\sum_x P(y|x) \cdot P(x)}$.

Note that the posterior is a probability distribution, i.e., $\sum_x P(x|y) = 1$

Bayes Law

in machine learning: access uncertainty of model parameters $\boldsymbol{w}$ using observations $\mathcal{D}$:

$$p(\boldsymbol{w}|\mathcal{D}) \quad = \quad \frac{p(\mathcal{D}|\boldsymbol{w})p(\boldsymbol{w})}{p(\mathcal{D})}$$

prior prob: prob of $\boldsymbol{w}$ *before* observing the data

likelihood: how probable are the data $\mathcal{D}$ for different settings of the parameter vector $\boldsymbol{w}$, the likelihood can be viewed as a function of $\boldsymbol{w}$

posterior prob: prob of $\boldsymbol{w}$ *after* observing the data

Summarizing Bayes' theorem:

what you know about $\boldsymbol{w}$ after the data arrive is what you knew before $p(\boldsymbol{w})$ and what the data told you $p(\mathcal{D}|\boldsymbol{w})$

Bayes Law

- we can condition Bayes Law on another random variable $Z = z$

$$P(x|y,z) \quad = \quad \frac{P(y|x,z) \cdot P(x|z)}{P(y|z)}$$

  as long as $P(y|z) > 0$.

- similarly we can condition the rule for combining probabilities of independent random variables on $Z = z$:

$$P(x,y|z) = P(x|z)P(y|z)\,,$$

  which is known as **conditional independence**. This is equivalent to

$$\begin{aligned} P(x|z) \quad &= \quad P(x|z,y) \\ P(y|z) \quad &= \quad P(y|z,x) \end{aligned}$$

  Meaning: the variable y carries no information about x, if z is known (similar for x and y)

  Proof: Condition $P(x|y) = P(x,y)/P(y)$ on $Z = z$ leading to $P(x|y,z) = P(x,y|z)/P(y|z)$ and insert $P(x,y|z) = P(x|z)P(y|z)$ to give $P(x|y,z) = P(x|z)$ and similar for $P(y|z)$.

Expectation and covariance of a random variable

- the **expectation** of a random variable X is given by

$$\begin{aligned} E[X] &= \sum_x x\, p(x) \quad \text{(discrete)} \\ E[X] &= \int x\, p(x) dx \quad \text{(continuous)} \end{aligned}$$

- the **covariance** of a random variable X is given by

$$\mathrm{Cov}[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$$

Example: expectation and covariance of a linear function of a random variable $Y = aX + b$ are:

$$\begin{aligned} E[Y] &= E[aX + b] = aE[X] + b \\ \mathrm{Cov}[Y] &= \mathrm{Cov}[aX + b] = a^2 \mathrm{Cov}[X] \end{aligned}$$

This is familiar: $X \sim \mathcal{N}(\mu, \sigma^2)$ and $Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

Entropy

- the **entropy** of a probability distribution $p(x)$ is given by

$$H_p(x) \quad = \quad E[-\log_2 p(x)]$$

which resolves to

$$\begin{aligned} H_p(x) &= -\sum_x p(x)\,\log_2 p(x) \quad \text{(discrete)} \\ H_p(x) &= -\int p(x)\,\log_2 p(x) dx \quad \text{(continuous)} \end{aligned}$$

The entropy is the expected information that the value of x caries.
$p(x)$ is the probability of observing x

Interpretations of probability

- **frequency interpretation** (**objective** probability)
  - probabilities describe frequencies of outcomes in random experiments
- **Bayesian interpretation** (**subjective** probability)
  - probabilities describe degrees of belief in propositions
  - probabilities can be used to describe assumptions and inferences
  - degrees of beliefs can be mapped onto probabilities if they satisfy some consistency rules (**Cox axioms**)

Important:

You cannot do inferences without making assumption

In this class we take the Bayesian viewpoint