

# Intelligent Robotics Systems

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# Robot motion and perception

# News from the robotics world

clip

# Robot motion

# Motion models

- **state-transition pdf** or **motion model**:

$$p(x_t|u_t, x_{t-1})$$

  is the pdf for the transition from state $x_{t-1}$ to state $x_t$ performing action $u_t$

- **prediction step** in Bayes filter algorithm

$$\bar{bel}(x_t) = \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

- **next**: derive motion models for specific robots

# Kinematic motion models

- ignore dynamics
- In practice, one often finds two types of kinematic motion models:

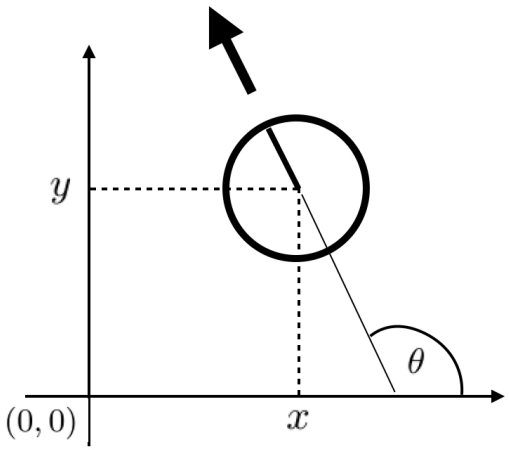
  **odometry-based**

  **velocity-based**

- Odometry-based models for systems that are equipped with wheel encoders
- Velocity-based when no wheel encoders are available

# Kinematic motion models

- kinematic motion model of a rigid robot: six DOF (three Cartesian coordinates and three Euler angles)
- mobile robot in a planar environment: three DOF
  **robot pose** = **location** $(x, y)$ and **orientation** $\theta$

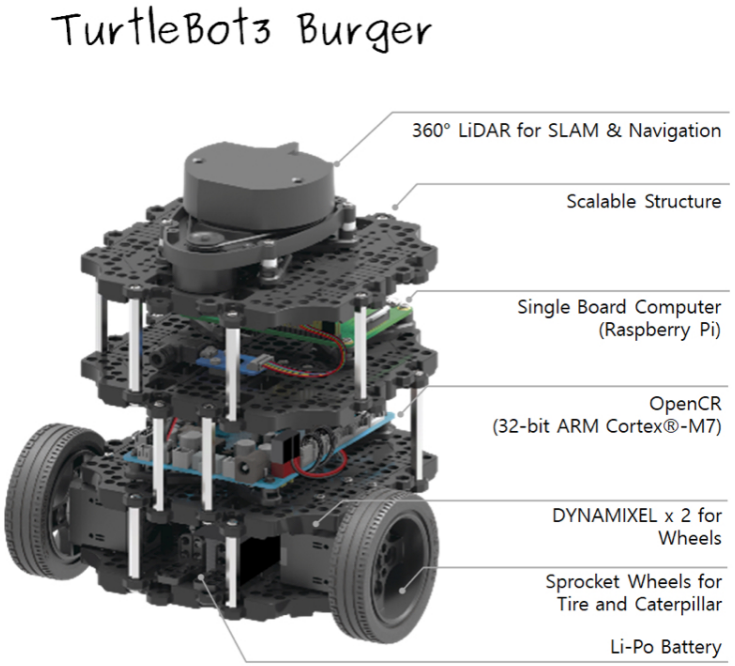


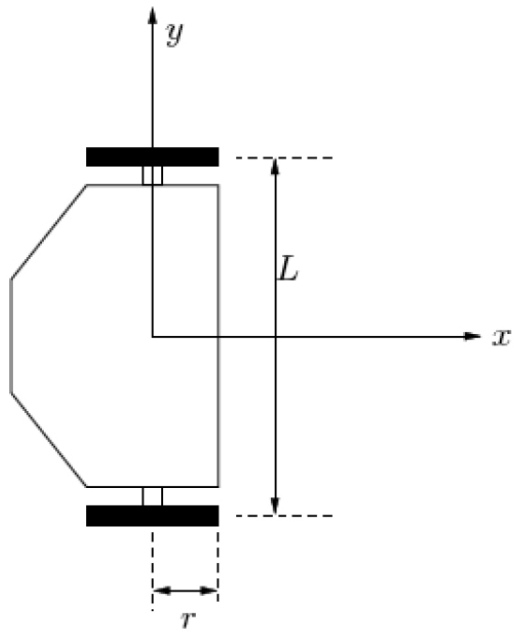

state: $\boldsymbol{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$

remark: orientation is called **bearing** or **heading direction**

$$\theta = 0: \quad \text{robot moves in } x\text{−direction}$$
$$\theta = \pi/2: \quad \text{robot moves in } y\text{−direction}$$

# Velocity-based motion model: Differential drive



# Differential drive: motion model

motion model:

$$\begin{aligned} \dot{x} &= \frac{r}{2}(u_l + u_r)\cos\theta \\ \dot{y} &= \frac{r}{2}(u_l + u_r)\sin\theta \\ \dot{\theta} &= \frac{r}{L}(u_l - u_r) \end{aligned}$$

$u = (u_l, u_r)$: action/control
$=$ angular wheel velocities (in rad/s)
$L$: distance between the two wheels
$r$: radius of wheel

# Differential drive: motion model

motion model:

it is sometimes preferable to transform the action space:
$(u_l, u_r) \Rightarrow (u_{tr}, u_{rot})$

$$\begin{aligned} u_{tr} &= (u_l + u_r)/2 \\ u_{rot} &= (u_l - u_r) \end{aligned}$$

$u_{tr}$ : angular velocity inducing translation
$u_{rot}$ : angular velocity inducing rotation

$$\begin{aligned} \dot{x} &= u_{tr} \cos\theta \\ \dot{y} &= u_{tr} \sin\theta \\ \dot{\theta} &= \frac{r}{L} u_{rot} \end{aligned}$$

# Differential drive: modes of operation

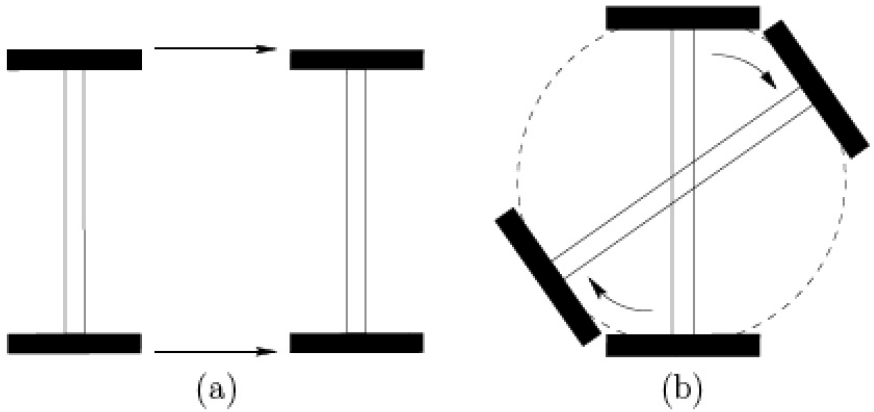



Figure 13.3: (a) Pure translation occurs when both wheels move at the same angular velocity; (b) pure rotation occurs when the wheels move at opposite velocities.

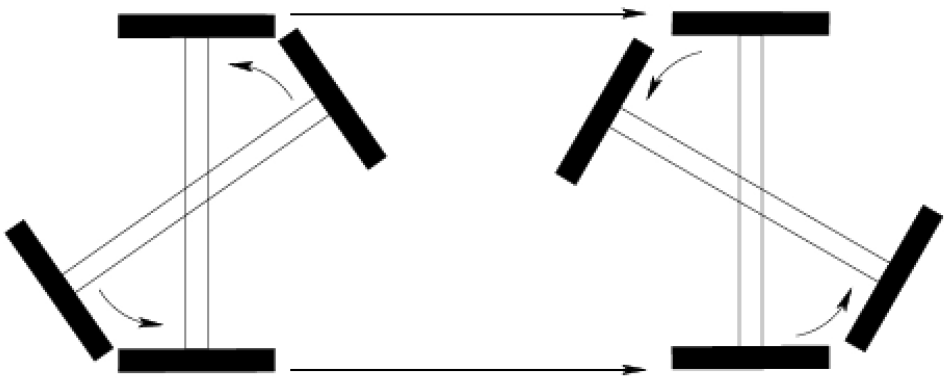


Figure 13.4: The shortest path traversed by the center of the axle is simply the line segment that connects the initial and goal positions in the plane. Rotations appear to be cost-free.

# Differential drive: discretized motion model

- robot moves from $(x, y, \theta)$ to $(x', y', \theta')$ in time interval $\Delta t$
- motion command $u = (u_l, u_r)$

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \frac{r}{2}(u_l + u_r)\cos\theta\Delta t \\ \frac{r}{2}(u_l + u_r)\sin\theta\Delta t \\ \frac{r}{L}(u_l - u_r)\ \Delta t \end{pmatrix}$$

# Differential drive: real motion model

- real motion is noisy
- actual velocities $(\hat{u}_l, \hat{u}_r)^T$ differ from the commanded ones $(u_l, u_r)^T$:

$$\begin{pmatrix} \hat{u}_l \\ \hat{u}_r \end{pmatrix} = \begin{pmatrix} u_l \\ u_r \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix},$$

  where $\epsilon_1$ and $\epsilon_2$ are Gaussian random variables with zero mean:

$$\epsilon_1 \sim \mathcal{N}(0, \alpha_1|u_l| + \alpha_2|u_r|)$$
$$\epsilon_2 \sim \mathcal{N}(0, \alpha_3|u_l| + \alpha_4|u_r|)$$

  and $\alpha_i \geq 0$ $(i = 1, \ldots, 4)$ are robot-specific error parameters.

- real motion model

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \frac{r}{2}(\hat{u}_l + \hat{u}_r)\cos\theta\Delta t \\ \frac{r}{2}(\hat{u}_l + \hat{u}_r)\sin\theta\Delta t \\ \frac{r}{L}(\hat{u}_l - \hat{u}_r)\ \Delta t \end{pmatrix}$$

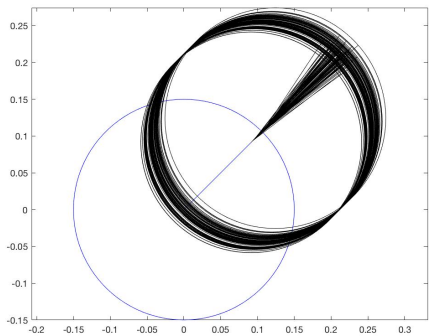
# Differential drive: sampling algorithm

- sample poses $(x', y', \theta')$ from a pose $(x, y, \theta)$ and control $u = (u_l, u_r)$

1: **Algorithm sample differential drive** $(u_t, x_{t-1})$
$(u_l, u_r) = u_t$, $(x, y, \theta) = x_{t-1}$
2: $\hat{u}_l = u_l + \text{sample}(0, \alpha_1|u_l| + \alpha_2|u_r|)$
3; $\hat{u}_r = u_r + \text{sample}(0, \alpha_3|u_l| + \alpha_4|u_r|)$
4: $x' = x + \frac{r}{2}(\hat{u}_l + \hat{u}_r)\cos\theta\Delta t$
5: $y' = y + \frac{r}{2}(\hat{u}_l + \hat{u}_r)\sin\theta\Delta t$
6: $\theta' = \theta + \frac{r}{L}(\hat{u}_l - \hat{u}_r)\Delta t$
7: return $x_t = (x', y', \theta')$

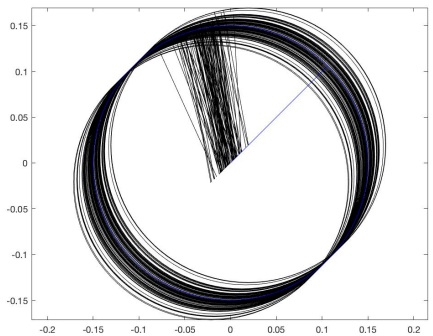
# Sampling algorithm - velocity motion model

robots: $N = 100, L = 0.3, r = 0.05$
initial pose: $x_0 = (0, 0, \pi/4)$



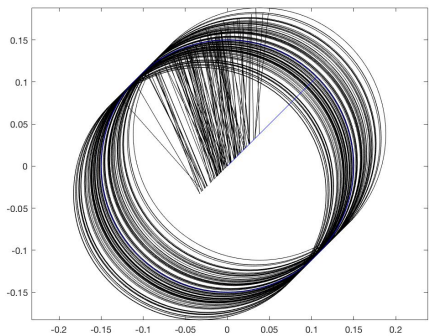
motor commands:
$u_l = 3$
$u_r = 3$

noise parameters:
$\alpha_1 = 0.05$
$\alpha_2 = 0.05$
$\alpha_3 = 0.05$
$\alpha_4 = 0.05$



motor commands:
$u_l = 3$
$u_r = -3$

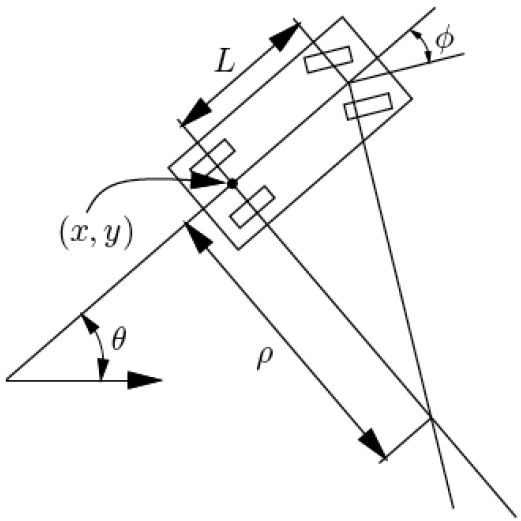
noise parameters:
$\alpha_1 = 0.05$
$\alpha_2 = 0.05$
$\alpha_3 = 0.05$
$\alpha_4 = 0.05$



motor commands:
$u_l = 3$
$u_r = -3$

noise parameters:
$\alpha_1 = 0.05$
$\alpha_2 = 0.05$
$\alpha_3 = 0.15$
$\alpha_4 = 0.15$

# Velocity-based motion model: Simple car model




motion model:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\phi \end{aligned}$$

derivation:

$$\begin{aligned} ds &= \rho d\theta \\ \rho &= L/\tan\phi \\ &\Rightarrow \\ d\theta &= \frac{ds}{L}\tan\phi \quad \text{(similar triangles!)} \end{aligned}$$

action: $u = (v, \phi)$

$L$: length of car

# Velocity-based motion model: Simple car model

motion model - version 2:

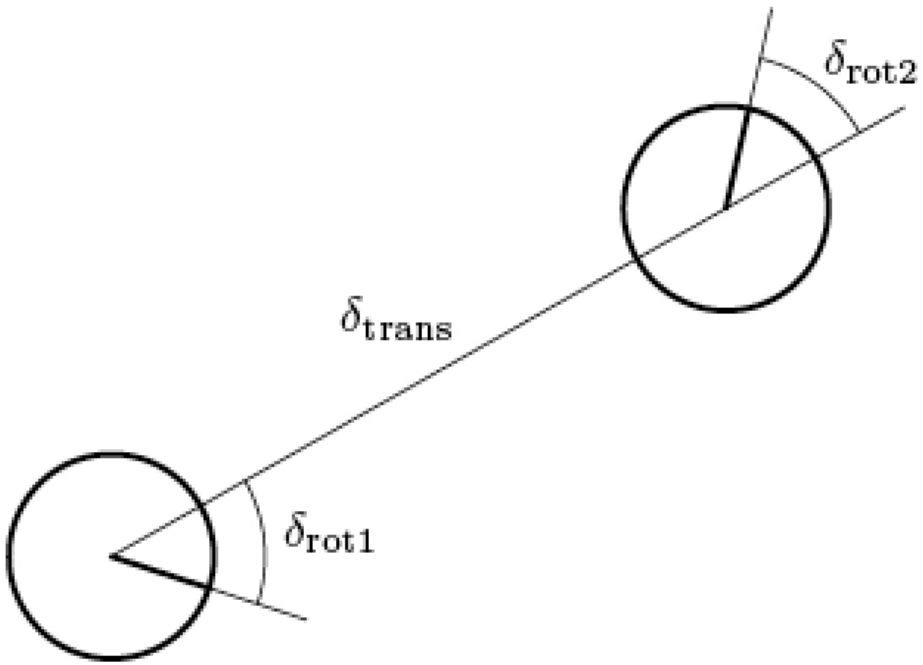
$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\phi \\ \dot{\phi} &= \omega \end{aligned}$$

action: $u = (v, \omega)$ (velocity-based)
motion model - version 3:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\phi \\ \dot{\phi} &= \omega \\ \dot{v} &= a \end{aligned}$$

action: $u = (a, \omega)$

# Odometry motion model




The robot motion is modeled is modeled by a rotation $\delta_{rot1}$, a translation $\delta_{trans}$ and another rotation $\delta_{rot2}$.
**Homework**: PR, p.132-139

# Summary

from deterministic to probabilistic motion models:

1. identify (kinematic) state and actions
2. derive deterministic (kinematics) motion model from geometry and physics
3. discretize motion model
4. add noise to action variables (often Gaussian white noise)

# Motion and maps

- robot motion in **free space**: $p(x_t|u_t, x_{t-1})$
- robot motion **in an environment** described by a **map** $m$: $p(x_t|u_t, x_{t-1}, m)$

  If $m$ carries information relevant to pose estimation then:

$$p(x_t|u_t, x_{t-1}) \neq p(x_t|u_t, x_{t-1}, m)$$

Approximation: $p(x_t|u_t, x_{t-1}, m) = \eta p(x_t|u_t, x_{t-1}) \cdot p(x_t|m)$
Proof:

$$\begin{aligned}
p(x_t|m, u_t, x_{t-1}) &\overset{\text{Bayes}}{=} \eta \cdot p(m|x_t, u_t, x_{t-1}) \cdot p(x_t|u_t, x_{t-1}) \\
&\overset{\text{Markov}}{=} \eta \cdot p(m|x_t) \cdot p(x_t|u_t, x_{t-1}) \\
&\overset{\text{Bayes}}{=} \eta \cdot \frac{p(x_t|m)p(m)}{p(x_t)} \cdot p(x_t|u_t, x_{t-1}) \\
&= \tilde{\eta} \cdot \frac{p(x_t|m) \cdot p(x_t|u_t, x_{t-1})}{p(x_t)} \\
&= \tilde{\tilde{\eta}} \cdot p(x_t|m) \cdot p(x_t|u_t, x_{t-1})
\end{aligned}$$

# Motion and maps

**Algorithm motion_model_with_map**$(x_t, u_t, x_{t-1}, m)$**:**

*return* $p(x_t \mid u_t, x_{t-1}) \cdot p(x_t \mid m)$

**Algorithm sample_motion_model_with_map**$(u_t, x_{t-1}, m)$**:**

*do*

$x_t =$ **sample_motion_model**$(u_t, x_{t-1})$

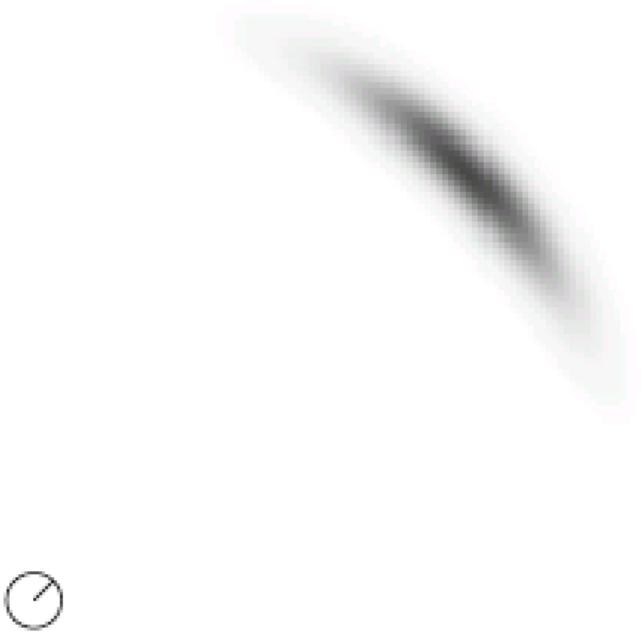
$\pi = p(x_t \mid m)$

*until* $\pi > 0$

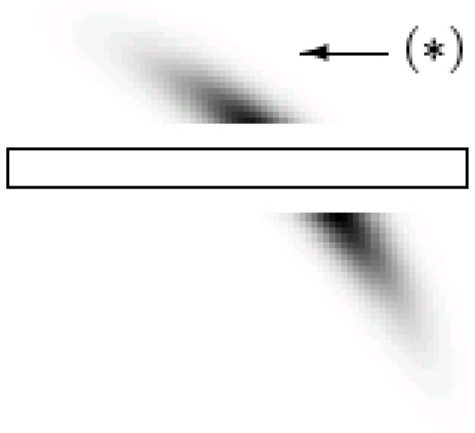
*return* $\langle x_t, \pi \rangle$

# Motion and maps

(a) $p(x_t \mid u_t, x_{t-1})$



(b) $p(x_t \mid u_t, x_{t-1}, m)$



# Robot perception

# Measurement models

- **Measurement models** or **sensor models**:

$$p(z_t|x_t)$$

  is the pdf for observing $z_t$ in state $x_t$
- **measurement step (correction)** in Bayes filter algorithm

$$bel(x_t) = \int p(z_t|x_t)\bar{bel}(x_{t-1})dx_{t-1}$$

- **next**: derive measurement models for specific sensors

# Measurement and maps

- measurement model in **free space**:

$$p(z_t|x_t)$$

- measurements in an **environment** described by a map:

$$p(z_t|x_t, m)$$

# Maps

- a **map** is a list of objects in the environment along with their properties (locations, signatures, etc)

$$m = \{m_1, m_2, \ldots, m_n, \ldots m_N\}$$

two types of maps:

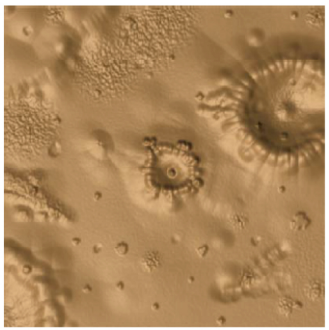
1. **location-based** maps
2. **feature-based** maps

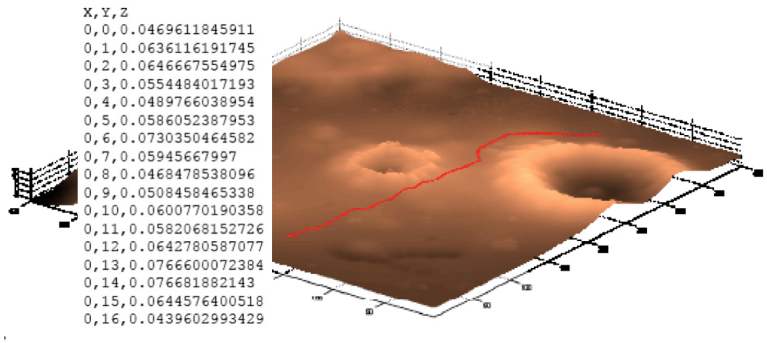
# Location-based maps

- in **location-based** maps the element $m_n$ specifies the location of the element. In planar map one writes $m_{x,y}$.
- loction-based maps are **volumetric**, since they give a label to every point in the environment
- volumetric maps contain information not only about the object in the map, but also about the absence of objects (free space)

# Location-based maps - examples

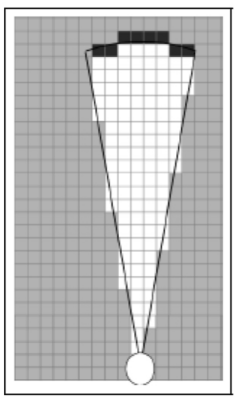
- **digital elevation map** (DEM)
  $m_{x,y} \Rightarrow (x, y, z)$






- **occupancy grid map**
  $m_{x,y} \Rightarrow (x, y, o)$, where $o = 0$ free and $o = 1$ occupied

occupancy grid maps




architectural blue-print

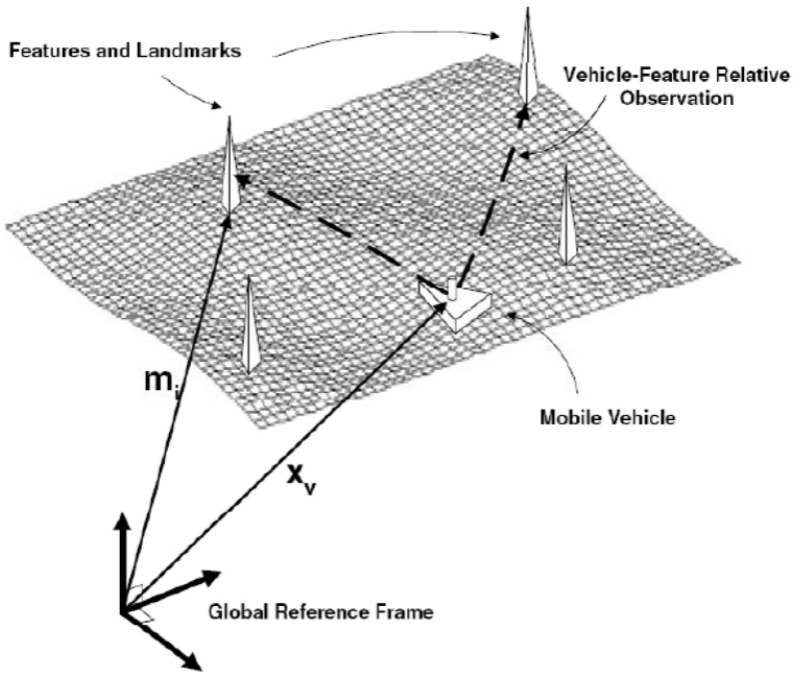
# Feature-based maps

- features are distinctive elements of geometric primitives of the environment
- in feature-based maps the element $m_k$ contains the property (or properties) of the feature and its location on the map:

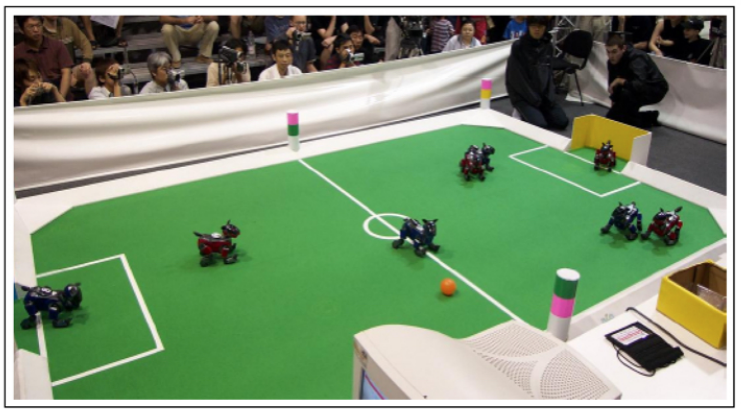
$$m_n = \Big( \underbrace{m_{j,x}, m_{j,y}}_{\text{location of feature } j} , \underbrace{s_j}_{\text{property of feature } j} \Big)$$

- feature-based maps only specify the shape of the environment at specific locations (not volumetric).
- features-based maps are popular in robotics. Features can be extracted from measurements $z_t$ using vision at no costs. This can be formalized with a feature extractor function $f(z_t)$
- reduction of computational complexity: highdimensional measurement space $\Rightarrow$ lowdimensional feature space
- features often corresponds to distinct objects or places in the environment: **landmarks**

# Feature-based maps - examples




RoboCup

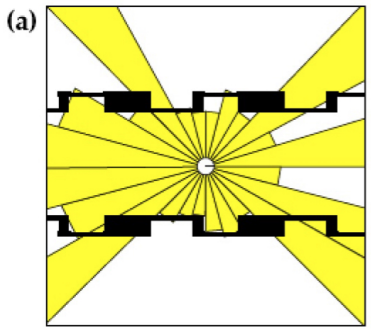


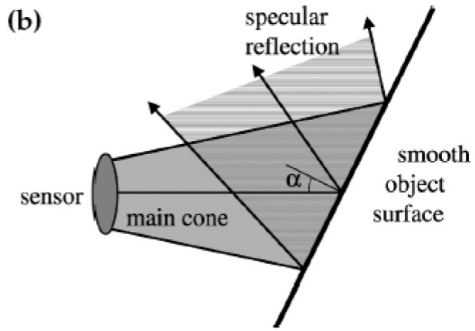
six landmarks

# Sensors - Range scanners

most mobile robots have range scanners

- **sonar range scanners** - range is measured along a cone

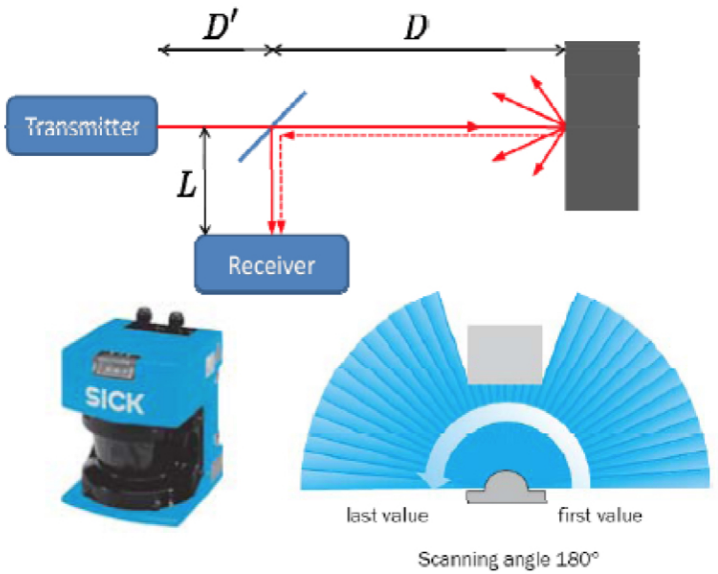


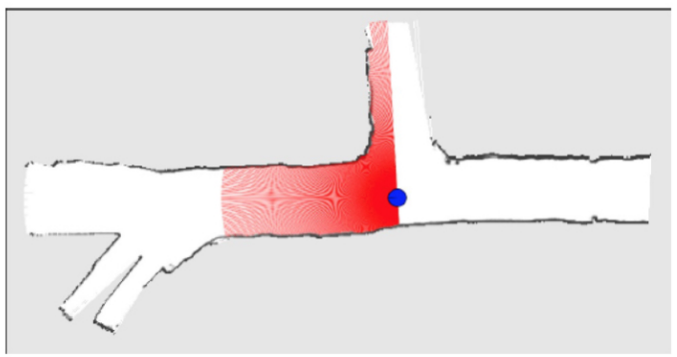




$$z_t = \{z_t^1, \ldots, z_t^k\}$$

range values (scalars)

- **laser range scanners** - range is measured along a beam



$$z_t = \{z_t^1, \ldots, z_t^k\}$$

range values (scalars)

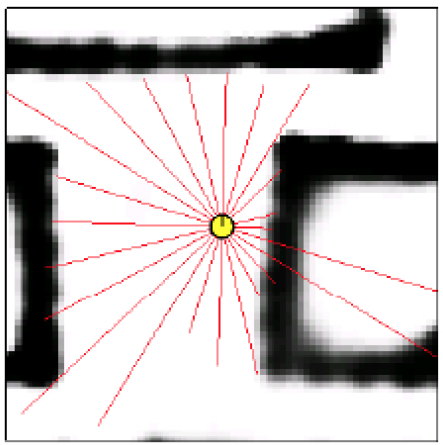
# Measurement model

- scan $z$ consists of $K$ measurements

$$z_t = \{z_t^1, \ldots, z_t^K\}$$

- individual measurements are independent given the robot position $x_t$ and map $m$

$$p(z_t|x_t, m) = \prod_{k=1}^{K} p(z_t^k|x_t, m)$$



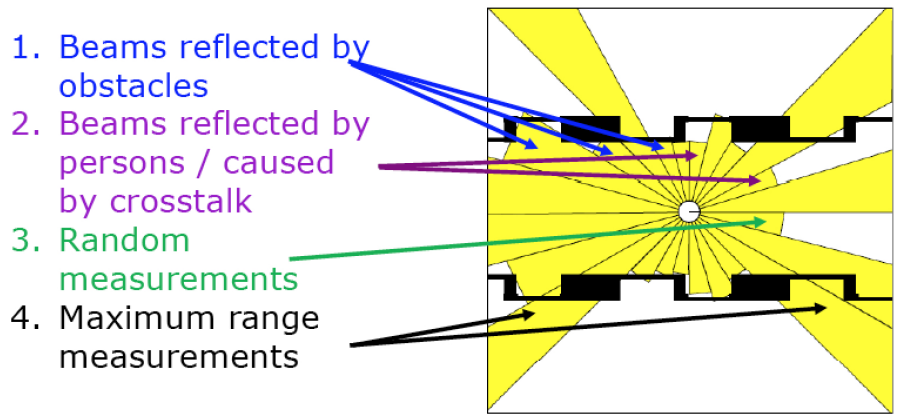
$z_t^k$ is an individual measurement (e.g., one range value)
$m$ is a map

# Beam models of range finders

- measurement model $p(z_t^k | x_t, m)$ of range finders

four types of measurement errors:

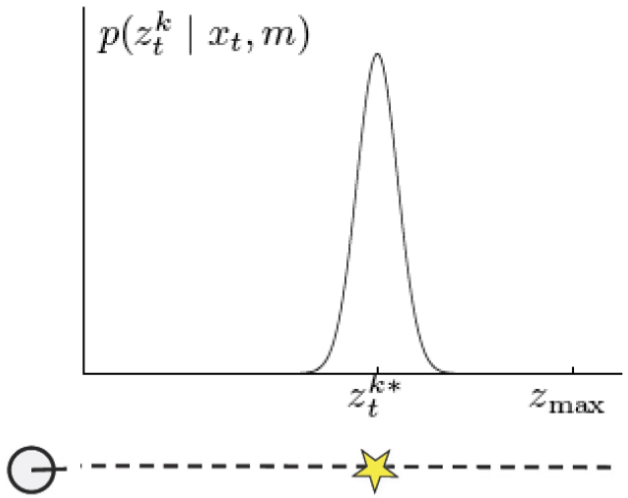
1. small measurement noise
2. errors due to unexpected objects
3. errors due to failures to detect objects
4. random unexplained noise



# Beam models of range finders - $p(z_t^k|x_t, m)$

## 1. measurement noise

(a) Gaussian distribution $p_{\text{hit}}$

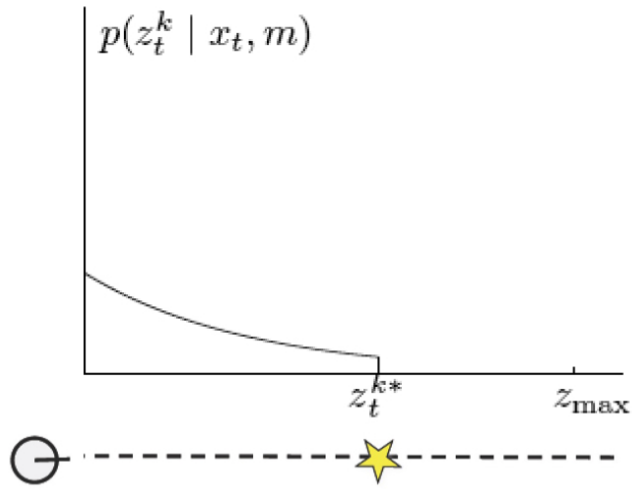



$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) = \frac{1}{\sqrt{2\pi\sigma_{\text{hit}}^2}}\, e^{-\frac{1}{2}\frac{(z_t^k - z_t^{k*})^2}{\sigma_{\text{hit}}^2}}$$

## 2. unexpected obstacles

(b) Exponential distribution $p_{\text{short}}$



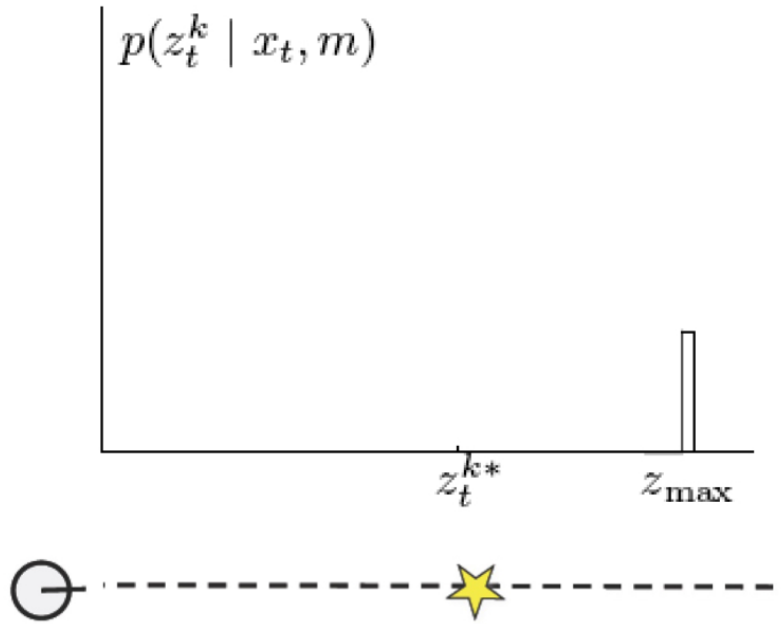

$$p_{\text{short}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \lambda_{\text{short}}\, e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

$z_t^k$: true range of the object extracted from a map (location or feature-based map)

# Beam models of range finders - $p(z_t^k|x_t, m)$

## 3. failures

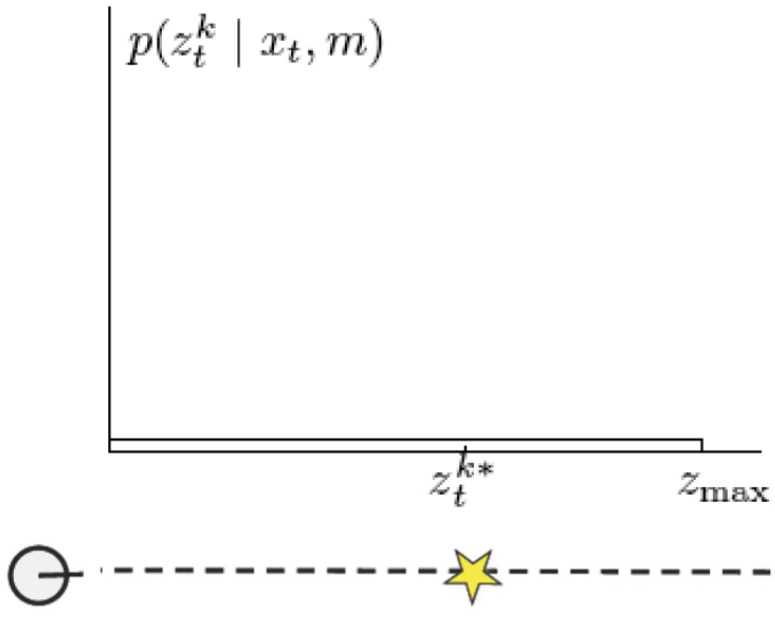
(c) Uniform distribution $p_{\max}$




$$p_{\max}(z_t^k \mid x_t, m) = I(z = z_{\max}) = \begin{cases} 1 & \text{if } z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

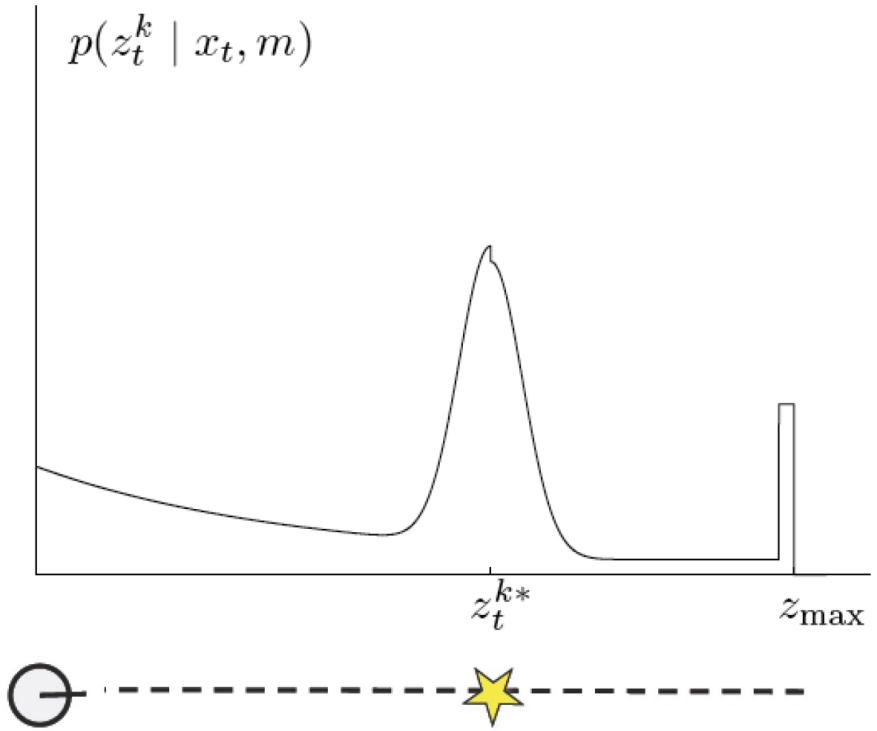
## 4. random measurements

(d) Uniform distribution $p_{\text{rand}}$




$$p_{\text{rand}}(z_t^k \mid x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \leq z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

# Beam models of range finders - $p(z_t^k|x_t, m)$

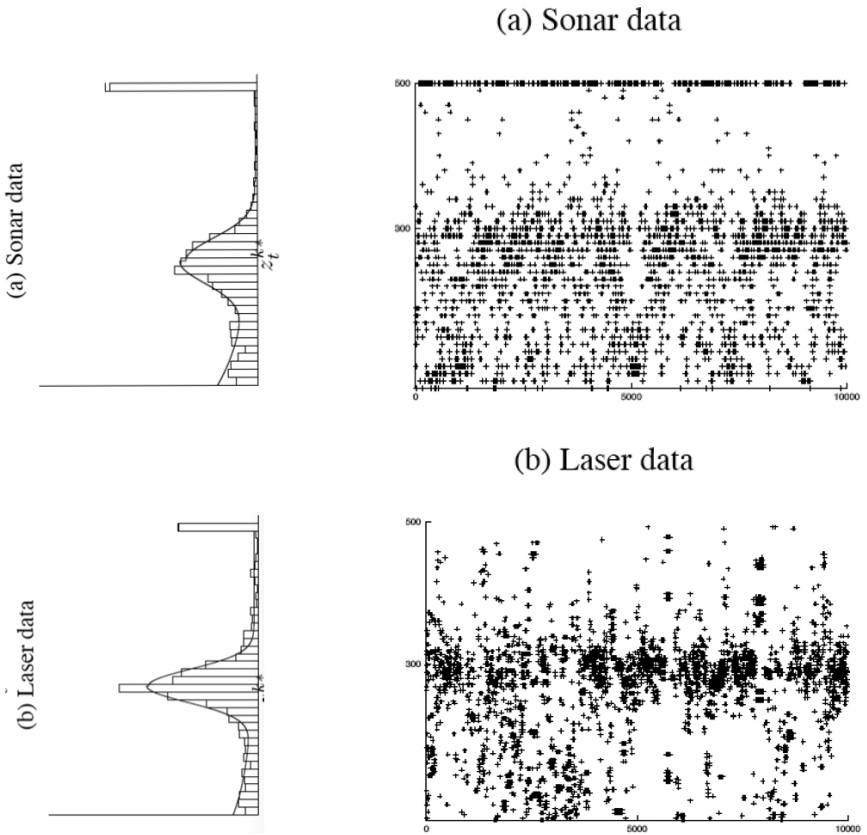



final measurement pdf = mixture of the four distributions

$$p(z_t^k \mid x_t, m) = \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\max} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{short}}(z_t^k \mid x_t, m) \\ p_{\max}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{pmatrix}$$

# Beam models - raw measurement data and fitted pdf

measured distance for expected distance of $z_t^{k*} = 300$ cm




Optimization techniques are used to choose the intrinsic model parameters (e.g., EM algorithm)

# Beam measurement model

**Algorithm beam_range_finder_model**$(z_t, x_t, m)$**:**

$q = 1$

*for* $k = 1$ *to* $K$ *do*

*compute* $z_t^{k*}$ *for the measurement* $z_t^k$ *using ray casting*

$$p = z_{\text{hit}} \cdot p_{\text{hit}}(z_t^k \mid x_t, m) + z_{\text{short}} \cdot p_{\text{short}}(z_t^k \mid x_t, m)$$
$$+ z_{\text{max}} \cdot p_{\text{max}}(z_t^k \mid x_t, m) + z_{\text{rand}} \cdot p_{\text{rand}}(z_t^k \mid x_t, m)$$

$q = q \cdot p$

*return* $q$

**Input**: map $m$, robot pose $x_t = (x, y, \theta)$, complete range scan $z_t = \{z_t^1, z_t^2, \ldots, z_t^K\}$

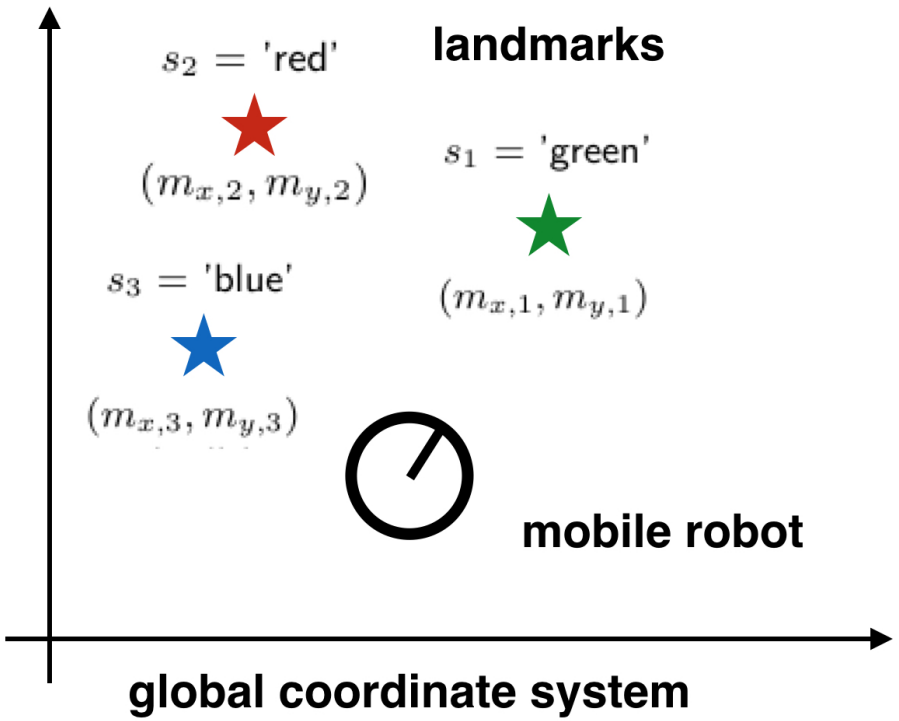
**Output**: numerical value of measurement pdf $q = p(z_t|, x_t, m)$

# Feature-based measurement models

**Given**:

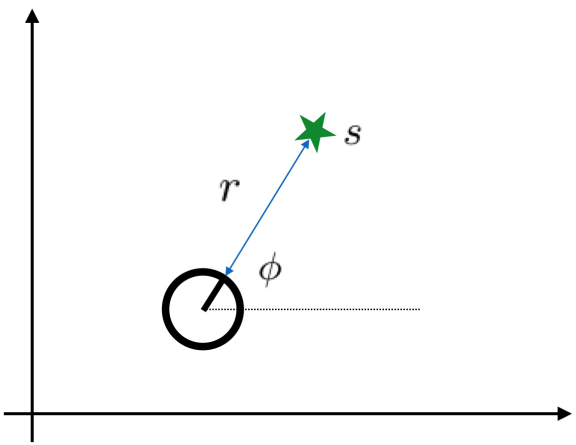
1. a **map** with a list of features (landmarks)
   $m = (m_1, m_2, \ldots, m_n, \ldots, m_N)$

   with $m_n = ( \underbrace{m_{j,x}, m_{j,y}}_{\text{location of feature } j} , \underbrace{s_j}_{\text{property of feature } j} )$



# Feature-based measurement models

2. a sensor can measure the **range** and **bearing** of the landmarks with respect to the local robot coordinate frame
3. the feature extractor $f(z_t)$ can provide one or more **signatures** (height, color, binary code, etc)




measurements:

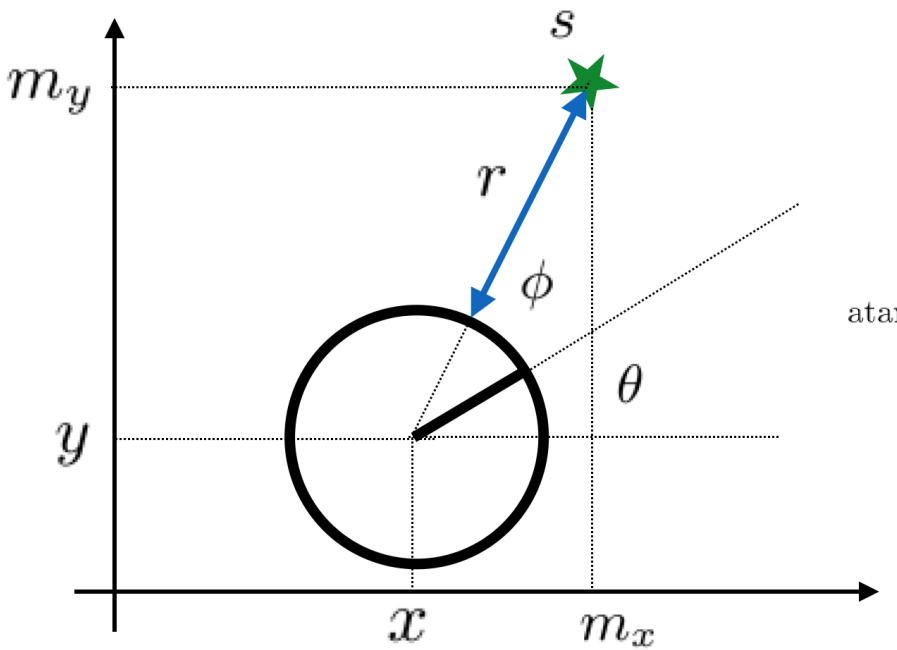
$$f(z_t) = \{f_t^1, f_t^2, \dots\} = \{ \begin{pmatrix} r_t^1 \\ \phi_t^1 \\ s_t^1 \end{pmatrix}, \begin{pmatrix} r_t^2 \\ \phi_t^2 \\ s_t^2 \end{pmatrix}, \dots \}$$

measurement model:

$$p(f(z_t)|x_t, m) = \prod_i p(r_t^i, \phi_t^i, s_t^i | x_t, m)$$

next: derive the measurements from the given pose $x_t$ and map $m$ ...

# Feature-based measurement models




$$\text{atan2}(y,x) = \begin{cases} \text{atan}(y/x) & \text{if } x > 0 \\ \text{sign(y)}\ (\pi - \text{atan}(|\text{y/x}|)) & \text{if } x < 0 \\ 0 & \text{if } x = y = 0 \\ \text{sign(y)}\ \pi/2 & \text{if } x = 0, y \neq 0 \end{cases}$$

*i-th* feature at time $t$ corresponds to *j-th* landmark in the map:

$$\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ s_j \end{pmatrix} + \begin{pmatrix} \epsilon_{\sigma_r^2} \\ \epsilon_{\sigma_\phi^2} \\ \epsilon_{\sigma_s^2}, \end{pmatrix} \quad \text{robot pose : } \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

$$\epsilon \sim \mathcal{N}$$

# Feature-based measurement models

- Key problem: data association. Who is what?
- We assume that a **correspondence variable** exists, able to uniquely identifying the landmark:

$$c_t^i \in \{1, 2, \ldots, N+1\}, \quad N : \text{total number of landmarks}$$

if $c_t^i = j \leq N$ then the $i$-th feature corresponds to the $j$-th landmark
if $c_t^i = N+1$ then the $i$th feature does not corresponds to any feature in the map $m$

# Feature-based measurement models

Now we can compute the feature-based measurement pdf:
$p(f_t^i | c_t^i, x_t, m)$

**Algorithm landmark_model_known_correspondence**$(f_t^i, c_t^i, x_t, m)$**:**

$f_t^i = (r_t^i \;\; \phi_t^i \;\; s_t^i)^T$

$j = c_t^i$

$\hat{r} = \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2}$

$\hat{\phi} = \text{atan2}(m_{j,y} - y, m_{j,x} - x) \text{ - } \theta$

$q = \textbf{prob}(r_t^i - \hat{r}, \sigma_r^2) \cdot \textbf{prob}(\phi_t^i - \hat{\phi}, \sigma_\phi^2) \cdot \textbf{prob}(s_t^i - s_j, \sigma_s^2)$

*return* $q$

**Input**: map $m$, pose $x_t = (x, y, \theta)$, feature measurement $f_t^i$, correspondence variable $c_t^i$
**Output**: numerical value of measurement pdf $q = p(f_t^i | c_t^i, x_t, m)$

# MC Localization revisited

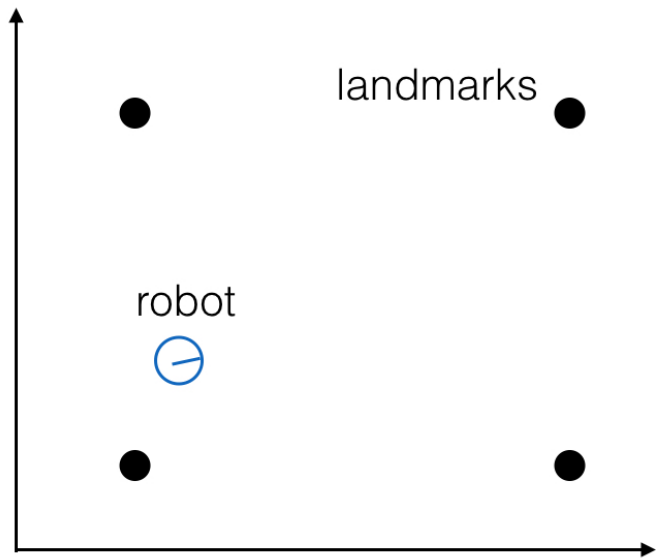
MCL algorithm for landmark-based localization

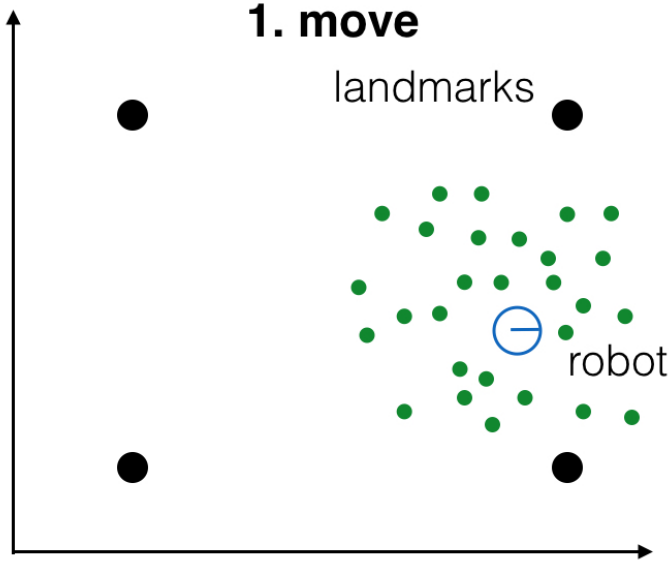
1: **Algorithm MCL**($\mathcal{X}_{t-1}, u_t, z_t, m$)**:**
2: $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
3: for $m = 1$ to $M$ do
4: $x_t^{[m]} =$ **sample_motion_model**$(u_t, x_{t-1}^{[m]})$
5: $w_t^{[m]} =$ **measurement_model**$(z_t, x_t^{[m]}, m)$
6: $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
7: endfor
8: for $m = 1$ to $M$ do
9: draw $i$ with probability $\propto w_t^{[i]}$
10: add $x_t^{[i]}$ to $\mathcal{X}_t$
11: endfor
12: return $\mathcal{X}_t$

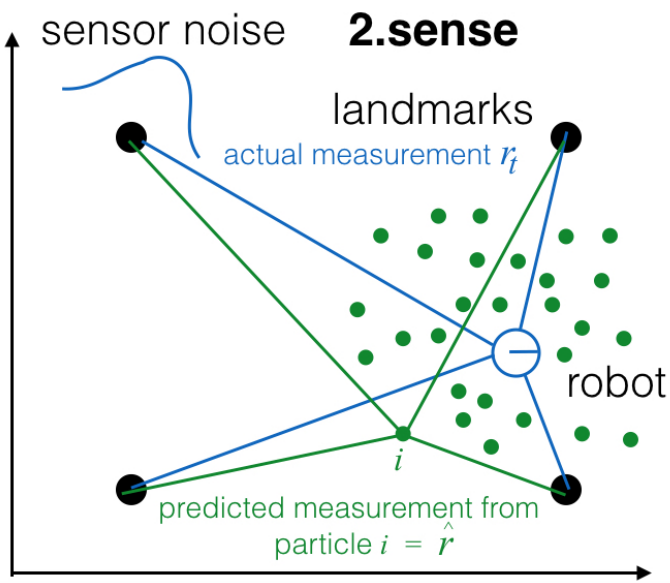
The algorithm **landmark_model_known_correspondance** provides the likelihood model used in line 5 to weigh the predicted samples.

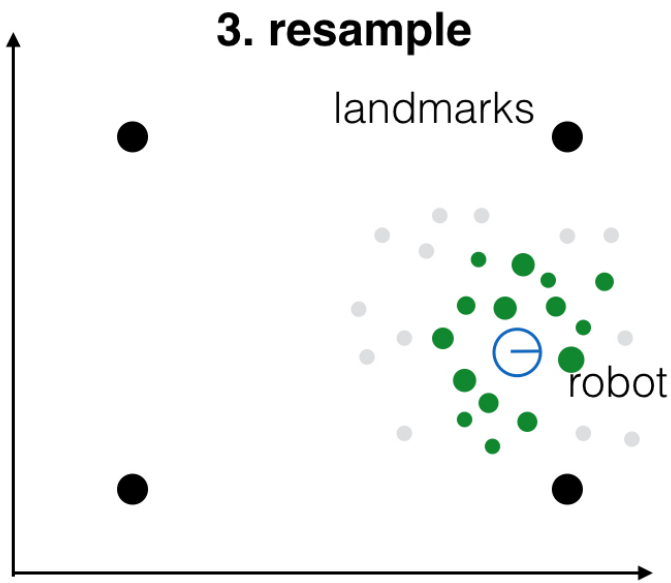
# MC Localization revisited

One cycle in an MCL algorithm for landmark-based localization



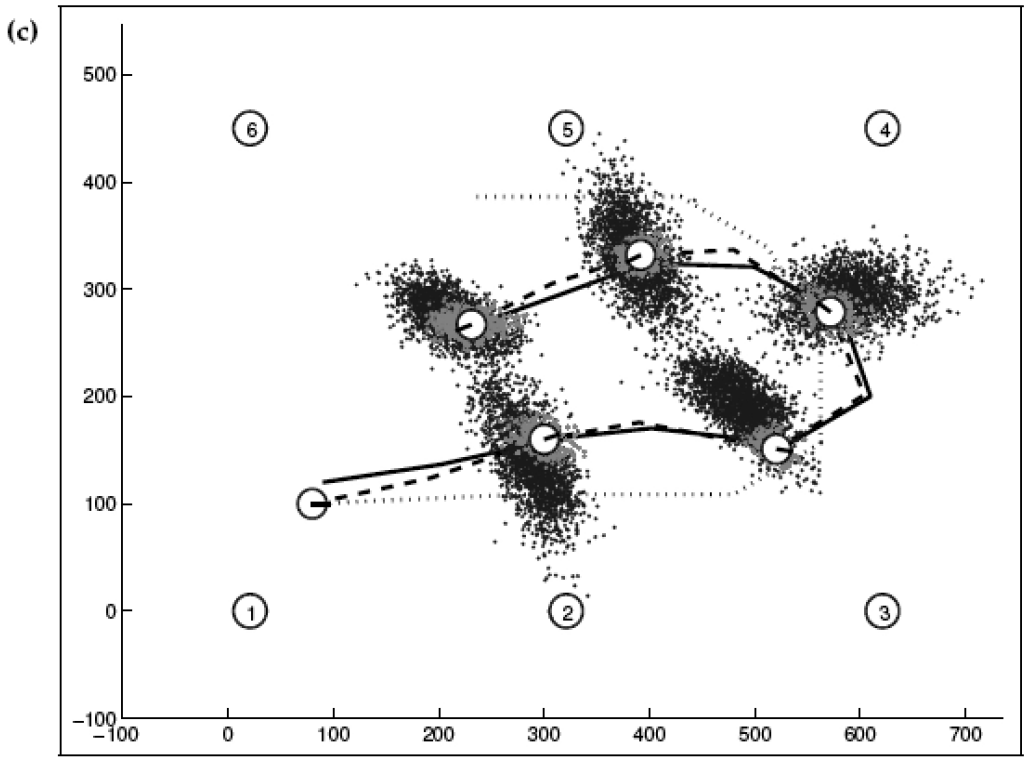



# MC Localization revisited - example

MCL algorithm for landmark-based localization




six landmarks: $1, 2, \ldots, 6$

$\cdots$: path from commanded control

—: true trajectory

– –: mean path estimate by MCL algorithm

# MC Localization revisited

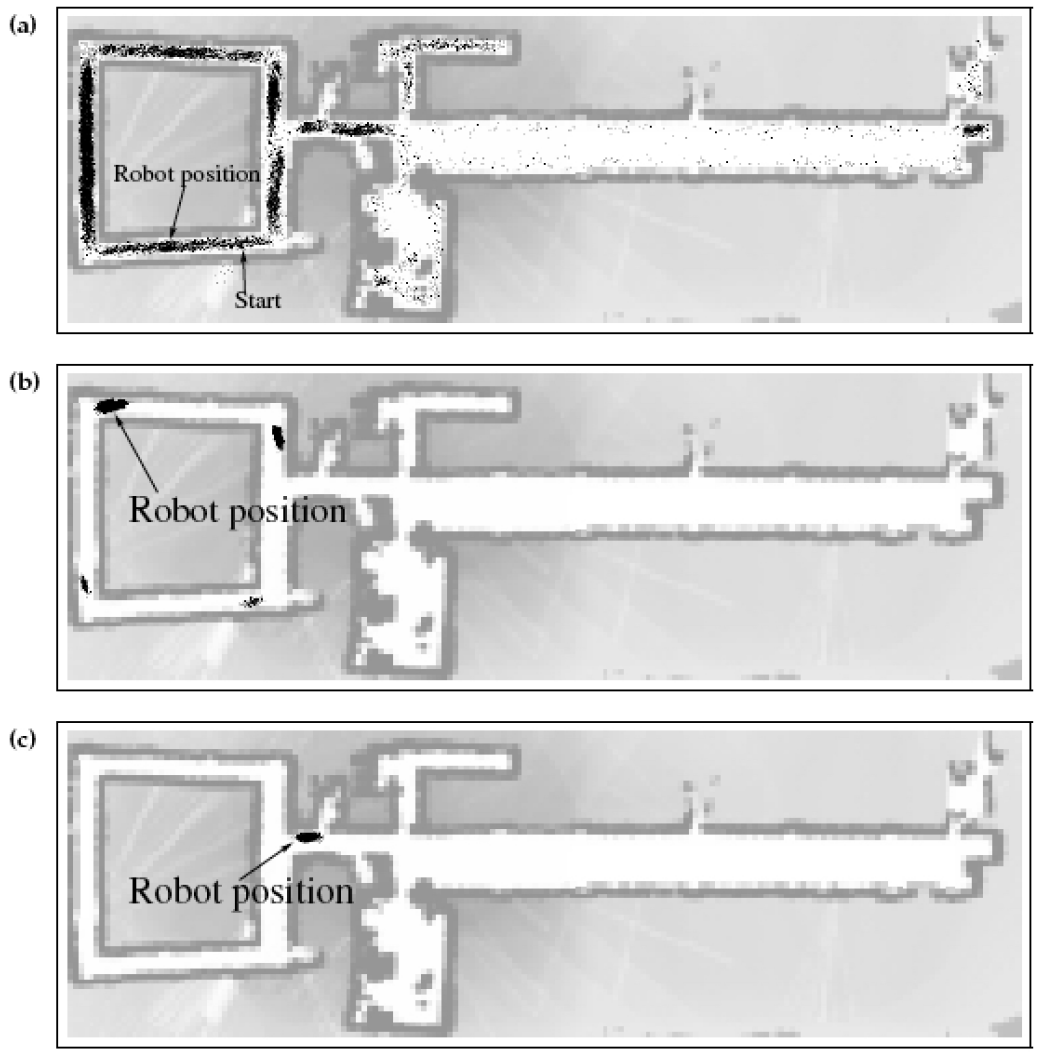
MCL algorithm using an array of sonar range finders

1: **Algorithm MCL**$(\mathcal{X}_{t-1}, u_t, z_t, m)$**:**
2: $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$
3: for $m = 1$ to $M$ do
4: $x_t^{[m]} = \textbf{sample\_motion\_model}(u_t, x_{t-1}^{[m]})$
5: $w_t^{[m]} = \textbf{measurement\_model}(z_t, x_t^{[m]}, m)$
6: $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
7: endfor
8: for $m = 1$ to $M$ do
9: draw $i$ with probability $\propto w_t^{[i]}$
10: add $x_t^{[i]}$ to $\mathcal{X}_t$
11: endfor
12: return $\mathcal{X}_t$

The algorithm **beam_range_finder_model** provides the likelihood model used in line 5 to weigh the predicted samples.

# MC Localization revisited - example

MCL algorithm using an array of sonar range finders



# Acknowledgements - additional resources


- Lecture Notes: *Robot Mapping* by Cyrill Stachniss
- Lecture Notes: *Probabilistic Fundamentals in Robotics* by Basilio Bona