Intelligent Robotics Systems

Exercise 1

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1. Calculation of Kalman filter steps (25 pts)

The motion and measurement model of a one-dimensional system are given by the following set of equations:

$$\begin{aligned} x_k &= x_{k-1} + \epsilon_t \quad \text{with} \quad \epsilon_t = \mathcal{N}(0,1) \\ z_k &= x_k + \delta_t \quad \text{with} \quad \delta_t = \mathcal{N}(0,2) \end{aligned}$$

where $\mathcal{N}(a,b)$ denotes a Gaussian random variable with mean a and variance b.
The following measurements are taken:

$$z_1 = 2, z_2 = 3$$

The initial state is given by

$$\begin{aligned} \mu_0 &= 1 \\ \Sigma_0 &= 10 \end{aligned}$$

a.) Calculate two Kalman filter steps ($k = 1, 2$) by completing the following table:

k	$\bar{\mu}_k$	$\bar{\Sigma}_k$	K_k	μ_k	Σ_k
1					
2					

b.) Explain what these parameters describe.

c.) What is the steady-state covariance matrix $\Sigma_\infty := \lim_{k\to\infty} \Sigma_k$?

2. State estimation using a Kalman filter (75 pts)

Implement a Kalman filter for the example of a moving car presented in lecture 2: A car is moving with constant acceleration a perturbed by noise with zero mean and variance σ_a^2 along a straight road. Noisy measurements (with variance σ^2) of the position x are taken. Estimate the location and velocity of the car.

For the implementation of the Kalman filter set the total simulation to $T = 10\,\text{s}$ with a time step of $\Delta t = 0.1\,\text{s}$. Assume that the mean acceleration of the car is $a = 1.5\text{m/s}^2$.

a.) Define first the state vector for the problem and derive the motion and measurement models. Provide all matrices needed for the implementation of the Kalman filter.

b.) The position measurements of the sensor are provided in an attached text file ('data.txt'). Read the

data and plot the graph of the measurements in a Matlab figure. Provide an estimate of the car position by averaging the measurement data (e.g., by using the smooth function in Matlab). Provide an estimate of the car velocity and plot the result in a different figure.
Hint: Matlab provides a function dlmread to read ascii files.

c.) Add to the figures the exact position and exact velocity using the dynamics derived in part a.) assuming that the process noise is $\sigma_a^2 = 0.25$ and the initial state is $x_0 = \mu_0 = (0, 0)^T$.

d.) Implement a Kalman filter assuming that the process noise is $\sigma_a^2 = 0.25$, measurement noise is $\sigma^2 = 100$ and the initial state is given by

$$x_0 = \mu_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Plot the Kalman filter estimates for position and velocity in the corresponding figures that you generated in part b.). Compare your Kalman estimate with the estimate that you obtained in part b.).
Run the Kalman filter again but now assume a measurement noise of $\sigma^2 = 1$. Which differences do you observe? Explain.

e.) Plot the probability density functions (pdfs) as a function of position for (i) the prediction step before measurement update (in green), (ii) the measurement model (in blue) and (iii) the Kalman position estimate (in red) for three different times $t = 0, 5, 10$ s. Add the exact location of the car by a vertical line. For visualization use a different figure for every time step and normalize the maximum of the unimodal pdfs to 1.Hint: Matlab provides a function normpdf to encode a Gaussian pdf.