Subject A : Free Electrons in Crystals (chapter 7)

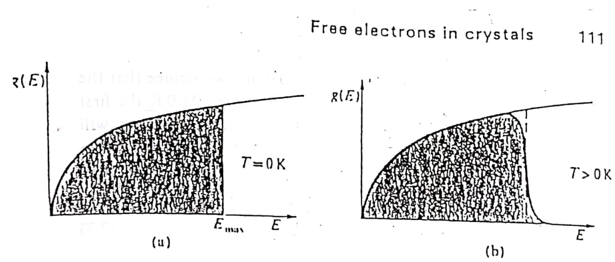
1. Energy levels are discrete -> due to the Pauli exclusion principle only one electron may inhabit an energy level.
2. For a set of states where all the states are occupied by electrons - A closed shell which is very tightly bound to the nuclei
3. There may be a few remaining electrons outside the shell - these are loosely bound to the nuclei - the **free electrons** to a first order approximation
4. When an atom loses a free electron - it becomes a positively charged ion - This is ignored
5. Neighboring atoms may share those free electrons.
6. A free electron may be viewed as a 'particle in a box' which derives the standing wave solution for the *Schrödinger* equation. $E = \frac{\hbar^2 k^2}{2m}$ and the momentum is $m\nu = hk$
7. The number of electron states up to a certain k-state value is given by $G(k) = \frac{Vk^3}{3\pi^2} = \frac{V(2mE)^{3/2}}{3\pi^2 h} \Rightarrow g(E) = \frac{dG}{dE} = \frac{V(2m)^{3/2}E^{1/2}}{2\pi^2\hbar^3} = \frac{3}{2}G(E)/E$ which is the density of states per unit energy - This complies with Heisenbergs uncertainty principle.
8. Counting the number of electrons in **non-metals** up to the state of E_{max} energy $N = \frac{V(2mE_{max})^{2/3}}{3\pi^2\hbar^3}$, this module does not apply to metals, and their conductivity attributes since the temperature is absent.
9. Thermal energy can only affect electrons in energy states close to E_{max}, inhabiting the "outer shell" as was described in (2).
10. 

FIG. 7.2. The density of states $g(E)$ as a function of energy for the free-electron model. (a) At 0 K all states are occupied up to E_{max}. (b) At higher temperatures the occupation of states in the region of E_{max} is smeared out.
11. A better treatment includes the Fermi-Dirac function which gives the probability that an electron has an energy E at temperature T $f_{FD}(E) = \left\{exp\left(\frac{E-E_F}{K_B T}\right)+1\right\}^{-1}$ so the number of electrons occupying the states between E and $E + dE$ is given by$N(E) = \int g(E) f_{FD}(E)\, dE$

12. E_F is the Fermi energy, defined by equating the total number of occupied states to the actual number of free electrons.

    (a) In metals we assume that E_F is equal to E_{max}

13. $\langle E_{tot} \rangle = \frac{1}{N} \int_0^\infty E \cdot g(E) f_{FD}(E) \, dE$

14. At a temperature of $0K$ we derive that $\langle E \rangle = \frac{3}{5} E_F$ which is a few eV, this changes very little with temperature.

15. <u>Specific Heat</u> - states the amount of heat required for a single unit of mass of a substance to be raised by one degree of temperature.

    (a) The contribution of the electrons to the total specific heat of a metal is undetectable in room temperature.

    (b) Taking into account only electrons with energy level E_F we obtain the following $c_e = \frac{1}{2} \pi^2 \frac{N}{E_F} K_B^2 T$ which is the specific heat of the electrons at temperature T. We note that this result is linear with temperature.

    (c) The Lattice specific heat has the form of AT^3 and therefore at low temperatures (liquid Helium region) the electrons are significant in the total specific heat of the metal.

16. We recall that matter is divided according to its reaction to an external magnetic field:

    (a) Diamagnetism - Opposes the applied field, and is present (to a changing extent) in all materials.

    (b) Paramagnetism/Ferromagnetism - Which aligns with the magnetic field.

    (c) Metals show a weak paramagnetic effect which is not temperature dependent (unlike other materials). This can be explained with the free electron theory:

        i. It arises from the magnetic moment, associated with the spin of the free electrons.

        ii. In the presence of a magnetic field B the spin may have either of two orientations; parallel (up) or anti-parallel (down)

        iii. parallel alignment is more energy efficient, therefore there will be a net magnetic moment parallel to B - which causes the effect to be temperature independent.

Subject B: Electrical Conductivity an Band Theory (chapter 8)

1. The free electrons are lesser bound to the nuclei and therefore when applying an external field they move according to the force induced by it, while the electrons in lower energy states are tightly bound, and do not change states when a field is applied.

2. We cannot assume the electrons moving as "electron in-a-box" when accounting for the path of the electron. We must take into account the fact that electrons are moving through a region containing a regularly arranged lattice of positive ions.

   (a) These ions produce a pattern of periodically changing electric field which affects the path of the electrons.

   (b) If the lattice is perfectly arranged an electron is attracted to the ions as in one direction is it will be in the opposite - hence no net change to the movement of the electron.

   (c) In reality there are two types of lattice imperfections:

      i. Thermal vibrations, preventing the atoms of being completely static and arranged$\Rightarrow$ ρ_1 which at low temperatures $\propto T^5$ and at higher temperatures $\propto T$

      ii. The presence of impurity atoms and other point defects$\Rightarrow$ ρ_0 the residual resistance

      iii. with ρ_i being the contribution to resistivity$\Rightarrow$ $\rho = \rho_0 + \rho_1 \rightarrow$ Mattheissen's rule.

   (d) The purity of a sample can be measured as the residual resistance ratio $\frac{\rho_{RT}}{\rho_0}$

3. Both heat and electrical current are transported by the free electrons. But the temperature dependance of the thermal resistivity W is not the same as that of the electrical resistivity ρ.

   (a) This is because charge transported by the electron is constant where as for heat conductivity, the thermal energy which can be transported, depends on the specific heat, which is temperature dependent.

   (b) The Wiedman-Franz law: $\frac{\kappa}{\sigma T} \equiv L$ the Lorentz constant.

4. Thermal resistivity is broken down into two components:

   (a) W_1 - the resistivity component due to thermal vibrations, which we can deduce from the WF law in low temperatures $\propto T^4$ and in high temperature $\propto Const$

   (b) W_0 -due to purities of the material $\propto \frac{1}{T}$

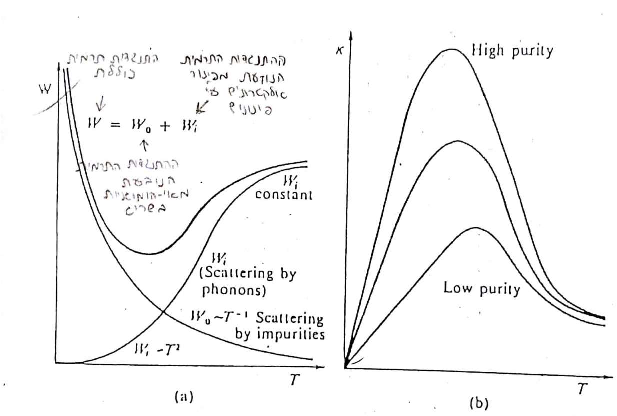
5. 

FIG. 8.3. Conduction of heat by electrons. (a) The electronic thermal resistivity W is the sum of two contributions, W_0 which increases as the temperature is reduced is caused by impurity scattering and W_i is due to electrons being scattered by phonons. (b) The thermal conductivity, κ, as a function of temperature for specimens of different purities.

6. In order to obtain a profound understanding of the nature of electrical conductors we must solve Schrodingers equation with a periodic potential: $V(r) = V(r + na) \Rightarrow -\frac{\hbar^2}{2m}\frac{d^2\psi}{dr^2} = \psi(E - V(r))$.

7. The solution to this equation (which is difficult to derive by hand accurately) lead us to conclude that there are specific energy levels which are either forbidden/permitted → These we call *Energy Bands*

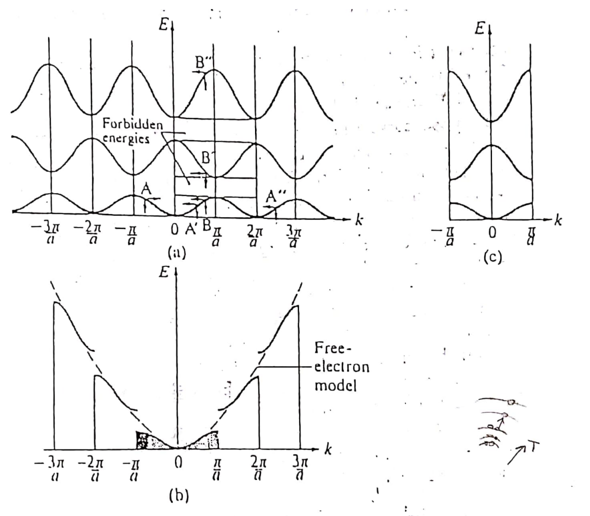
8. 

FIG. 8.4. The relationship between electron energy and wave vector for the simple band theory. (a) The extended-zone diagram which shows the full solution. (b) The more commonly used diagram which incorporates sections from the various curves of (a). (c) The reduced zone scheme in which the curves of (b) are folded back into the first zone.

9. When an electric field is applied in order to drive a current, and the atom

has an even amount of electrons so that there is an energy gap to the next permitted band - this material will be an insulators.

(a) If the energy gap is not too large, $1eV$ or less, a few electrons may be able to cross this gap, the amount of excited electrons will increase with temperature, therefore conduction properties may appear.

(b) In conductors, the electrical resistivity increases with temperature (see 2.c), therefore the conductivity will decrease.

(c) In general, atoms with even number of electrons are insulators, and odd number of electrons are conductors (Magnesium, zinc and mercury are conductors even though they hold an even amount of electrons, due to band overlapping).

10.

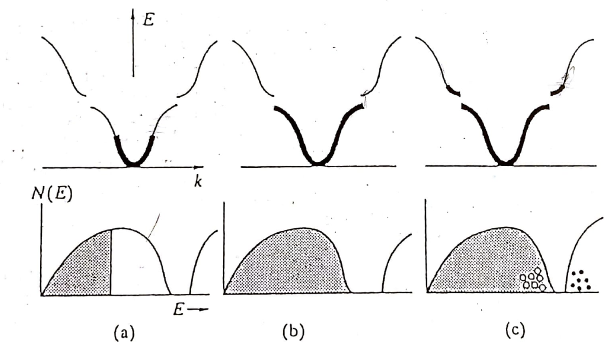


FIG. 8.5. E–k curves (upper) and density of states curves (lower) for (a) an electrica conductor; (b) an insulator (c) a semiconductor.

11. The velocity of the electrons is derived from the group velocity $v_g = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk}$ which is the derivative of the upper graph in the above figure.

(a) Towards the end of the band, although energy is increasing the speed decreases. This is because the 'strength' of the interaction of the electron with the periodic ionic potential has become significant; this can be viewed as kinetic vs. potential energy.

(b) This does mean that the velocity of a single electron decreases, but rather the probability of motion in the $\pm$ directions are equally probable, hence the expectation value shall be 0 $\big($although $\langle v^2 \rangle \neq 0\big)$

Subject C: Superconductivity (chapter 14)

1. Super conductivity is a state of zero resistance (empirically $< 10^{-25} ohm \cdot m$). Certain elements can become superconductors at very low temperatures (0-9K) which we denote T_c which varies between materials.

2. The actual purity specimen may influence the width of T_c, which holds a $\pm$ of $10^{-1} - 10^{-3}K$.

3. Superconductivity is also affected by an external magnetic fields. When a magnetic field is applied in a magnitude of a certain critical value, B_c (which is metal-temperature specific) the specimen is restored to it's normal resistance.
   (a) $B_c = B_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right]$ where B_0 is dependent of T_c
   (b) If $T_c \sim 1K \Rightarrow B_c \sim 10^{-2}T$
4. If a superconductor such that it produces a magnetic field equivalent to B_c the resistance of the sample will be restored$\Rightarrow$ critical current density.
5. At a superconducting state, applying a magnetic field $B_{ext} < B_c$ a persistent current will be induced within the specimen, and will completely cancel out B_{ext} resulting in $B_{int} = 0$, although the external field will induce a non zero value inside the specimen up to some finite depth.
6. When $B_{ext} > B_c$ normal resistance is restored in the specimen, $\rightarrow B_{int} = B_{ext}$, this is due to the persistent current dying out as result of the resistance.
7. Reducing $B_{ext} < B_c$ as B_{ext} passes B_c persistent current are induced, so that $B_{int} = 0$. This expulsion of the internal magnetic field is called the $Meisner\ Effect$.
8. Instead of varying B_{ext} we could vary T.
   (a) $B_{int} = \mu_0 H + \mu_0 m_v = B_{ext} + \mu_0 m_v$ where m_v is magnetic moment per unit volume.
   (b) At a superconducting state $\mu_0 m_v = -B_{ext}$ which is a perfect diamagnet.

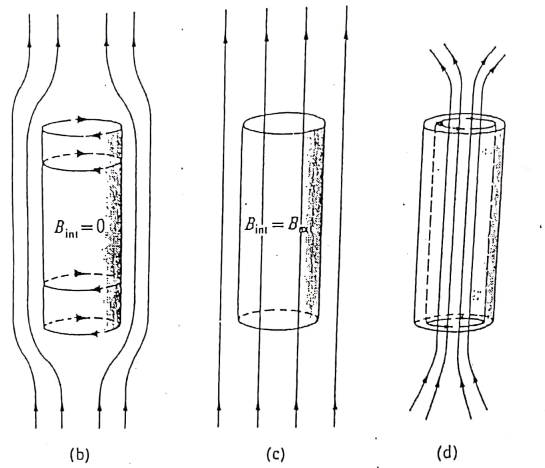
9. 

FIG. 14.3. $B_{int} = 0$ and the Meissner effect. (a) Phase diagram showing the region (shaded) where $B_{int} = 0$. The field is expelled at B_c on the paths CB_cA and DB_cE. (b) $B_{int} = 0$ due to the cancellation of B_{ext} by the field generated by the circulating currents on the surface of the specimen. (c) $B_{int} = B_{ext}$ when $B_{ext} > B_c$. (d) The magnetic field within a hollow cylinder is trapped when it becomes superconducting. It is produced by the circulating currents as indicated.

10. Superconductors can be divided to two superconducting types, based on how they transition from the superconducting state to the normal state when an external magnetic field exceeds B_c

    (a) Type 1 transition practically simultaneously the resistance returns and the diamagnetic moment becomes zero$\Rightarrow B_{int} = B_{ext}$

    (b) Type 2 is more gradual - at field B_{c1} filaments of the material become normal, and the specimen enters a mixed state, until reaching B_{c2} where all filaments become normal.

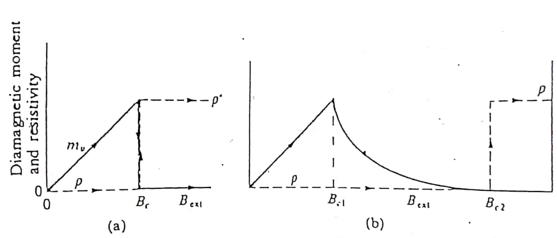
11. 

FIG. 14.4. The destruction of superconductivity by a magnetic field, B_{ext}. (a) In type-I materials there is a sharp transition at $B_{ext} = B_c$ and there the diamagnetic moment, m_v, drops to zero and the resistivity, ρ, rises to its normal value. (b) In type-II materials the field begins to penetrate at $B_{ext} = B_{c1}$ and m_v starts to decrease, but the resistance does not return. The transition is not complete until $B_{ext} = B_{c2}$.

12. As seen in B.2.c.ii there is a component of the resistance which is not temperature dependent (ρ_0) - then we ask how it is possible to reach zero resistivity.

    (a) electrons pair up due to a special type of attraction, a pair can only be scattered if the energy involved is sufficient to break the pair up.

(b) In general this energy will not be available, and so the electron pair passes on, undeviated by impurities.

13. Cooper Pairs

    (a) Such a pair is achieved by indirect interaction, caused by the way a positive ion in a lattice responds to the passage of electrons in its vicinity.

    (b) Suppose an electron passing close to an ion, there will be an attraction which will slightly modify the vibration of the ion, this in turn could interact with a second electron nearby, which will also be attracted by the ion.

    (c) The Net affect is an apparent attractive force between the two electrons, described in field theory as the exchange of a virtual phonon q.

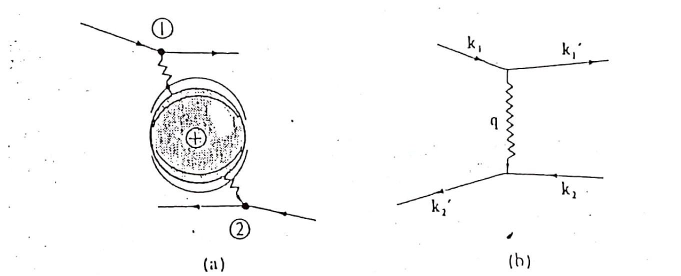


FIG. 14.6. (a) The pairing interaction between electrons occurs because the motion of electron 1 modifies the vibration of the ion and this in turn interacts with electron 2. The over-all effect can be a net attractive force between the electrons. It should be noted that, although the two electrons are here shown fairly close together, in reality they can be a coherence length apart (section 14.17)—this may be many *thousand* atomic spacings! (b) The pairing can be considered to be due to the emission of a phonon of wave vector q by electron 1, followed by its absorption by electron 2 (eqn (14.3)). k, k' are the electron wave vectors before and after the interaction.

14.

15. Since the energy involved is small

    (a) The pairs may be broken by thermal activation. The pairs will begin to form at T_c and as it is reduced, more pairs will be able to remain stable.

    (b) At $0K$ all electrons will be paired.

    (c) Superconductivity is not a property of the atom itself, but depends on the crystal lattice arrangement.

16. Since an electron pair has a lower energy than two normal electrons, there is an energy gap between the paired and unpaired states - denoted 2Δ (the energy needed to excite each electron is Δ).

17. Each electron may pair with another electron:

(a) In the state of zero current flowing, the most probable coupling is that of electrons with opposite and equal wave vectors (opposite direction and equal magnitude), and opposite spins.

(b) This state has a zero net momentum.

(c) If a current is flowing, **<u>How does this influence coupling???? Shift?</u>**

(d) In order for coupling to occur there must be vacant states for the electrons to transfer to $(k_1' + k_2' = k_1 + k_2)$.

Subject D: Energy Gap Measurements; Infrared and tunneling experiments

1.

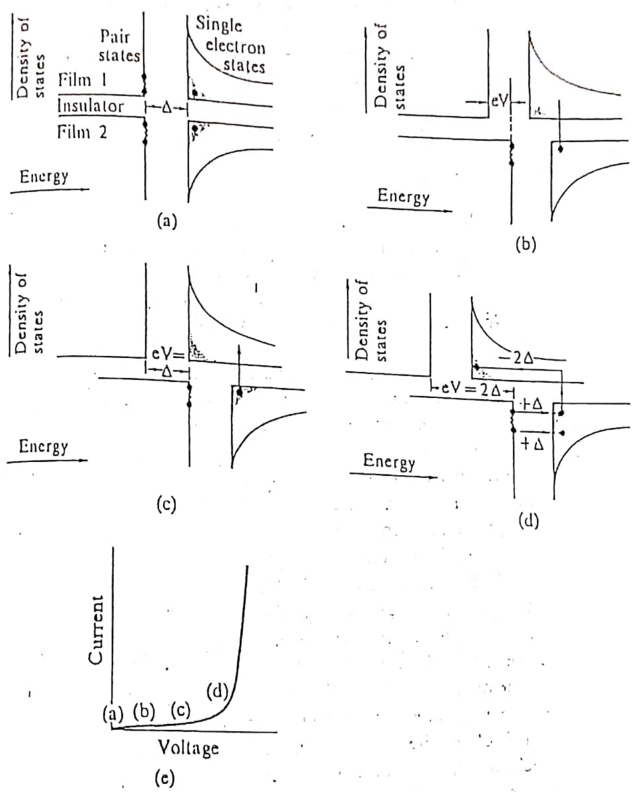


FIG. 14.16. Tunnelling between identical superconductors. (a) The potential, V, across the junction is zero. (b) For low V, the small number of single electrons present can tunnel from 2 to 1. (c) When the pair level of 2 is opposite the edge of the single electron band of 1 pair tunnelling cannot occur because energy is needed to separate the pair into single electrons. (d) When the relative energy displacement of the two films is 2Δ, the paired electrons can split up and tunnel with conservation of energy. (e) The voltage-current characteristic of the device. Note the increase in tunnelling current corresponding to (d).