1 Analysis Framework; Asymptotic notation; Analysis of non-recursive algorithms

1.1 Input size measures

For each of the following algorithms, indicate (i) a natural size metric for its inputs, (ii) its basic operation, and (iii) whether the basic operation count can be different for inputs of the same size:

a. Computing the sum of n numbers

b. Computing $n!$

c. Finding the largest element in a list of n numbers

d. Euclid's algorithm

e. Sieve of Eratosthenes

f. Pen-and-pencil algorithm for multiplying two n-digit decimal integers

1.2 Sequential search

Use the most appropriate notation among O, Θ, Ω to indicate the time efficiency class of sequential search:

a. in the worst case.

b. in the best case.

c. in the average case.

Consider a variation of sequential search that scans a list to return the number of occurrences of a given search key in the list. Does its efficiency differ from the efficiency of classic sequential search?

1.3 Asymptotic notation

For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

a. $(n^2 + 1)^{10}$

b. $\sqrt{10n^2 + 7n + 3}$

c. $2n \cdot \lg(n + 2)^2 + (n + 2)^2 \lg(\frac{n}{2})$

d. $2^{n+1} + 3^{n-1}$

e. $\lfloor \log_2 n \rfloor$

1.4 Analysis of iterative algorithm

Consider the following algorithm:

ALGORITHM *Secret*$(A[0..n-1])$
//Input: An array $A[0..n-1]$ of n real numbers
$minval \leftarrow A[0]$; $maxval \leftarrow A[0]$
for $i \leftarrow 1$ **to** $n-1$ **do**
    if $A[i] < minval$
        $minval \leftarrow A[i]$
    if $A[i] > maxval$
        $maxval \leftarrow A[i]$
return $maxval - minval$

a. What does this algorithm compute?

b. What is its basic operation?

c. How many times is the basic operation executed?

d. What is the efficiency class of this algorithm?

e. Suggest an improvement, or a better algorithm altogether, and indicate its efficiency class. If you cannot do it, try to prove that, in fact, it cannot be done.