The Sound of Wine

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Research Question.

“How does the resonant frequency observed when a wine glass rim is excited depend on the volume of wine within the glass?”

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WEB SPECIAL

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Man who shatters glass with voice

By FPJ Bureau

Fig. 1: “New Delhi: Jaime Vendera can break wine glasses with his raw voice, but the US-based vocal coach doesn’t consider himself special and gives credit to years of hard work for achieving the unique skill” (Bureau).

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Introduction

Jamie Vandera's glass shattering skills, although visually impressive involve nothing more than the application of basic physics principles. The high frequency oscillation of air molecules provided by his voice, resonate with the natural frequency of the empty wine glass. Under such conditions, there is a large scale transfer of energy, leading to the ultimate shattering of the glass.

Jamie Vandera's skills are clearly unusual, however most people have produced similar resonance by rubbing their finger around the rim of a wine glass. The principle remains the same. If the rubbing of the wine glass produces frequencies similar to those of the natural frequency of the glass, there is large scale energy transference to the glass as well as the air inside it, producing the familiar singing wine glass effect. There are subtle differences between the two phenomena. In the case of the singing wine glass, the focus is shifted from the vibration of the glass itself to the vibration of the air within the glass, since it is this oscillation that one hears.

It is frustratingly difficult to make wine glasses sing in a cheap dinner party. For resonance to occur, high quality lead crystal glass is required. It is also observed that the thickness of the glass affects the ability to create resonance. Thicker rimmed glasses are difficult to excite. Whether the dinner party is good or bad, it is observed that as the wine is drunk and the glass emptied, the resonant frequency lowers. This implies that there is an inverse relationship between the volume of unconsumed wine and the resonant frequency when the rim is excited. The nature of this relationship is the subject of this experimental project. The research question addressed is: **"How does the resonant frequency observed when a wine glass rim is excited depend on the volume of wine within the glass?"**

There is a possibility that the resonant frequency will be affected by the morphology of the glass. It seems likely that a convex, bulbous glass vessel might produce different resonant frequencies to a cylindrical tumbler. In this paper, the study will be restricted to a cylindrical wine glass, since this simplifies the mathematical treatment. The liquid used will be water rather than wine. Excitations are produced by wetting the finger and making complete circular rotations of the rim. The mechanism by which this motion produces a constant frequency will be discussed. An understanding of the resonance of wine glasses extends beyond middle class dinner parties and can be extended to areas as diverse as acoustical engineering where resonance is desired and structural analysis in architecture, where resonance is often unwelcome.

Exploration of background ideas.

When a guest in a dinner party rubs the rim of his glass, his finger vibrates with a distinct frequency. The production of this frequency is explained by a process referred to as stick and slip. In the initial phase, the force applied to the finger is increased until friction reaches its maximum static value, at which point the finger starts to slide with a corresponding reduction in the applied force. This leads to the finger coming to a halt. The force is then increased until once again the finger slides. This process is repeated at a microscopic level many times every second and gives rise to a frequency of motion which drives the oscillations of the glass. Increasing or decreasing the speed and pressure of the finger changes this frequency. When the frequency matches the natural resonant frequency of the glass, large scale energy transference occurs and the glass resonates. If the glass is full of wine, then the resonance of the glass is transferred to the liquid. Air molecules in contact with the liquid surface and the glass acquire the same vibrational frequency which is transferred to the ear. It is necessary to understand the relationship between the fundamental frequency of the glass and the volume of wine. It is easiest to first consider the case of an empty cylindrical vessel. The upper rim of the vessel vibrates in distinctive modes. The fundamental mode is shown in figure 2 below.

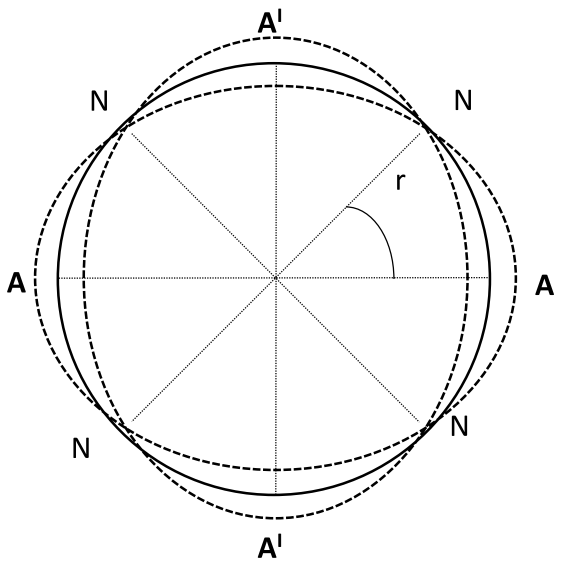

Fig. 2: The fundamental mode
of an oscillating wine glass
viewed from above.

Key to understanding the fundamental mode is the high Young's modulus of glass. This means that glass is essentially inextensible under moderate stresses, such as those present in the singing wine glass. A consequence of this is that the circumferential length remains constant under oscillation. The fundamental mode archives this through alternative orthogonal distortions as shown in figure 3. This mode of vibration is made easier to understand by analogy with an origami fortune teller which is frequently used by young children to predict their futures.

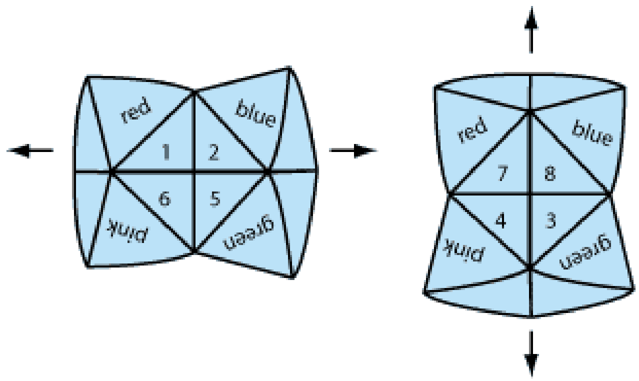

Fig. 3: The motion of the origami fortune teller toy (Origami).

This mode of vibration is characterized by four fixed nodal points labelled N, and anti-nodal points labelled A and A'. Alternate elliptical distortions occur in the y and x directions such that the circumferential length remains the same at all times. These distortions occur in a similar fashion to that of the origami fortune teller. The radial displacement at the nodes is constant and equal to r. At the antinodes, the radial displacement varies between values of (r + Δr) and (r - Δr). This paper will make several key assumptions. It will be assumed that the thickness of the glass is uniform and that the glass itself is cylindrical. These assumptions allow a mathematical treatment for the oscillation of the empty glass to be obtained. Oscillations under such conditions were studied by Rayleigh in his theory of "The Theory of Sound, Vol. 1" (Lord Rayleigh). A more recent treatment is by AP French and that will be followed in this project (A. P. French 688-694). Figure 4 shows a diagram of an empty wine glass oscillating in its fundamental mode.

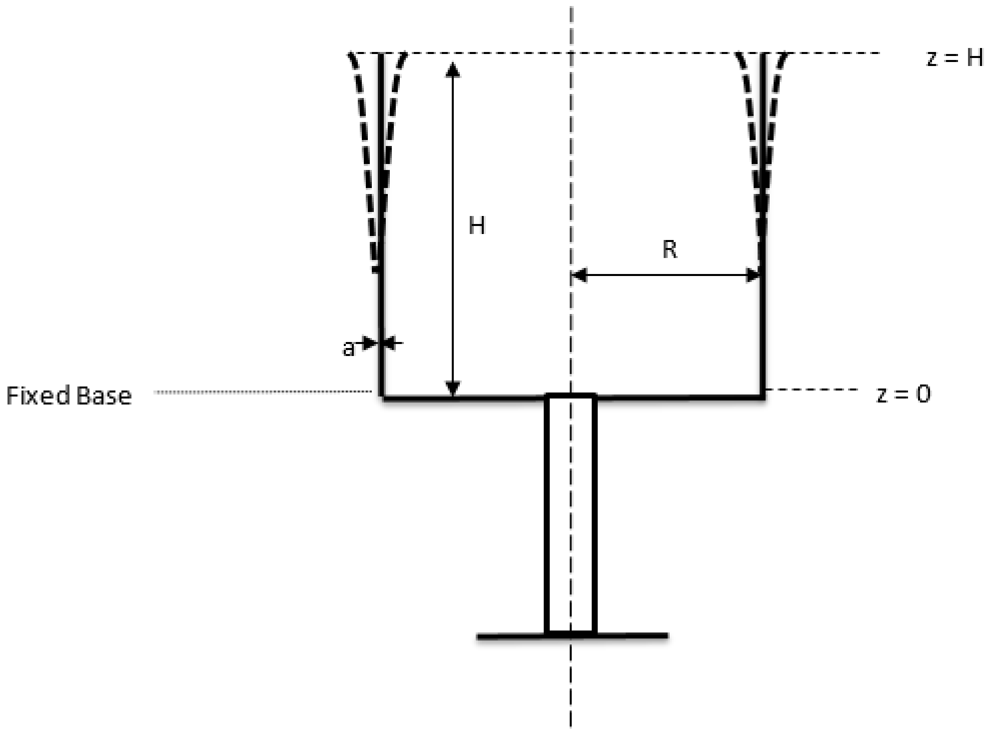

Fig. 4: The profile of an empty vibrating cylindrical wine glass with key parameters labelled.

The height of the wine glass is H and the undisturbed radius is R. The glass is assumed to have a uniform thickness given by a. The vertical coordinate of a glass element is described by z. The radial displacement is denoted as ΔR. This displacement is a sinusoidal function of time for all azimuthal coordinates, such that.

$$\Delta(R) = \Delta_0 \cos(\omega t) \quad [1]$$

Where Δ_0 represents the maximum radial displacement and occurs for azimuthal angles of ($\frac{\pi}{4}, \frac{3\pi}{4} \ldots$). The radial displacement is also a function of azimuthal angle and the vertical position z. Such that:

$$\Delta(t, \theta, z) = f(z)\Delta_0 Cos(\omega t)Cos(2\theta) \; [2]$$

In equation 2, the function f(z) is chosen such that f(0) = 0, and f(H) = 1. The nature of the angular frequency can be determined by considering the kinetic and potential energy of an oscillating element. The kinetic energy of a rim element is given by:

$$KE = A\left(\frac{d\Delta}{dt}\right)^2 \; [3]$$

The potential energy of the corresponding element is a function of the radial displacement:

$$PE = B(\Delta)^2 \quad [4]$$

In equations 3 and 4 (A and B) are constants. The total energy is given by:

$$E = A\left(\frac{d\Delta}{dt}\right)^2 + B(\Delta)^2 \; [5]$$

Substituting equation 1 into 5 produces:

$$E = A\Delta_0^2\omega^2 Sin^2(\omega t) + B\Delta_0^2 Cos^2(\omega t) \; [6]$$

As the rim oscillates, the total energy of the element is constant. This implies that: $\frac{dE}{dt} = 0$. This means that equation 6 can be differentiated and equated to 0. When this is done, it is seen that:

$$A\Delta_0^2\omega^2 2Sin^2(\omega t)Cos(\omega t) = B\Delta_0^2 2Cos(\omega t)\omega Sin(\omega t) \; [7]$$

Although this equation seems complicated at first sight, it simplifies to:

$$\omega^2 = \frac{B}{A} \ [8]$$

Equation 8 is interesting in that it infers that the angular frequency of the fundamental mode depends on the ratio of the potential energy of a rim element to its kinetic energy. This will become important when considering the effect of adding wine to the glass. It should be remembered that the treatment so far, applies to an empty wine glass. Evaluation of the constants B and A is beyond the scope of this experimental project, but results in the following relationship:

$$v_0 = \frac{1}{2\pi}\left(\frac{3Y}{5\rho_g}\right)^{1/2}\frac{a}{R^2} \quad [9]$$

In this equation, Y is the Young's modulus, ρ_g is the density of the glass and a is the thickness of the glass. The equation predicts that the fundamental frequency of the empty glass is inversely proportional to the square of the radius R.

The treatment so far is restricted to the oscillation of an empty glass. One needs to consider the effect of adding wine to the resonant frequency described in equation 9. When wine is added, it vibrates in resonance with the glass and by this means, the kinetic energy of the system at a given moment in time, increases. The potential energy remains constant. This infers that the frequency decreases and that the greater the quantity of wine in the glass, the greater the effect that is observed. This is consistent with the increasing frequencies heard as the wine glass is emptied. To account for this, the kinetic energy term in equation 3 needs to be adjusted, whilst the potential term remains unchanged. Once again, the process is complicated but can be shown (A. P. French 688-694) to result in the following relationship:

$$\left(\frac{v_0}{v_h}\right)^2 \approx C + \frac{\alpha}{5}\frac{\rho_l R}{\rho_g A}\left(\frac{h}{H}\right)^4 [10]$$

Equation 10 appears complicated but tells a simple story. The dependent variable is seen to be the square of the fundamental frequency observed when the rim is excited. The independent variable is the inverse 4th root of the height of the wine denoted as (H). This agrees with observation since as the wine height is increased, the resonant frequency is observed to decrease. The equation predicts that the relationship between these variables is not one of direct proportionality but is linear in nature. The value of the intercept is unknown and is referred to as (C). The gradient of the graph is expected to be influenced by the Young's modulus of the glass and the glass thickness as well as other diverse physical parameters. Practical measurements of the resonant frequency for varying wine heights will be recorded and the linearity of equation 10 will be investigated.

Experimental challenges.

In order to test the complex relationship between the frequency of an excited rim of a wine glass and the height of the wine, a number of experimental challenges arise. The first challenge is that the theory is developed for a cylindrical wine glass. Hence, the first challenge was to find an ideal wine glass which approximates to cylindrical wine glasses. This was far more challenging than you might assume. The author visited many retail outlets in search of such wine glasses and found that most wine glasses found were exceptionally convex. Finally, the author alighted on reasonably cylindrical wine glasses made of bohemian crystals, purchased from a local store. A photograph of which is enclosed below.

Fig. 5: The wine glass made out of Bohemian glass and purchased from a local wine store.

The photograph shows that the wine glass is reasonably cylindrical however it is not perfectly cylindrical and that might cause the results to deviate somewhat from the theory discussed. Another major experimental challenge is that the frequencies and the height are going to have to be determined, to a very high degree of precision since the dependent and independent variables are compound variables. This means that the fractional errors, if any, on the frequency are going to be added and then doubled. Regarding the fractional errors on the height of the wine, those are going to be added and then multiplied by a factor of four. Hence, errors on the compound variables need to be reduced. This is done by measuring both frequencies (the frequency of the excited rim as the wine glass is empty (V_0), and the frequency filled with a certain height of wine (V_h)) using an advanced FFT (Fast Fourier Transformation) application that uses Fourier transformations to decompose the sounds into its individual components. This software solved the experimental challenge of having a large number of frequencies as it can easily isolate the fundamental frequency and measure it with a high degree of precision. Although many FFT software are available on the internet such as Audacity, the software used for the purposes of this paper was that of Spectrum View by Oxford Wave Research. This app was chosen for its clarity, ease of use and precision. A screenshot of the app is shown below for a particular frequency.

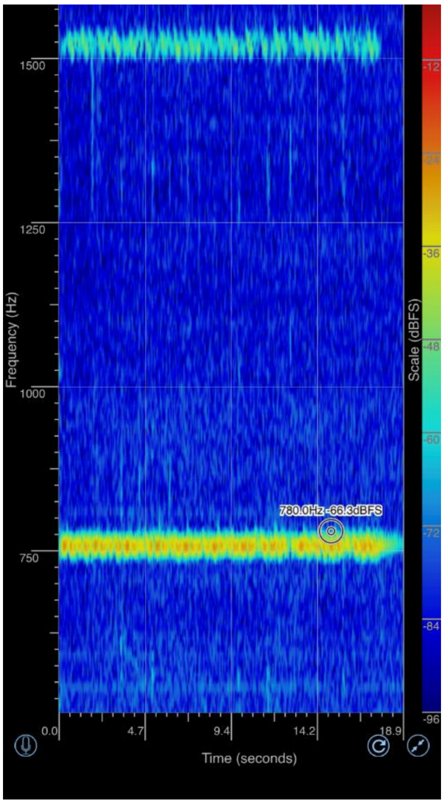

Fig. 6: FFT application used to measure a particular frequency of the glass. The vertical axis shows the frequency measured to a precision of 1Hz and the horizontal axis shows the time.

The screenshot shows that the frequencies are color coded. Dark red means a higher amplitude and orange and yellow mean lower amplitudes. The screenshot clearly shows the fundamental frequency as the dark red frequency with the greatest amplitude. In addition, the approach used in this paper was to zoom in on the frequency to show the bandwidth. This was done in order to take the average frequency by taking the difference between the maximum and minimum frequencies and dividing by two. Since the FFT application records at 100 samples every second and the author allowed it to be played for 3 seconds, the analysis contains 300 data sets and therefore it is unnecessary to take any repeat trials. Therefore, by using this application, the author has solved the primary requirements of identifying the fundamental frequency as the application measured it to a high degree of precision, showing a low bandwidth at all times.

Having discussed the challenges involved in measuring the dependent variable accurately, we now turn our attention to the challenges with the independent variable which was found to be a compound variable $(\frac{h}{H})^4$. The same challenges were faced with the independent variable in terms of the enlarged fractional error due to adding the errors since it is in compound format and then multiplying by a factor of four since the variable is raised to the 4^{th} power. This means once more, that it is essential to measure the height of the glass and the height of the wine in the glass to a high degree of precision. To minimize this, the wine glass selection was important. This is because not only is a cylindrical wine glass needed, but a tall one as well, to minimize the fractional error on the compound variable. However, the author found it difficult to find perfectly cylindrical wine glasses. The final wine glass selected after reviewing 20 different wine glasses was a tall wine glass that best approximates to being cylindrical and offered the greatest range of heights is shown in figure 5. Furthermore, using a meter ruler precise to 1cm is one possibility. However, the method used to measure the height of the wine to decrease the fractional errors on the height of the wine was using a Digital Vernier Caliper. With these problems solved, the procedure is now relatively simple. The glass is excited when empty and the frequency is recorded and then successively greater volumes of wine is added to the glass and the frequencies are recorded, the variables are then processed the graph is produced.

Raw and processed data.

f_o/hz	220	h_o/m	0.090
Δf_o/hz	10	Δh_o/m	0.001

Fig. 7: Constant data table.

f_{hmax}/hz	f_{hmin}/hz	f_h/hz	Δf/hz	$(f_o/f_h)^2$	$\Delta(f_o/f_h)^2$	h/m+/-0.001m	$(h/h_o)^4$	$\Delta(h/h_o)^4$
883.5	836.8	860	47	0.07	0.01	0.047	0.07	0.01
1051.9	1001.6	1030	50	0.05	0.01	0.021	0.003	0.001
1016	974	995	42	0.05	0.01	0.031	0.014	0.002
991	942.8	967	48	0.05	0.01	0.036	0.026	0.004
959.6	918.7	939	41	0.05	0.01	0.039	0.04	0.01
938.7	902.1	920	37	0.06	0.01	0.041	0.04	0.01
847.9	808.4	828	40	0.07	0.01	0.0513	0.11	0.01
706.7	663.2	685	44	0.10	0.02	0.0663	0.29	0.03
666.5	632.4	649	34	0.11	0.02	0.069	0.35	0.04
690.4	660.7	676	30	0.11	0.02	0.067	0.31	0.03
804.5	768.8	787	36	0.08	0.01	0.0543	0.13	0.02
785.8	745.8	766	40	0.08	0.02	0.058	0.17	0.02
757.1	718.9	738	38	0.09	0.02	0.06	0.20	0.02
704.5	674.3	689	30	0.10	0.02	0.065	0.27	0.03
650.6	621.5	636	29	0.12	0.02	0.071	0.39	0.04
730.6	699.5	715	31	0.09	0.02	0.0633	0.24	0.03
740.5	710.3	725	30	0.09	0.02	0.0616	0.22	0.02

Fig. 8: The table of raw and processed data for frequency emitted when the rim of the wine glass is excited versus the height of the wine.

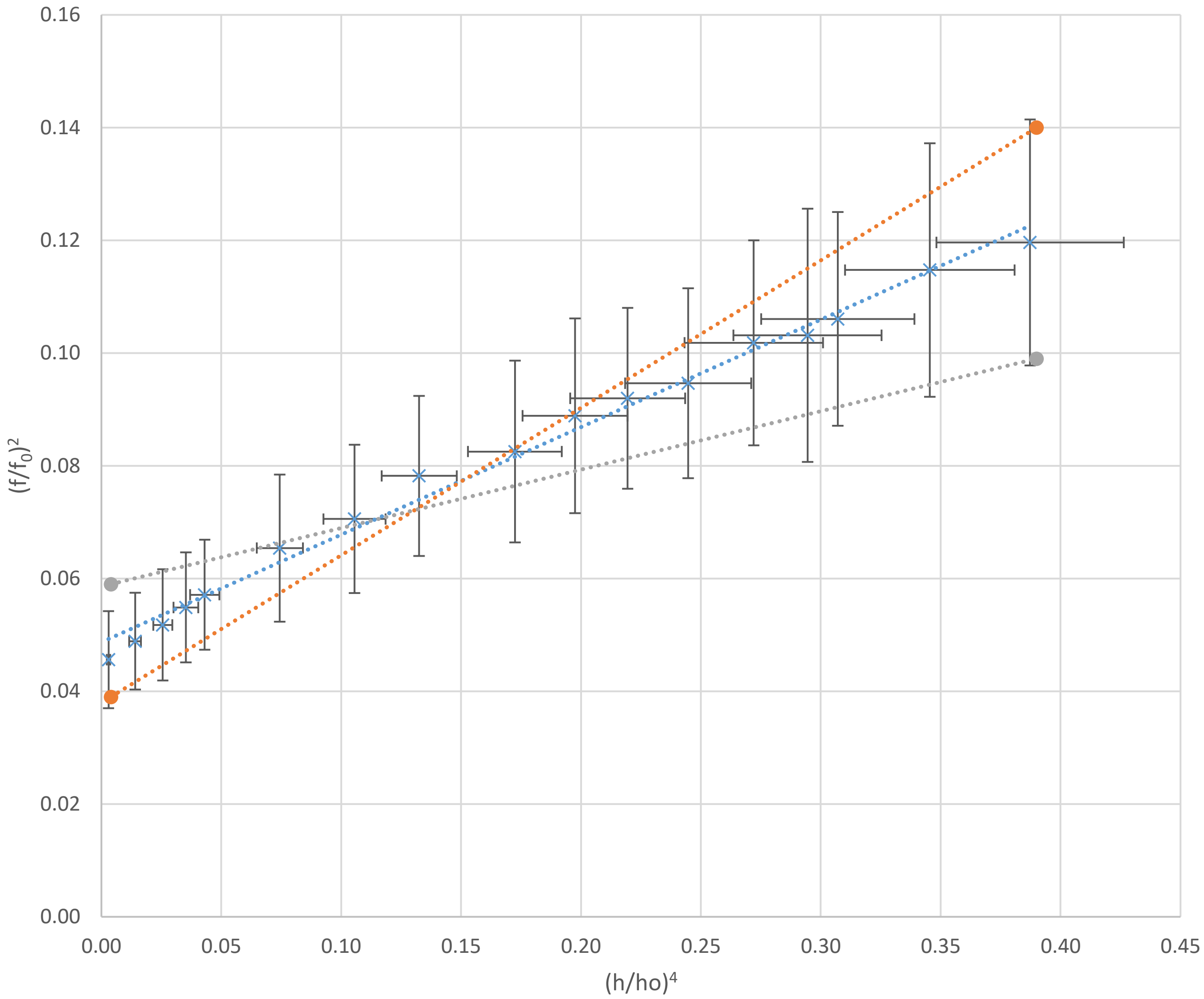

Fig 9: A graph of the square of the frequency of the filled wine glass compared to the fundamental frequency versus the 4th power of the height of the liquid compared to the height of the glass. The trend line is in blue. The gray and orange lines represent approximate maximum and minimum slopes.

Discussion

This project sets out to investigate the relationship between the frequency emitted when the rim of the wine glass is excited and the height of the wine in the glass. The theory was developed for a cylindrical wine glass. The wine glass chosen was reasonably but not perfectly cylindrical. The theory could not be answered with a single variable; hence a compound variable is considered. The theory predicted the relationship described by equation 10:

$$\left(\frac{v_0}{v_h}\right)^2 \approx C + \frac{\alpha}{5}\frac{\rho_l R}{\rho_g A}\left(\frac{h}{H}\right)^4$$

In order to test this relationship, a graph of $\left(\frac{v_0}{v_h}\right)^2$ versus $\left(\frac{h}{H}\right)^4$ was plotted. Before a conclusion can be made it is necessary to investigate the role of errors in the dependent and independent variables and their effect on the final graph. The frequencies were measured using the Oxford Wave Research FFT application. The fundamental frequency of the wine glass when its empty, appears in all compound variables. The frequency was measured as 220Hz with a bandwidth variation of +/-10Hz associated with the FFT application. This represents approximately 5% error on the fundamental frequency of the wine glass which will appear in all the compound variables. The value of the empty height of the wine glass is important as it will appear in all of the compound variables. This was measured with a Vernier caliper to the nearest millimeter. Although the Vernier caliper measures to the nearest tenth of a millimeter, there was uncertainty due to the slightly curved manufacturing of the glass and the hand-blown rim. Because it was a hand-blown glass, the rim was slightly uneven, meaning that although the height was measured with a Vernier caliper accurate to a tenth of a millimeter, the variation on the rim was observed to be approximately one millimeter. Therefore, the height of the wine glass was recorded to the nearest millimeter of 0.090m. This represents a pleasingly small error of 1%. This error is going to appear in all the compound variables. Having measured the base fundamentals, it is necessary to measure the frequency for varying heights of alcohol. The bigger the height, the smaller the fractional error on the variable. However, one cannot neglect small heights, otherwise the results will not reflect the entire range of heights. Therefore, a range of heights from about 21mm to 71mm were taken into account. The greatest fractional error was for the smallest height of 0.021m where the error of one millimeter corresponds to around 5%. The least significant error was for the greatest height of 0.071m, where the error of one millimeter corresponds to around 1%.

It is necessary to consider how this propagation applies to the variable plotting of $\left(\frac{h}{H}\right)^4$. To demonstrate the effect of the accumulation of errors, consider the case for the smallest height which gives the largest fractional error. For the smallest height of 0.021m. The value of $\left(\frac{h}{H}\right)^4$ for this measurement was 0.003. The error for this reading is the fractional error associated with the height which was approximately 5% plus the fractional error of the base height which was 1% giving a total error of 6%. This is multiplied by a factor of 4 due to the power term, giving a total error of 24%, corresponding to approximately 0.001. The height is recorded as 0.003 ±0.001.

Having discussed the errors on height, we can now consider the errors governing the frequency of the rim as it is excited. Again, we will consider the highest fractional error shown by the measurement of the lowest frequency which was 636Hz. The error for the empty frequency is 5%. For this particular value, the frequency was 636Hz with a bandwidth of ±29Hz which corresponds to an additional 5% error. Therefore, for this data set, the total cumulative error is 20%. The variable is recorded as 636 ±29Hz.

Having discussed the extent and role of errors, we can now consider the graph (Figure 9). A key feature of the graph is that the error bars are significant. The error bars on the frequency tend to increase as the ratio increases for reasons discussed earlier. Error bars for the compound variables $\left(\frac{h}{H}\right)^4$ also appear to do the same. The size of the error bars is reasonable considering that all values are multiplied by a factor of four or two. This suggests that the measurements are conducted to a high degree of precision. The graph shows the trend-line plotted in blue which passes comfortably through all the error bars and the major practical data lies very close to the trend-line. The size of the error bars means that little significance should be attached to this observation. The degree of scattering in the graph was very low which shows that random errors have been controlled. Maximum and minimum trend lines have been plotted. The graph also shows a clear intercept and linearity.

The three equations obtained from the graph are:

$$\left(\frac{v_0}{v_h}\right)^2 = 0.2617\left(\frac{h}{H}\right)^4 + 0.038 \quad [11]$$

$$\left(\frac{v_0}{v_h}\right)^2 = 0.1908\left(\frac{h}{H}\right)^4 + 0.0487 \quad [12]$$

$$\left(\frac{v_0}{v_h}\right)^2 = 0.1036\left(\frac{h}{H}\right)^4 + 0.0586 \quad [13]$$

Having discussed the graph, we can now discuss the extent at which the data supports the theoretical predictons. The theory had several key features. It was impossible to evaluate the gradient because of unknown parameters in the theory, such as the Young's modulus of the glass. Likewise, a quantitative evaluation of the intercept cannot be conducted.

Conclusion.

This investigation set out to explore the relationship between the resonant frequency of an excited wine glass and the depth of the wine. The theory behind wine glass resonance was explored. A complex relationship was found. It was predicted that the key variables were ratios relative to constant values. Linearity was predicted between the square of the fundamental frequency and the 4^{th} power of the height of the wine. The data supported this finding, however there was significant uncertainties present, allowing for the possibility of alternative plots to be achieved. In addition, it was not possible to compare the experimental gradient against a theoretical value due to uncertainty as to the value of the Young's modulus of the glass. The spread of the data suggested that random errors had been successfully controlled. The presence of potential systematic errors cannot be evaluated due to an unknown theoretical intercept. Further investigation into non cylindrical wine glasses might be of interest in generalizing the findings of this experimental project.

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