№ 3

1) $ay^3 + d = 0$

$a = 1,\ d = 8$ $\qquad \Delta(a) = 10^{-3};\ \Delta(d) = 10^{-3}$

$y^3 + 8 = 0$

$y^* = \cancel{-8(-2)^3} \sqrt[3]{-\frac{d}{a}}$

$$\Delta y = \frac{1}{3}\underbrace{\left(-\frac{d}{a}\right)}_{-8}^{-2/3} \cdot \Delta\left(\frac{-d}{a}\right) =$$

$$= \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{-\Delta d \cdot a + \Delta a\, d}{a^2} =$$

$$= \frac{1}{12} \cdot \frac{10^{-3} \cdot 8 - 10^{-3} \cdot 1}{1} = \underbrace{\frac{1}{12} \cdot 7 \cdot 10^{-3}}_{}$$

22

$\underline{0{,}583} \cdot 10^{-3}$

Ответ:

2) $h_{opt} = ?$ $\qquad |u^{(5)}(t)| \le M_5$

$$u'(x) \approx \frac{u(x-2h) - 8u(x-h) + 8u(x+h) - u(x+2h)}{12}$$

$$u(x\pm h) = u(x) \pm h u'(x) + \frac{h^2}{2}u''(x) \pm \frac{h^3}{6}u''' + \frac{h^4}{24}u^{(4)} \pm$$
$$\pm \frac{h^5}{120}u^{(5)} + O(h^5)$$

$$u(x\pm 2h) = u(x) \pm 2h u' + 2h^2 u'' \pm \frac{4}{3}h^3 u''' + \frac{2h^4}{3}u^{(4)} \pm$$
$$\pm \frac{4}{15}h^5 u^{(5)} + O(h^5)$$

$$u(x-2h) - 8\, u(x-h) + 8\cdot u(x+h) - u(x+2h) =$$

$$= \underbrace{16 h u' + \frac{8h^3 u''}{3} + \frac{8h^5}{60}u^{(5)} - 4hu' -}$$
$$\underbrace{- \frac{8}{3}h^3 u''' - \frac{8}{45}h^5 u^{(5)}}$$

$$u^{(1)}(x) = \frac{\sim}{12h} = u' - \frac{u^{(5)}h^4}{30}$$

$$\varepsilon_{мет} = u'(x) - u^{(1)}(x) = \left|\frac{u^{(5)}h^4}{30}\right| \le \underbrace{\frac{h^4}{30}M_5}_{\text{4ый пор. ап.}}$$

$$\varepsilon_{round} = \frac{18\Delta u\cdot 12h - (u(x-2h) - 8u(x-h) + 8u(x+h) - u(x+2h))\cdot 12\Delta h}{12^2 h^2}$$

$$\approx \frac{3\Delta y}{2h}$$

$$\varepsilon = \frac{3\Delta y}{2h} + \frac{M_5 h^4}{30}$$

$$\varepsilon'_h = -\frac{3\Delta y}{2h^2} + \frac{4}{30} M_5 h^3 = 0$$

$$\Downarrow$$

$$h^5 = \frac{45}{4} \Delta y \cdot \frac{1}{M_5}$$

$$\boxed{h = \sqrt[5]{\frac{45\Delta y}{4M_5}}}$$

№4

$$x^2 - 2x + 1 - \lambda = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4(1-\lambda)}}{2} = \frac{2 \pm 2\sqrt{\lambda}}{2} =$$

$$= 1 \pm \sqrt{\lambda}$$

$$x^2 - 2x + 1 - \lambda + \varepsilon = 0$$

$$x_{1,2} = 1 \pm \sqrt{\lambda + \varepsilon}$$

$$\sqrt{\lambda} - \sqrt{\lambda + \varepsilon} \le 10^{-4}$$

$$\sqrt{\lambda}\left(1 - \frac{\sqrt{\lambda - \varepsilon}}{\sqrt{\lambda}}\right) \simeq \sqrt{\lambda}\left(\frac{\varepsilon}{2\lambda}\right) \le 10^{-4}$$

$$\varepsilon = 2\sqrt{\lambda}\cdot 10^{-4} = 2\cdot 2{,}5\cdot 10^{-7}$$

$$\underline{\underline{5\cdot 10^{-7}}}$$

Ответ:

№5

x_n ; $5x_{n+1} - x_n = 4$

x_0 - с погр. 10^{-6}

$$5\lambda^{n+1} - \lambda^n = 0$$

$$\lambda = \frac{1}{5}$$

$$x_n = A\left(\frac{1}{5}\right)^n + C$$

$$x_0 = A + C$$

$$x_1 = \frac{A}{5} + C$$

$$5x_1 - x_0 = \cancel{\frac{4A}{5}} \cancel{= 4} \quad A + 5C - A - C = 4$$

$$4C = 4$$

$$\Downarrow$$

$$C = 1$$

$$x_n = A\left(1/5\right)^n + 1 =$$

$$= (x_0 - 1)\left(\frac{1}{5}\right)^n + 1$$

$$\frac{\widetilde{x_n}}{x_n} = 1 + \frac{\Delta x_n}{x_n} = 1 + \frac{\Delta x_0 5^{-n}}{(x_0 - 1)5^{-n} + 1} \approx 1 + \frac{\Delta x_0}{x_0 - 1}$$

$x_0 = 1$: $\varepsilon(x_n) \to 1$