Process Optimization Project Report

Module 3.1: Quantifying Environmental Load for Sustainable "Rendezvous"

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Declaration of Tool Usage

I declare that in completing this assignment:

- I used an LLM-based tool (Gemini) for assistance in:
  - Brainstorming model formulations and exploring nonlinear crowding effects.
  - Formatting the mathematical expressions and generating LaTeX code.
  - Verifying the symbolic differentiation of the proposed objective function.
- I understand the submitted solution fully.
- I can explain and justify every part of my code and reasoning.
- I have verified all results independently.

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1 Introduction

The "Rendezvous" cultural festival at IIT Delhi presents a significant logistical and environmental challenge. As part of the Sustainability Task Force, our objective in Module 3.1 is to develop a robust mathematical model to quantify the total environmental load (E) of the festival. This model balances exploration of complex system dynamics (crowding, economies of scale) with the exploitation of rigorous optimization techniques to identify trade-offs and guide sustainable decision-making.

2 Nomenclature

The variables and parameters used in the mathematical model are defined in Table 1.

Table 1: Nomenclature Table

Symbol	Description	Units	Type
N	Number of attendees	persons	Decision Var
S	Number of food stalls/vendors	count	Decision Var
A	Total event activities	activity-hours	Decision Var
E	Total environmental load	kg CO_2-eq	Objective Fn
α_1	Base per-capita impact coefficient	kg CO_2-eq/person	Parameter
α_2	Base per-stall impact coefficient	kg CO_2-eq/stall	Parameter
α_3	Base per-activity impact coefficient	kg CO_2-eq/activity-hr	Parameter
γ_{NS}	Congestion penalty coefficient	kg CO_2-eq · stall/person^2	Parameter
β_N	Crowding nonlinearity coefficient	kg CO_2-eq/person$^{1.3}$	Parameter
β_S	Stall scaling nonlinearity coefficient	kg CO_2-eq/stall$^{1.2}$	Parameter
β_A	Activity scaling nonlinearity coefficient	kg CO_2-eq/activity-hr$^{0.8}$	Parameter
ϵ	Grid emission factor (India)	kg CO_2/kWh	Parameter
τ	Average event duration per attendee	hours	Parameter
P_{light}	Lighting power per attendee	kW/person	Sub-parameter
P_{stall}	Power per food stall	kW/stall	Sub-parameter
P_{stage}	Power per activity stage	kW/stage	Sub-parameter
ϕ_{waste}	Base waste generation per person	kg/person/day	Sub-parameter
k_{crowd}	Crowding waste amplification factor	dimensionless	Sub-parameter
ξ	Sustainability Efficiency Metric	kg CO_2-eq/(person·hr)	Derived Var

3 Assumptions and Justifications

To develop a tractable yet expressive model, the following assumptions are made based on Life Cycle Assessment (LCA) principles and chemical engineering process economics (Letterman, 1980):

1. **A1: Nonlinear Crowding Effects (Diseconomies of Scale)**. *Justification*: Environmental impact does not scale linearly with attendee density. As crowd density increases, waste collection efficiency drops, littering increases (overflowing bins), and resource competition leads to higher per-capita waste generation. We model this with a superlinear term N^{β_1} where $\beta_1 > 1$.

2. **A2: Economie of Scale in Infrastructure**. *Justification*: High-fixed-cost infrastructure (generators, stages) becomes more efficient per unit of activity as scale increases.

However, the total load still grows. For activities A, we use a sublinear power term $A^{0.8}$ for base energy load, reflecting standard equipment scaling laws (0.6-factor rule).

3. **A3: Congestion-Induced Waste Leakage**. *Justification*: When the ratio of attendees to service points (stalls S) is high ($N/S \gg$ capacity), "leakage" occurs in the form of unsegregated waste and littering. This is modeled as a penalty term proportional to N^2/S.

4. **A4: Sub-component Decomposition**. *Justification*: The total load E is the sum of Energy (E_{energy}), Waste (W_{total}), and Emissions ($E_{emissions}$), adhering to ISO 14040 LCA framework boundaries (Cradle-to-Grave for site operations).

4 Mathematical Model Formulation

4.1 Objective Function Construction

We propose a multi-variable objective function $E(N, S, A)$ comprising three components: Linear Base Load (E_{base}), Nonlinear Scale Effects (E_{scale}), and Interaction/Congestion Penalties (E_{inter}).

$$E(N, S, A) = E_{base} + E_{scale} + E_{inter} \tag{1}$$

Substituting the terms:

$$E(N, S, A) = (\alpha_1 N + \alpha_2 S + \alpha_3 A) + (\beta_N N^{1.3} + \beta_S S^{1.2} + \beta_A A^{0.8}) + \left(\gamma_{NS}\frac{N^2}{S}\right) \tag{2}$$

Where:

- $\alpha_1 N$: Direct resource consumption per attendee.
- $N^{1.3}$: The superlinear crowding term.
- $\gamma_{NS}\frac{N^2}{S}$: The **Congestion Penalty**. As $S \to 0$ for fixed N, environmental load explodes due to waste management failure. As $S \to \infty$, this term vanishes (perfect service), but the direct cost $\alpha_2 S$ increases. This creates a trade-off.

4.2 Sub-Component Integration

The aggregate function E is derived from specific engineering estimates:

$$E_{energy} = \epsilon \cdot \tau \cdot (P_{light} N + P_{stall} S + P_{stage} A) \tag{3}$$

$$W_{total} = \phi_{waste} \cdot N \cdot (1 + k_{crowd}\frac{N}{S}) \tag{4}$$

$$CO2e = E_{transp} + E_{gen} + E_{mat} \tag{5}$$

Equation (4) explicitly shows how waste generation rate depends on the crowding ratio N/S.

4.3 Parameter Estimation for Rendezvous, IIT Delhi

To ground the model in a realistic scenario, we estimate parameter values for the "Rendezvous" cultural festival at IIT Delhi, held on the institute's 325-acre (131 ha) green campus. Rendezvous attracts approximately 160,000 attendees over 4 days (∼40,000 per day), features ∼100 food/vendor stalls and 15–20 parallel activity stages [1, 2, 3, 9]. The estimated values are presented in Table 2.

Table 2: Estimated Parameter Values for Rendezvous, IIT Delhi

Parameter	**Value**	**Justification**	**Source**
Base Impact Coefficients (Linear Terms)			
α_1	2.5 kg CO_2-eq/person	35% of total 7 kg/person/day on-site	[9]
α_2	18 kg CO_2-eq/stall	Energy (60 kWh × 0.727) + materials	[2]
α_3	12 kg CO_2-eq/act-hr	25 kW stage, adj. for 50% renewables	[2]
Nonlinear Scaling Coefficients			
β_N	0.002	Urban superlinear scaling (∼1.15–1.3)	[1]
β_S	0.5	Supply-chain logistics fragmentation	[7]
β_A	5.0	Six-tenths rule, exponent 0.8	[10, 4]
Congestion Penalty			
γ_{NS}	0.0005	$S^* \approx 210$ stalls at N=40,000	[9]
Physical / Engineering Constants			
ϵ	0.727 kg CO_2/kWh	Indian grid FY 2023–24	[2]
τ	6 hours	Avg. student attendance at campus fest	[8]
Sub-Component Parameters			
P_{light}	0.05 kW/person	LED area lighting, 5 W/m^2	[2]
P_{stall}	7 kW/stall	Midpoint of 5–12 kW food truck range	[2]
P_{stage}	25 kW/stage	Sound (15) + LED (5) + AV (5) kW	[2]
ϕ_{waste}	0.65 kg/person/day	Delhi per-capita MSW	[3]
k_{crowd}	0.002	Waste ×1.75 at $N/S \approx 400$	[9]

Key Observation: With these values, the congestion term $\gamma_{NS}N^2/S = 0.0005 \times 40000^2/100 =$ 8,000 kg CO_2-eq dominates the environmental load at full capacity, underscoring the critical importance of sufficient stall infrastructure. The optimal stall count $S^* \approx N\sqrt{\gamma_{NS}/\alpha_2} = 40000\sqrt{0.0005/18} \approx 210$ stalls.

4.4 Region of Interest Analysis

Based on the official layout plan (Region.pdf) and campus map, the festival's primary "Region of Interest" (ROI) spans a central band covering approximately **93 acres** (37.6 hectares), representing about 29% of the total 320-acre IIT Delhi campus. This region encompasses the high-activity zones in the West (Nalanda/SAC), Center (Sports Complex), and East (Academic Core to Main Gate).

For the purpose of spatial modeling in Module 3.2, this ROI is divided into a fine grid of **155 cells**, each approximately **0.6 acres** (50m × 50m) in size, to allow for high-resolution crowd density analysis. The breakdown of sub-zones is detailed in Table **??**.

Table 3: Refined Region of Interest Sub-Zones for Rendezvous

Zone	**Description**	**Area (acres)**	**Type**
West	SAC, OAT, Nalanda, Parking	22	Event Venue
Center	Sports Complex (Indoor/Outdoor)	28	Open Ground
East	Academic Core, LHC to Main Gate	35	Academic
Circulation	Connecting Pathways	8	Roads
	Total ROI	**93**	—

5 Optimization Analysis

5.1 Unconstrained Optimization of E

We seek to minimize $E(N, S, A)$ over the domain $N, S, A \geq 0$.

Gradient Check:

$$\frac{\partial E}{\partial N} = \alpha_1 + 1.3\beta_N N^{0.3} + \frac{2\gamma_{NS} N}{S} > 0 \quad \text{(always positive for } N, S > 0\text{)} \tag{6}$$

$$\frac{\partial E}{\partial A} = \alpha_3 + 0.8\beta_A A^{-0.2} > 0 \tag{7}$$

Since the gradient components for productivity variables (N, A) are strictly positive, the unconstrained global minimum lies at the trivial solution:

$$(N^*, S^*, A^*) = (0, 0, 0) \tag{8}$$

Insight: Minimizing total environmental footprint to zero implies having no festival. This formulation, while chemically accurate, does not capture the *value* of the event.

5.2 Sub-Optimization: Optimal Stall Count

However, for a **fixed number of attendees** N, there exists an optimal infrastructure level S^*. Differentiating E with respect to S:

$$\frac{\partial E}{\partial S} = \alpha_2 + 1.2\beta_S S^{0.2} - \gamma_{NS}\frac{N^2}{S^2} \tag{9}$$

Setting $\frac{\partial E}{\partial S} = 0$:

$$\alpha_2 + 1.2\beta_S S^{0.2} = \gamma_{NS}\frac{N^2}{S^2} \tag{10}$$

The LHS represents the *Marginal Environmental Cost* of adding a stall (embodied carbon, power). The RHS represents the *Marginal Environmental Benefit* of reducing congestion (less litter). The unique intersection $S^*(N)$ defines the optimal resource allocation for any crowd size. Approximating for small nonlinearities ($\beta_S \approx 0$):

$$S^* \approx N\sqrt{\frac{\gamma_{NS}}{\alpha_2}} \tag{11}$$

This square-root law suggests that infrastructure should scale linearly with crowd size to maintain constant congestion levels (N/S), but considering base costs, an optimal ratio emerges.

5.3 Sustainability Efficiency (Part e)

To address the trivial minimum problem, we refine the objective to minimize **Impact Intensity** or maximize **Sustainability Efficiency** (ξ):

$$\text{minimize } \xi(N, S, A) = \frac{E(N, S, A)}{N \cdot \tau} \tag{12}$$

This function represents the environmental cost per "attendee-hour".

$$\xi = \frac{\alpha_1}{\tau} + \frac{\alpha_2 S}{N\tau} + \frac{\alpha_3 A}{N\tau} + \frac{\gamma_{NS} N}{S\tau} + \dots \tag{13}$$

Minimizing ξ with respect to N now involves trade-offs between fixed environmental costs (stalls, stages) which benefit from economies of scale (larger N), and crowding costs ($\gamma_{NS}N/S$) which penalize scale. This formulation yields a non-trivial measure of the "greenest" event scale.

6 Preliminary Insights and Discussion

- **Crowding is Environmentally Expensive**: The term $\gamma_{NS} N^2/S$ is the dominant driver of inefficiency at large scales. Investing in sufficient infrastructure (S) is not just operationally necessary but environmentally critical to prevent waste leakage.
- **Economies of Scale**: The sublinear energy term for activities ($A^{0.8}$) suggests that consolidating entertainment into fewer, high-quality events is greener than many small, fragmented setups.
- **Critical Ratio**: The analysis reveals a critical Attendee-to-Stall ratio $(N/S)_{crit} = \sqrt{\alpha_2/\gamma_{NS}}$. Operating above this ratio causes environmental load to spike due to congestion.

References

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