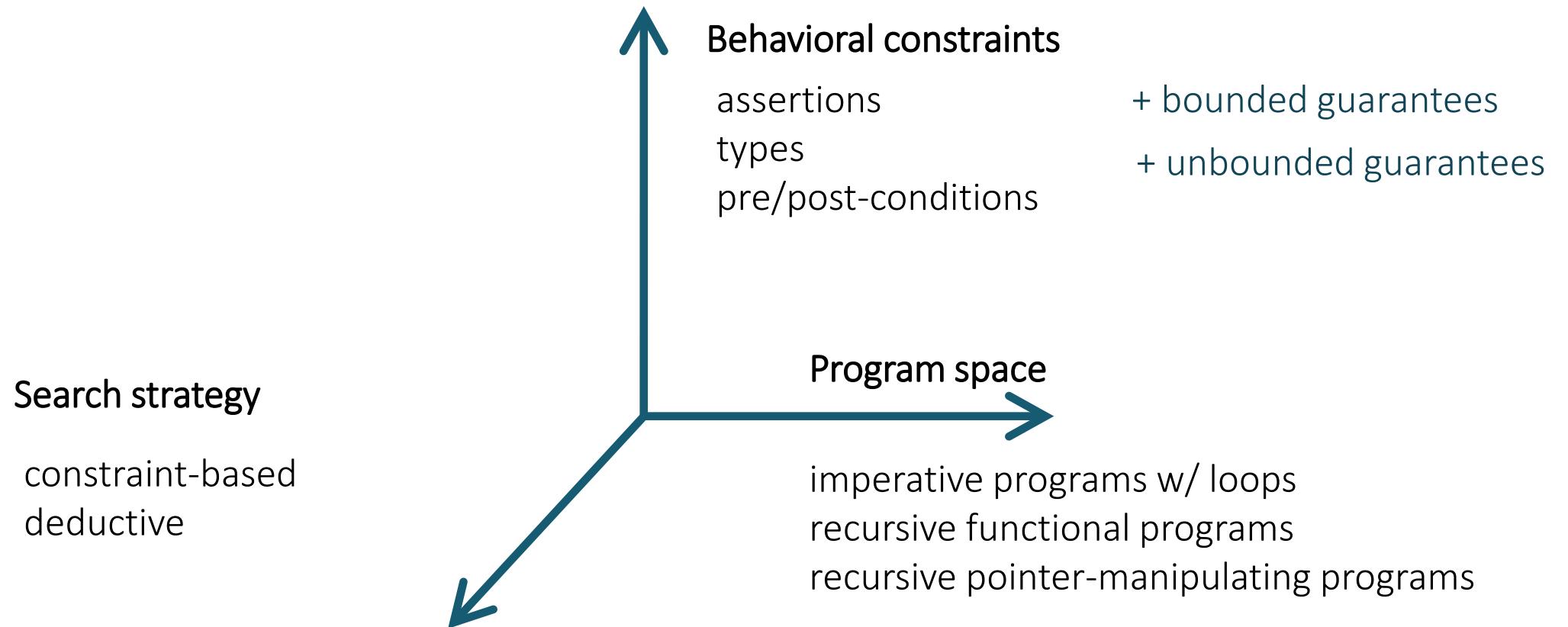


Lecture 11

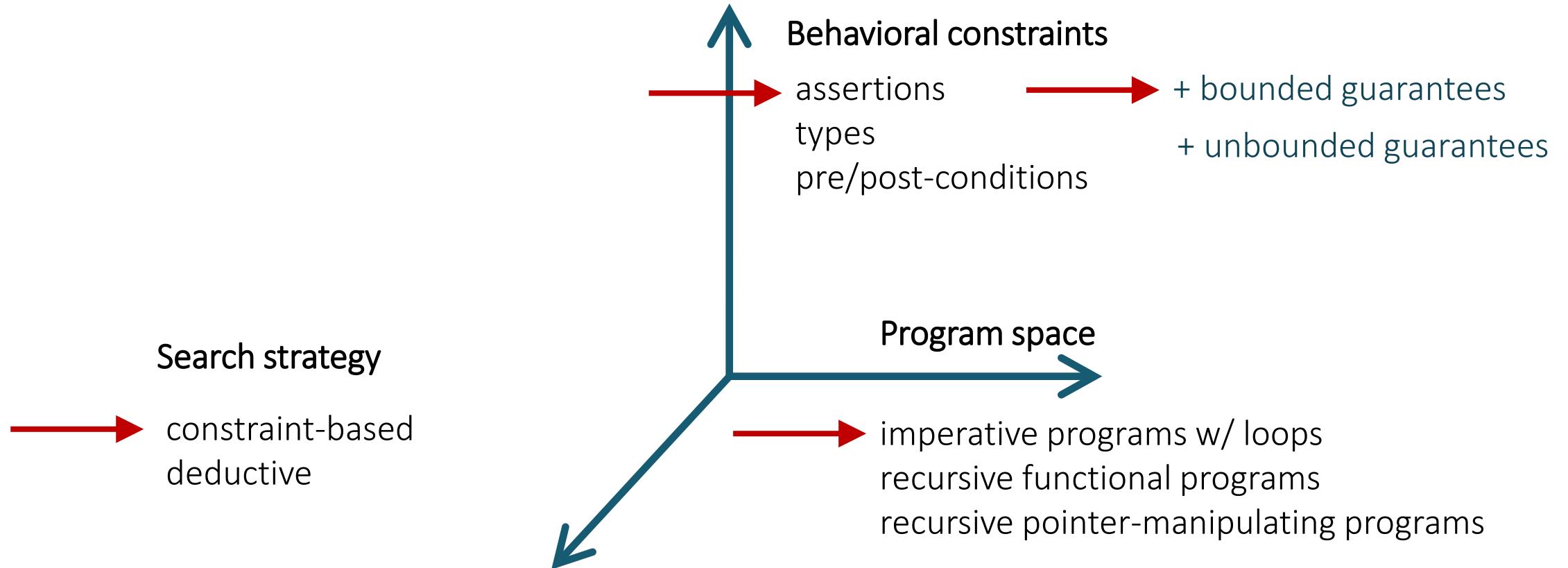
Type-Driven Synthesis

Nadia Polikarpova

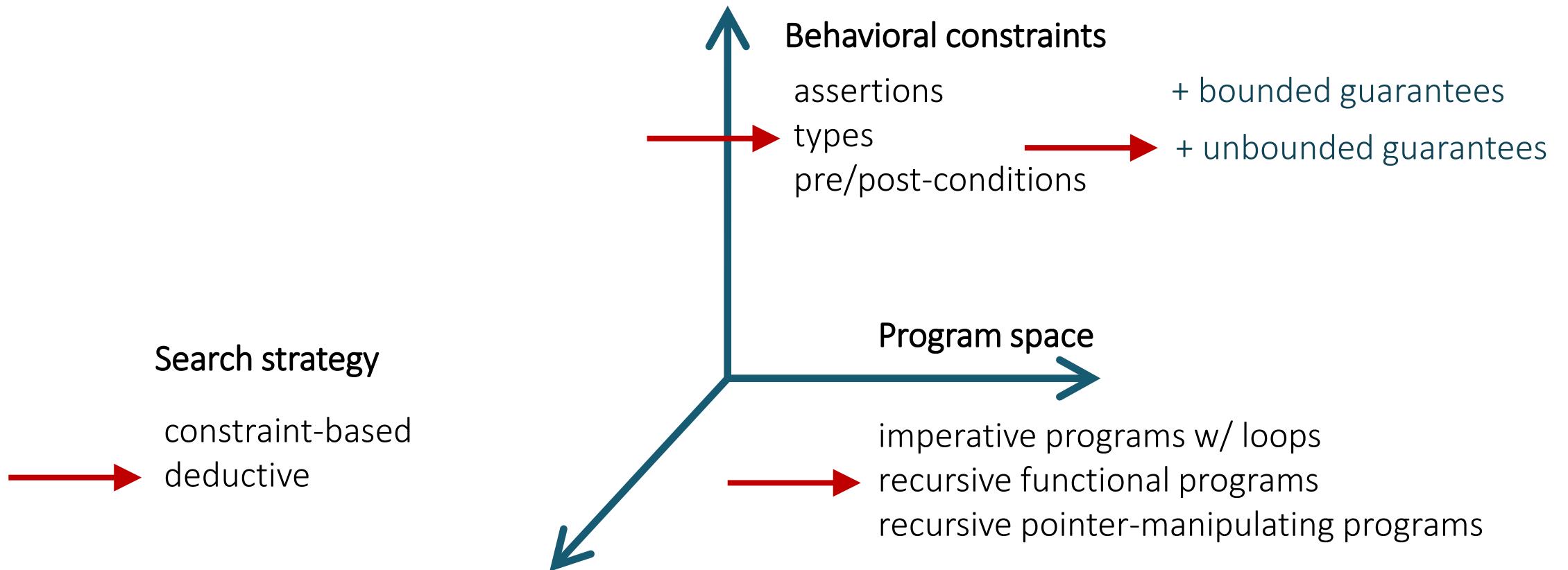
Module II



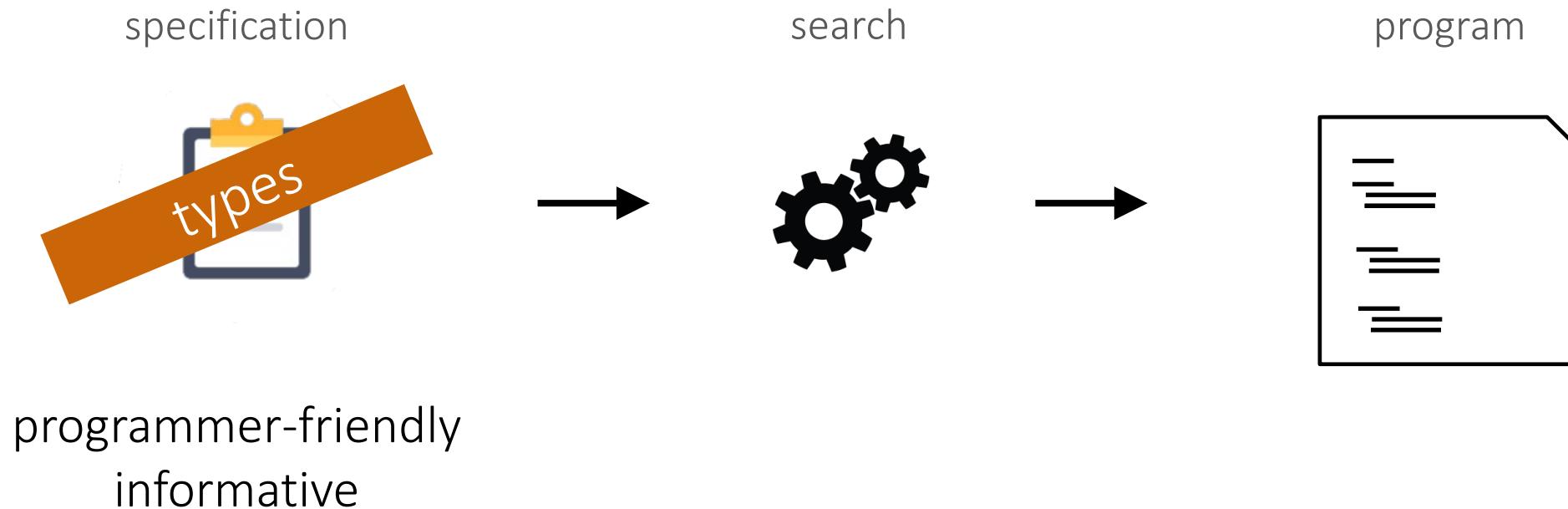
Last week



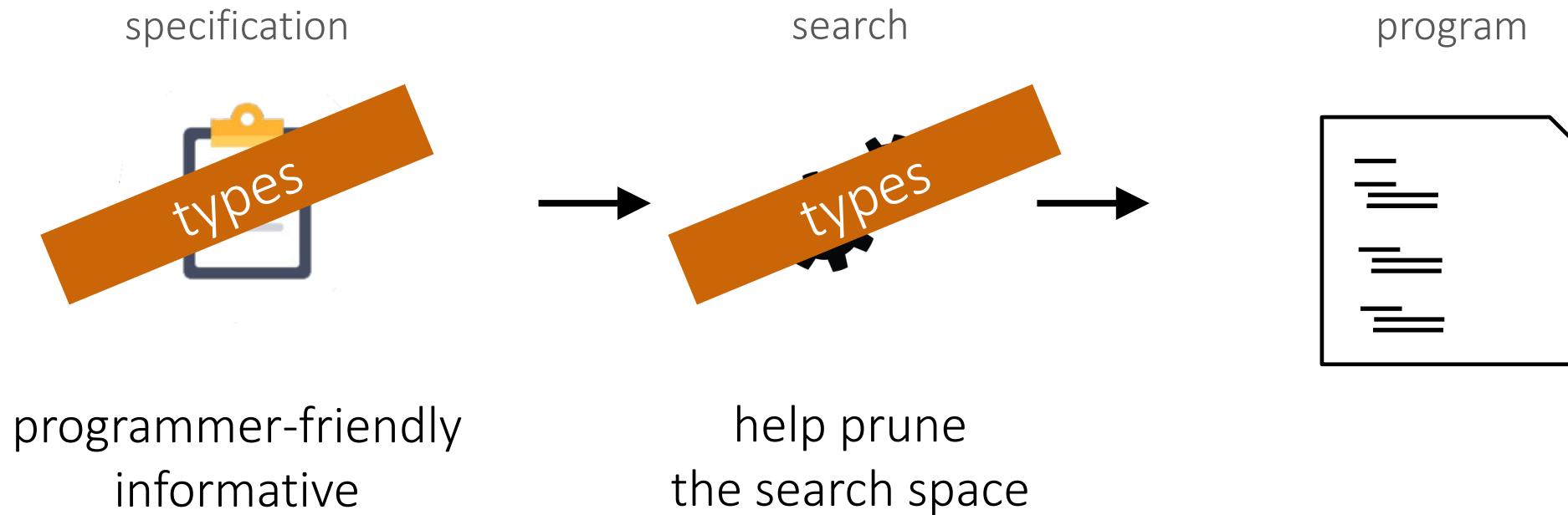
This week



Type-driven program synthesis



Type-driven program synthesis



Which program do I have in mind?

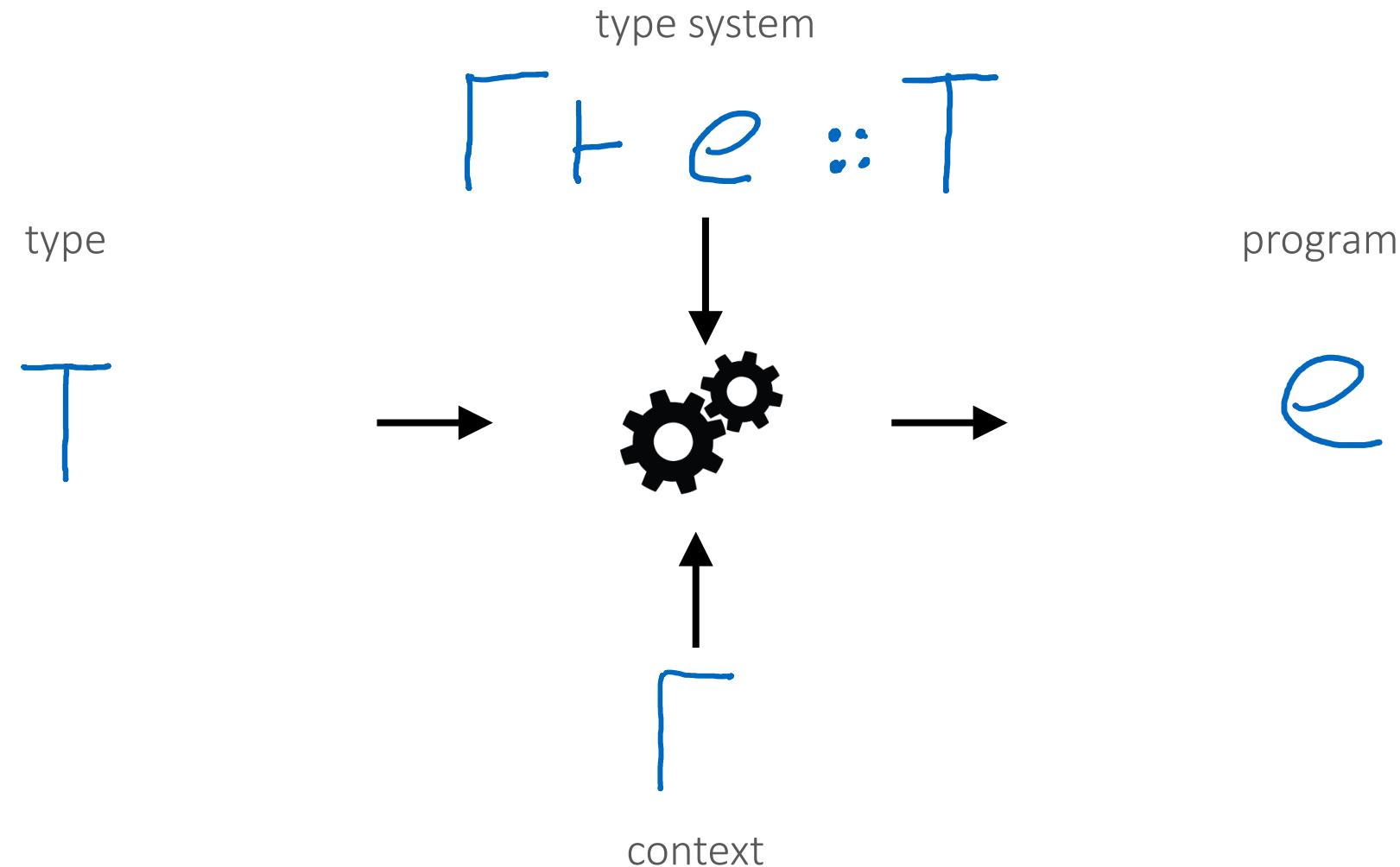
Char -> String -> [String]

split string at custom separator

a -> Int -> [a]

list with n copies of input value

Type-driven program synthesis



This week

intro to type systems

enumerating well-typed terms

bidirectional type systems

synthesis with types and examples

polymorphic types

refinement types

synthesis with refinement types

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What is a type system?

Deductive system for proving facts about programs and types

Defined using *inference rules* over *judgments*

typing judgement

program / term
↓
context $\rightarrow \Gamma \vdash e :: T \leftarrow$ type

“under context Gamma, term e has type T”

A simple type system: syntax

$$e ::= 0 \mid e + \mid x \mid e e \mid \lambda x. e \quad \text{-- expressions}$$

example program: increment by two

$$\lambda x. (x + 1) + 1$$

A simple type system: syntax

$e ::= 0 \mid e + \mid x \mid e e \mid \lambda x. e$ -- expressions

$T ::= \text{Int} \mid T \rightarrow T$ -- types

$\Gamma ::= \cdot \mid x:T, \Gamma$ -- contexts

Inference rules = typing rules

$$\frac{t\text{-zero}}{\Gamma \vdash 0 :: \text{Int}}$$

$$\frac{t\text{-succ} \quad \Gamma \vdash e :: \text{Int}}{\Gamma \vdash e + 1 :: \text{Int}}$$

$$\frac{t\text{-var} \quad x : T \in \Gamma}{\Gamma \vdash x :: T}$$

$$\frac{t\text{-abs} \quad \Gamma, x : T_1 \vdash e :: T_2}{\Gamma \vdash \lambda x. e :: T_1 \rightarrow T_2}$$

$$\frac{t\text{-app} \quad \begin{array}{c} \Gamma \vdash e_1 :: T' \rightarrow T \\ \Gamma \vdash e_2 :: T' \end{array}}{\Gamma \vdash e_1 e_2 :: T}$$

Typing derivations

A derivation of $\Gamma \vdash e :: T$ is a tree where

1. the root is $\Gamma \vdash e :: T$
2. children are related to parents via inference rules
3. all leaves are axioms

Typing derivations

let's build a derivation of

$$\cdot \vdash \lambda x. x + 1 :: \text{Int} \rightarrow \text{Int}$$

we say that $\lambda x. x + 1$ is **well-typed** in the empty context
and has type $\text{Int} \rightarrow \text{Int}$

Typing derivations

$$\begin{array}{c} \frac{}{\Gamma \vdash 0 :: \text{Int}} \quad \text{t-zero} \\ \frac{\Gamma \vdash e :: \text{Int}}{\Gamma \vdash e + 1 :: \text{Int}} \quad \text{t-plus} \\ \frac{t-var}{\Gamma \vdash x :: T} \quad \frac{}{\Gamma, x : T_1 \vdash e :: T_2} \quad \text{t-abs} \\ \frac{\Gamma \vdash e_1 :: T' \rightarrow T \quad \Gamma \vdash e_2 :: T'}{\Gamma \vdash e_1 e_2 :: T} \quad \text{t-app} \end{array}$$

$\cdot \vdash \lambda x. x + 1 :: \text{Int} \rightarrow \text{Int}$

Typing derivations

is $(\lambda x. x) + 1$ well-typed (in the empty context)?

no! no way to build a derivation of $\cdot \vdash (\lambda x. x) + 1 :: _$

we say that $(\lambda x. x) + 1$ is **ill-typed**

Let's add lists!

$e ::= \dots | [] | e:e | \text{match } e \text{ with } [] \rightarrow e | x:x \rightarrow e$

$T ::= \text{Int} | \text{List} | T \rightarrow T$

Example program: head with default

$$\lambda x. \text{match } x \text{ with } nil \rightarrow 0 \mid y: ys \rightarrow y$$

Typing rules

$$\frac{t\text{-nil}}{\Gamma \vdash [] :: \text{List}}$$

$$\frac{t\text{-cons} \quad \Gamma \vdash e_1 :: \text{Int} \quad \Gamma \vdash e_2 :: \text{List}}{\Gamma \vdash e_1 : e_2 :: \text{List}}$$

what should the t-match rule be?

$$\frac{t\text{-match} \quad \begin{array}{c} \Gamma \vdash e_0 :: \boxed{1} \\ \Gamma \vdash e_1 :: \boxed{2} \quad \Gamma \vdash \boxed{4} \quad + e_2 :: \boxed{3} \end{array}}{\Gamma \vdash \text{match } e_0 \text{ with } [] \rightarrow e_1 \mid x:xs \rightarrow e_2 :: T}$$

Typing rules

$$\frac{t\text{-nil}}{\Gamma \vdash [] :: \text{List}}$$

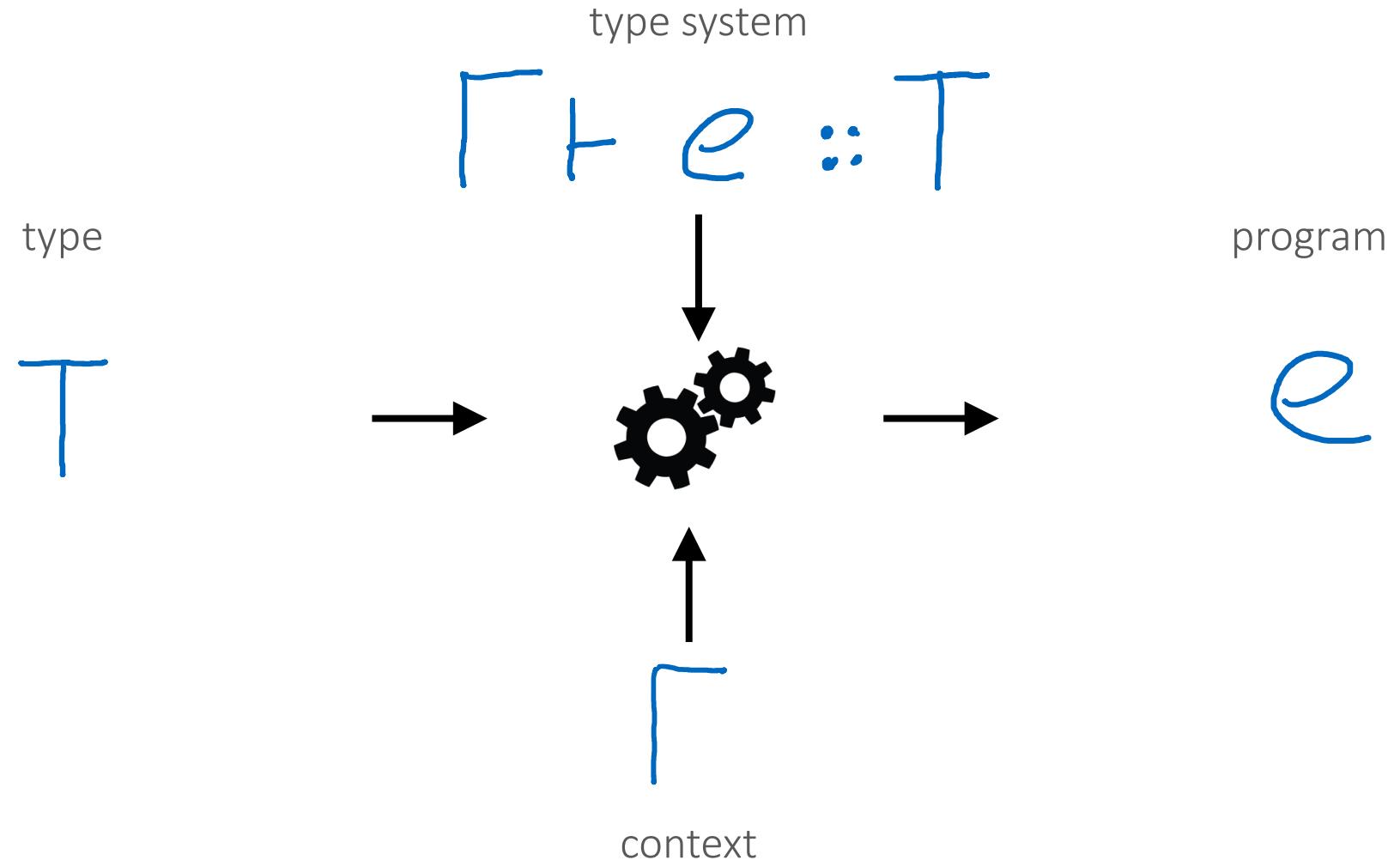
$$\frac{t\text{-cons} \quad \Gamma \vdash e_1 :: \text{Int} \quad \Gamma \vdash e_2 :: \text{List}}{\Gamma \vdash e_1 : e_2 :: \text{List}}$$

$$\frac{t\text{-match} \quad \Gamma \vdash e_0 :: \text{List} \quad \Gamma \vdash e_1 :: T \quad \Gamma, x:\text{Int}, xs:\text{List} \vdash e_2 :: T}{\Gamma \vdash \text{match } e_0 \text{ with } [] \rightarrow e_1 \mid x:xs \rightarrow e_2 :: T}$$

Example: head with default

- $\vdash \lambda x. \text{match } x \text{ with } nil \rightarrow 0 \mid y: ys \rightarrow y :: \text{List} \rightarrow \text{Int}$

Type system → synthesis



This week

intro to type systems

enumerating well-typed terms

synthesis with types and examples

polymorphic types

refinement types

synthesis with refinement types

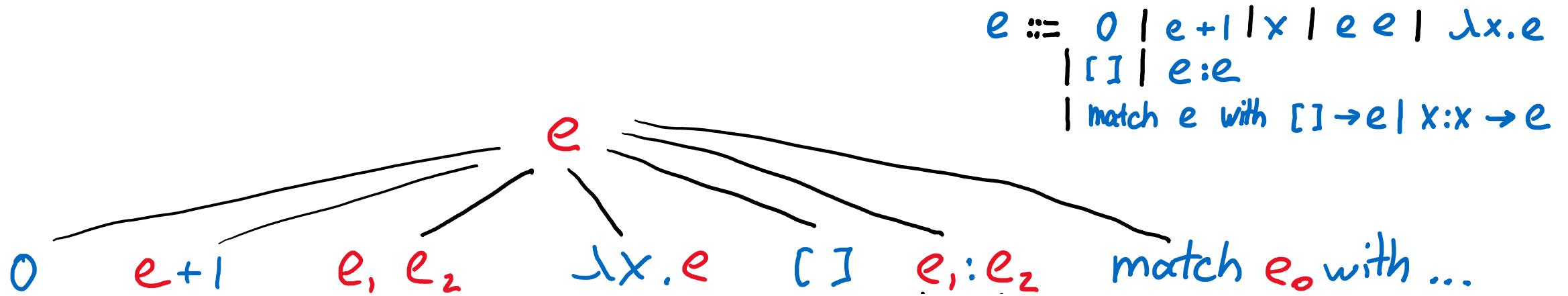
Enumerating well-typed terms

how should I enumerate all terms of type $\text{List} \rightarrow \text{List}$?
(up to depth 2, in the empty context)

naïve idea: syntax-guided enumeration

1. enumerate all terms *generated by the grammar*
2. type-check each term and throw away ill-typed ones

Syntax-guided enumeration



31 complete programs enumerated
only 2 have the type $\text{List} \rightarrow \text{List}$!
can we do better?

Enumerating well-typed terms

how should I enumerate all terms of type $\text{List} \rightarrow \text{List}$?
(up to depth 2, in the empty context)

better idea: type-guided enumeration
enumerate all derivations *generated by the type systems*
extract terms from derivations (well-typed by construction)

Synthesis as proof search

input: synthesis goal $\Gamma \vdash ? :: T$

output: derivation of $\Gamma \vdash e :: T$ for some e

search strategy: top-down enumeration of derivation trees

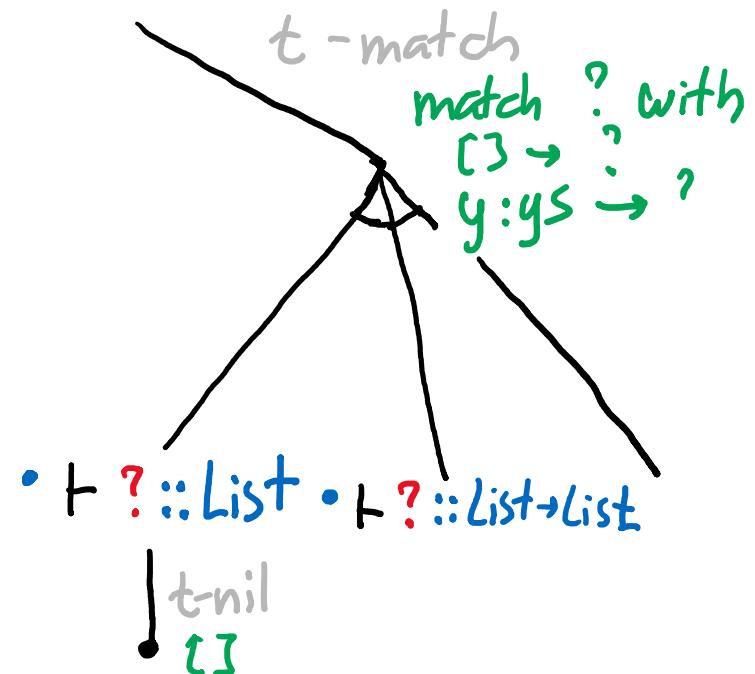
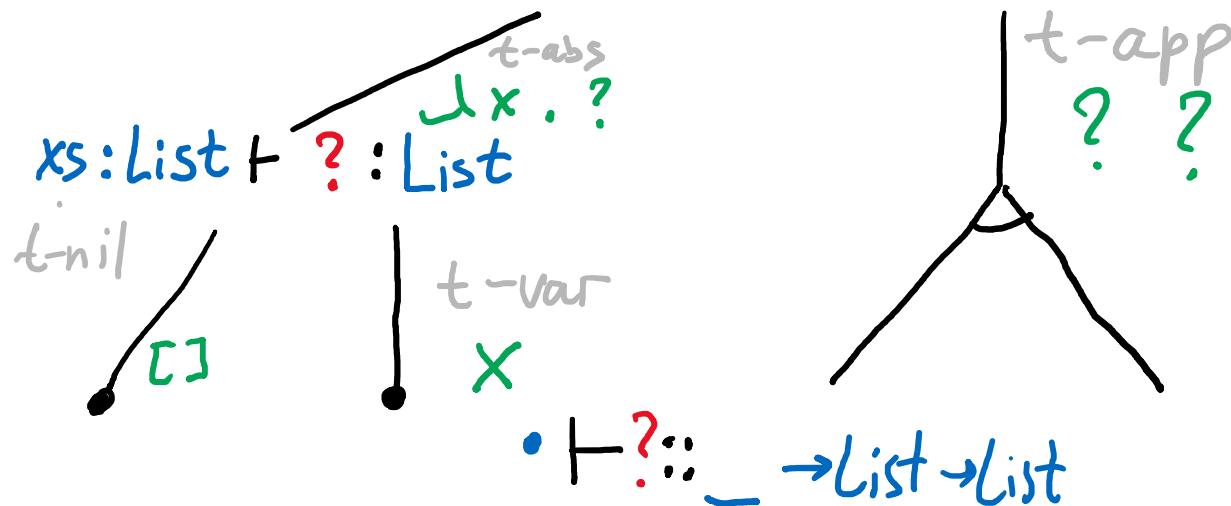
like syntax-guided top-down enumeration but
derivation trees instead of ASTs
typing rules instead of grammar

Type-guided enumeration

only 2 programs fully constructed!
all other programs *rejected early*

$$\begin{array}{c}
 \frac{x:T \in \Gamma}{\Gamma \vdash x : T} \quad t-zero \quad \frac{\Gamma \vdash e : Int}{\Gamma \vdash 0 : Int} \quad t+ \frac{\Gamma \vdash e : Int}{\Gamma \vdash e + 1 : Int} \\
 \\[10pt]
 t-abs \quad \frac{\Gamma, x:T_1 \vdash e : T_2}{\Gamma \vdash \lambda x. e : T_1 \rightarrow T_2} \quad t-app \quad \frac{\Gamma \vdash e_1 : T' \rightarrow T \quad \Gamma \vdash e_2 : T'}{\Gamma \vdash e_1 e_2 : T} \\
 \\[10pt]
 t-nil \quad \frac{}{\Gamma \vdash [] : List} \quad t-cons \quad \frac{\Gamma \vdash e_1 : Int \quad \Gamma \vdash e_2 : List}{\Gamma \vdash e_1 :: e_2 : List} \\
 \\[10pt]
 t-match \quad \frac{\Gamma \vdash e_0 : List \quad \Gamma \vdash e_1 : T \quad \Gamma, x:Int, xs:List \vdash e_2 : T}{\Gamma \vdash \text{match } e_0 \text{ with } [] \rightarrow e_1 \mid x:xs \rightarrow e_2 : T}
 \end{array}$$

- $\vdash ? : List \rightarrow List$



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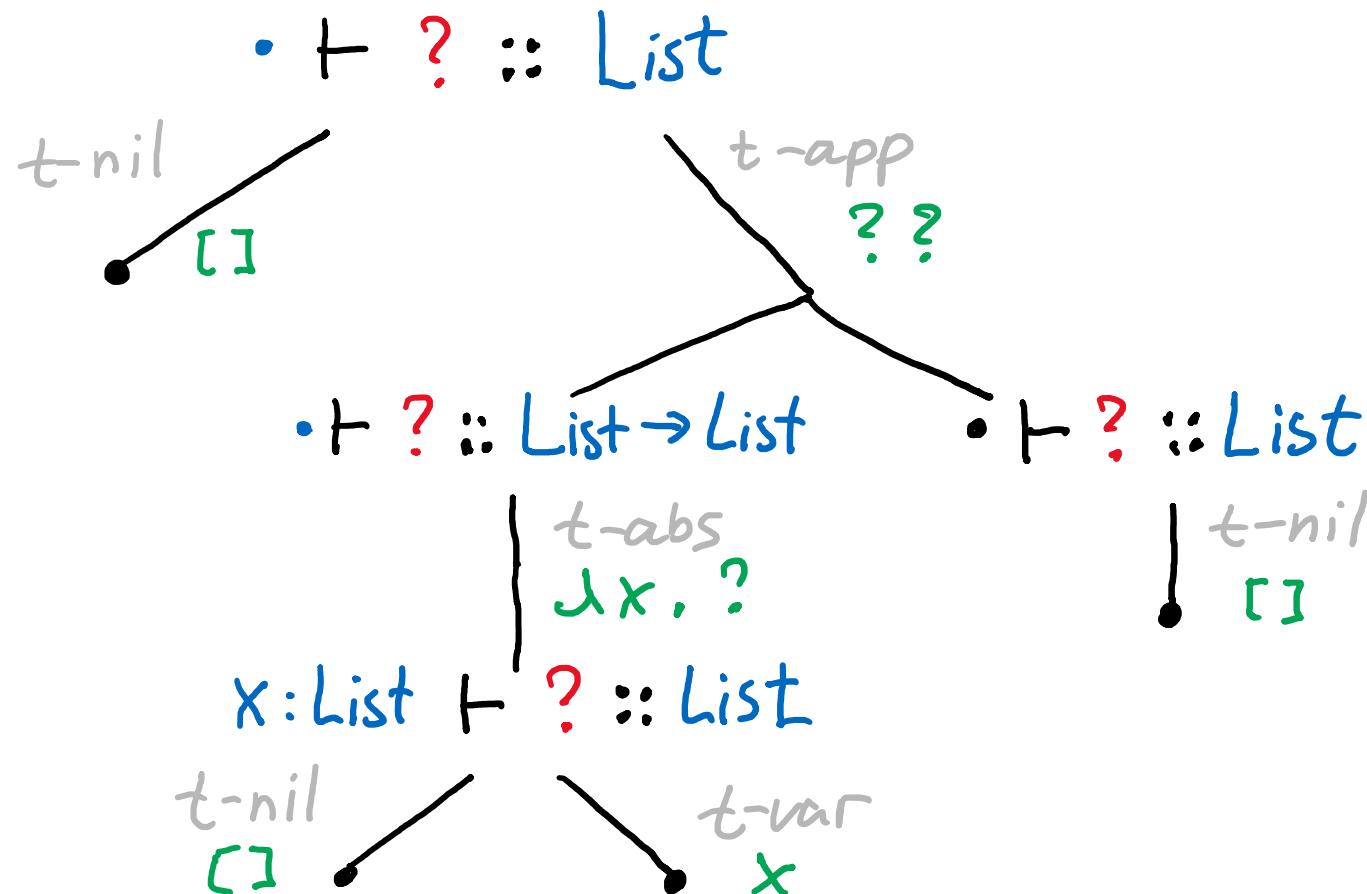
refinement types

synthesis with refinement types

Bidirectional type system

Makes top-down propagation of types explicit
Helps with equivalence reduction

What's wrong with this search?



Enumerated 3 programs:

nil

$(\lambda x. x) \text{ nil}$

$(\lambda x. \text{nil}) \text{ nil}$

They are all equivalent!

Redundant programs

$$(\lambda x. e_1) e_2 \xrightarrow{\beta\text{-reduction}} e_1 [x := e_2]$$

Generating programs on the left
is a waste of time!

$$\begin{array}{c} \text{match } [] \text{ with} \\ [] \rightarrow e_1, \\ y:ys \rightarrow e_2 \end{array} \xrightarrow{\text{match-nil}} e_1$$

Idea: only generate programs *in
normal form*

$$\begin{array}{c} \text{match } e_1 : e_2 \text{ with} \\ [] \rightarrow e_3, \\ y:ys \rightarrow e_4 \end{array} \xrightarrow{\text{match-cons}} e_4 [y := e_1, ys := e_2]$$

Restrict type system
to make redundant programs *ill-typed*

Normal-form programs

$e ::= x \mid ei$

elimination forms

$i ::= 0 \mid i+1 \mid \lambda x. i \mid [] \mid i:i$
| match e with [] → i | x:x → i

introduction forms

$B ::= \text{Int} \mid \text{List}$

base types

$T ::= B \mid T \rightarrow T$

types

Bidirectional typing judgments

$$\Gamma \vdash i \Leftarrow T$$

“under context Gamma, i checks against type T”

$$\Gamma \vdash e \Rightarrow T$$

“under context Gamma, e generates type T”

[Pierce, Turner. Local Type Inference. 2000]

Bidirectional typing rules

$$\text{e-var} \frac{x:T \in \Gamma}{\Gamma \vdash x : T}$$

$$\text{e-app} \frac{\Gamma \vdash e \Rightarrow T' \rightarrow T \quad \Gamma \vdash i \Leftarrow T'}{\Gamma \vdash e i \Rightarrow T}$$

$$\text{i-e} \frac{\Gamma \vdash e \Rightarrow B}{\Gamma \vdash e \Leftarrow B}$$

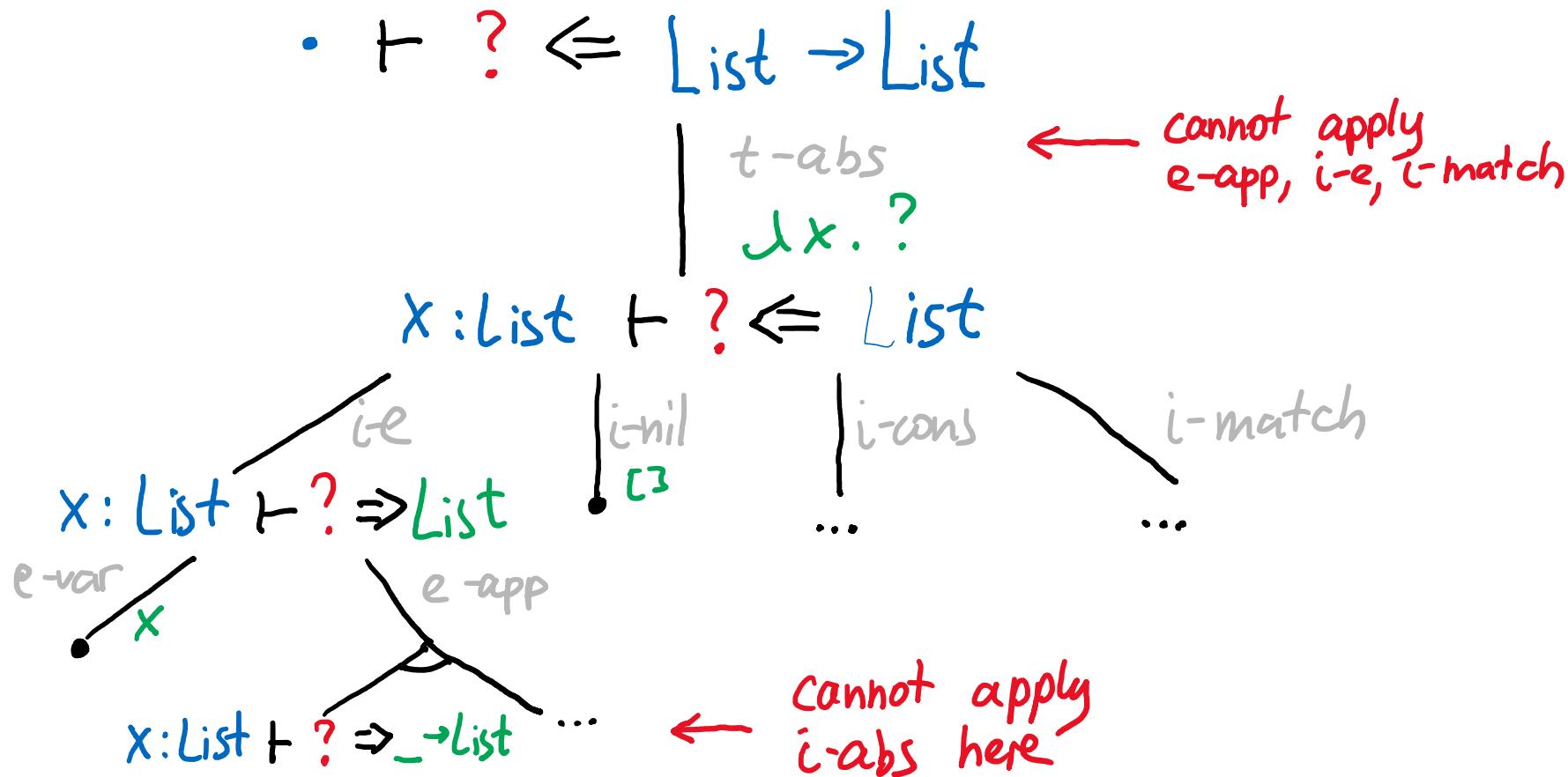
$$\text{i-nil} \frac{}{\Gamma \vdash [] \Leftarrow \text{List}}$$

$$\text{i-cons} \frac{\Gamma \vdash i_1 \Leftarrow \text{Int} \quad \Gamma \vdash i_2 \Leftarrow \text{List}}{\Gamma \vdash i_1 : i_2 \Leftarrow \text{List}}$$

$$\text{i-abs} \frac{\Gamma, x:T_1 \vdash i \Leftarrow T_2}{\Gamma \vdash \lambda x. i \Leftarrow T_1 \rightarrow T_2}$$

$$\text{i-match} \frac{\Gamma \vdash e \Rightarrow \text{List} \quad \Gamma \vdash i_1 \Leftarrow B \quad \Gamma, y:\text{Int}, ys:\text{List} \vdash i_2 \Leftarrow B}{\Gamma \vdash \text{match } e \text{ with } [] \Rightarrow i_1 \mid y:ys \Rightarrow i_2 \Leftarrow B}$$

Type-guided enumeration



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synthesis with refinement types

Simple types are not enough

specification

“duplicate every
element in a list”

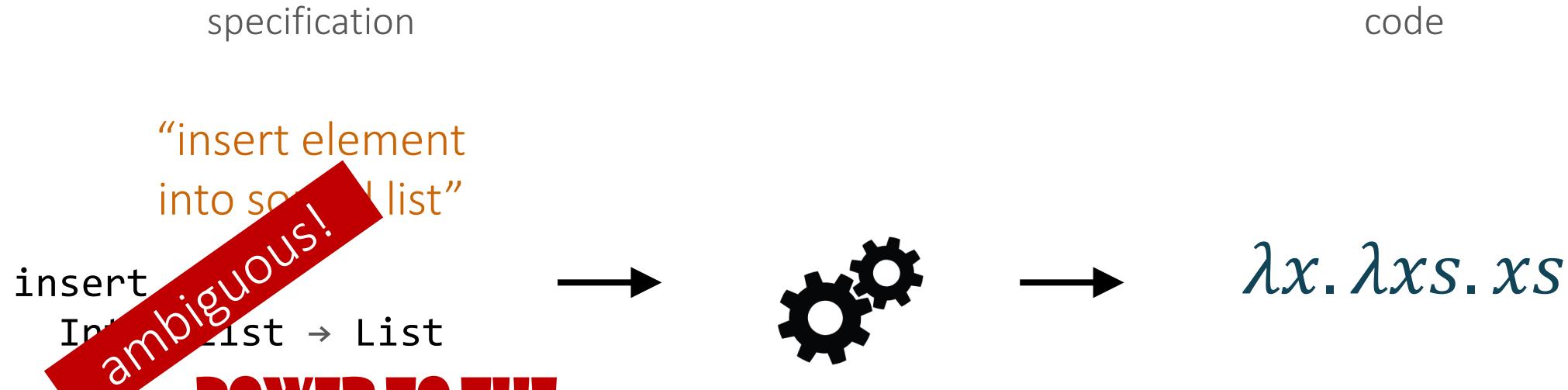
stutter :: List \rightarrow List \rightarrow



code

$\lambda xs. xs$

Simple types are not enough



Type-driven synthesis in 3 easy steps

1. Annotate types with extra specs
examples, logical predicates, resources, ...
2. Design a type system for annotated types
propagate as much info as possible from conclusion to premises
3. Perform type-directed enumeration as before

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Type + examples

specification

“duplicate every element in a list”

$\text{List} \rightarrow \text{List}$

Myth



fix $f(xs)$. match xs with
 $nil \rightarrow nil$
 | $y: ys \rightarrow y: y: f(ys)$

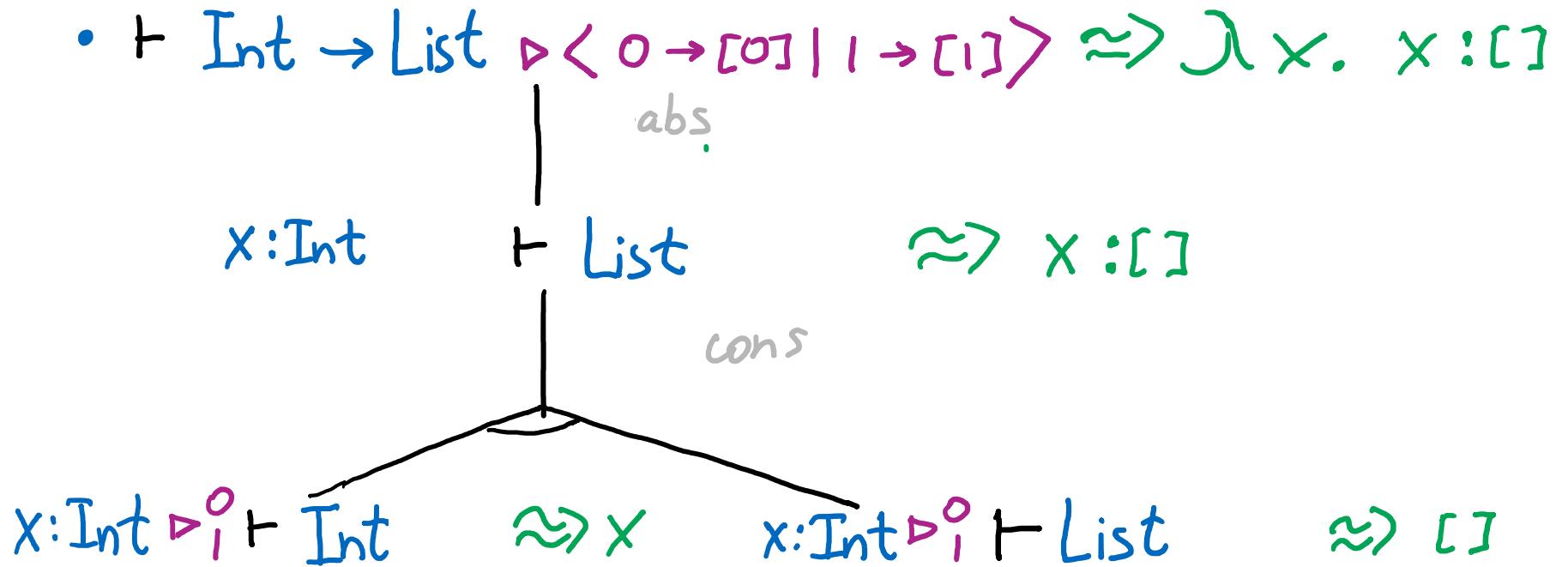
▷ $\langle [] \rightarrow [] | [0] \rightarrow [0,0] | [1,0] \rightarrow [1,1,0,0] \rangle$

[Osera, Zdancewic , Type-and-Example-Directed Program Synthesis. 2015]

Types + examples: syntax

$v ::= 0 \mid v+1 \mid [] \mid v:v \mid \overline{v \rightarrow v}$	values
$X ::= \begin{matrix} v_1 \\ \vdots \\ v_n \end{matrix}$	vectors of examples
$R ::= T \triangleright X$	type refined with examples
$\Gamma ::= \cdot \mid x:R, \Gamma$	context

Example: singleton



no search! simply propagate the spec top-down

Type-driven synthesis in 3 easy steps

1. Annotate types with examples
2. Design a type system for annotated types
3. Perform type-directed enumeration as before

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Polymorphic types

$\forall \alpha. \alpha \rightarrow \text{List } \alpha$

Polymorphic types for synthesis

• $\vdash ? :: \text{Int} \rightarrow \text{List Int}$

$\lambda x. \text{nil}$

$\lambda x. [0], \lambda x. [1], \dots$

$\lambda x. [x]$

$\lambda x. [\text{double } 0], \lambda x. [\text{dec } 0]$

$\lambda x. [0,0], \lambda x. [0,1], \dots$

$\lambda x. [x, x]$

• $\vdash ? :: \forall d. d \rightarrow \text{List } d$

which of these programs
match the polymorphic type?

Polymorphic types for synthesis

• $\vdash ? :: \text{Int} \rightarrow \text{List Int}$

$\lambda x. \text{nil}$

$\lambda x. [0], \lambda x. [1], \dots$

$\lambda x. [x]$

$\lambda x. [\text{double } 0], \lambda x. [\text{dec } 0]$

$\lambda x. [0,0], \lambda x. [0,1], \dots$

$\lambda x. [x, x]$

• $\vdash ? :: \forall d. d \rightarrow \text{List } d$

1. $\lambda x. \text{nil}$

eliminate ambiguity!

2. $\lambda x. [x]$

prune the search!

3. $\lambda x. [x, x]$

Polymorphic types

$$B ::= \text{Int} \mid \text{List } B \mid \lambda$$

base types

$$T ::= B \mid T \rightarrow T$$

types

$$S ::= T \mid \forall \lambda. S$$

type schemas (polytypes)

$$\Gamma ::= \bullet \mid x : S, \Gamma \mid \lambda \lambda, \Gamma$$

contexts

Judgments

$$\Gamma \vdash i \leqslant S$$

type checking:

“under context Gamma, i checks against a schema S”

$$\Gamma \vdash e \Rightarrow T$$

type inference:

“under context Gamma, e generates type T”

Typing rules

$$\text{i-hil} \frac{}{\Gamma \vdash [] \Leftarrow \text{List } B}$$

$$\text{i-cons} \frac{\Gamma \vdash i_1 \Leftarrow B \quad \Gamma \vdash i_2 \Leftarrow \text{List } B}{\Gamma \vdash i_1 : i_2 \Leftarrow \text{List } B}$$

$$\text{i-gen} \frac{\Gamma, d \vdash i \Leftarrow S}{\Gamma \vdash i \Leftarrow \forall d. S}$$

$$\text{e-var} \frac{x : \overline{\forall d. T} \in \Gamma}{\Gamma \vdash x \Rightarrow \overline{[\alpha \mapsto T']} T}$$

how do we guess T' ?
Hindley-Milner type inference!

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Refinement types

Nat

base types

max :: $x: \text{Int} \rightarrow y: \text{Int} \rightarrow \{ v: \text{Int} \mid x \leq v \wedge y \leq v \}$

dependent
function types

xs :: $\{ v: \text{List Nat} \}$

polymorphic
datatypes

Refinement types: measures

```
data List α where
  Nil   :: { List α | Len v = 0 }
  Cons  :: x: α → xs: List α
            → { List α | Len v = Len xs + 1 }
```

syntactic sugar:

```
measure Len :: List α → Int
  Len Nil = 0
  Len (Cons _ xs) = Len xs + 1
```

example: duplicate every element in a list

```
stutter :: ??
```

Refinement types: sorted lists

```
data SList α where
  Nil   :: SList α
  Cons  :: x: α → xs: SList {α | x ≤ v }
                           → SList α
```

example: insert an element into a sorted list

```
insert :: ??
```

Refinement types

$$B ::= \text{Int} \mid \text{List } B \mid d$$

base types

$$T ::= \{\nu:B. \mid \varphi\} \mid x:T \rightarrow T$$

types

$$S ::= T \mid \#d. S$$

type schemas (polytypes)

$$\Gamma ::= \cdot \mid x:S, \Gamma \mid d, \Gamma$$

contexts

Example: increment

$$\begin{aligned} \text{Nat} &= \{\nu: \text{Int} \mid \nu \geq 0\} \\ \Gamma &= [\text{inc}: y: \text{Int} \rightarrow \{\nu: \text{Int} \mid \nu = y + 1\}] \end{aligned}$$

we need subtyping!

$$\Gamma \vdash \lambda x. \text{inc } x \Leftarrow \text{Nat} \rightarrow \text{Nat}$$

Subtyping

intuitively: T' is a subtype of T if all values of type T' also belong to T

written $T' <: T$

e.g. $\text{Nat} <: \text{Int}$ or $\{\nu: \text{Int} \mid \nu = 5\} <: \text{Nat}$

$$\text{sub-base} \quad \frac{[\![\Gamma]\!] \wedge \phi' \Rightarrow \phi}{\Gamma \vdash \{\nu: B \mid \phi'\} <: \{\nu: B \mid \phi\}}$$

$$\text{sub-fun} \quad \frac{\Gamma \vdash T_1 <: T'_1 \quad \Gamma; x: T_1 \vdash T'_2 <: T_2}{\Gamma \vdash x: T'_1 \rightarrow T'_2 <: x: T_1 \rightarrow T_2}$$

$\text{Pos} <: \text{Nat}$ 

$\text{Int} \rightarrow \text{Int} <: \text{Int} \rightarrow \text{Nat}$ 
 $\text{Int} \rightarrow \text{Int} <: \text{Nat} \rightarrow \text{Int}$ 
 $x: \text{Int} \rightarrow \{\nu: \text{Int} \mid \nu = x + 1\} <: \text{Nat} \rightarrow \text{Nat}$ 

Typing rules

$$\frac{i : e \quad \Gamma \vdash e \Rightarrow T \quad \Gamma \vdash T \leq \{B | \varphi\}}{\Gamma \vdash e \Leftarrow \{B | \varphi\}}$$

$$e\text{-app} \frac{\Gamma \vdash e \Rightarrow y:T_1 \rightarrow T_2 \quad \Gamma \vdash i \Leftarrow T_1}{\Gamma \vdash e \ i \Rightarrow [y \mapsto i]T_2}$$

Example: increment

$$\Gamma = [\text{inc}: y: \text{Int} \rightarrow \{v: \text{Int} \mid v = y + 1\}]$$

$$\begin{array}{c}
 \frac{\text{e-var}}{\Gamma, x: \text{Nat} \vdash \text{inc} \Rightarrow y: \text{Int} \rightarrow \{ \text{Int} \mid v = y + 1 \}}
 \quad
 \frac{\text{e-var} \quad \frac{\Gamma, x: \text{Nat} \vdash x \Rightarrow \text{Nat}}{\Gamma, x: \text{Nat} \vdash x \Leftarrow \text{Int}} \quad \frac{\Gamma, x: \text{Nat} \vdash \text{Nat} <: \text{Int}}{\Gamma, x: \text{Nat} \vdash x <: \text{Int}}}{\Gamma, x: \text{Nat} \vdash \text{inc } x \Rightarrow \{ \text{Int} \mid v = x + 1 \} \quad \Gamma, x: \text{Nat} \vdash \{ \text{Int} \mid v = x + 1 \} <: \text{Nat}}
 \\
 \frac{\text{e-app} \quad \text{i.e.}}{\Gamma, x: \text{Nat} \vdash \text{inc } x \Leftarrow \text{Nat}}
 \\
 \frac{\text{i-abs}}{\Gamma \vdash \lambda x. \text{inc } x \Leftarrow \text{Nat} \rightarrow \text{Nat}}
 \end{array}$$

subtyping constraints

$\text{Nat} <: \text{Int}$

$x: \text{Nat} \vdash \{v: \text{Int} \mid v = x + 1\} <: \text{Nat}$

implications

$v \geq 0 \Rightarrow \text{true}$

$x \geq 0 \wedge v = x + 1 \Rightarrow v \geq 0$

SMT solver: VALID!

Refinement type checking

idea: separate type checking into
subtyping constraint generation and **subtyping constraint solving**

1. Generate a constraint for every subtyping premise in derivation
2. Reduce subtyping constraints to implications
3. Use SMT solver to check implications

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Synthesis from refinement types

specification

“duplicate every
element in a list”

```
stutter ::  
  xs:List a →  
  {v:List a | len v =  
    2 * len xs}
```



code

```
match xs with  
  Nil → Nil  
  Cons h t →  
    Cons h (Cons h (stutter t))
```

[Polikarpova, Kuraj, Solar-Lezama, Program Synthesis from Polymorphic Refinement Types. 2016]

Synthesis from refinement types

specification

“insert element
into sorted list”

```
insert :: x:a →  
  xs:SList a →  
  {v:SList a | elems v =  
    elems xs ∪ {x}}
```



code

```
match xs with  
  Nil → Cons x Nil  
  Cons h t →  
    if x ≤ h  
      then Cons x xs  
      else Cons h (insert x t)
```

Type-driven synthesis in 3 easy steps

1. Annotate types with logical predicates
2. Design a type system for annotated types
3. Perform type-directed enumeration as before

Type-directed enumeration for insert

```
x:a → xs:SList a →  
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert = ??
```

Type-directed enumeration

```
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert x xs = ??
```

context:
x: a
xs: SList a

Type-directed enumeration

```
{v:SList a | elems v = elems xs ∪ {x}}
```

context:
x: a
xs: SList a



```
insert x xs =  
  match xs with  
    Nil → ??  
    Cons h t → ??
```

Type-directed enumeration

```
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert x xs =
  match xs with
    Nil → ???
    Cons h t → ???
```

context:
x: a
xs: SList a
elems xs = {}

Type-directed enumeration

```
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert x xs =  
  match xs with  
    Nil → Nil  
    Cons h t → ??
```



context:
x: a
xs: SList a
elems xs = {}

Constraints:
 $\forall x: \{\} = \{\} \cup \{x\}$

SMT solver: INVALID!

The hard part: application

```
x:a → xs:SList a →  
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert x xs =  
  match xs with  
    Nil → Cons x Nil  
    Cons h t →  
      Cons h (insert x ??)
```

should this program be rejected?
yes!
cannot guarantee output is sorted!

Round-trip type-checking (RTTC)

$$\Gamma \vdash i \leqslant S$$

type checking:

“under context Gamma, i checks against schema S”

$$\Gamma \vdash e \leqslant T \Rightarrow T'$$

type strengthening:

“under context Gamma, e checks against type T and generates a stronger type T’”

RTTC rules

$$\text{ie } \frac{\Gamma \vdash e \Leftarrow \{B|\varphi\} \Rightarrow T}{\Gamma \vdash e \Leftarrow \{B|\varphi\}}$$

$$e-\forall \Gamma \frac{x:T' \in \Gamma \quad \Gamma \vdash T' \leq : T}{\Gamma \vdash x \Leftarrow T \Rightarrow T'}$$

$$e\text{-app} \frac{\Gamma \vdash e \Leftarrow \perp \Rightarrow T \Rightarrow y:T_1 \rightarrow T_2 \quad \Gamma \vdash i \Leftarrow T_1}{\Gamma \vdash e i \Leftarrow T \Rightarrow [y \mapsto i]T_2}$$

The hard part: application

elems will depend on the missing part...

but sortedness we can already check!

```
{v:SList a | elems v = elems xs ∪ {x}}
```



```
insert x xs =
  match xs with
    Nil → Cons x Nil
    Cons h t →
      Cons h (insert x ??)
```

The hard part: application

$$\{v:a \mid h \leq v\}$$


```
insert x xs =  
  match xs with  
    Nil → Cons x Nil  
    Cons h t →  
      Cons h (insert x ???)
```



context:
x: a
xs: SList a
h: a
t: SList {a| $h \leq v$ }

Constraints:
 $\forall x, h: h \leq x$

insert :: $x:\tau \rightarrow$
 $xs:SList \tau \rightarrow$
 $SList \tau$

SMT solver: INVALID!

Synquid: contributions

Unbounded correctness guarantees

Round-trip type system to reject incomplete programs

- + GFP Horn Solver

Refinement types can express complex properties in a simple way

- handles recursive, HO functions
- automatic verification for a large class of programs due to polymorphism
(e.g. sorted list insert)

Synquid: limitations

User interaction

- refinement types can be large and hard to write
- components need to be annotated (how to mitigate?)

Expressiveness limitations

- some specs are tricky or impossible to express
- cannot synthesize recursive auxiliary functions

Condition abduction is limited to liquid predicates

Cannot generate arbitrary constants

No ranking / quality metrics apart from correctness

Synquid: questions

Behavioral constraints? Structural constraints? Search strategy?

- Refinement types
- Set of components + built-in language constraints
- Top-down enumerative search with type-based pruning

Typo in the example in Section 3.2

- $\{B_0 \mid \perp\} \rightarrow \{B_1 \mid \perp\} \rightarrow \{\text{List Pos} \mid \text{len } v = \underline{25}\}$

Can RTTC reject these terms?

`inc ?? :: {Int | v = 5}`

- where `inc :: x:Int → {Int | v = x + 1}`
- NO! don't know if we can find `?? :: {Int | v + 1 = 5}`

`nats ?? :: List Pos`

- where `nats :: n:Nat → {List Nat | len v = n}`
`Nat = {Int | v >= 0}, Pos = {Int | v > 0}`
- YES! `n:Nat → {List Nat | len v = n}` not a subtype of
`_ → List Pos`

`duplicate ?? :: {List Int | len v = 5}`

- where `duplicate :: xs>List a → {List a | len v = 2*(len xs)}`
- YES! using a consistency check ($\text{len } v = 2 * (\text{len } xs) \wedge \text{len } v = 5 \rightarrow \text{UNSAT}$)