From Spectrum Pooling to Space Pooling: Opportunistic Interference Alignment in MIMO Cognitive Networks

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Abstract

We describe a non-cooperative interference alignment (IA) technique which allows an opportunistic multiple input multiple output (MIMO) link (secondary) to harmlessly coexist with another MIMO link (primary) in the same frequency band. We assume perfect channel knowledge in the primary receiver and transmitter such that capacity is achieved by transmiting along the spatial directions (SD) associated with the singular values of its channel matrix using a water-filling power allocation (PA) scheme. Often, power limitations lead the primary transmitter to leave some of its SD unused. We show that the opportunistic link can transmit its own data if it is possible to align the interference produced on the primary link with such unused SDs. We provide both a processing scheme to perform IA and a PA scheme which maximizes the transmission rate of the opportunistic link. We determine the asymptotes of the achievable transmission rates of the opportunistic link in the regime of large numbers of antennas to show that depending on the signal-to-noise ratio and the numbers of transmit and receive antennas of the primary and opportunistic links, both systems can achieve transmission rates of the same order.

EDICS: MSP-APPL, WIN-APPL, WIN-PHYL.

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I. Introduction

The concept of cognitive radio is well-known by now. The main idea is to let a class of radio devices, called secondary systems, opportunistically access certain portions of spectrum left unused by other radio devices, called primary systems, at a given time or geographical area [2]. These pieces of unused spectrum, known as white-spaces, appear mainly when either transmissions in the primary network are sporadic, i.e., there are periods over which no transmission takes place, or there is no network infrastructure for the primary system in a given area. In the case of dense networks, a white-space might be a rare and short-lasting event. As a matter of fact, the idea of cognitive radio as presented in [2] (i.e., spectrum pooling), depends on the existence of such white-spaces [3]. Indeed, in the absence of those spectrum holes, secondary systems are unable to transmit and at the same time, guarantee no additional interference on the primary systems. One solution to this situation has been provided recently under the name of interference alignment (IA) [4]. Basically, IA refers to the construction of signals such that the resulting interference signal lies in a subspace orthogonal to the one spanned by the signal of interest at each receiver [5]. The IA concept was introduced almost simultaneously by several authors [6], [7], [4], [8]. After such seminal works, IA has become an important tool to study the interference channel, namely its degrees of freedom [5], [4], [9]. The feasibility and implementation issues of IA regarding mainly the required channel state information (CSI) has been also extensively studied [10], [11], [12], [13].

In this paper we study an IA scheme named opportunistic IA (OIA). This scheme was initially introduced in [1]. The idea behind OIA can be briefly described as follows. The primary link is modeled by a single-user MIMO channel since it must operate free of any additional interference produced by secondary systems. Then, assuming perfect CSI at both transmitter and receiver ends, capacity is achieved by implementing a water-filling power allocation (PA) scheme [14] over the spatial directions associated with the singular values of its channel transfer matrix. Interestingly, even if the primary transmitters maximize their transmission rates, power limitations generally lead them to leave some of their spatial directions (SD) unused. The unused SD can therefore be reused by another system operating in the same frequency band. Indeed, an opportunistic transmitter can send its own data to its respective receiver by processing its signal in such a way that the interference produced on the primary link impairs only the unused SDs. Hence, these spatial resources can be very useful for a secondary system when the available spectral resources are fully exploited over a certain period in a geographical area. As we will show, the existence of the so called unused SD in the primary link highly depends on the channel realization between primary terminals and their power budget.

This paper is structured as follows. First, we introduce our system model in Sec. II, which consists of an interference channel with MIMO links. Then, our aim in Sec. III is twofold. First, we provide an analysis of the feasibility of the OIA scheme. For this purpose, we study the existence and the average number of transmit opportunities (SD left unused by the primary system) as a function of the number of antennas at both the primary and secondary terminals. Second, we describe the proposed interference alignment technique and power allocation (PA) policy at the secondary transmitter. In Sec. IV, we use tools from random matrix theory for large systems to analyze the achievable transmission rate of the opportunistic transmitter when no optimization is performed over its input covariance matrix. We illustrate our theoretical results by simulations in Sec. V. Therein, we show that our approach allows the secondary link to achieve transmission rates of the same order as those of the primary link. Finally, in Sec. VI we state our conclusions and provide possible extensions of this work.

II. SYSTEM MODEL

Notations. In the sequel, matrices and vectors are respectively denoted by boldface upper case symbols and boldface lower case symbols. An $N \times K$ matrix with ones in its main diagonal and zeros in its off-diagonal entries is denoted by $\mathbf{I}_{N\times K}$, while the identity matrix of size N is simply denoted by $\mathbf{I}_N$. An $N \times K$ matrix with zeros in all its entries (null matrix) is denoted by $\mathbf{0}_{N\times K}$. Matrices $\mathbf{X}^T$ and $\mathbf{X}^H$ are the transpose and Hermitian transpose of matrix $\mathbf{X}$, respectively. The determinant of matrix $\mathbf{X}$ is denoted by $|\mathbf{X}|$. The expectation operator is denoted by $\mathbb{E}\left[.\right]$. The indicator function associated with a given set $\mathcal{A}$ is denoted by $\mathbb{1}_{\mathcal{A}}(.)$, and defined by $\mathbb{1}_{\mathcal{A}}(x) = 1$ (resp. 0) if $x \in \mathcal{A}$ (resp. $x \notin \mathcal{A}$). The Heaviside step function and the Dirac delta function are respectively denoted by $\mu(\cdot)$ and $\delta(\cdot)$. The symbols $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of non-negative integers, real numbers, and complex numbers, respectively. The subsets $[0, +\infty)$ and $(-\infty, 0]$ are denoted by $\mathbb{R}^+$ and $\mathbb{R}^-$, respectively. The operator $[x]^+$ with $x \in \mathbb{R}$ is equivalent to the operation $\max(0, x)$. Let $\mathbf{A}$ be an $n \times n$ square matrix with real eigenvalues $\lambda_{A,1}, \ldots, \lambda_{A,n}$. We define the empirical eigenvalue distribution of $\mathbf{A}$ by $F_A^{(n)}(\cdot) \triangleq \frac{1}{n}\sum_{i=1}^{n}\mu(\lambda - \lambda_{A,i})$, and, when it exists, we denote $f_A^{(n)}(\lambda)$ the associated eigenvalue probability density function, where $F_{\mathbf{A}}(\cdot)$ and $f_{\mathbf{A}}(\cdot)$ are respectively the associated limiting eigenvalue distribution and probability density function when $n \to +\infty$.

We consider two unidirectional links simultaneously operating in the same frequency band and producing mutual interference. Both transmitters are equipped with N_t antennas while both receivers use N_r antennas. The first transmitter-receiver pair $(\mathrm{Tx}_1, \mathrm{Rx}_1)$ is the primary link. The pair $(\mathrm{Tx}_2, \mathrm{Rx}_2)$ is an opportunistic link subject to the strict constraint that the primary link must transmit at a rate equivalent to its single-user capacity. The transmitters send independent messages and no cooperation between them is allowed, i.e., there is no message conferencing between the transmitters and each transmitter sends private messages. This scenario is known as the MIMO interference channel (IC) [15], [16].

In this paper, we assume the channel transfer matrices between different nodes to be fixed over the whole duration of the transmission. The channel transfer matrix from transmitter $j \in \{1,2\}$ to receiver $i \in \{1,2\}$ is an $N_r \times N_t$ matrix denoted by $\mathbf{H}_{ij}$ which corresponds to the realization of a random matrix with independent and identically distributed (i.i.d.) complex Gaussian circularly symmetric entries with zero mean and variance $\frac{1}{N_t}$, which implies

$$\mathrm{Trace}\left(\mathbb{E}\left[\mathbf{H}_{ij}\,\mathbf{H}_{ij}^H\right]\right) = N_r. \tag{1}$$

The ζ_i symbols simultaneously transmitted by transmitter i, denoted by $s_{i,1}, \ldots, s_{i,\zeta_i}$, are represented by the vector $\mathbf{s}_i = (s_{i,1}, \ldots, s_{i,\zeta_i})^T$. In our model, transmitter i processes its symbols using a matrix $\mathbf{V}_i$ to construct its transmitted signal $\mathbf{V}_i\mathbf{s}_i$. Therefore, the matrix $\mathbf{V}_i$ is called pre-processing matrix. Following a matrix notation, the primary and secondary received signals, represented by the $N_r \times 1$ column-vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, can be written as

$$\begin{pmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{V}_1\mathbf{s}_1 \\ \mathbf{V}_2\mathbf{s}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{pmatrix}, \tag{2}$$

where $\mathbf{n}_i$ is an N_r-dimensional vector representing noise effects at receiver i with entries modeled by an additive white Gaussian noise (AWGN) process with zero mean and variance σ_i^2, i.e., $\mathbb{E}\left[\mathbf{n}_i\mathbf{n}_i^H\right] = \sigma_i^2\mathbf{I}_{N_r}$. At transmitter i, the $\zeta_i \times \zeta_i$ power allocation matrix $\mathbf{P}_i$ is defined by the input covariance matrix $\mathbf{P}_i = \mathbb{E}\left[\mathbf{s}_i\mathbf{s}_i^H\right]$. Without loss of optimality [14] we assume that $\mathbf{P}_i = \mathbb{E}\left[\mathbf{s}_i\mathbf{s}_i^H\right]$ is a diagonal matrix. Choosing $\mathbf{P}_i$ therefore means selecting a given PA policy. The power constraints on the transmitted signals $\mathbf{V}_i\mathbf{s}_i$ can be written as

$$\forall i \in \{1,2\}\,, \quad \mathrm{Trace}\left(\mathbf{V}_i\mathbf{P}_i\mathbf{V}_i^H\right) \leqslant N_t\, p_{i,\max}. \tag{3}$$

We consider, without any loss of generality, that both transmitters are limited by the same maximum transmit power $N_t\, p_{\max}$, i.e., $\forall i \in \{1,2\}$, $p_{i,\max} = p_{\max}$.

At receiver $i \in \{1,2\}$, the signal $\mathbf{r}_i$ is processed using an $N_r \times N_r$ matrix $\mathbf{D}_i$ to form the N_r-dimensional vector $\mathbf{y}_i = \mathbf{D}_i\mathbf{r}_i$. All along this paper, we refer to $\mathbf{D}_i$ as the post-processing matrix at receiver i. Regarding channel knowledge assumptions at the different nodes, we assume that the primary terminals (transmitter and receiver) have perfect knowledge of the matrix $\mathbf{H}_{11}$ while the secondary terminals have perfect knowledge of all channel transfer matrices $\mathbf{H}_{ij}$, $\forall (i,j) \in \{1,2\}^2$. One might ask whether this setup is higly demanding in terms of information assumptions. In fact, there are several technical arguments making this setup relatively realistic: (a) in some contexts channel reciprocity can be exploited to acquire CSI at the transmitters; (b) feedback channels are often available in wireless communications [11], and (c) learning mechanisms [12] can be exploited to iteratively learn the required CSI. In any case, the perfect information assumptions provide us with an upper bound on the achievable transmission rate for the secondary link.

III. Interference Alignment Strategy

In this section, we describe how both links introduced in Sec. II can simultaneously operate under the constraint that no additional interference is generated by the opportunistic transmitter on the primary receiver. First, we revisit the transmitting scheme implemented by the primary system [14], then we present the concept of transmit opportunity, and finally we introduce the proposed opportunistic IA technique.

A. Primary Link Performance

According to our initial assumptions (Sec. II) the primary link must operate at its highest transmission rate in the absence of interference. Hence, following the results in [17], [14], the optimal pre-processing and post-processing schemes for the primary link are given by the following theorem.

Theorem 1 (Telatar-1995 [17]): Let $\mathbf{H}_{11} = \mathbf{U}_{H_{11}} \mathbf{\Lambda}_{H_{11}} \mathbf{V}_{H_{11}}^H$, *with* $\mathbf{\Lambda} = \mathrm{diag}\left(\lambda_{H_{11},1}, \ldots, \lambda_{H_{11},\zeta_2}\right)$, *be a singular value decomposition (SVD) of the* $N_r \times N_t$ *channel transfer matrix* $\mathbf{H}_{11}$. *The primary link achieves capacity by choosing* $\mathbf{V}_1 = \mathbf{V}_{H_{11}}$, $\mathbf{D}_1 = \mathbf{U}_{H_{11}}^H$, $\mathbf{P}_1 = \mathrm{diag}(p_{1,1}, \ldots, p_{1,N_t})$, *where*

$$\forall i \in \{1, \ldots, N_t\}, \quad p_{1,i} = \left[\beta - \frac{\sigma_1^2}{\lambda_{H_{11}^H H_{11},i}}\right]^+, \tag{4}$$

with, $\mathbf{\Lambda}_{H_{11}^H H_{11}} = \mathbf{\Lambda}_{H_{11}}^H \mathbf{\Lambda}_{H_{11}} = \mathrm{diag}\left(\lambda_{H_{11}^H H_{11},1}, \ldots, \lambda_{H_{11}^H H_{11},N_t}\right)$ *and the constant* β *(water-level) is set to saturate the power constraint (3).*

B. Transmit Opportunities

When implementing its capacity-achieving transmission scheme, the primary transmitter sees an equivalent channel which consists of $N = \min(N_r, N_t)$ parallel sub-channels with channel gains $\lambda_{H_{11}^H H_{11},i}$, with $i \in \{1, \ldots, \min(N_t, N_r)\}$. Each sub-channel can be interpreted as a spatial direction from the primary transmitter to the primary receiver. Interestingly, (4) shows that some of these sub-channels can be left unused: the sub-channel i is said to be unused by the primary transmitter when $p_{1,i} = 0$. These unused sub-channels or SD are named transmit opportunities (TO).

Definition 2 (Transmit Opportunities): Let $\lambda_{H_{11}^H H_{11},1}, \dots \lambda_{H_{11}^H H_{11},\min(N_t,N_r)}$ *be the eigenvalues of the matrix* $\mathbf{H}_{11}^H \mathbf{H}_{11}$ *and* β *be the water-level in (Th. 1). The opportunistic terminal is said to have* S *transmit opportunities if there exists a set* $\mathcal{S} \subset \{1, \dots, \min(N_t, N_r)\}$ *such that* $|\mathcal{S}| = S$*, and for all* $s \in \mathcal{S}$*,* $\lambda_{H_{11}^H H_{11},s} \neq 0$ *and* $p_{1,s} = 0$*, i.e.,*

$$S = \sum_{i=1}^{\min(N_t,N_r)} \mathbb{1}_{\left\{\left[0, \frac{\sigma_1^2}{\beta}\right]\right\}}(\lambda_{H_{11}^H H_{11},i}). \tag{5}$$

As $p_{\max} > 0$, the primary link transmits at least over sub-channel $i^* = \arg \max\limits_{n \in \{1,\dots,\min(N_t,N_r)\}} \{\lambda_{H_{11}^H H_{11},n}\}$ regardless of its SNR. Then, we always have that $S \leqslant \min(N_t, N_r) - 1$. A natural question arises as to whether the number of TOs is sufficiently high for the secondary link to achieve a significant transmission rate. In order to provide an element of response to this question we propose a method to find an approximation of the number of TOs per transmit antenna, s_∞. First, we define the following regime.

Definition 3 (Regime of Large Numbers of Antennas): The regime of large numbers of antennas (RLNA) is defined as follows:

- $N_t \to +\infty$;
- $N_r \to +\infty$;
- $\lim\limits_{N_t,N_r \to +\infty} \dfrac{N_t}{N_r} = \alpha < +\infty$.

In the RLNA, s_∞ is defined by

$$s_\infty \triangleq \lim_{N_t,N_r \to +\infty} \frac{S}{N_t}. \tag{6}$$

As a preliminary step toward determining the expression of s_∞, we first show how to find the asymptotic water-level β_∞ in the RLNA. First, recall from the water-filling solution (4) and the power constraint (3) that

$$\frac{1}{N_t} \sum_{i=1}^{N_t} p_{1,i} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left[\beta - \frac{\sigma_1^2}{\lambda_{H_{11}^H H_{11},i}}\right]^+. \tag{7}$$

Define the real function q by

$$q(\lambda) = \left| \begin{array}{ll} 0, & \text{if } \lambda = 0, \\ \left[\beta - \frac{\sigma_1^2}{\lambda}\right]^+, & \text{if } \lambda > 0, \end{array} \right. \tag{8}$$

which is continuous and bounded on $\mathbb{R}^+$. Equation (7) can be rewritten as

$$\frac{1}{N_t} \sum_{i=1}^{N_t} q(\lambda_i) = \int_{-\infty}^{\infty} q(\lambda)\ f_{H_{11}^H H_{11}}^{(N_t)}(\lambda)\ d\lambda \tag{9}$$

where $f^{(N_t)}_{H_{11}^H H_{11}}$ is the probability density function associated with the empirical eigenvalue distribution $F^{(N_t)}_{H_{11}^H H_{11}}$ of $\mathbf{H}_{11}^H \mathbf{H}_{11}$. In the RLNA, the empirical eigenvalue distribution $F^{(N_t)}_{H_{11}^H H_{11}}$ converges almost surely to the deterministic limiting eigenvalue distribution $F_{H_{11}^H H_{11}}$, known as the Marčenko-Pastur law [18] whose associated density is

$$f_{H_{11}^H H_{11}}(\lambda) = \left(1 - \frac{1}{\alpha}\right)\delta(\lambda) + \frac{\sqrt{(\lambda - a)^+ (b - \lambda)^+}}{2\pi\alpha\lambda}, \tag{10}$$

where $a = (1 - \sqrt{\alpha})^2$ and $b = (1 + \sqrt{\alpha})^2$.

Note that the Marčenko-Pastur law has a bounded real positive support $\{\{0\} \cup [a, b]\}$ and q is continuous and bounded on $\mathbb{R}^+$. Consequently, in the RLNA, we have the almost sure convergence of (9), i.e.,

$$\int_{-\infty}^{\infty} q(\lambda)\ f^{(N_t)}_{H_{11}^H H_{11}}(\lambda)\ d\lambda \xrightarrow{a.s.} \int_{-\infty}^{\infty} q(\lambda) f_{H_{11}^H H_{11}}(\lambda) d\lambda \tag{11}$$

Thus, in the RLNA (Def. 3), the water-level β_∞ is the unique solution [19] to the equation

$$\int_{\max(\frac{\sigma_1^2}{\beta}, a)}^{b} \left(\beta - \frac{\sigma_1^2}{\lambda}\right) \frac{\sqrt{(\lambda - a)(b - \lambda)}}{2\pi\alpha\lambda} d\lambda - p_{\max} = 0, \tag{12}$$

and it does not depend on any specific realization of the channel transfer matrix $\mathbf{H}_{11}$, but only on the maximum power $p_{\max}$ and the receiver noise power σ_1^2.

We now give the expression of the asymptotic number of TOs per transmit antenna, s_∞. From (5), we have

$$\begin{aligned} s_\infty &= \lim_{N_t \to +\infty, N_r \to +\infty} \frac{1}{N_t} \sum_{i=1}^{N_t} \mathbb{1}_{\left\{\left[0, \frac{\sigma_1^2}{\beta}\right]\right\}}(\lambda_{H_{11}^H H_{11}, i}) \\ &= \lim_{N_t \to +\infty, N_r \to +\infty} \int_{-\infty}^{\infty} \mathbb{1}_{\left\{\left[0, \frac{\sigma_1^2}{\beta}\right]\right\}}(\lambda)\ f^{(N_t)}_{H_{11}^H H_{11}}(\lambda)\ d\lambda \\ &\xrightarrow{a.s.} \int_{a}^{\min(\frac{\sigma_1^2}{\beta}, b)} \frac{\sqrt{(\lambda - a)(b - \lambda)}}{2\pi\alpha\lambda}\ d\lambda. \end{aligned} \tag{13}$$

Thus, given the asymptotic water-level β_∞ for the primary link, the asymptotic number of TOs per transmit antenna is given by the following expression

$$s_\infty = \int_{a}^{\min(\frac{\sigma_1^2}{\beta_\infty}, b)} \frac{\sqrt{(\lambda - a)(b - \lambda)}}{2\pi\alpha\lambda}\ d\lambda. \tag{14}$$

In Fig. 1, we plot the asymptotic estimation of s_∞ for several values of the fraction $\alpha = \frac{N_t}{N_r}$ for the case of $N_t \in \{4, 8\}$. The conclusions from this figure are twofold. First, it shows that the asymptotic approximation is a valid estimation even in the case where the numbers of antennas are relatively small for medium and large SNRs (> 0 dBs). More importantly, it shows that even for small numbers of antennas, it is likely to find a significant number of TOs that can be exploited by opportunistic transmitters. This gives us a strong motivation for designing algorithms capable of exploiting these opportunities.

C. Pre-processing Matrix

In this section, we define the interference alignment condition to be met by the secondary transmitter and determine a pre-processing matrix satisfying this condition.

Definition 4 (IA condition): Let $\mathbf{H}_{11} = \mathbf{U}_{H_{11}} \mathbf{\Lambda}_{H_{11}} \mathbf{V}_{H_{11}}^H$ *be an SVD of* $\mathbf{H}_{11}$ *and*

$$\mathbf{R} = \sigma_1^2 \mathbf{I}_{N_r} + \mathbf{U}_{H_{11}}^H \mathbf{H_{12}} \mathbf{V}_2 \mathbf{P}_2 \mathbf{V}_2^H \mathbf{H}_{12}^H \mathbf{U}_{H_{11}}, \tag{15}$$

be the covariance matrix of the co-channel interference (CCI) plus noise signal in the primary link. The opportunistic link is said to satisfy the IA condition if its opportunistic transmission is such that the primary link achieves the transmission rate of the equivalent single-user system, which translates mathematically as

$$\log_2 \left| \mathbf{I}_{N_r} + \frac{1}{\sigma_1^2} \mathbf{\Lambda}_{H_{11}} \mathbf{P}_1 \mathbf{\Lambda}_{H_{11}}^H \right| = \log_2 \left| \mathbf{I}_{N_r} + \mathbf{R}^{-1} \mathbf{\Lambda}_{H_{11}} \mathbf{P}_1 \mathbf{\Lambda}_{H_{11}}^H \right|. \tag{16}$$

Our objective is first to find a pre-processing matrix $\mathbf{V}_2$ that satisfies the IA condition and then, to tune the PA matrix $\mathbf{P}_2$ and post-processing matrix $\mathbf{D}_2$ in order to maximize the transmission rate for the secondary link.

Lemma 1 (Pre-processing matrix $\mathbf{V}_2$*): Let* $\mathbf{H}_{11} = \mathbf{U}_{H_{11}} \mathbf{\Lambda}_{H_{11}} \mathbf{V}_{H_{11}}^H$ *be an ordered SVD of* $\mathbf{H}_{11}$*, with* $\mathbf{\Lambda}_{H_{11}} = \left(\lambda_{H_{11},1}, \ldots, \lambda_{H_{11},\min(N_r,N_t)} \right)$ *such that* $\lambda_{H_{11},1}^2 \geqslant \lambda_{H_{11},2}^2 \geqslant \ldots \geqslant \lambda_{H_{11},\min(N_r,N_t)}^2$*. Let also the* $N_r \times N_t$ *matrix* $\tilde{\mathbf{H}} \stackrel{\triangle}{=} \mathbf{U}_{H_{11}}^H \mathbf{H}_{12}$ *have a block structure,*

$$\tilde{\mathbf{H}} = \begin{array}{r c} & \overset{N_t}{\longleftrightarrow} \\ N_r - S \updownarrow & \\ S \updownarrow & \end{array} \!\!\!\!\! \begin{pmatrix} \tilde{\mathbf{H}}_1 \\ \tilde{\mathbf{H}}_2 \end{pmatrix}. \tag{17}$$

The IA condition (Def. 4) is satisfied independently of the PA matrix $\mathbf{P}_2$*, when the pre-processing matrix* $\mathbf{V}_2$ *satisfies the condition:*

$$\tilde{\mathbf{H}}_1 \mathbf{V}_2 = \mathbf{0}_{(N_r - S) \times \zeta_2}. \tag{18}$$

Proof: See Appendix A. ■

Another solution to the IA condition was given in [1], namely $\mathbf{V}_2 = \mathbf{H_{12}}^{-1} \mathbf{U}_{H_{11}} \bar{\mathbf{P}}_1$ for a given diagonal matrix $\bar{\mathbf{P}}_1 = \mathrm{diag}\left(\bar{p}_{1,1}, \ldots, \bar{p}_{1,N_t}\right)$, with $\bar{p}_{1,i} = \left[\frac{\sigma_2^2}{\lambda_{H_{11}^2}} - \beta \right]^+$, where β is the water-level of the primary system (*Th. 1*). However, such a solution is more restrictive than (18) since it requires $\mathbf{H}_{12}$ to be invertible and does not hold always hold for the case when $N_r \neq N_t$.

Plugging $\mathbf{V}_2$ from (18) into (15) shows that to guarantee the IA condition (4), the opportunistic transmitter has to avoid allocating power to the sub-channels being used by the primary transmitter, i.e., the non-zero entries of the diagonal of matrix $\mathbf{\Lambda}_{H_{11}}\mathbf{P}_1\mathbf{\Lambda}_{H_{11}}^H$. That is the reason why we refer to our technique as OIA.

From Lemma 1, it appears that the columns of matrix $\mathbf{V}_2$ have to belong to the null space $\mathrm{Ker}(\tilde{\mathbf{H}}_1)$ of $\tilde{\mathbf{H}}_1$ and therefore to the space spanned by the $\dim\mathrm{Ker}(\tilde{\mathbf{H}}_1) = N_t - \mathrm{rank}(\tilde{\mathbf{H}}_1)$ last columns of matrix $\mathbf{V}_{\tilde{H}_1}$, where $\tilde{\mathbf{H}}_1 = \mathbf{U}_{\tilde{H}_1}\mathbf{\Lambda}_{\tilde{H}_1}\mathbf{V}_{\tilde{H}_1}^H$:

$$\mathbf{V}_2 \in \mathrm{Span}\left(\mathbf{v}_{\tilde{H}_1}^{(N_t-\mathrm{rank}(\tilde{\mathbf{H}}_1)+1)}, \ldots, \mathbf{v}_{\tilde{H}_1}^{(N_t)}\right). \tag{19}$$

Here, for all $i \in \{1, \ldots, N_t\}$, the column vector $\mathbf{v}_{\tilde{H}_1}^{(i)}$ represents the i^{th} column of matrix $\tilde{\mathbf{H}}_1$ from the left to the right.

In the following we assume that the ζ_2 columns of the matrix $\mathbf{V}_2$ form an orthonormal basis of the corresponding subspace (19), and thus, $\mathbf{V}_2^H\mathbf{V}_2 = \mathbf{I}_{\zeta_2}$. Moreover, we would like to point out that:

- When $N_r - S \leqslant N_t$, $\mathrm{rank}(\tilde{\mathbf{H}}_1) \leq N_r - S$ and $\dim\mathrm{Ker}(\tilde{\mathbf{H}}_1) \geq N_t - (N_r - S)$ with equality if and only if $\tilde{\mathbf{H}}_1$ is full row-rank. This means that there are always at least $N_t - (N_r - S)$ non-null orthogonal vectors in $\mathrm{Ker}(\tilde{\mathbf{H}}_1)$, and thus, $\zeta_2 = \dim\mathrm{Ker}(\tilde{\mathbf{H}}_1)$. Consequently, $\mathbf{V}_2$ can always be chosen to be different from the null matrix $\mathbf{0}_{N_t\times\zeta_2}$.
- When, $N_t < N_r - S$, $\mathrm{rank}(\tilde{\mathbf{H}}_1) \leq N_t$ and $\dim\mathrm{Ker}(\tilde{\mathbf{H}}_1) \geq 0$, with equality if and only if $\tilde{\mathbf{H}}_1$ is full column-rank. This means that there are non-zero vectors in $\mathrm{Ker}(\tilde{\mathbf{H}}_1)$ if and only if $\tilde{\mathbf{H}}_1$ is not full column-rank. Consequently, $\mathbf{V}_2$ is a non-zero matrix if and only if $\tilde{\mathbf{H}}_1$ is not full column-rank, and again $\zeta_2 = \dim\mathrm{Ker}(\tilde{\mathbf{H}}_1)$.

Note that by processing $\mathbf{s}_2$ with $\mathbf{V}_2$ the resulting signal $\mathbf{V}_2\mathbf{s_2}$ becomes orthogonal to the space spanned by a *subset* of rows of the cross-interference channel matrix $\tilde{\mathbf{H}} = \mathbf{U}_{H_{11}}^H\mathbf{H}_{12}$. This is the main difference between the proposed OIA technique and the classical zero-forcing beamforming (ZFBF) [20], for which the transmit signal must be orthogonal to the whole row space of matrix $\tilde{\mathbf{H}}$. In the next section we tackle the problem of optimizing the post-processing matrix $\mathbf{D}_2$ to maximize the achievable transmission rate for the opportunistic transmitter.

D. Post-processing Matrix

Once the pre-processing matrix $\mathbf{V}_2$ has been adapted to perform IA according to (19), no harmful interference impairs the primary link. However, the secondary receiver undergoes the CCI from the primary transmitter. We recall that the opportunistic receiver has full CSI of all channel matrices $\mathbf{H}_{i,j}$. Then, the joint effect of the CCI and noise signals can be seen as a colored Gaussian noise with covariance matrix

$$\mathbf{Q} = \mathbf{H}_{21}\mathbf{V}_{H_{11}}\mathbf{P}_1\mathbf{V}_{H_{11}}^H\mathbf{H}_{21}^H + \sigma_2^2\mathbf{I}_{N_r}. \tag{20}$$

Given an input covariance matrix $\mathbf{P}_2$, the mutual information between the input $\mathbf{s}_2$ and the output $\mathbf{y}_2 = \mathbf{D}_2\mathbf{r}_2$ is

$$\begin{aligned} R_2(\mathbf{P}_2, \sigma_2^2) = \ & \log_2 \left| \mathbf{I}_{N_r} + \mathbf{D}_2\mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H\mathbf{D}_2^H \left(\mathbf{D}_2\mathbf{Q}\mathbf{D}_2^H\right)^{-1} \right| \\ & \leqslant \log_2 \left| \mathbf{I}_{N_r} + \mathbf{Q}^{-\frac{1}{2}}\mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H\mathbf{Q}^{-\frac{1}{2}} \right|, \end{aligned} \tag{21}$$

where equality is achieved by a whitening post-processing filter $\mathbf{D}_2 = \mathbf{Q}^{-\frac{1}{2}}$ [21]. i.e., the mutual information between the transmitted signal $\mathbf{s}_2$ and $\mathbf{r}_2$, is the same as that between $\mathbf{s}_2$ and $\mathbf{y}_2 = \mathbf{D}_2\mathbf{r}_2$. Note also that expression (21) is maximized by a zero-mean circularly-symmetric complex Gaussian input $\mathbf{s}_2$ [14].

E. Power Allocation Matrix Optimization

In this section, we are interested in finding the input covariance matrix $\mathbf{P}_2$ which maximizes the achievable transmission rate for the opportunistic link, $R_2(\mathbf{P}_2, \sigma_2^2)$ assuming that both matrices $\mathbf{V}_2$ and $\mathbf{D}_2$ have been set up as discussed in Sec. III-C and III-D, respectively. More specifically, the problem of interest in this section is:

$$\begin{aligned} \max_{\mathbf{P}_2} \quad & \log_2 \left| \mathbf{I}_{N_r} + \mathbf{Q}^{-\frac{1}{2}}\mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H\mathbf{Q}^{-\frac{1}{2}} \right| \\ \text{s.t.} \quad & \text{Trace}\left(\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\right) \leqslant p_{\max}. \end{aligned} \tag{22}$$

Before solving the optimization problem (OP) in (22), we briefly describe the uniform PA scheme (UPA). The UPA policy can be very useful not only to relax some information assumptions and decrease computational complexity at the transmitter but also because it corresponds to the limit of the optimal PA policy in the high SNR regime.

1) Uniform Power Allocation: In this case, the opportunistic transmitter does not perform any optimization on its own transmit power. It rather uniformly spreads its total power among the previously identified TOs. Thus, the PA matrix $\mathbf{P}_2$ is assumed to be of the form

$$\mathbf{P}_{2,UPA} = \gamma\mathbf{I}_{\zeta_2}, \tag{23}$$

where the constant γ is chosen to saturate the transmit power constraint (3),

$$\gamma = \frac{N_t\, p_{\max}}{\text{Trace}\left(\mathbf{V}_2\mathbf{V}_2^H\right)} = \frac{N_t p_{\max}}{\zeta_2}. \tag{24}$$

2) Optimal Power Allocation: Here, we tackle the OP formulated in (22). For doing so, we assume that the columns of matrix $\mathbf{V}_2$ are unitary and mutually orthogonal. We define the matrix $\mathbf{K} \triangleq \mathbf{Q}^{-\frac{1}{2}}\mathbf{H}_{22}\mathbf{V}_2$, where $\mathbf{K}$ is an $N_r \times \zeta_2$ matrix. Let $\mathbf{K} = \mathbf{U}_K\mathbf{\Lambda}_K\mathbf{V}_K^H$ be an SVD of matrix $\mathbf{K}$, where the matrices $\mathbf{U}_K$ and $\mathbf{V}_K$ are unitary matrices with dimensions $N_r \times N_r$ and $\zeta_2 \times \zeta_2$ respectively. The matrix $\mathbf{\Lambda}_K$ is an $N_r \times \zeta_2$ matrix with at most $\min\{N_r, \zeta_2\}$ non-zero singular values in its main diagonal. The entries in the diagonal of the matrix $\mathbf{\Lambda}_K$ are denoted by $\lambda_{K,1}, \ldots, \lambda_{K,\min\{Nr,\zeta_2\}}$. Finally, the original OP (22) can be rewritten as

$$\begin{array}{ll} \arg\max\limits_{\mathbf{P}_2} & \log_2 \left|\mathbf{I}_{N_r} + \mathbf{\Lambda}_K\mathbf{V}_K^H\mathbf{P}_2\mathbf{V}_K\mathbf{\Lambda}_K^H\right| \\ \text{s.t.} & \text{Trace}\left(\mathbf{P}_2\right) = \text{Trace}\left(\mathbf{V}_K^H\mathbf{P}_2\mathbf{V}_K\right) \leqslant N_t\, p_{\max}. \end{array} \tag{25}$$

Here, we define the square matrices of dimension ζ_2,

$$\tilde{\mathbf{P}}_2 \triangleq \mathbf{V}_K^H\mathbf{P}_2\mathbf{V}_K, \tag{26}$$

and $\mathbf{\Lambda}_{K^HK} \triangleq \mathbf{\Lambda}_K\mathbf{\Lambda}_K = \text{diag}\left(\lambda_{K,1}^2, \ldots, \lambda_{K,\zeta_2}^2\right)$. Using the new variables $\tilde{\mathbf{P}}_2$ and $\mathbf{\Lambda}_{K^HK}$, we can write that

$$\begin{aligned} \left|\mathbf{I}_{N_r} + \mathbf{\Lambda}_K\mathbf{V}_K^H\mathbf{P}_2\mathbf{V}_K\mathbf{\Lambda}_K^H\right| &= \left|\mathbf{I}_{\zeta_2} + \mathbf{\Lambda}_{K^HK}\tilde{\mathbf{P}}_2\right| \\ &\leqslant \prod_{i=1}^{\zeta_2}\left(1 + \lambda_{K,i}^2\tilde{p}_{2,i}\right), \text{[22]}, \end{aligned} \tag{27}$$

where $\tilde{p}_{2,i}$, with $i \in \{1, \ldots, \zeta_2\}$ are the entries of the main diagonal of matrix $\tilde{\mathbf{P}}_2$. Note that in (27) equality holds if $\tilde{\mathbf{P}}_2$ is a diagonal matrix [22]. Thus, choosing $\tilde{\mathbf{P}}_2$ to be diagonal maximizes the transmission rate. Hence, the OP simplifies to

$$\begin{array}{ll} \max\limits_{\tilde{p}_{2,1}\ldots\tilde{p}_{2,\zeta_2}} & \sum\limits_{i=1}^{\zeta_2}\log_2\left(1 + \lambda_{K,i}^2\tilde{p}_{2,i}\right) \\ \text{s.t.} & \sum\limits_{i=1}^{\zeta_2}\tilde{p}_{2,i} \leqslant p_{\max}, \end{array} \tag{28}$$

The simplified optimization problem (28) has eventually a water-filling solution of the form

$$\forall i \in \{1, \ldots, \zeta_2\}, \quad \tilde{p}_{2,i} = \left[\beta_2 - \frac{1}{\lambda_{K,i}^2}\right]^+, \tag{29}$$

where, the water-level β_2 is determined to saturate the power constraints in the optimization problem (28). Once the matrix $\tilde{\mathbf{P}}_2$ (26) has been obtained using water-filling (29), we define the optimal PA matrix $\mathbf{P}_{2,OPA}$ by

$$\mathbf{P}_{2,OPA} \quad = \quad \text{diag}\left(\tilde{p}_{2,i}, \ldots, \tilde{p}_{2,\zeta_2}\right), \tag{30}$$

while the left and right hand factors, $\mathbf{V}_K$ and V_K^H, of matrix $\tilde{\mathbf{P}}_2$ in (26) are included in the pre-processing matrix:

$$\mathbf{V}_{2,OPA} = \mathbf{V}_2\mathbf{V}_K. \tag{31}$$

In the next section, we study the achievable transmission rates of the opportunistic link.

IV. Asymptotic Transmission Rate of the Opportunistic Link

In this section, we analyze the behavior of the opportunistic rate per antenna

$$\bar{R}_2(\mathbf{P}_2, \sigma_2^2) \triangleq \frac{1}{N_r}\log_2\left|\mathbf{I}_{N_r} + \mathbf{Q}^{-1}\mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H\right| \tag{32}$$

in the RLNA. Interestingly, this quantity can be shown to converge to a limit, the latter being independent of the realization of $\mathbf{H}_{22}$. The usefulness of this limit has been shown in many papers (see references in [23]). In the present work, we essentially use this limit to conduct a performance analysis of the system under investigation but it is important to know that it can be further exploited, for instance, to prove some properties, or simplify optimization problems [24]. A key transform for analyzing quantities associated with large systems is the Stieltjes transform.

Definition 5: Let $\mathbf{X}$ *be an* $n\times n$ *random matrix with empirical eigenvalue distribution function* $F_X^{(n)}$*. We define the Stieltjes transform associated with the distribution* $F_X^{(n)}$*, for* $z \in \mathbb{C}^+ = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$*:*

$$G_X(z) \triangleq \int \frac{1}{\lambda - z}\mathrm{d}F_X^{(n)}(\lambda). \tag{33}$$

By exploiting the Stieltjes transform and results from random matrix theory for large systems, it is possible to find the limit of (32) in the RLNA. The corresponding result is as follows.

Proposition 6: [Asymptotic Transmission Rate of the Opportunistic Link] *Define the matrices*

$$\mathbf{M}_1 \triangleq \mathbf{H}_{12}\mathbf{V}_{H_{11}}\mathbf{P}_1\mathbf{V}_{H_{11}}^H\mathbf{H}_{12}^H \tag{34}$$

$$\mathbf{M}_2 \triangleq \mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H \tag{35}$$

$$\mathbf{M} \triangleq \mathbf{M}_1 + \mathbf{M}_2, \tag{36}$$

and consider the system model described in Sec. II with a primary link using the configuration $(\mathbf{V}_1, \mathbf{D}_1, \mathbf{P}_1)$ *described in Sec. III-A, and a secondary link with the configuration* $(\mathbf{V}_2, \mathbf{D}_2, \mathbf{P}_2)$ *described in Sec. III-C, III-D, with* $\mathbf{P}_2$ *any PA matrix independent from the noise level* σ_2^2*. Then, in the RLNA (Def. 3), under the assumption that* $\mathbf{P}_1$ *and* $\mathbf{V}_2\mathbf{P}_2\mathbf{P}_2^H$ *have a limiting eigenvalue distributions* F_{P_1} *and* $F_{V_2P_2V_2^H}$

with compact support, the asymptotic transmission rate per antenna of the opportunistic link $(\mathrm{Tx}_2\text{-}\mathrm{Rx}_2)$ *is*

$$\bar{R}_{2,\infty}(p_{\max}, \sigma_2^2) = \frac{1}{\ln 2} \int_{\sigma_2^2}^{+\infty} G_{M_1}(-z) - G_M(-z)\,\mathrm{d}z, \tag{37}$$

where, $G_M(z)$ *and* $G_{M_1}(z)$ *are the Stieltjes transforms of the limiting eigenvalue distribution of matrices* $\mathbf{M}$ *and* $\mathbf{M}_1$*, respectively.* $G_M(z)$ *and* $G_{M_1}(z)$ *are obtained by solving the fixed point equations (with unique solution when* $z \in \mathbb{R}^-$ *[25]):*

$$G_{M_1}(z) = \frac{-1}{z - g(G_{M_1}(z))} \tag{38}$$

and

$$G_M(z) = \frac{-1}{z - g(G_M(z)) - h(G_M(z))}, \tag{39}$$

respectively, where the functions $g(u)$ *and* $h(u)$ *are defined as follows*

$$g(u) \triangleq \mathbb{E}\left[\frac{p_1}{1 + \frac{1}{\alpha} p_1 u}\right], \tag{40}$$

$$h(u) \triangleq \mathbb{E}\left[\frac{p_2}{1 + \frac{1}{\alpha} p_2 u}\right]. \tag{41}$$

with p_1 *and* p_2 *random variables with distribution* F_{P_1} *and* $F_{V_2 P_2 V_2^H}$*.*

Proof: For the proof, see Appendix C. ■

The (non-trivial) result in Prop. 6 holds for any power allocation matrix $\mathbf{P}_2$ independent of σ_2^2. In particular, the case of the uniform power allocation policy perfectly meets this assumption. This also means that it holds for the optimum PA policy in the high SNR regime. For low and medium SNRs, the authors have noticed that the matrix $\mathbf{P}_{2,OPA}$ is in general not independent of σ_2^2. This is because $\mathbf{P}_2$ is obtained from a water-filling procedure. The corresponding technical problem is not trivial and is therefore left as an extension of the present work.

V. Numerical Results

A. Comparison between IA and ZFBF

We compare our IA scheme with the zero-forcing beamforming (ZFBF) scheme [20]. Within this scheme, the pre-processing matrix $\mathbf{V}_2$, denoted by $\mathbf{V}_{2,ZFBF}$, satisfies the condition

$$\mathbf{H}_{12}\mathbf{V}_{2,ZFBF} = \mathbf{0}_{N_r,\zeta_2}, \tag{42}$$

which implies that ZFBF is feasible only in some particular cases regarding the rank of matrix $\mathbf{H}_{12}$. For instance, when $N_t \leqslant N_r$ and $\mathbf{H}_{12}$ is full column rank, the pre-processing matrix is the null matrix, i.e.,

$\mathbf{V}_{2,ZFBF} = \mathbf{0}_{N_t,\zeta_2}$ and thus, no transmission takes place. Conversely, in the case of OIA when $N_t \leqslant N_r$, it is still possible to opportunistically transmit with a non-null matrix $\mathbf{V}_2$ in two cases as shown in Sec. III-C:

- if $N_r - S < N_t$,
- or if $N_r - S \geq N_t$ and $\tilde{\mathbf{H}}_1$ is not full column rank.

Another remark is that when both primary and secondary receivers come close, the opportunistic link will observe a significant power reduction since both the targeted and nulling directions become difficult to be distinguished. This power reduction will be less significant in the case of OIA since it always holds that $\zeta_2 = \mathrm{rank}(\mathbf{V}_2) \geqslant \mathrm{rank}(\mathbf{V}_{2,ZFBF})$ thanks to the existence of the additional TOs. Strict equality holds only when $S = 0$. As discussed in Sec. III-B, the number of TOs (S) is independent of the position of one receiver with respect to the other. It rather depends on the channel realization $\mathbf{H}_{11}$ and the SNR of the primary link.

In the following, we consider the cases where $N_t \geqslant N_r$, $N_t = N_r$, and $N_t < N_r$.

1) More antennas at the Transmitter: In Fig. 2, we consider the case where $\alpha = \frac{5}{4}$, with $N_r \in \{3, 9\}$. In this case, we observe that even for a small number of antennas, the OIA technique is superior to the classical ZFBF. Moreover, the higher the number of antennas, the higher the difference between the performance of both techniques. An important remark here is that, at high SNR, the performance of ZFBF and OIA is almost identical, since the number of TOs tends to zero as shown in Fig. 1. Similarly, the UPA and OPA schemes perform identically at high SNR.

2) Same Number of Antennas at Receivers and Transmitters: Now, consider the case where $N_t = N_r$. In this case, ZFBF is feasible only when the matrix $\mathbf{H}_{12}$ is not full row rank. Hence, since the required condition is a very rare event, we focus only on the OIA solution. In Fig. 3, we plot the transmission rate for the case where $N_r = N_t \in \{3, 6, 9\}$. We observe that at high SNR for the primary link and small number of antennas, the uniform PA performs similarly as the optimal PA. For a higher number of antennas and low SNR in the primary link, the difference between the uniform and optimal PA is significant.

3) More Antennas at the Receiver: The case when $N_r > N_t$ is a very special case. Here, an opportunistic transmission takes place only if $N_r - N_t \leqslant S$ and $\tilde{\mathbf{H}}_{11}$ is not full column rank. Note also that, in this case, no transmission takes place using the classical ZFBF technique. In Fig. 4, we plot the achievable transmission rate of the opportunistic link, when the difference $N_r - N_t = 1$, with $N_r \in \{6, 9\}$. As in the previous case, when there exists few TOs (high-medium SNR levels of the primary link. See Fig. 1), the achievable transmission rate of the opportunistic link tends to zero.

B. Asymptotic Transmission Rate

In Fig. 5, we plot both primary and secondary transmission rates for a given realization of matrices $\mathbf{H}_{i,j}$ $\forall(i,j) \in \{1,2\}^2$. We also plot the asymptotes obtained from Prop. 6 considering UPA in the secondary link and the optimal PA of the primary link (4). We observe that in both cases the transmission rate converges rapidly to the asymptotes even for a small number of antennas. This shows that Prop. 6 constitutes a good estimation of the achievable transmission rate for the secondary link even for finite number of antennas. We use Prop. 6 to compare the asymptotic transmission rate of the secondary and primary link. The asymptotic transmission rate of the primary receiver corresponds to the capacity of a single user $N_t \times N_r$ MIMO link whose asymptotes are provided in [26]. From Fig. 5, it becomes evident how the secondary link is able to achieve transmission rates of the same order as the primary link depending on both its own SNR and that of the primary link.

VI. Conclusions

In this paper, we proposed a technique to recycle spatial directions left unused by a primary MIMO link, so that they can be re-used by secondary links. Interestingly, the number of spatial directions can be evaluated analytically and shown to be sufficiently high to allow a secondary system to achieve a significant transmission rate. We provided a signal construction technique to exploit those spatial resources and a power allocation policy which maximizes the opportunistic transmission rate. Based on our asymptotical analysis, we show that this technique allows a secondary link to achieve transmission rates of the same order as those of the primary link, depending on their respective SNRs. To mention few interesting extensions of this work, we recall that our solution concerns only two MIMO links. The case where there exists several opportunistic devices and/or several primary devices remains to be studied. Additionally, a lot of work remains to be done to soften the CSI conditions.

Appendix A
Proof of Lemma 1

Here we prove *Lemma 1* which states that : if a matrix $\mathbf{V}_2$ satisfies the condition $\tilde{\mathbf{H}}_1\mathbf{V}_2 = \mathbf{0}_{(N_r-S)\times\zeta_2}$ then it meets the IA condition (4).

Proof: Let $\mathbf{H}_{11} = \mathbf{U}_{H_{11}}\mathbf{\Lambda}_{H_{11}}\mathbf{V}_{H_{11}}^H$ be an ordered SVD of $\mathbf{H}_{11}$, with $\mathbf{\Lambda}_{H_{11}} = \left(\lambda_{H_{11},1},\ldots,\lambda_{H_{11},\min(N_r,N_t)}\right)$ such that $\lambda_{H_{11},1}^2 \geqslant \lambda_{H_{11},2}^2 \geqslant \ldots \geqslant \lambda_{H_{11},\min(N_r,N_t)}^2$. Given that the singular values of the matrix $\mathbf{H}_{11}$ are sorted, we can write the matrix $\mathbf{\Lambda}_{H_{11}}\mathbf{P}_1\mathbf{\Lambda}_{H_{11}}^H$ as a block matrix,

$$\mathbf{\Lambda}_{H_{11}}\mathbf{P}_1\mathbf{\Lambda}_{H_{11}}^H = \begin{pmatrix} \mathbf{\Psi} & \mathbf{0}_{(N_r-S)\times S} \\ \mathbf{0}_{S\times(N_r-S)} & \mathbf{0}_{S\times S} \end{pmatrix}, \tag{43}$$

where the diagonal matrix $\mathbf{\Psi}$ with dimension (N_r-S) is $\mathbf{\Psi} = \text{diag}\left(\lambda_{H_{11},1}^2\, p_{1,1}, \ldots, \lambda_{H_{11},N_r-S}^2\, p_{1,N_r-S}\right)$.

Now let us split the interference-plus-noise covariance matrix (15) as:

$$\mathbf{R} = \begin{array}{r} \\ N_r - S\updownarrow \\ S\updownarrow \end{array} \begin{array}{c} \begin{array}{cc} \overset{N_r-S}{\longleftrightarrow} & \qquad\qquad \overset{S}{\longleftrightarrow} \end{array} \\ \begin{pmatrix} \mathbf{R}_1 + \sigma_1^2\mathbf{I}_{N_r-S} & \mathbf{R}_2 \\ \mathbf{R}_2^H & \mathbf{R}_3 + \sigma_1^2\mathbf{I}_S \end{pmatrix} \end{array}, \tag{44}$$

where $(\mathbf{R}_1 + \mathbf{I}_{N_r-S})$ and $(\mathbf{R}_3 + \mathbf{I}_{N_r-S})$ are invertible Hermitian matrices, and matrices $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ are defined from (15) and (17) as

$$\mathbf{R}_1 \triangleq \tilde{\mathbf{H}}_1\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\tilde{\mathbf{H}}_1^H, \tag{45}$$

$$\mathbf{R}_2 \triangleq \tilde{\mathbf{H}}_1\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\tilde{\mathbf{H}}_2^H, \tag{46}$$

$$\mathbf{R}_3 \triangleq \tilde{\mathbf{H}}_2\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\tilde{\mathbf{H}}_2^H. \tag{47}$$

Now, by plugging expressions (43) and (44) in (16), the IA condition can be rewritten as follows:

$$\begin{gathered} \log_2\left|\sigma_2^2\mathbf{I}_{N_r-S} + \mathbf{\Psi}\right| - \log_2\left|\sigma_2^2\mathbf{I}_{Nr}\right| = \\ \log_2\left|\mathbf{R}_1 + \sigma_1^2\mathbf{I}_{N_r-S} + \mathbf{\Psi}\right| - \log_2\left|\mathbf{R}_1 + \sigma_1^2\mathbf{I}_{N_r-S}\right| - \\ \log_2\left(\frac{\left|\mathbf{R}_3+\sigma_1^2\mathbf{I}_S-\mathbf{R}_2^H(\mathbf{R}_1+\sigma_1^2\mathbf{I}_{N_r-S})^{-1}\mathbf{R}_2\right|}{\left|\mathbf{R}_3+\sigma_1^2\mathbf{I}_S-\mathbf{R}_2^H(\mathbf{R}_1+\sigma_1^2\mathbf{I}_{N_r-S}+\mathbf{\Psi})^{-1}\mathbf{R}_2\right|}\right) \qquad . \end{gathered} \tag{48}$$

Note that there exists several choices for the submatrices $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ allowing the equality in (48) to be met. We see that a possible choice in order to meet the IA condition is $\mathbf{R}_1 = \mathbf{0}$, $\mathbf{R}_2 = \mathbf{0}$, independently of the matrix $\mathbf{R}_3$. Hence, from (45) and (46) we have $\mathbf{R}_1 = \mathbf{0}$ and $\mathbf{R}_2 = \mathbf{0}$, by having matrix $\mathbf{V}_2$ satisfying the condition $\tilde{\mathbf{H}}_1\mathbf{V}_2 = \mathbf{0}_{N_r-S\times\zeta_2}$, for any given PA matrix $\mathbf{P}_2$, which concludes the proof. ■

APPENDIX B
DEFINITIONS

In this appendix, we present useful definitions and previous results used in the proofs of Appendix C.

Definition 7: Let $\mathbf{X}$ be an $n \times n$ random matrix with empirical eigenvalue distribution function $F_X^{(n)}$. We define the following transforms associated with the distribution $F_X^{(n)}$, for $z \in \mathbb{C}^+ = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$:

$$\text{Stieltjes transform: } G_X(z) \triangleq \int_{-\infty}^{\infty} \frac{1}{t-z} \mathrm{d}F_X^{(n)}(t), \tag{49}$$

$$\Upsilon_X(z) \triangleq \int_{-\infty}^{\infty} \frac{zt}{1-zt} \mathrm{d}F_X^{(n)}(t), \tag{50}$$

$$\text{S-transform: } S_X(z) \triangleq \frac{1+z}{z} \Upsilon_X^{-1}(z), \tag{51}$$

where the function $\Upsilon_X^{-1}(z)$ is the reciprocal function of $\Upsilon_X(z)$, i.e.,

$$\Upsilon_X^{-1}(\Upsilon_X(z)) = \Upsilon_X(\Upsilon_X^{-1}(z)) = z. \tag{52}$$

From Eq. (33) and Eq. (50), we obtain the following relationship between the function $\Upsilon_X(z)$ (named η-transform in [27]) and the Stieltjes transform $G_X(z)$,

$$\Upsilon_X(z) = -1 - \frac{1}{z} G_X\left(\frac{1}{z}\right). \tag{53}$$

APPENDIX C
PROOF OF PROPOSITION 6

In this appendix, we provide a proof of Prop. 6 on the asymptotic expression of the opportunistic transmission rate per antenna, defined by

$$\bar{R}_{2,\infty}(\mathbf{P}_2, \sigma^2) \triangleq \lim_{\substack{N_t, N_r \to \infty \\ \frac{N_t}{N_r} \to \alpha < \infty}} \bar{R}_2(\mathbf{P}_2, \sigma^2)$$

. First, we list the steps of the proof and then we present a detailed development for each of them:

1) Step 1: Express $\frac{\partial \bar{R}_{2,\infty}(\mathbf{P}_2, \sigma_2^2)}{\partial \sigma_2^2}$ as function of the Stieltjes transforms $G_{M_1}(z)$ and $G_M(z)$,
2) Step 2: Obtain $G_{M_1}(z)$,
3) Step 3: Obtain $G_M(z)$,
4) Step 4: Integrate $\frac{\partial \bar{R}_{2,\infty}(\mathbf{P}_2, \sigma_2^2)}{\partial \sigma_2^2}$ to obtain $\bar{R}_{2,\infty}(\mathbf{P}_2, \sigma_2^2)$.

Fist Step: Express $\frac{\partial \bar{R}_{2,\infty}(\mathbf{P}_2, \sigma_2^2)}{\partial \sigma_2^2}$ as a function of the Stieltjes transforms $G_{M_1}(z)$ and $G_M(z)$.

Using (21) and (20), the opportunistic rate per receive antenna $\bar{R}_2$ can be re-written as follows

$$\begin{aligned} \bar{R}_2(\mathbf{P}_2, \sigma^2) &= \frac{1}{N_r} \log_2 \left| \mathbf{I}_{N_r} + \mathbf{Q}^{-\frac{1}{2}} \mathbf{H}_{22} \mathbf{V}_2 \mathbf{P}_2 \mathbf{V}_2^H \mathbf{H}_{22}^H \mathbf{Q}^{-\frac{1}{2}} \right| \\ &= \frac{1}{N_r} \log_2 \left| \sigma_2^2 \mathbf{I}_{N_r} + \mathbf{M}_1 + \mathbf{M}_2 \right| \\ &\quad - \frac{1}{N_r} \log_2 \left| \sigma_2^2 \mathbf{I}_{N_r} + \mathbf{M}_1 \right|, \end{aligned} \tag{54}$$

with $\mathbf{M}_1 \triangleq \mathbf{H}_{21}\mathbf{V}_{H_{11}}\mathbf{P}_1\mathbf{V}_{H_{11}}^H\mathbf{H}_{21}^H$, $\mathbf{M}_2 \triangleq \mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H$, and $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$. Matrices $\mathbf{M}$ and $\mathbf{M}_1$ are Hermitian Grammian matrices with eigenvalue decomposition $\mathbf{M} = \mathbf{U}_M\mathbf{\Lambda}_M\mathbf{U}_M^H$ and $\mathbf{M}_1 = \mathbf{U}_{M_1}\mathbf{\Lambda}_{M_1}\mathbf{U}_{M_1}^H$, respectively. Matrix $\mathbf{U}_M$ and $\mathbf{U}_{M_1}$ are $N_r \times N_r$ unitary matrices, and $\mathbf{\Lambda}_M = \mathrm{diag}(\lambda_{M,1}, \ldots, \lambda_{M,N_r})$ and $\mathbf{\Lambda}_{M_1} = (\lambda_{M_1,1}, \ldots, \lambda_{M_1,N_r})$ are square diagonal matrices containing the eigenvalues of the matrices $\mathbf{M}$ and $\mathbf{M}_1$ in decreasing order. Expression (54) can be written as

$$\begin{aligned}
\bar{R}_2(\mathbf{P}_2, \sigma_2^2) &= \frac{1}{N_r}\sum_{i=1}^{N_r} \log_2\left(\sigma_2^2 + \lambda_{M,i}\right) - \frac{1}{N_r}\sum_{i=1}^{N_r} \log_2\left(\sigma_2^2 + \lambda_{M_1,i}\right) \\
&= \int \log_2\left(\lambda + \sigma_2^2\right) \mathrm{d}F_M^{(N_r)}(\lambda) - \int \log_2\left(\lambda + \sigma_2^2\right) \mathrm{d}F_{M_1}^{(N_r)}(\lambda) \\
&\xrightarrow{a.s} \int \log_2\left(\lambda + \sigma_2^2\right) \mathrm{d}F_M(\lambda) - \int \log_2\left(\lambda + \sigma_2^2\right) \mathrm{d}F_{M_1}(\lambda),
\end{aligned} \tag{55}$$

where $F_M^{(N_r)}$ and $F_{M_1}^{(N_r)}$ are respectively the empirical eigenvalue distributions of matrices $\mathbf{M}$ and $\mathbf{M}_1$ of size N_r, that converge almost surely to the asymptotic eigenvalue distributions F_M and F_{M_1}, respectively. F_M and F_{M_1} have a compact support. Indeed the empirical eigenvalue distribution of Wishart matrices $\mathbf{H}_{ij}\mathbf{H}_{ij}^H$ converges almost surely to the compactly supported Marčenko-Pastur law, and by assumption, matrices $\mathbf{V}_i\mathbf{P}_i\mathbf{V}_i^H$, $i = 1, 2$ have a limit eigenvalue distribution with a compact support. Then by *Lemma 5* in [28], the asymptotic eigenvalue distribution of $\mathbf{M}_1$ and $\mathbf{M}_2$ have a compact support. The logarithm function being continuous, it is bounded on the compact supports of the asymptotic eigenvalue distributions of $\mathbf{M}_1$ and $\mathbf{M}$, therefore, the almost sure convergence in (55) could be obtained by using the bounded convergence theorem [29].

From (55), the derivative of the asymptotic rate $\bar{R}_{2,\infty}(\mathbf{P}_2, \sigma^2)$ with respect to the noise power σ_2^2 can be written as

$$\begin{aligned}
\frac{\partial}{\partial \sigma_2^2}\bar{R}_{2,\infty}(\mathbf{P}_2, \sigma_2^2) &= \frac{1}{\ln 2}\left(\int \frac{1}{\sigma_2^2 + \lambda}\mathrm{d}F_M(\lambda) - \int \frac{1}{\sigma_2^2 + \lambda}\mathrm{d}F_{M_1}(\lambda)\right) \\
&= \frac{1}{\ln 2}\left(G_M\left(-\sigma_2^2\right) - G_{M_1}\left(-\sigma_2^2\right)\right).
\end{aligned} \tag{56}$$

where $G_M(z)$ and $G_{M_1}(z)$, with $z \in \mathbb{C}^+ = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$, are the Stieltjes transforms of the asymptotic eigenvalue distributions F_M and F_{M_1}, respectively.

Step 2: Obtain $G_{M_1}(z)$

Matrix $\mathbf{M}_1$ can be written as

$$\mathbf{M}_1 = \sqrt{\alpha}\mathbf{H}_{21}\mathbf{V}_{H_{11}}\frac{\mathbf{P}_1}{\alpha}\mathbf{V}_{H_{11}}^H\mathbf{H}_{21}^H\sqrt{\alpha}. \tag{57}$$

The entries of the $N_r \times N_t$ matrix $\sqrt{\alpha}\mathbf{H}_{21}$ are zero-mean i.i.d. complex Gaussian with variance $\frac{\alpha}{N_t} = \frac{1}{N_r}$, thus $\sqrt{\alpha}\mathbf{H}_{21}$ is bi-unitarily invariant. Matrix $\mathbf{V}_{H_{11}}$ is unitary, consequently $\sqrt{\alpha}\mathbf{H}_{21}\mathbf{V}_{H_{11}}$ has the same distribution as $\sqrt{\alpha}\mathbf{H}_{21}$, in particular its entries are i.i.d. with mean zero and variance $\frac{1}{N_r}$. From (4), $\frac{\mathbf{P}_1}{\alpha}$ is diagonal, and by assumption it has a limit eigenvalue distribution $F_{\frac{\mathbf{P}_1}{\alpha}}$. Thus we can apply *Lemma 1.1* in [25] to $\mathbf{M}_1$, in the particular case where $\mathbf{A} = \mathbf{0}_{N_r}$ to obtain the Stieltjes transform of the asymptotic eigenvalue distribution of matrix $\mathbf{M}_1$

$$\begin{aligned} G_{M_1}(z) &= G_{\mathbf{0}}\left(z - \alpha \int \frac{\lambda}{1 + \lambda G_{M_1}(z)} dF_{\frac{P_1}{\alpha}}(\lambda)\right) \\ &= G_{\mathbf{0}}\left(z - \alpha \int_{-\infty}^{\infty} \frac{\lambda}{1 + \lambda G_{M_1}(z)} \alpha f_{P_1}(\alpha\lambda) d\lambda\right) \\ &= G_{\mathbf{0}}\left(z - \int_{-\infty}^{\infty} \frac{p_1}{1 + \frac{p_1}{\alpha} G_{M_1}(z)} f_{P_1}(p_1) dp_1\right) \\ &= G_{\mathbf{0}}\left(\, z - g(G_{M_1}(z))\,\right) \end{aligned} \tag{58}$$

where the function $g(u)$ is defined by

$$g(u) \triangleq \int_{-\infty}^{\infty} \frac{p_1}{1 + \frac{p_1}{\alpha} u} f_{P_1}(p_1) dp_1 = \mathbb{E}\left[\frac{p_1}{1 + \frac{1}{\alpha} p_1 u}\right]. \tag{59}$$

where the random variable p_1 follows the cumulative distribution function (c.d.f) F_{P_1}.

The square null matrix $\mathbf{0}$ has an asymptotic eigenvalue distribution $F_{\mathbf{0}}(\lambda) = \mu(\lambda)$. Thus, its Stieltjes transform is

$$G_{\mathbf{0}}(z) = \int_{-\infty}^{\infty} \frac{1}{\lambda - z} \delta(\lambda) \mathrm{d}\lambda = -\frac{1}{z}. \tag{60}$$

Then, using expressions (58) and (60), we obtain

$$G_{M_1}(z) = \frac{-1}{z - g(G_{M_1}(z))}. \tag{61}$$

Expression (61) is a fixed-point equation with unique solution when $z \in \mathbb{R}^-$ [25].

Step 3: Obtain $G_M(z)$ Recall that

$$\mathbf{M} \triangleq \mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H + \mathbf{H}_{21}\mathbf{V}_{H_{11}}\mathbf{P}_1\mathbf{V}_{H_{11}}^H\mathbf{H}_{21}^H \tag{62}$$

To obtain the Stieltjes transform G_M, we apply *Lemma 1.1* in [25] as in Step 2:

$$G_M(z) = G_{M_2}\left(\, z - g(G_M(z))\,\right). \tag{63}$$

To obtain the Stieltjes transform G_{M_2} of the asymptotic eigenvalue distribution function of the matrix $\mathbf{M}_2 = \mathbf{H}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^H\mathbf{H}_{22}^H$, we first express its S-transform:

$$\begin{aligned} S_{M_2}(z) &= S_{H_{22}V_2P_2V_2^H H_{22}^H}(z) \\ &= \left(\tfrac{z+1}{z+\alpha}\right) S_{H_{22}^H H_{22} V_2 \mathbf{P}_2 V_2^H}\left(\tfrac{z}{\alpha}\right) \text{ by \textit{Lem. 1 in [28]}} \\ &= \left(\tfrac{z+1}{z+\alpha}\right) S_{H_{22}^H H_{22}}\left(\tfrac{z}{\alpha}\right) S_{V_2\mathbf{P}_2V_2^H}\left(\tfrac{z}{\alpha}\right) \quad \text{by \textit{Lem. 6} in [30]} \\ &= S_{H_{22}H_{22}^H}(z)\ S_{V_2\mathbf{P}_2V_2^H}\left(\tfrac{z}{\alpha}\right) \quad \text{by \textit{Lem. 1} in [28]} \\ &= \left(\tfrac{1}{1+\frac{z}{\alpha}}\right) S_{V_2\mathbf{P}_2V_2^H}\left(\tfrac{z}{\alpha}\right) \quad \text{by \textit{Lem. 6} in [30]} \end{aligned}$$

The S-transforms $S_{M_2}(z)$ and $S_{V_2\mathbf{P}_2V_2^H}\left(\frac{z}{\alpha}\right)$ in expression (64) can be written as functions of their η-transforms:

$$S_{M_2}(z) = \frac{1+z}{z}\Upsilon_{M_2}^{-1} \quad \text{From (51)} \tag{64}$$

$$\begin{aligned} S_{V_2P_2V_2^H}\left(\frac{z}{\alpha}\right) &= \frac{1+\frac{z}{\alpha}}{\frac{z}{\alpha}}\Upsilon_{V_2P_2V_2^H}^{-1}\left(\tfrac{z}{\alpha}\right) \quad \text{From (51)} \\ &= \frac{\alpha+z}{z}\Upsilon_{V_2P_2V_2^H}^{-1}\left(\tfrac{z}{\alpha}\right) \end{aligned} \tag{65}$$

Then, plugging (64) and (65) into (64) yields

$$\Upsilon_{M_2}^{-1}(z) = \left(\frac{\alpha}{1+z}\right)\Upsilon_{V_2P_2V_2^H}^{-1}\left(\frac{z}{\alpha}\right) \tag{66}$$

Now, using the relation (53) between both the η-transform and the Stieltjes transform, we write

$$G_{M_2}(z) = \left(\frac{-1}{z}\right)\left(\Upsilon_{M_2}\left(\frac{1}{z}\right)+1\right), \tag{67}$$

and from (63), we obtain

$$G_M(z) = \left(\frac{-1}{z-g(G_M(z))}\right)\left(\Upsilon_{M_2}\left(\frac{1}{z-g(G_M(z))}\right)+1\right). \tag{68}$$

We handle (68) to obtain $G_M(z)$ as a function of the $\Upsilon_{V_2P_2V_2^H}(z)$

$$G_M(z) = \left(-\frac{1}{z-g(G_M(z))}\right)\left(1+\alpha\Upsilon_{V_2P_2V_2^H}\left(-\frac{G_M(z)}{\alpha}\right)\right). \tag{69}$$

From the definition of the η-transform (50), it follows that

$$\Upsilon_{V_2P_2V_2^H}\left(-\frac{G_M(z)}{\alpha}\right) = \int \frac{-\frac{G_M(z)}{\alpha}t}{1+\frac{G_M(z)}{\alpha}t} dF_{V_2P_2V_2^H}(t) \tag{70}$$

Using (70) in (69), we have

$$G_M(z) = \left(-\frac{1}{z-g(G_M(z))}\right)(1-G_M(z)\,h(G_M(z))\,) \tag{71}$$

with the function $h(u)$ defined as follows

$$h(u) \triangleq \int \frac{t}{1+\frac{u}{\alpha}t} dF_{V_2P_2V_2^H}(t) = \mathbb{E}\left[\frac{p_2}{1+\frac{1}{\alpha}p_2 u}\right]$$

where the random variable p_2 follows the distribution $F_{V_2P_2V_2^H}$.

Factorizing $G_M(z)$ in (71) finally yields

$$G_M(z) = \frac{-1}{z - g(G_M(z)) - h(G_M(z))} \tag{72}$$

Expression (72) is a fixed point equation with unique solution when $z \in \mathbb{R}_-$ [25].

Fourth Step: Integrate $\frac{\partial \bar{R}_2(\mathbf{P}_2,\sigma_2^2)}{\partial \sigma_2^2}$ to obtain $\bar{R}_2(\mathbf{P}_2,\sigma_2^2)$ in the RLNA.

From (56), we have that

$$\frac{\partial}{\partial \sigma_2^2}\bar{R}_{2,\infty}(\mathbf{P}_2,\sigma_2^2) = \frac{1}{\ln 2}\left(G_M\left(-\sigma_2^2\right) - G_{M_1}\left(-\sigma_2^2\right)\right). \tag{73}$$

Moreover, it is know that if $\sigma_2^2 \to \infty$ no reliable communication is possible and thus, $\bar{R}_{2,\infty}(p_{\max},\infty) = 0$. Hence, the asymptotic rate of the opportunistic link can be obtained by integrating expression (73)

$$\bar{R}_{2,\infty}(p_{\max},\sigma_2^2) = \frac{-1}{\ln 2}\int_{\sigma_2^2}^{\infty}\left(G_M\left(-z\right) - G_{M_1}\left(-z\right)\right) \mathrm{d}z, \tag{74}$$

which ends the proof.

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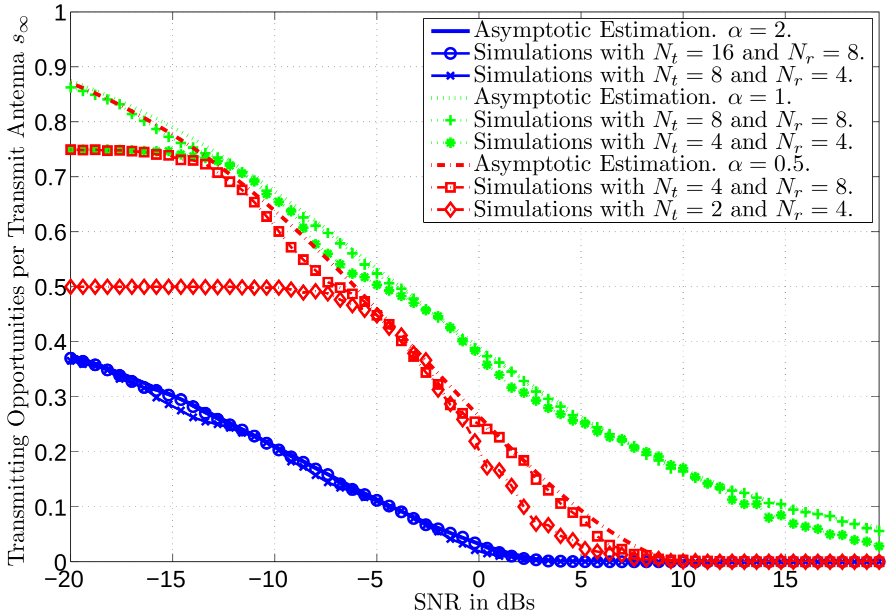

Figure 1. Fraction of transmit opportunities observed when using a given channel realization $\mathbf{H}_{11}$ (Eq. 5) and estimated value (Eq. 14) when $N_r \in \{4, 8\}$ and $\alpha = \frac{N_t}{N_r}$ as function of the $\mathrm{SNR} = \frac{p_{\max}}{\sigma_1^2}$, with $\sigma_1 = \sigma_2$.

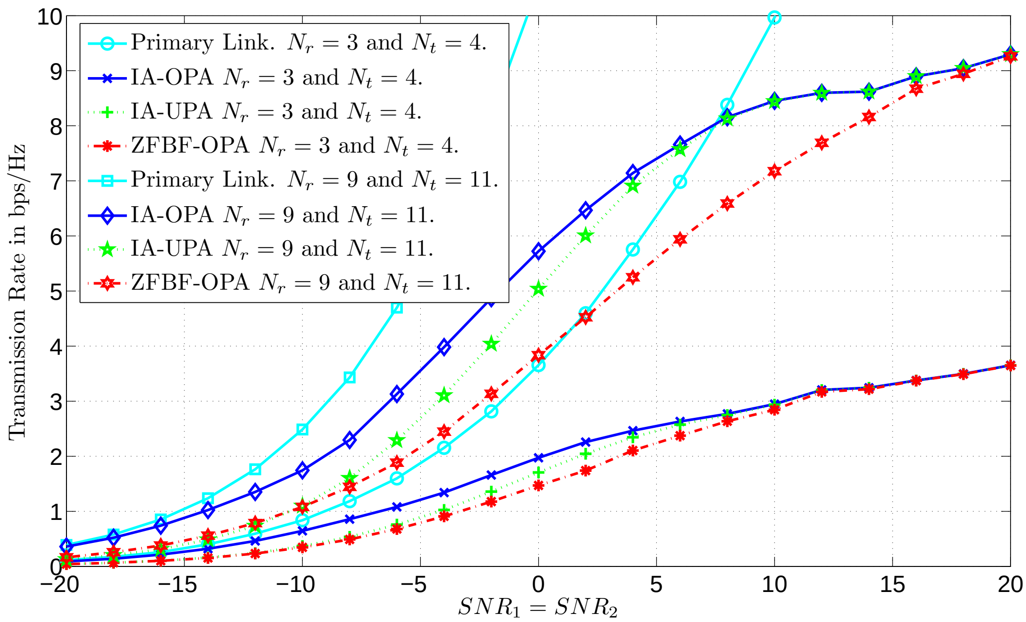

Figure 2. Transmission rate of the opportunistic link as a function of the $\mathrm{SNR}_1 = \frac{p_{\max}}{\sigma_1^2}$ of the primary link when IA and ZFBF are implemented. The number of antennas satisfy $\alpha = \frac{N_t}{N_r} \approx \frac{5}{4}$, with $N_r \in \{3, 9\}$ and $\mathrm{SNR}_i = \frac{p_{\max}}{\sigma_i^2}$, with $\sigma_1 = \sigma_2$.

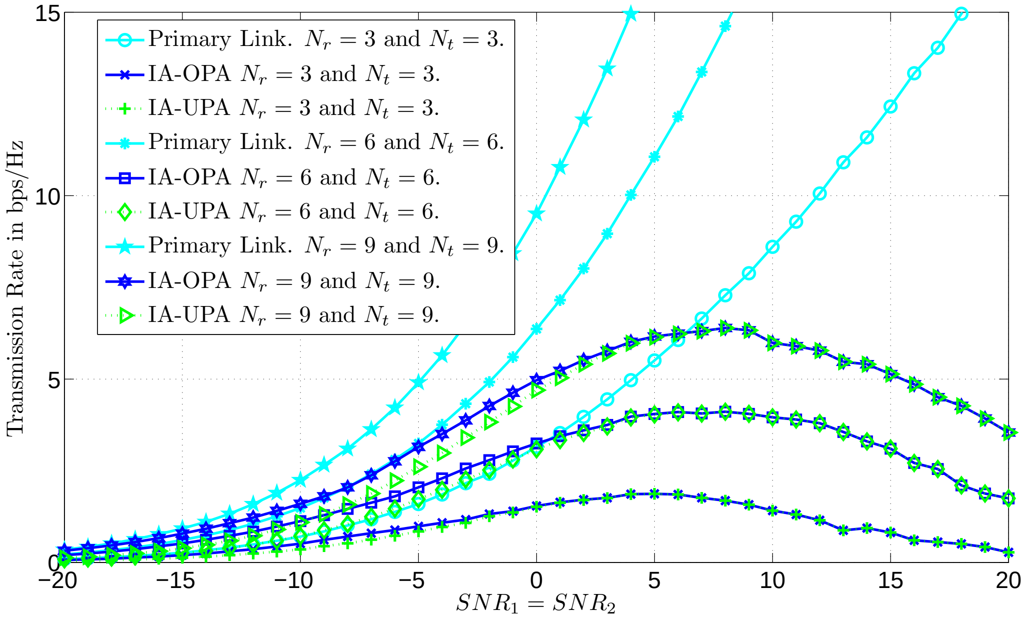

Figure 3. Transmission rate of the opportunistic link as a function of the $\mathrm{SNR}_i = \frac{p_{\max}}{\sigma_i^2}$, with $\sigma_1 = \sigma_2$. The number of antennas satisfy $N_t = N_r$, with $N_r \in \{3, 9\}$.

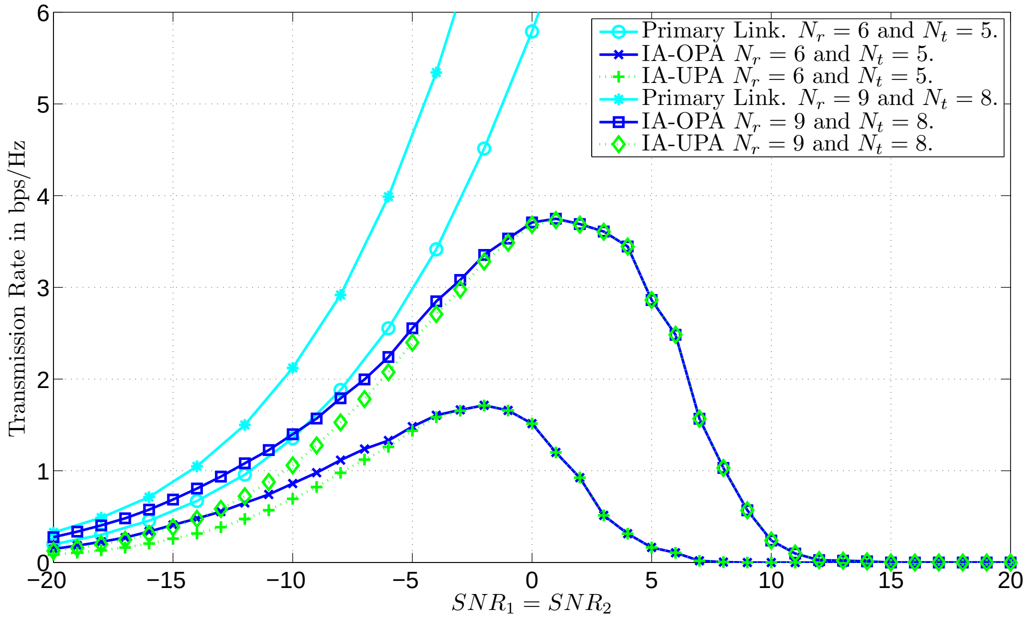

Figure 4. Transmission rate of the opportunistic link as a function of the $\mathrm{SNR}_i = \frac{p_{\max}}{\sigma_i^2}$, with $\sigma_1 = \sigma_2$. The number of antennas satisfy $N_t = N_r - 1$, with $N_r \in \{3, 9\}$.

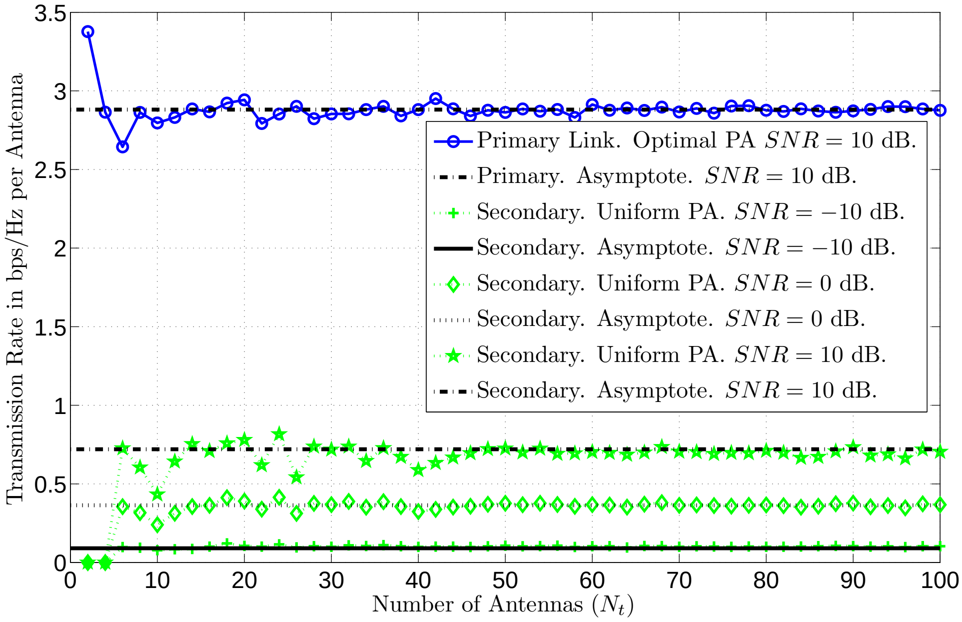

Figure 5. Asymptotic transmission rates per antenna of the opportunistic link observed when using a specific channel realization ($\mathbf{H}_{ij}\ \forall (i,j) \in \{1,2\}^2$) and using Prop. 6 as a function of the number of antennas, when $N_r = N_t$ using uniform PA at different SNR levels. $\mathrm{SNR}_i = \frac{p_{\max}}{\sigma_i^2}$.