On the Non-Coherent Wideband Multipath Fading Relay Channel

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***Abstract*—We investigate the multipath fading relay channel in the limit of a large bandwidth, and in the non-coherent setting, where the channel state is unknown to all terminals, including the relay and the destination. We propose a hypergraph model of the wideband multipath fading relay channel, and show that its min-cut is achieved by a non-coherent peaky frequency binning scheme. The so-obtained lower bound on the capacity of the wideband multipath fading relay channel turns out to coincide with the block-Markov lower bound on the capacity of the wideband frequency-division Gaussian (FD-AWGN) relay channel. In certain cases, this achievable rate also meets the cut-set upper-bound, and thus reaches the capacity of the non-coherent wideband multipath fading relay channel.**

I. Introduction

The general relay channel is among the smallest building blocks of communication networks, yet its capacity is still an open problem. Bounds on the capacity of the general relay channel, and the capacity of some particular classes of relay channels, have been derived in the past [1]. In particular in [2], the expression of the cut-set upper bound from [1], and the generalized block-Markov lower bound were derived for the case of the frequency-division additive white Gaussian noise (FD-AWGN) relay channel, where the source and the relay transmit in different bands. However, despite a plethora of recent works proposing cooperative strategies for wireless relaying networks and studying their performance in the high SNR regime, the capacity of the multipath fading relay channel remains unknown. Specifically, few works [3] analyzed the fading relay channel in the low SNR regime.

This paper focuses on analyzing the multipath fading relay channel in the non-coherent setting, where neither the source, nor the relay, nor the destination have channel state information (CSI), and in the wideband regime, alternatively named low SNR regime. Indeed, in the wideband regime, power is shared among a large number of degrees of freedom, making the SNR per degree of freedom low. Thus the wideband regime is power limited, but not interference limited on the contrary to the high SNR regime. In the wideband regime, the capacity of the point-to-point AWGN channel [4] and the capacity of the point-to-point non-coherent multipath fading channel [5] were shown to be both equal to the received SNR: $C_{Fading} = C_{AWGN} = \frac{P}{N_0} = \lim_{W\to\infty} W\log(1+\frac{P}{WN_0})$. Moreover, in the wideband limit of fading channels, spread-spectrum signals were shown to achieve poor performance, whereas peaky signals in time and frequency, such as low duty-cycle FSK, along with non-coherent detection, were shown to be capacity optimal [6]. The capacity of the point-to-point multiple input multiple output (MIMO) channel in the wideband limit was addressed in [7]. In particular, for the SIMO channel with two receive antennas with respective gains 1 and a^2, the capacity is $C_{SIMO} = (1+a^2)\frac{P}{N_0}$. Results on multiple user channels in the wideband limit include, the capacity region of the AWGN Broadcast Channel (BC) [8], for which time-sharing was shown to be optimal, and the capacity region of the AWGN Multiple Access Channel (MAC) [9], for which FDMA allows all sources to achieve their point-to-point interference-free capacity to the destination.

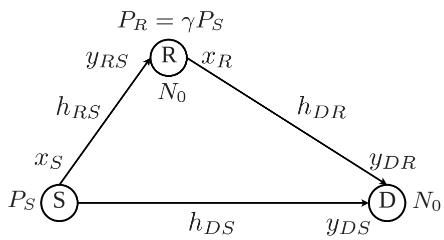
Some observations can be drawn from previous works on point-to-point and multiple user channels in the wideband regime: the capacity in the multipath fading case is the same as in the AWGN case, it can be reached in a non-coherent setting, and interference is not an issue. Coming back to the non-coherent multipath fading relay channel in the wideband regime, two questions naturally arise

- Can the FD-AWGN lower bound [2] be achieved in the non-coherent multipath fading case?
- Can the cut-set upper-bound [1] be reached?

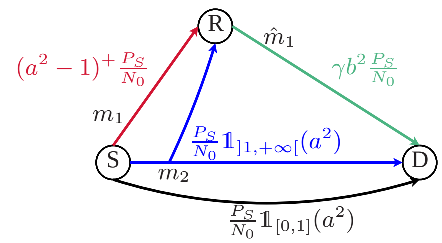
Note that in the wideband regime, considering the FD channel is relevant and meets the relay half-duplex constraint. This paper addresses these questions through three main contributions:

1) A hypergraph model of the wideband multipath fading relay channel is proposed.
2) The hypergraph min-cut is shown to be achieved in the non-coherent wideband multipath fading relay channel by a peaky frequency-binning scheme.
3) The hypergraph min-cut is shown to coincide with the generalized block-Markov lower bound on the capacity of the wideband FD-AWGN relay channel, and in certain channel configurations with the cut-set upper-bound, in which case it is equal to capacity.

The rest of the paper is organized as follows. In Section II, the hypergraph model of the wideband multipath fading relay channel is described, and the achievable hypergraph min-cut is compared with bounds on the capacity of the wideband FD-AWGN relay channel. The non-coherent scheme achieving the hypegraph min-cut is described in Section III, while its correspondence with the hypegraph model is detailed in Section IV, leading to the concluding Section V.



(a) Multipath fading relay channel



(b) Hypergraph model of the wideband multipath fading relay channel. *For the sake of clearness,* $(1-2T_d/T_c)$ *is not displayed in edge capacities.*

Fig. 1. Wideband fading relay channel

II. System model and main results

Notations: $\mathbb{N}$ and $\mathbb{R}$ denote the sets of non-negative integers, and real numbers, respectively. Let $m \in \mathbb{N}$, the set of non-negative integers less or equal to m is denoted $\mathbb{N}_m \triangleq \{0, \ldots, m\}$. The subset $[0, +\infty[$ of $\mathbb{R}$ is denoted by $\mathbb{R}^+$. Let $x \in \mathbb{R}$, $(x)^+ \triangleq \max\{0, x\}$. Let S be a set, the indicator function is defined by $\mathbb{1}_S(x) = 1$ if $x \in S$, $\mathbb{1}_S(x) = 0$ if $x \notin S$. $\Pr\{A\}$ is the probability of event A, $\mathrm{E}[\cdot]$ is the statistical expectation operator, and X is $\mathcal{CN}(\mu, \sigma^2)$ means that X is a circularly symmetric complex Gaussian random variable with mean μ and variance σ^2.

A. Wideband multipath fading relay channel

Consider the three-node network in Figure 1(a), where the source S, the relay R and the destination D are equipped with a single antenna. Source and relay are assumed to have average power constraints in the time-continuous channel model of P_S and $P_R = \gamma P_S$ Joules/s respectively. We assume that S, R and D have no channel state information (CSI), thus the multipath channel is considered in the non-coherent regime. In order to respect the half-duplex constraint at the relay, we assume that S and R transmit in two different frequency bands of respective width W_S and W_R. During each temporal block of duration T, S transmits a new codeword which R and D receive in the first frequency band; R performs some transformation on the signal received from S in the previous block and relays it to D in the second frequency band; D decodes a new codeword by processing the signals it received from S and R.

As in [6] the continuous-time multipath fading channel between transmitter $u \in \{S, R\}$ and receiver $v \in \{R, D\}$ is represented by the impulse response $h_{vu}(t) = \sum_{l=1}^{L_{vu}} a_{vu,l}(t)\delta(t - d_{vu,l}(t))$, where L_{vu} is the number of paths, and $a_{vu,l}(t)$ and $d_{vu,l}(t)$ are the gain and delay of path l at time t. For the sake of simplicity, we assume that all channels h_{vu}, $u \in \{S, R\}$, $v \in \{R, D\}$ have similar coherence-time T_c and delay-spread T_d. Moreover we consider a block-fading model where the processes $\{a_{vu,l}(t)\}$ and $\{d_{vu,l}(t)\}$ have constant values $\{a_{vu,l}(nT_c)\}$ and $\{d_{vu,l}(nT_c)\}$ over intervals $[nT_c, (n+1)T_c[$. Furthermore, the processes $\{a_{vu,l}(nT_c)\}$ and $\{d_{vu,l}(nT_c)\}$ are assumed to be independent, stationary and ergodic. Finally, let $a, b \in \mathbb{R}^+$, we assume a non-symmetric network, with stationary total channel gains $\sum_{l=1}^{L_{DS}} \mathrm{E}[|a_{DS,l}(0)|^2] = 1$, $\sum_{l=1}^{L_{RS}} \mathrm{E}[|a_{RS,l}(0)|^2] = a^2$, $\sum_{l=1}^{L_{DR}} \mathrm{E}[|a_{DR,l}(0)|^2] = b^2$.

A signal $x_u(t)$ transmitted in channel $h_{vu}(t)$ leads a received signal $y_{vu}(t) = \sum_{l=1}^{L_{vu}} a_{vu,l}(t)x_u(t - d_{vu,l}(t)) + z_v(t)$, where $z_v(t)$ is a white Gaussian noise process with power spectral density $N_0/2$.

As the band grows large, the capacity of the point-to-point non-coherent wideband multipath fading channel is equal to the received SNR [6]. Thus, the capacities of the point-to-point wideband channels between the source and the destination, the source and the relay, and the relay and the destination are respectively $C_{DS} = \frac{P_S}{N_0}$, $C_{RS} = a^2\frac{P_S}{N_0}$, and $C_{DR} = b^2\frac{P_R}{N_0}$.

B. Hypergraph model and main results

In this section, we introduce a hypergraph model of the wideband multipath fading relay channel, and gather our main results in *Theorem 1*. More precisely, we show that the hypergraph min-cut is achieved by a non-coherent relaying scheme based on peaky signals, which is described in details in Section III, and we compare the hypergraph min-cut with bounds on the capacity of the FD-AWGN relay channel.

The proposed hypergraph model of the wideband relay channel is depicted in Figure 1(b). A hyperedge connects a transmitting node to several receiving nodes. A message transmitted over a hyperedge at a rate below its capacity can be decoded reliably by all the receiving nodes. Messages transmitted over disjoint hyperedges are independent. This hypergraph model of the relay channel is motivated by the broadcast nature of the wireless link: when a source transmits a signal over the wireless link, several receiving nodes can overhear the signal and extract some of the information transmitted by the source. The hypergraph model allows to clarify the correlation between the pieces of information decoded at different receiving nodes, by breaking the wireless link from a transmitting node into a set of hyperedges carrying independent messages. In Figure 1(b), the blue hyperedge represents a reliable channel from the source to both the relay and the destination with capacity $\frac{P_S}{N_0}\mathbb{1}_{]1,+\infty[}(a^2)$, while the red and black edges represent extra reliable channels to the relay only with capacity $(a^2-1)^+\frac{P_S}{N_0}$, and to the destination only with capacity $\frac{P_S}{N_0}\mathbb{1}_{[0,1]}(a^2)$, respectively. Note that the black channel cannot coexist simultaneously with the red and blue channels. Finally, the green edge represents a reliable channel from the relay to the destination with capacity $\gamma b^2\frac{P_S}{N_0}$.

Theorem 1: Consider the non-coherent wideband multipath fading relay channel, described in Section II. When the system bandwidth $W_S + W_R$ grows large,

1) a lower bound on the capacity is provided by the min-cut on the hypergraph model

$$R = \min\left\{\max\{1, a^2\}, (1 + b^2\gamma)\right\} \frac{P_S}{N_0}\left(1 - 2\frac{T_d}{T_c}\right). \quad (1)$$

2) this achievable rate (1) is equal to the wideband limit of the generalized block Markov lower bound of the FD-AWGN channel [2] with the same received SNRs in the point-to-point source-destination, source-relay, and relay-destination channels when the channel is underspread ($T_d \ll T_c$).
3) in the case where $a^2 \geq 1 + b^2\gamma$ and $T_d \ll T_c$, this achievable rate (1) is equal to the FD-AWGN cut-set upper-bound $(1+b^2\gamma)\frac{P_S}{N_0}$, and it is therefore the capacity of the non-coherent wideband multipath fading channel.

The proof of 1) in *Theorem 1* is provided in [10]. We now address 2) and 3). The cut-set upper bound, and the generalized block-Markov lower bound on the capacity of the FD-AWGN relay channel were derived in [2]. When the system bandwidth grows large, the cut-set upper bound converges to

$$C_{FD-AWGN} \leq \min\left\{(1 + a^2), (1 + \gamma b^2)\right\} \frac{P_S}{N_0}, \quad (2)$$

and the generalized block-Markov lower bound converges to

$$C_{FD-AWGN} \geq \min\left\{\max\{1, a^2\}, (1 + \gamma b^2)\right\} \frac{P_S}{N_0}. \quad (3)$$

Comparing (1) and (3) shows that the lower bounds on the capacity of the non-coherent multipath fading relay channel and the FD-AWGN channel coincide in the wideband limit when the channel is underspread ($T_d \ll T_c$). This justifies 2) in *Theorem 1*, and shows that the hypergraph model is also valid in the FD-AWGN case.

In the case where $a^2 \geq 1 + b^2\gamma$ and the channel is underspread ($T_d \ll T_c$), the bounds (1), (3) and (2) coincide. The capacity of the multipath fading relay channel with infinite bandwidth cannot exceed the cut-set upper bound of the infinite bandwidth AWGN relay channel. We can then conclude that $(1 + b^2\gamma)\frac{P_S}{N_0}$ is the capacity of the non-coherent wideband multipath fading channel in that case, as stated in 3) in *Theorem 1*.

The multipath fading achievable rate (1) and the FD-AWGN cut-set upper-bound (2) are plotted in Figure 3 in blue and red respectively, in the case where $1 < \gamma b^2$.

III. Relaying Scheme Achieving the Min-Cut

In this section, we describe the non-coherent scheme which achieves the hypergraph min-cut in the multipath fading case.

A. Peaky Signaling at Source

Let M_S, M_R and M_D be positive integers such that the codebook size at source is $M_S = M_R M_D$. Consider a couple of independent random integers (m_1, m_2) such that $m_1 \in \mathbb{N}_{M_R-1} \triangleq \{0, \ldots, M_R - 1\}$, $m_2 \in \mathbb{N}_{M_D-1} \triangleq \{0, \ldots, M_D-1\}$. Then the Euclidian division theorem ensures that there exists a unique source message $m \in \{0, \ldots, M_S-1\}$

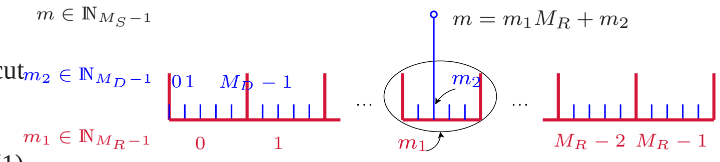


Fig. 2. Binning $m = (m_1, m_2)$

such that $m = m_1 M_D + m_2$. The representation of m as a couple (m_1, m_2) has a binning interpretation. Indeed, the M_S messages can be grouped into M_R bins of $M_D = \frac{M_S}{M_R}$ messages. The integer m_1 represents the bin index of message m, while m_2 is the index of message m within bin m_1, as illustrated in Figure 2. For $m_1 \in \mathbb{N}_{M_R-1}$, the m_1-th bin is denoted bin_{m_1} and contains the M_D messages $\text{bin}_{m_1} = \{m_1 M_D, \ldots, m_1 M_D + M_D - 1\}$.

During the first block of the cooperative transmission scheme, the source transmits a message m using the peaky-signaling scheme in [6], which was shown to achieve capacity in the wideband regime, and that we recall briefly in this section. The transmission scheme is based on Frequency Shift Keying (FSK) and low-duty cycle, and is therefore peaky both in frequency and time. We denote by $\theta \in]0, 1]$ the duty-factor, representing the fraction of time during which the source actually transmits power. If the source transmits power during T_s, then the time separating two successive transmissions is T_s/θ. Using FSK the transmitted signal corresponding to the m-th message is given in the baseband by a sinusoid at frequency f_m with power P_S/θ

$$x_S(t) = \begin{cases} \sqrt{\frac{P_S}{\theta}} \exp(j2\pi f_m t) & ,0 \leq t \leq T_s \\ 0 & ,T_s \leq t \leq T_s/\theta, \end{cases} \quad (4)$$

where the transmission duration is chosen to be shorter than the coherence time $T_s \leq T_c$. Frequencies f_m are taken to be integer multiples of $1/(T_s - 2T_d)$, leading to a minimum bandwidth $W_S = M_S/(T_s - 2T_d)$ for a codebook size M_S.

During the interval $[T_d, T_s - T_d]$ the processes $\{a_{RS,l}(t)\}$ and $\{d_{RS,l}(t)\}$ are constant, thus the signal received by the relay, when message m is sent, is given by

$$\begin{aligned} y_{RS}(t) &= \sum_{l=1}^{L_{RS}} a_{RS,l} \sqrt{\frac{P_S}{\theta}} \exp(j2\pi f_m (t - d_{RS,l})) + z_R(t) \\ &= G_{RS} x_S(t) + z_R(t), \end{aligned}$$

where $G_{RS} = \sum_{l=1}^{L_{RS}} a_{RS,l} \exp(-j2\pi f_m d_{RS,l})$ is the complex gain of the source-relay channel during $[T_d, T_s - T_d]$.

Similarly, we define the complex gain for the source-destination channel $G_{DS} = \sum_{l=1}^{L_{DS}} a_{DS,l} \exp(-j2\pi f_m d_{DS,l})$ and the signal received by the destination during $[T_d, T_s - T_d]$, $y_{DS}(t) = G_{DS} x_S(t) + z_D(t)$. The source repeats the transmission of a symbol N times over N disjoint time intervals $\frac{T_s}{\theta}$ to obtain diversity. Both the relay and the destination receive the N signals corresponding to message m during the first temporal block of total duration $T = \frac{NT_s}{\theta}$.

To transmit a codeword carrying $\ln M_S$ nats of information, an average power P_S is used, and the source rate is given by $R \triangleq \frac{\theta}{NT_s} \ln M_S$. Note that the source rate can be written $R = R_1 + R_2$, with $R_1 \triangleq \frac{\theta}{NT_s} \ln M_R$ and $R_2 \triangleq \frac{\theta}{NT_s} \ln M_D$.

B. Processing at Relay

Upon reception of the N source signals, the relay first decodes the bin index $\hat{m}_1$, then it forwards $\hat{m}_1$ to the destination using peaky signaling. Note that if the number of bins was set to $M_R = 1$ single bin, this would render the relay unused, and correspond to a direct transmission from S to D.

Phase 1: Decoding $\hat{m}_1$ by correlating
The relay correlates the n^{th} received signal against each frequency $k \in \mathbb{N}_{M_S - 1}$, forming the correlations

$$R_{RS,k}(n) \triangleq \frac{1}{\sqrt{N_0(T_s - 2T_d)}} \int_{T_d}^{T_s - T_d} y_{RS}(t) exp(-j2\pi f_k t) dt$$
$$= \delta_{km} \sqrt{\frac{P_S(T_s - 2T_d)}{\theta N_0}} G_{RS}(n) + W_k(n), \quad (5)$$

where $G_{RS}(n)$ is the complex gain in interval n, $\{W_k(n)\}_n$ are i.i.d. circularly symmetric complex Gaussian random variables with unit-variance. By modeling assumption, $\{G_{RS}(n)\}_n$ are i.i.d. complex random variables. Assuming a large number of paths, $\{G_{RS}(n)\}_n$ can be modeled by i.i.d. circularly symmetric complex Gaussian random variables with 0-mean and variance a^2. Then for each k, $\{R_k(n)\}$ are i.i.d. $\mathcal{CN}(0, \sigma_k^2)$ with variances

$$\sigma_k^2 = 1 + \delta_{km} \frac{a^2 P_S(T_s - 2T_d)}{\theta N_0} \ , k \in \mathbb{N}_{M_S - 1}. \quad (6)$$

Note that $\sigma_k^2 = 1$ for all $k \neq m$. The relay decoder builds the decision variables

$$S_{RS,k} = \frac{1}{N} \sum_{n=1}^{N} |R_{RS,k}(n)|^2, \quad (7)$$

which for all $k \neq m$ are i.i.d. These decision variables are compared with the threshold $A_R = 1 + (1 - \epsilon) \frac{a^2 P_S(T_s - 2T_d)}{\theta N_0}$, with $\epsilon \in]0, 1[$ to determine the set $\mathcal{S}_R$ of bins containing at least one frequency above threshold

$$\mathcal{S}_R \triangleq \{k \in \mathbb{N}_{M_R - 1} : \ \exists \, l \in \text{bin}_k \text{ s.t. } S_{RS,l} \geq A_R\}. \quad (8)$$

If $\mathcal{S}_R$ only contains a single bin k, the relay decodes $\hat{m}_1 = k$, otherwise it declares an error.

Phase 2: Forwarding the bin index $\hat{m}_1$
If the relay has not declared an error at the end of Phase 1, then it forwards the bin index $\hat{m}_1$ to the destination using peaky FSK in the second frequency band, with duty cycle θ and frequencies multiple of $1/(T_s - 2T_d)$. Similarly to the source, the relay repeats N times the transmission of $\hat{m}_1$ over disjoint intervals of duration $\frac{T_s}{\theta}$ for diversity. In the n-th interval, during the fraction θ of time where the relay signal is non-null, the signal is given by $x_R(t) = \sqrt{\frac{P_R}{\theta}} \exp(j2\pi f_{\hat{m}_1} t)$.

During the interval $[(N + n - 1)\frac{T_s}{\theta} + T_d, (N + n)\frac{T_s}{\theta} - T_d]$, of length $(T_s - 2T_d)$, the signal received by the destination, corresponding to the n-th relay signal, can be written $y_{DR}(t) = G_{DR} x_R(t) + z_D(t)$, where $G_{DR} = \sum_{l=1}^{L_{DR}} a_{DR,l} \exp(-j2\pi f_{\hat{m}_1} d_{DR,l})$ is the complex gain of the relay-destination channel. To transmit a codeword carrying $\ln M_R$ nats of information, a minimum bandwidth $W_R = M_R/(T_s - 2T_d)$ and an average power P_R are used, and the relay rate is given by $R_1 = \frac{\theta}{N T_s} \ln M_R$.

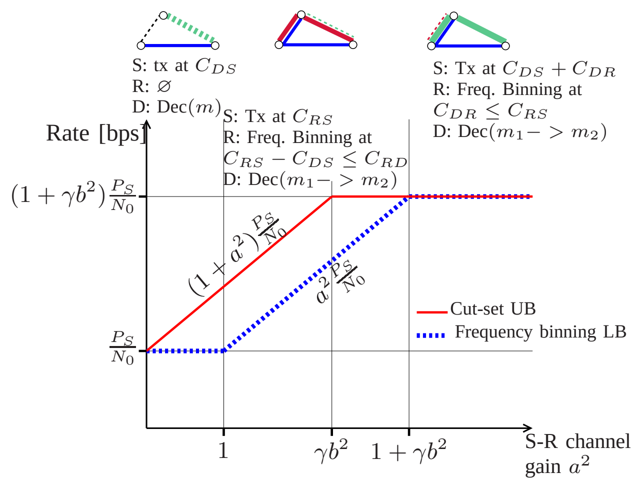


Fig. 3. Bounds on the capacity of the non-coherent wideband multipath fading relay channel

C. Decoding at Destination

At the end of the second phase, the destination has received $2N$ signals corresponding to the same message m, half coming from the source, and half being the retransmissions from the relay. The destination first processes the signal from the relay to decode the bin index m_1, then the signal from the source to decode the remaining index m_2.

Step 1: Decoding the bin index $\hat{\hat{m}}_1$
Similarly to (5), the destination correlates the N signals from the relay against each of the M_R frequencies in the second band, to form the correlations $R_{DR,k}(n)$, for $n \in \mathbb{N}_N$, and $k \in \mathbb{N}_{M_R - 1}$, given by

$$R_{DR,k}(n) = \delta_{k\hat{m}_1} \sqrt{\frac{P_R(T_s - 2T_d)}{\theta N_0}} G_{DR}(n) + W_{R,k}(n),$$

where $\{W_{R,k}(n)\}$ are i.i.d. $\mathcal{CN}(0, 1)$. Assuming a large number of paths, $\{G_{DR}(n)\}_n$ are modeled by i.i.d. $\mathcal{CN}(0, b^2)$ random variables. Then, for each $k \in \mathbb{N}_{M_R - 1}$, the variables $\{R_k(n)\}_n$ are i.i.d. $\mathcal{CN}(0, \sigma_{R,k}^2)$ with variances $\sigma_{R,k}^2 = 1 + \delta_{k\hat{m}_1} \frac{b^2 P_R(T_s - 2T_d)}{\theta N_0}$. The destination compares the decision variables $S_{DR,k} = \frac{1}{N} \sum_{n=1}^{N} |R_{DR,k}(n)|^2$ with the threshold $B_R = 1 + (1 - \epsilon_1) \frac{b^2 P_R(T_s - 2T_d)}{\theta N_0}$ and builds the set $\mathcal{S}_1 = \{k \in \mathbb{N}_{M_R - 1} : \ S_{DR,k} \geq B_R\}$. If $|\mathcal{S}_1| = 1$, the destination decodes $\hat{\hat{m}}_1$, otherwise it declares an error.

Step 2: Decoding the index $\hat{m}_2$
If the destination has not declared an error at the end of Step 1, it can proceed with the decoding by processing the signal it received from the source in the previous block. The destination uses $\hat{\hat{m}}_1$ to locate the bin of M_D frequencies containing the source message m in the signal y_{DS}. The destination correlates the N messages it received from the source against the M_D frequencies in $\text{bin}_{\hat{\hat{m}}_1} = \{\hat{\hat{m}}_1 M_D, \ldots, \hat{\hat{m}}_1 M_D + M_D - 1\}$ to form the correlations $R_{DS,l}(n)$, for $n \in \mathbb{N}_N$, and $l \in \text{bin}_{\hat{\hat{m}}_1}$

$$R_{DS,l}(n) = \delta_{lm} \sqrt{\frac{P_S(T_s - 2T_d)}{\theta N_0}} G_{DS}(n) + W_{S,l}(n), \quad (9)$$

where $\{W_{S,l}(n)\}_n$ are $\mathcal{CN}(0, 1)$, and for each l, the variables $\{R_{DS,l}(n)\}$ are i.i.d. $\mathcal{CN}(0, \sigma_{S,l}^2)$ with variance $\sigma_{S,l}^2 = 1 +$

$\delta_{lm}\frac{P_S(T_s-2T_d)}{\theta N_0}$, $l \in \text{bin}_{\hat{\hat{m}}_1}$. It should be pointed out that the relayed signal allows the destination to reduce the dimension of the space in which it looks for the source message m. More precisely, the relayed message allows the destination to reduce the number of noisy frequencies, to which it needs to compare the signal y_{DS}, from $M_S = M_R M_D$ to M_D. This observation is critical in the wideband regime where performance is mainly impaired by noise. For $l \in \text{bin}_{\hat{\hat{m}}_1}$, the destination builds the decision variables $S_{DS,l} = \frac{1}{N}\sum_{n=1}^{N}|R_{DS,l}(n)|^2$. By comparing them with $B_S = 1 + (1-\epsilon_2)\frac{P_S(T_s-2T_d)}{\theta N_0}$, it builds the set $\mathcal{S}_2 = \{l \in \text{bin}_{\hat{\hat{m}}_1} \ : \ S_{DS,l} \geq B_S\}$. If $|\mathcal{S}_2| = 1$, the destination decodes $\hat{m}_2$, otherwise it declares an error.

If the destination decoder passes Steps 1 and 2 without declaring an error, the destination forms the final decoded message $\hat{m} = \hat{\hat{m}}_1 M_D + \hat{m}_2$.

IV. Hypergraph interpretation

In this section, we give the correspondence between the min-cut achieving scheme in Section III, and the hypergraph model in Figure 1(b). The relaying scheme in Section III is a form of selective decode-and-forward, where the minimum amount of relayed information depends on the quality of the source-relay channel $C_{RS} = a^2\frac{P_S}{N_0}$ with respect to the channels $C_{DS} = \frac{P_S}{N_0}$ and $C_{DR} = b^2\gamma\frac{P_S}{N_0}$. Indeed, the amount of information forwarded by the relay is parameterized by the value of M_R, relatively to $M_S = M_R M_D$. Three different regimes can be identified, as shown in Figure 3.

Regime $a^2 \leq 1$: in this regime $C_{RS} \leq C_{DS}$, the source-destination channel is more reliable than the source-relay channel. S transmits directly to D at capacity $C_{DS} = \frac{P_S}{N_0}$ without using the relay. This is equivalent to setting the number of bins to a single bin, $M_R = 1$, containing all messages $M_D = M_S$. The achievable rate $R = C_{DS} = \frac{P_S}{N_0}$ is given by the capacity of the black source-destination hyperedge.

Regime $1 < a^2 \leq 1 + b^2\gamma$: in this regime $C_{DS} < C_{RS} \leq C_{DS} + C_{DR}$, the source-relay channel is stronger than the source-destination channel but weaker than the cut on the multiple-access (MA) side. S transmits m at rate $R = C_{RS} = a^2\frac{P_S}{N_0}$, by splitting m into submessages m_2 sent on the blue hyperedge at rate $R_2 = C_{DS} = \frac{P_S}{N_0}$, and m_1 sent on the red hyperedge at rate $R_1 = C_{RS} - C_{DS} = (a^2-1)\frac{P_S}{N_0}$. R decodes and reliably forwards the bin index $\hat{m}_1$ to D on the green hyperedge since $R_1 \leq C_{DR} = b^2\gamma\frac{P_S}{N_0}$. D will use the signals from R and S to decode the remaining index $\hat{m}_2$. The number of bins is chosen $M_R \in]1, M_S[$ such that M_D matches the capacity of the source-destination channel, and M_R can be handled by the source-relay and relay-destination channels. The achievable rate $R = C_{RS} = a^2\frac{P_S}{N_0}$ is given by the sum of the capacities of the red and blue hyperedges.

Regime $1 + b^2\gamma < a^2$: in this regime $C_{DS} + C_{DR} < C_{RS}$, the source-relay channel is better than the multiple-access cut. S transmits at a rate equal to the capacity of the MA cut $R = C_{DS} + C_{DR} = (1+b^2\gamma)\frac{P_S}{N_0}$, by splitting m into submessages m_2 sent on the blue hyperedge at rate $R_2 = C_{DS} = \frac{P_S}{N_0}$, and m_1 sent on the red hyperedge at rate $R_1 = C_{DR} = \gamma b^2\frac{P_S}{N_0} \leq C_{RS} = a^2\gamma\frac{P_S}{N_0}$. R decodes and forwards the bin index $\hat{m}_1$ to D. The destination uses the signals from S and R to decode the remaining $\hat{m}_2$. The number of bins $M_R \in]1, M_S[$ matches the capacity of the relay-destination channel, and $M_D \in]1, M_S[$ matches the capacity of the source-destination channel. The achievable rate $R = C_{DS} + C_{DR} = (1+b^2\gamma)\frac{P_S}{N_0}$ is given by the sum of the capacities of the green and blue hyperedges.

In these three regimes, the achievable rate is the hypergraph min-cut. The relationship between the rate achieved by the peaky binning scheme and the hypergraph min-cut appears as a simple tool to derive achievable rates, and the corresponding transmission schemes, in larger wideband relaying networks.

V. Conclusion

We propose a hypergaph model of the relay channel in the wideband limit, and show that its min-cut can be achieved not only in the FD-AWGN case, but also in the non-coherent multipath fading case thanks to a relaying scheme combining peaky signals and binning. In certain channel configurations, the so-obtained achievable rate also coincides with the cut-set upper-bound, and thus is equal to the capacity of the non-coherent wideband multipath fading relay channel.

In the remaining cases, where the achievable rate does not coincide with the cut-set upper bound, a question remains open: can the gap to the cut-set upper-bound be closed? If the capacity of the relay-destination channel was infinite, as in the SIMO channel, the cut $(1+a^2)\frac{P}{N_0}$ could be achieved, and the gap closed. However, because of the relay power constraint and the destination noise, the relay cannot make its received signal perfectly available to the destination as in the SIMO case. This raises the question as to whether virtual MIMO gains can actually be achieved in the wideband regime.

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