The evolution of human cooperation: homophily, non-additive benefits, and higher-order relatedness

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Acknowledgements

Country:

I acknowledge the Turrbal and Yugara people and as the owners of this land. I pay respect to their Elders, past and present, and recognise QUT has always been a place of teaching, learning, and research.

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Humans are cooperative

- Introspection — ‘I am a moral being’
- Humans are a highly cooperative speces
- Eusocial insects– relatives
- Humans– cooperate with non-relatives, strangers

Why cooperate?

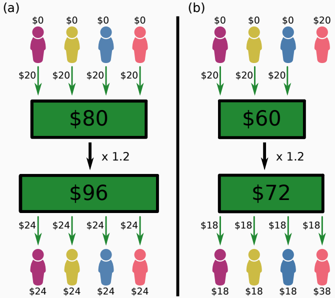
- Cooperate = help others at a cost to yourself

- Why help others at a cost to yourself?
- Seems to violate Darwinian logic
    - So others will help you in the future?
    - So you'll get a good reputation?
- Tricky to think about costs and benefits

- Game theory: put cooperation problem in its purest form so we can think about it clearly

Public goods game example

- Example:
  1. Public good that multiplies contributions by 1.2
  2. Everyone contributes $\rightarrow$ maximise total payoffs
  3. However, not contributing maximises individual payoff

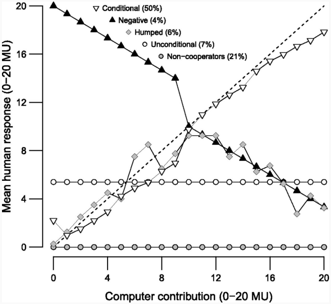


- Never makes sense to contribute
  - Returns are split equally
  - Marginal per-capita return $= 1.2/4 = 0.3 < 1$
    - 30c return for every $1 contributed

How do people really behave in linear PGGs?

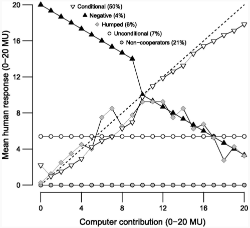
- Example: Burton-Chellew *et al.* (2016, PNAS)
  - Elicited contributious in PGG
  - Played against a computer
  - Computer play presumably removed fairness/empathy considerations

- Contribution level depends on contribution of others
- Similar results in other studies
- People genuinely seem believe this is payoff maximising!



Why do people make this mistake?

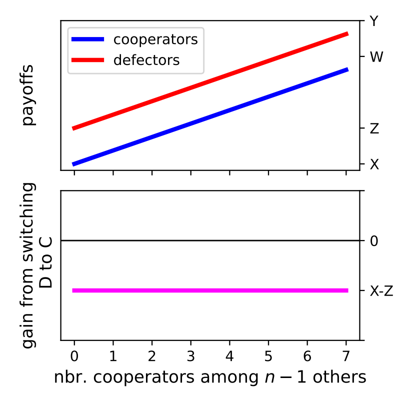
- Deeply unnatural scenario
- Previous work has focused on two ‘mistakes’:
    1. Mistake one-shot game for iterated game
    2. Mistake anonymous game for one with reputation concerns



My focus: Mistaking a linear game for a nonlinear one

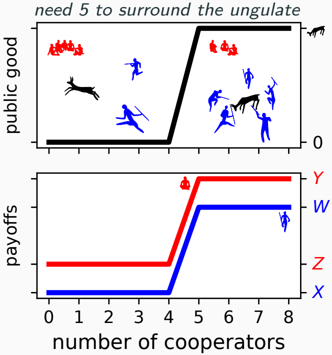
Linear public goods game

- In a linear game:
  - Benefit increases at constant rate with nbr. cooperators
  - No matter how many cooperators in the group, always lose by switching from D to C
- *n*-player generalisation of the Prisoner's Dilemma



Nonlinear public goods game

- Claim: sigmoid benefit functions relevant to our early history
- Defectors still get higher payoffs than Cooperators
- However, if your tribe is one short of the threshold, you should cooperate

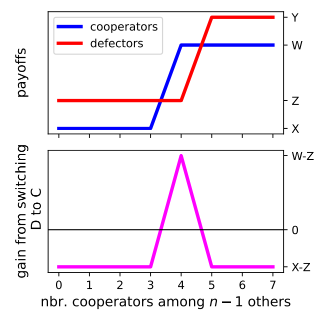


Nonlinear public goods game

- Claim: sigmoid benefit functions relevant to our early history
- Defectors still get higher payoffs than Cooperators

- However, if your tribe is one short of the threshold, you should cooperate
- Pink: contribution depends on contribution of others
- Key: chance to be 'pivotal':
  - if cooperators rare, don't cooperate
  - if cooperators common, might get higher payoffs if you're a cooperator

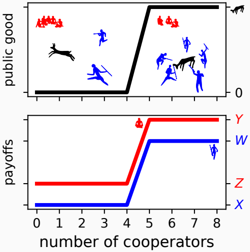
need 5 to surround the ungulate

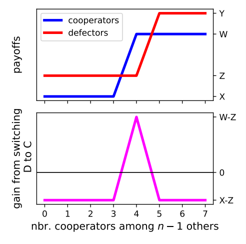


Nonlinear public goods game: evolutionary perspective

Translate game-theoretic → evolutionary perspective

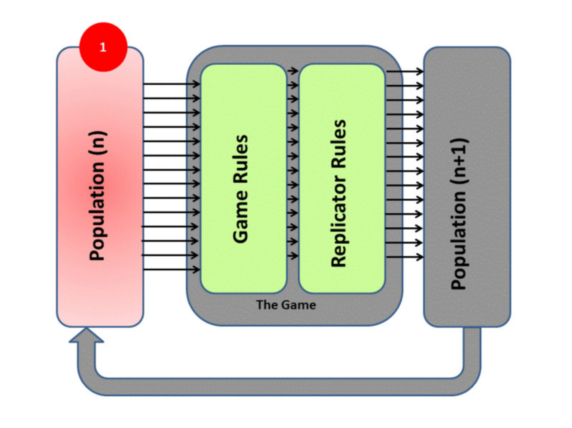
- In game-theoretic perspective:
    - if cooperators rare, shouldn't cooperate
    - if cooperators common, might get higher payoffs if you also cooperate
- Evolutionary perspective:
    - if cooperators rare (invasion), cooperation can't succeed
    - if cooperators common, cooperation might persist





Replicator dynamics overview

Population with genetically-encoded strategy (e.g., cooperate/defect)



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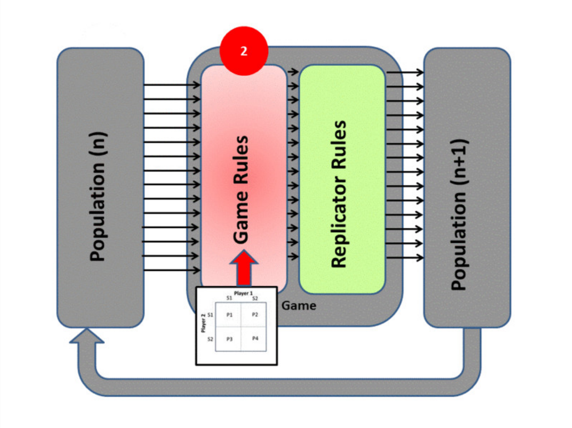
Replicator dynamics overview

Play game, receive payoffs



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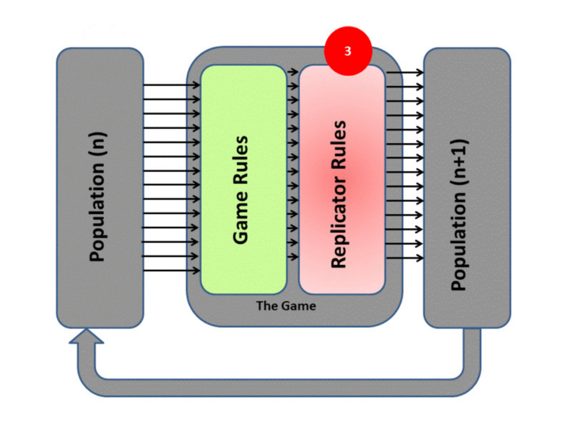
Replicator dynamics overview

High payoffs $\rightarrow$ more offspring



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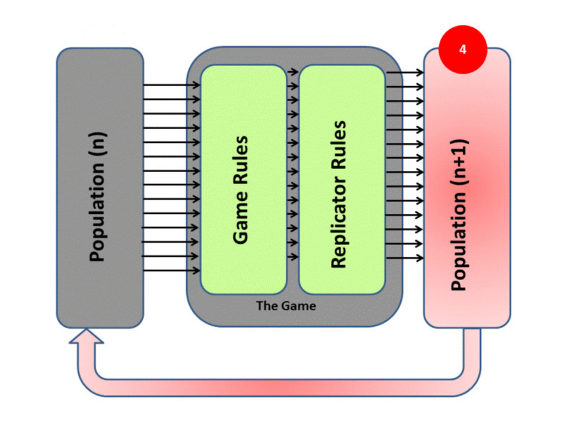
Replicator dynamics overview

New strategy frequencies in population



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Replicator dynamics

Change in proportion of x-strategists:

expected payoff to x-strategists

m is nbr. strategies

$$\dot{p}_x = \boxed{p_x} \left(\boxed{\bar{\pi}_x} - \boxed{\sum_{i=1}^{m} p_i \bar{\pi}_i} \right)$$

proportion of x-strategists

expected payoff in population

- growth rate proportional to how much better x-strategists' payoffs are compared to average

Replicator dynamics when groups formed randomly

$$\dot{p}_x = p_x \left(\overbrace{\bar{\pi}_x}^{\text{expected payoff to } x\text{-strategists}} - \sum_{i=1}^{m} p_i \bar{\pi}_i \right)$$

- $\boldsymbol{e}_x$: indicator, focal plays strategy x (below: 1 when cooperator)
- g_{nf}: non-focal strategy distribution (below: nbr. cooperators among nonfocals)

For prehistoric-hunt game:

$$\begin{aligned}\bar{\pi}_C &= \sum_{g_{\mathrm{nf}}=0}^{n-1} \overbrace{\pi(e_C, g_{\mathrm{nf}})}^{C\text{'s payoff when } g_{\mathrm{nf}} \text{ non-focals are } C} \; \overbrace{\mathbb{P}[G_{\mathrm{nf}} = g_{\mathrm{nf}}]}^{\text{prob. } g_{\mathrm{nf}} \text{ non-focals are } C}, \\ &= \sum_{g_{\mathrm{nf}}=0}^{n-1} \pi(e_C, g_{\mathrm{nf}}) \; \overbrace{\binom{n-1}{g_{\mathrm{nf}}} p_C^{g_{\mathrm{nf}}} (1 - p_C)^{n-1-g_{\mathrm{nf}}}}^{\text{binomial}}\end{aligned}$$

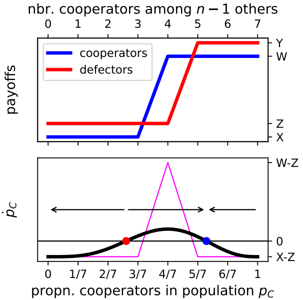
Two main results about nonlinear games

Recommend: Peña et al. (2014, J Theor Biol)

Two main known results:

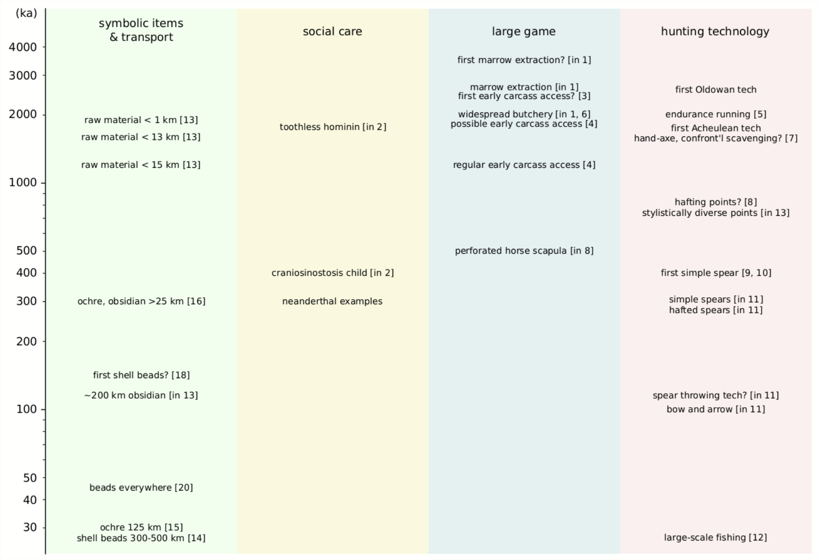
1. Cooperation can be sustained
    - Do people 'mistake' linear games for a nonlinear ones?
2. But cooperation cannot invade
    - Imagine a small nbr. of cooperators invading defectors...

So how did cooperation even get started?



What if, instead of randomly formed groups, groups tend to form with family members? Then invading Cooperators more likely to be grouped with other Cooperators.

Claim: Interactions with family more frequent in the past

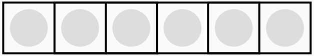


Genetically homophilic group formation

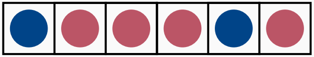
colours = strategies; vertical lines divide families

Random group formation

- Infinite population, no members from the same family



- Each strategy a random draw from the population



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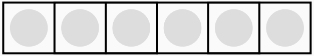

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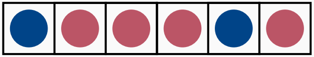
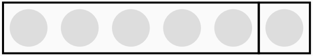
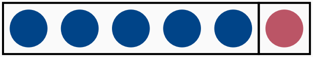


Homophilic group formation

- Individuals prefer to group with family members



- Members in the same family have the same strategy



- Rare invading Cooperators grouped with other Cooperators

Replicator dynamics with homophilic group formation

expected payoff to x-strategists

$$\dot{p}_x = p_x \left(\bar{\pi}_x - \sum_{i=1}^{m} p_i \bar{\pi}_i \right)$$

proportion of x-strategists

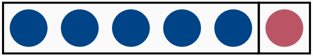
expected payoff in population

but now expected payoff:

no longer binomial

$$\bar{\pi}_C = \sum_{g_{\mathrm{nf}}=0}^{n-1} \pi(e_C, g_{\mathrm{nf}}) \; \mathbb{P}[G_{\mathrm{nf}} = g_{\mathrm{nf}} \mid G_0 = e_C]$$

nonfocal strategy distribution depends on focal's strategy



Hisashi's equation

Ohtsuki (2014, Phil Trans R Soc):

$$\dot{p}_1 = \sum_{g_{\mathrm{nf}}=0}^{n-1} \sum_{\ell=g_{\mathrm{nf}}}^{n-1} (-1)^{\ell-g_{\mathrm{nf}}} \binom{\ell}{g_{\mathrm{nf}}} \binom{n-1}{\ell}$$

relatedness terms

$$\Big[(1-\rho_1)\,\rho_{\ell+1}\,\pi(e_1, g_{\mathrm{nf}}) - \rho_1\,(\rho_\ell - \rho_{\ell+1})\,\pi(e_2, g_{\mathrm{nf}})\Big]$$

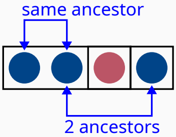
payoff terms

ρ_ℓ: probability that ℓ players sampled from the group without replacement have strategy 1.

Hisashi's equation

ρ_ℓ: probability that ℓ players sampled from the group without replacement have strategy 1.

E.g., prob. 2 individuals have strategy 1



$$\rho_2 = \overbrace{\theta_{2\to1}}^{\text{prob. same ancestor}} \overbrace{p_1}^{\text{prob. strategy-1}} + \underbrace{\theta_{2\to2}}_{\text{prob. two ancestors}} \underbrace{p_1^2}_{\text{prob. both strategy-1}}$$

In general:

$$\rho_\ell = \overbrace{\sum_{m=1}^{\ell}}^{\text{sum over nbr. ancestors } m} \overbrace{\theta_{\ell\to m}}^{\text{prob. } \ell \text{ sampled have } m \text{ common ancestors}} \underbrace{p_1}_{\text{propn. strategy-1 in populatn}}{}^{m}$$

Linear PGG is a function of dyadic relatedness $\theta_{2\to1}$ only

- If the PGG is linear, then only need dyadic relatedness (many papers)

$$\dot{p}_1 = f(\underbrace{\theta_{2\to1}}_{\text{dyadic relatedness, Hamilton's } r})$$

  - because the n-player game payoff can be written as a sum of payoffs in 2-player games

$$\pi^{(n)}(e_x, g_{\mathrm{nf}}^{(n)}) \equiv \sum_{g_{\mathrm{nf}}^{(2)}} \pi^{(2)}(e_x, g_{\mathrm{nf}}^{(2)})$$

- However, if the payoff function is nonlinear, higher-order relatedness coefficients needed (e.g., $\theta_{3\to1}, \theta_{3\to2}, \theta_{4\to1}$, etc.)
- How to get them?

How do we calculate the higher-order relatedness terms?

From probability $F_{\boldsymbol{q}}$ of group family-size distribution $\boldsymbol{q}$

For example, calculate $\theta_{2\to 1}$ for each $\boldsymbol{q}$.

	partition $\boldsymbol{q}$	$\theta_{2\to 1} \mid \boldsymbol{q}$	explanation
$F_{[4]}$		1	Any 2 will have a common ancestor.
$F_{[3,1]}$		$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$	Both must be blue (family size 3).
$F_{[2,2]}$		$1 \times \frac{1}{3} = \frac{1}{3}$	Choose any, then its 1 family member.
$F_{[2,1,1]}$		$\frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$	Only possible in the partition of 2.
$F_{[1,1,1,1]}$		0	Not possible.

So if we can calculate $F_{\boldsymbol{q}}$, then we can calculate $\theta_{l\to m}$

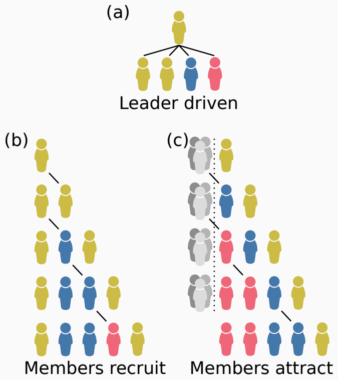
Homophilic group-formation models

(a) Leader driven:

- The leader is chosen at random from the population.
- Leader recruits/attracts kin with probability h and nonkin with probability $1 - h$.
- Group family size distribution

$$F_{[\ell,1,\ldots,1]} = \binom{n-1}{\ell-1} h^{\ell-1}(1-h)^{n-\ell}.$$

$h =:$ genetic homophily



Homophilic group-formation models

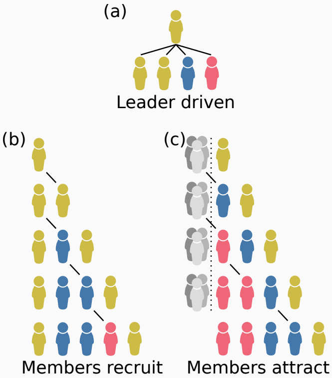
(b) Members recruit:

- All group members have an equal chance to recruit the next member.
- Equation in Kristensen *et al.* (2022)

(c) Members attract:

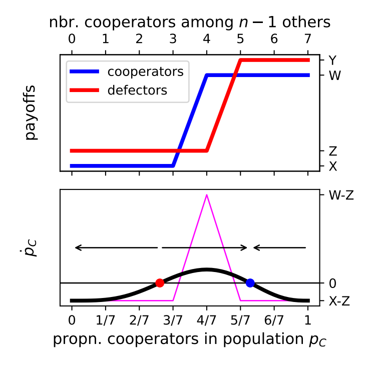
- Outsiders attracted to kin
- But also attracted to the group as a whole
- Use Ewens' formula (Ewen 1972).

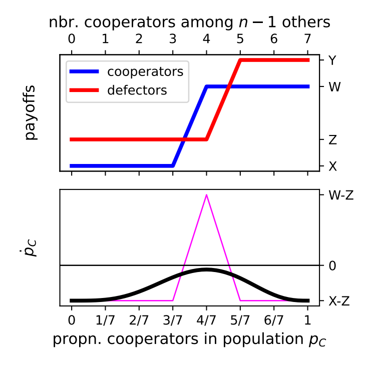
$h =:$ genetic homophily



NOTE: can be interpreted in terms of 'matching rules', i.e., strategy homophily *sensu* Jensen & Rigos (2018, Int J Game Theory)

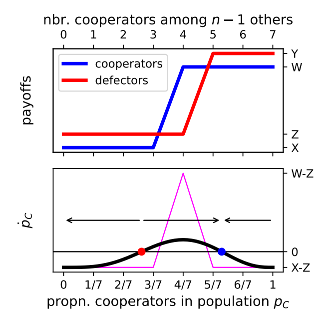
Recall no-homophily result: cooperation can (sometimes) persist but it can never invade:

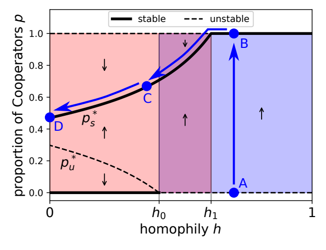




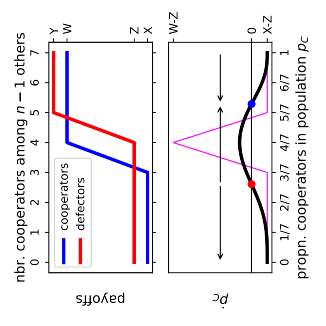
We want to go backwards in time — increase homophily — and see if cooperation can invade.

Results

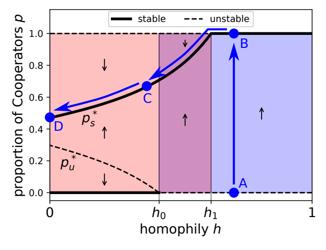

nbr. cooperators among $n-1$ others
0 1 2 3 4 5 6 7
payoffs
cooperators
defectors
Y
W
Z
X
$\dot{p}_C$
W-Z
0
X-Z
0 1/7 2/7 3/7 4/7 5/7 6/7 1
propn. cooperators in population p_C


stable
unstable
proportion of Cooperators p
1.0
0.8
0.6
0.4
0.2
0.0
B
C
D
A
p_s^*
p_u^*
0
h_0
h_1
1
homophily h

Results



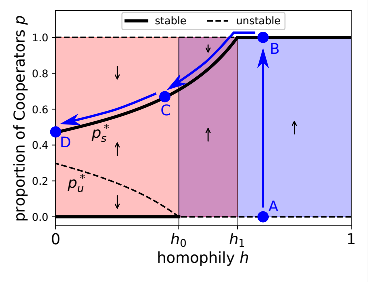
nbr. cooperators among $n-1$ others
payoffs
cooperators
defectors
Y
W
Z
X
W-Z
0
X-Z
$\dot{p}_C$
propn. cooperators in population p_C
0
1/7
2/7
3/7
4/7
5/7
6/7
1

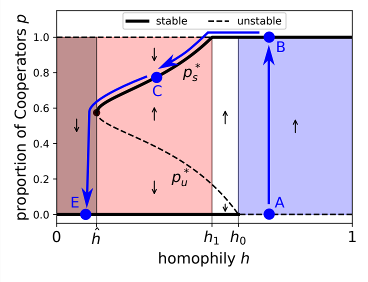


stable
unstable
proportion of Cooperators p
homophily h
p_s^*
p_u^*
A
B
C
D
0
h_0
h_1
1
0.0
0.2
0.4
0.6
0.8
1.0

Results

- Cooperation cannot invade a threshold game
  - Also true for sigmoid games in general (Peña *et al.*, 2014)
- Can arise through historical homophily





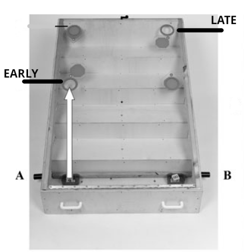
- For cooperation to persist, either:
  - Parameters such that it can be sustained in a well-mixed population
  - Some degree of homophily maintained

Many discrete strategies

- So far, 2 strategies; natural extension, *m* strategies
- Discrete strategies:
  - I could have modelled cooperate and defect as *degree* of cooperation — one continuous strategy
  - However, some strategies are naturally discrete
    - e.g., conditioning on the actions of others
  - Shared intentionality (Genty et al., 2020; Tomasello, 2020):
    - form a collective 'we' with a jointly optimised goal
    - make a joint commitment (!?) to the goal
    - coordinate our actions towards achieving it

Commitment

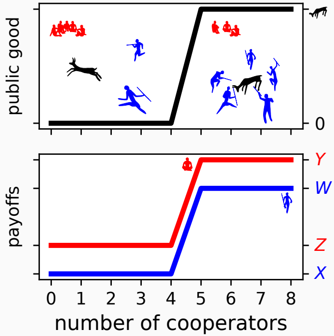
- Commitment is a norm: one should do what one promised
- Commitment distinguishes us from other apes
    - Experimental situations where one individual receives their reward early
    - 3.5-year-old children continue contributing until their partner also gets their reward (Hamann et al., 2012)
    - chimpanzees don't distinguish between continuing to help in an existing collaboration versus starting a new one (Greenberg et al., 2010)



Collaborative Condition

Commitment and coordination

- In the threshold game, hunters are a bit stupid
  - Cooperator will run off to do the hunt by themselves
- But people don't really behave this way– they coordinate
  - If we were in this situation, we'd have a conversation
  - That's also how people behave experimentally (e.g., Van de Kragt *et al.* (1983, Am Pol Sci Rev))
- Plus, coordination improves the evolutionary prospects for cooperation!



Coordination

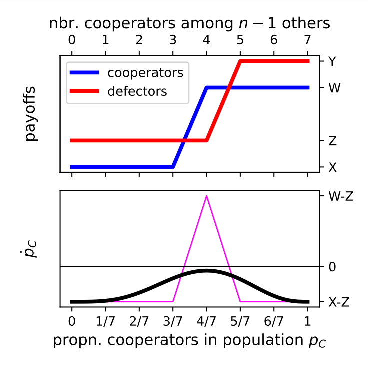
- Newton (2017 Games Econ Behav) ‘shared intentionality’ evolves under fairly general conditions in a public goods game

Jonathan Newton

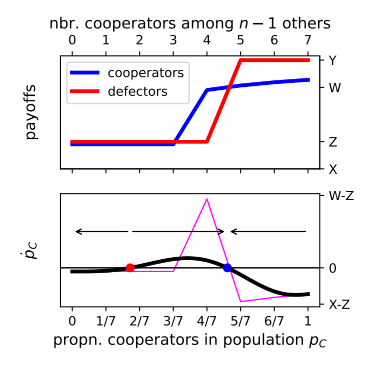
Coordination in a threshold-game example

- Extend the threshold game:
  - Coordinating cooperators draw straws to decide who will contribute
  - The ability to coordinate entails a small cognitive cost ε

old threshold game

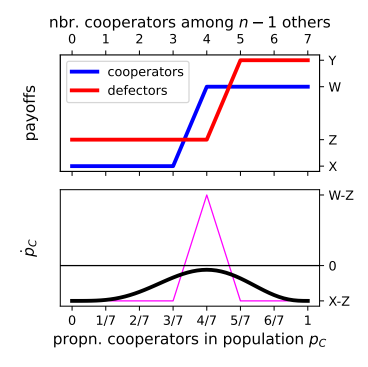


coordinated cooperation game

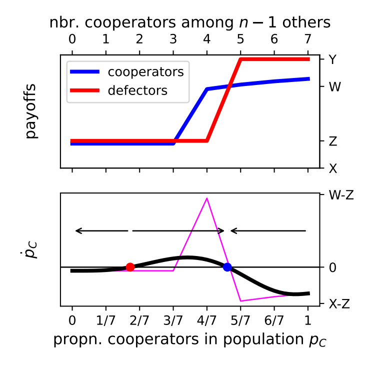


Coordination in a threshold game example

old threshold game



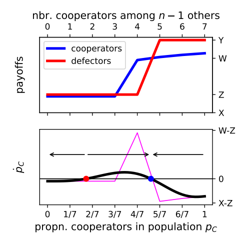
coordinated cooperation game



- Sustains cooperation where it could not otherwise be sustained
- Can't invade, but we already know we can overcome this with homophily

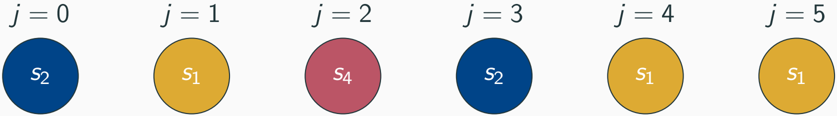
Coordination improves the prospects for coordination

- Coordination improves the prospects for coordination
- Coordination can even sustain cooperation in a linear game!
- ... wait
  - It never makes sense to contribute in the linear game
  - It's true the Defectors can't invade, but what about a type who participates in the lottery but doesn't follow through?
- Need to include another new strategy: Liars



New notation

- Subscripts: 0 = focal player; nf = nonfocal players; a = all players
- $\boldsymbol{G}$ random variable for strategy composition, takes values $\boldsymbol{g}$



- Players: $\boldsymbol{g}_0 = (0, 1, 0, 0)$, $\boldsymbol{g}_1 = (1, 0, 0, 0)$, $\boldsymbol{g}_2 = (0, 0, 0, 1), \ldots$
- Whole-group: $\boldsymbol{g}_{\mathrm{a}} = (3, 2, 0, 1)$
- Nonfocal: $\boldsymbol{g}_{\mathrm{nf}} = (3, 1, 0, 1)$
- $\boldsymbol{g}_j = \boldsymbol{e}_x$: player j plays strategy s_x (a 1 in the x-th position)

Many strategies

How does a trait change frequency over time?

George Robert Price

$$\overbrace{\Delta p_x}^{\text{dynamics of propn. of } s_x} = \mathbb{C}\mathrm{ov}[\underbrace{G_{0,x}}_{\text{focal's strategy indicator}}, \underbrace{W_0}_{\text{fitness of focal}}],$$

$$G_{0,x} = \begin{cases} 1 & \text{if focal strategy } s_x, \\ 0 & \text{otherwise.} \end{cases}$$

Many strategies

$$\underbrace{\boxed{\Delta p_x}}_{\text{dynamics of propn. of } s_x} = \mathbb{C}\mathrm{ov}[\underbrace{\boxed{G_{0,x}}}_{\text{1 if focal plays } s_x\text{; 0 otherwise}}, \underbrace{\boxed{W_0}}_{\text{fitness of focal}}],$$

$$W_0 = \underbrace{\boxed{s}}_{\text{survival probability}} + \overbrace{(1 - \boxed{s})}^{\text{vacancies}} \frac{1 + \overbrace{\boxed{\delta}}^{\text{selection strength}} \overbrace{\boxed{\Pi_0}}^{\text{focal payoff random variable}}}{1 + \boxed{\delta} \underbrace{\boxed{\overline{\pi}}}_{\text{avg. payoff in population}}}$$

Useful identity: $\mathbb{C}\mathrm{ov}[X, aY + b] = a\,\mathbb{C}\mathrm{ov}[X, Y]$

Substituting and rearranging:

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[\overbrace{\boxed{G_{0,x}}}^{\text{strategy indicator}}, \overbrace{\boxed{\Pi_0}}^{\text{focal payoff}}\right]$$

Other member accounting

focal's strategy indicator   focal payoff

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[G_{0,x} \, , \, \Pi_0 \right]$$

Payoff to the focal individual:

1 if focal plays s_i; 0 otherwise   payoff to s_i-player

$$\Pi_0 = \sum_{i=1}^{m} G_{0,i} \; \pi \left(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}} \right)$$

nonfocal strategy composition

Useful identity: $\mathbb{C}\mathrm{ov}[X, Y] = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$

$$\Delta p_x = \mathbb{E}\left[G_{0,x}\, \pi(\boldsymbol{e}_x, \boldsymbol{G}_{\mathrm{nf}}) \right] - p_x \sum_{i=1}^{m} \mathbb{E}\left[G_{0,i}\, \pi(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}}) \right]$$

nonfocal strategy composition

Other member accounting

nonfocal strategy composition

$$\Delta p_x = \mathbb{E}\left[G_{0,x}\,\pi(\boldsymbol{e}_x,\,\boxed{\boldsymbol{G}_{\mathrm{nf}}})\right] - p_x \sum_{i=1}^{m} \mathbb{E}\left[G_{0,i}\,\pi(\boldsymbol{e}_i,\,\boxed{\boldsymbol{G}_{\mathrm{nf}}})\right]$$

Let $\mathcal{G}_{\mathrm{nf}}$ be the set of all strat. compositions $\boldsymbol{g}_{\mathrm{nf}}$. Then expectations:

$$\begin{aligned}
\mathbb{E}\left[G_{0,i}\pi(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}})\right] &= \sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \mathbb{P}[\boldsymbol{G}_0 = \boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}}] \\
&= \sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \underbrace{\mathbb{P}[\boldsymbol{G}_0 = \boldsymbol{e}_i]}_{p_i}\, \mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i] \\
&= p_i \underbrace{\sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]}_{\overline{\pi}_i}
\end{aligned}$$

Recovered replicator eqn: $\Delta p_x \propto p_x \left(\overline{\pi}_x - \sum_{i=1}^{m} p_i \overline{\pi}_i\right) = p_x \left(\overline{\pi}_x - \overline{\pi}\right)$.

But $\mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]$ is not obvious: 

Whole-group accounting

Idea: draw a group at random, then draw a focal individual.

strategy indicator   focal payoff

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[G_{0,x} \, , \, \Pi_0 \right]$$

This time, focus on the whole-group distribution.

new payoff fnc wrt whole-group strategy composition

$$\Pi_0 = \sum_{i=1}^{m} G_{0,i} \; \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{a}})$$

Using a similar method to before involving covariance identities and re-arranging, we obtain

$$\Delta p_x = \sum_{\boldsymbol{g}_{\mathrm{a}} \in \mathcal{G}_{\mathrm{a}}} \left(\frac{g_{\mathrm{a},x}}{n} \hat{\pi}(\boldsymbol{e}_x, \boldsymbol{g}_{\mathrm{a}}) - p_x \sum_{i=1}^{m} \frac{g_{\mathrm{a},i}}{n} \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{a}}) \right) \mathbb{P}[\boldsymbol{G}_{\mathrm{a}} = \boldsymbol{g}_{\mathrm{a}}]$$

prob. of whole-group strategy composition

Prob. of whole-group strategy composition, $\mathbb{P}[\boldsymbol{G}_{\mathbf{a}} = \boldsymbol{g}_{\mathbf{a}}] =$?

colours = strategies; vertical lines divide families

$\mathbb{P}[\boldsymbol{G}_a =$  $]$

Prob. of whole-group strategy composition, $\mathbb{P}[G_{\mathrm{a}} = g_{\mathrm{a}}] =$?

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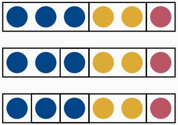
Prob. of whole-group strategy composition, $\mathbb{P}[G_{\mathrm{a}} = g_{\mathrm{a}}] =?$

colours = strategies; vertical lines divide families

$\mathbb{P}[\boldsymbol{G}_a =$  $]$

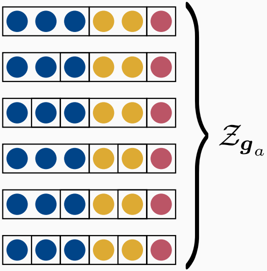


Prob. of whole-group strategy composition, $\mathbb{P}[G_{\mathrm{a}} = g_{\mathrm{a}}] =$?

colours = strategies; vertical lines divide families

$\mathbb{P}[\boldsymbol{G}_a =$  $]$

$$= \sum_{\boldsymbol{z} \in \mathcal{Z}_{\boldsymbol{g}_a}} \mathbb{P}[\boldsymbol{Z} = \boldsymbol{z}]$$

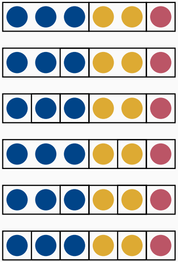


Prob. of whole-group strategy composition, $\mathbb{P}[G_\mathrm{a} = g_\mathrm{a}] = ?$

colours = strategies; vertical lines divide families

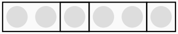
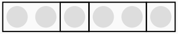
$\mathbb{P}[\boldsymbol{G}_a =$  $]$

$= \sum_{\boldsymbol{z} \in \mathcal{Z}_{\boldsymbol{g}_a}} \mathbb{P}[\boldsymbol{Z} = \boldsymbol{z}]$



$\mathcal{Z}_{\boldsymbol{g}_a}$

$\mathbb{P}[\boldsymbol{Z} =$  $]$

$= \mathbb{P}[$  $] \cdot \mathbb{P}[$  $\mid$  $]$

$F_{\boldsymbol{y}}$ $\qquad$ $C(\boldsymbol{z}) \cdot A(\boldsymbol{z}, \boldsymbol{p})$

Probability of strategywise family-size distribution:

get from homophilic group-formation model

$$\mathbb{P}[\boldsymbol{G}_\mathrm{a} = \boldsymbol{g}_\mathrm{a}] = \sum_{\boldsymbol{z} \in \mathcal{Z}_{\boldsymbol{g}_\mathrm{a}}} F_{\boldsymbol{y}} \; C(\boldsymbol{z}) \; A(\boldsymbol{z}, \boldsymbol{p})$$

prob. families' strategies

count of multiset permutations

nbr. families pursuing strategy s_i

$$A(\boldsymbol{z}, \boldsymbol{p}) = \prod_{i=1}^{m} p_i^{\|\boldsymbol{z}_i\|}$$

Analogous to the power terms in 2-strategy game, e.g.,

$$\rho_2 = \theta_{2\to1} \; p_1 + \theta_{2\to2} \; p_1^2$$

Whole-group accounting

Bringing it all together:

$$\Delta p_x \propto \underbrace{\sum_{\boldsymbol{g}_{\mathrm{a}} \in \mathcal{G}_{\mathrm{a}}}}_{\text{sum over group strategy compositions}} \left(\overbrace{\frac{g_{\mathrm{a},x}}{n}}^{\text{prob. focal pursues } s_x} \hat{\pi}(\boldsymbol{e}_x, \boldsymbol{g}_{\mathrm{a}}) - p_x \sum_{i=1}^{m} \frac{g_{\mathrm{a},i}}{n} \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{a}}) \right) \left(\overbrace{\sum_{\boldsymbol{z} \in \mathcal{Z}_{\boldsymbol{g}_{\mathrm{a}}}}}^{\text{over strategywise family-sizes}} C(\boldsymbol{z})\, A(\boldsymbol{z}, \boldsymbol{p})\, F_{\mathrm{sum}(\boldsymbol{z})} \right)$$

- Not as intuitive as the traditional replicator equation
  - $\Delta p_x \propto p_x \left(\overline{\pi}_x - \overline{\pi} \right)$
- Might be useful from computational perspective because we've split homophily calculations off from strategy identity
- Now it's clearer how to calculate $\mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]$

Aside: Payoff-matrix transformation example (2 players)

- Idea: transform payoffs so they take into account homophily
- Well-mixed game: $\dot{p}_i = p_i(\overline{\pi}_i - \overline{\pi}) = p_i((A\boldsymbol{p})_i - \boldsymbol{p}^T A \boldsymbol{p})$, where $a_{i,j} = \pi(\boldsymbol{e}_i, \boldsymbol{e}_j)$,

$$\overline{\boldsymbol{\pi}} = \begin{pmatrix} \overline{\pi}_1 \\ \vdots \\ \overline{\pi}_m \end{pmatrix} = \begin{array}{c c} & \xrightarrow{\text{nonfocal's strategy}} \\ \text{focal's strat.}\downarrow & \begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & & \vdots \\ a_{m,1} & \dots & a_{m,m} \end{pmatrix} \end{array} \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix} = \begin{pmatrix} a_{1,1}p_1 + \dots + a_{1,m}p_m \\ \vdots \\ a_{m,1}p_1 + \dots + a_{m,m}p_m \end{pmatrix}$$

- Now with homophily, dyadic relatedness $\theta_{2\to1}$

$$B = \underbrace{(1-\theta_{2\to1}) \begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & & \vdots \\ a_{m,1} & \dots & a_{m,m} \end{pmatrix}}_{i \text{ matched with random with prob. } 1-\theta_{2\to1}} + \underbrace{\theta_{2\to1} \begin{pmatrix} a_{1,1} & \dots & a_{1,1} \\ \vdots & & \vdots \\ a_{m,m} & \dots & a_{m,m} \end{pmatrix}}_{i \text{ matched with } i \text{ with prob. } \theta_{2\to1}}$$

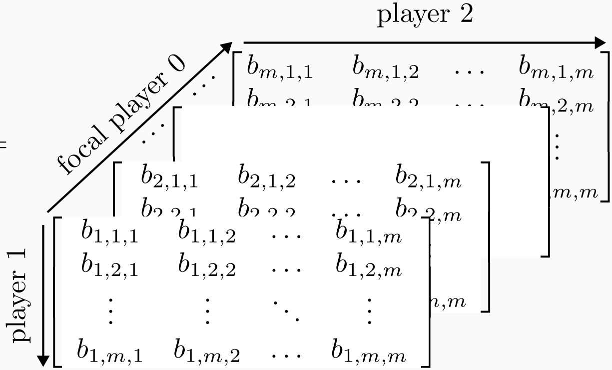
- Dynamics of A with homophily $\equiv$ dynamics of B well-mixed

$$\dot{p}_i = p_i((B\boldsymbol{p})_i - \boldsymbol{p}^T B \boldsymbol{p})$$

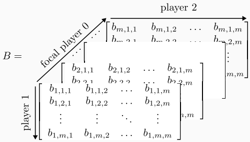
Aside: Payoff transformation n players

Seeking a solution to:

$B =$



Payoff transformation n players



$$b_{\boldsymbol{u}} = \sum_{\boldsymbol{q} \vdash n} \boxed{F_{\boldsymbol{q}}} \left(\sum_{q_0 \in \boldsymbol{q}} \frac{q_0}{n |\mathcal{J}_{q_0, \boldsymbol{q}}|} \left(\sum_{\boldsymbol{j} \in \mathcal{J}_{q_0, \boldsymbol{q}}} a_{\boldsymbol{u}_{\boldsymbol{j}}} \right) \right)$$

get from group-formation model

Code to calculate it on Github:

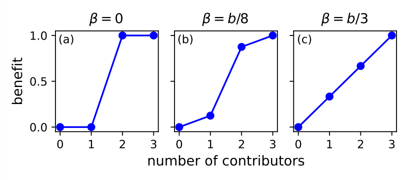
1. Numerically: `TransmatBase` class `functions/transmat_base.py`.
2. Symbolically: `functions/symbolic_transformed.py`.

But why would you want to do this?

- B is expensive to calculate, but matrix multiplication is optimised, can be worth the trade-off when finding steady states
- Use maths from well-mixed case, e.g., Jorge Peña's analysis techniques (example in appendix)

Coordinated cooperation

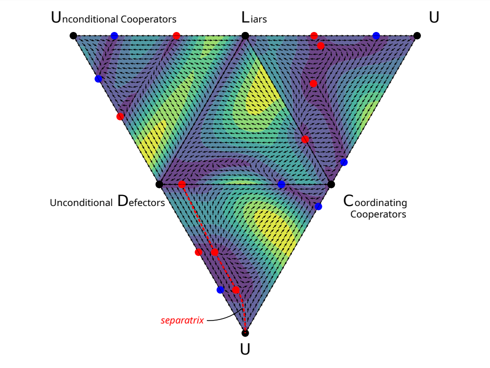
- Game with 4 strategies:
  1. D: unconditional Defector, never contributes
  2. C: Coordinating cooperator, hold lottery, follow through if chosen
     - Nbr. contributors $\tau =$ threshold, or inflection point if sigmoid
  3. L: Liar, participate in lottery, never contributes
  4. U: Unconditional cooperator, always contributes
- C and L pay cognitive cost ε regardless of game outcome
- U and C pay contribution cost c if contributing
- Explore the range from linear to threshold game



Example 3-player - symbolic analysis

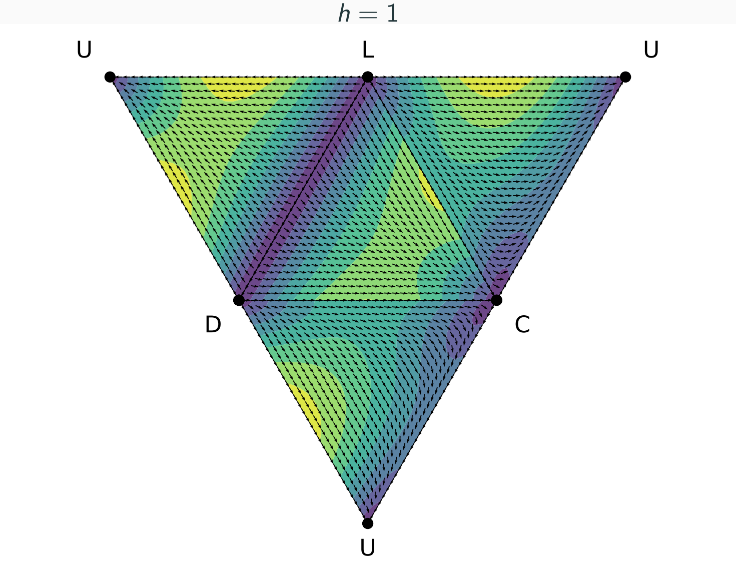
Example 8-player - numerical analysis

How to read results



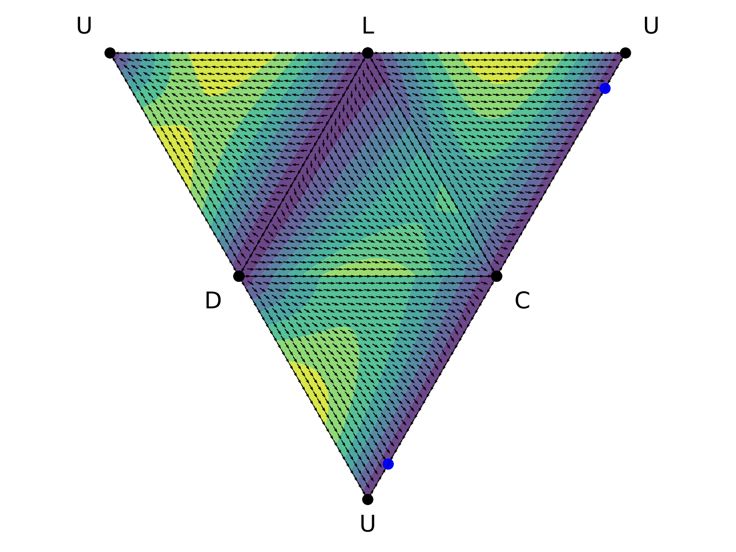
- Evolutionary dynamics for a given homophily level h
  - Dynamics inside a triangular pyramid
  - The points represent a population with just one strategy, lines 2 strategies, triangles 3
  - Blue points are stable in that dimension, red points unstable

Results 1: fairly nonlinear benefits function

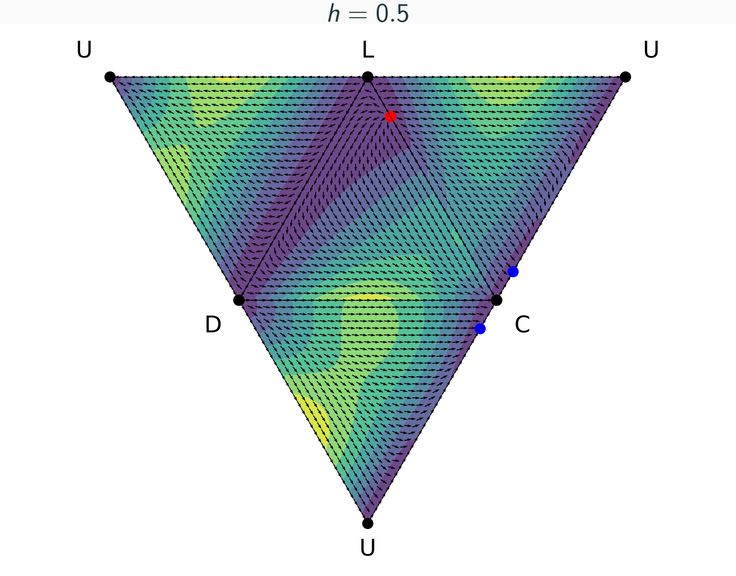


Results 1: fairly nonlinear benefits function

$h = 0.645$

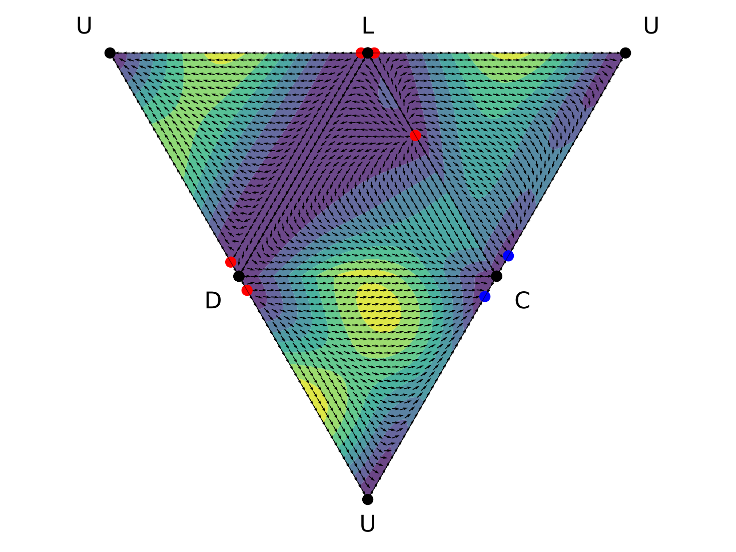


Results 1: fairly nonlinear benefits function



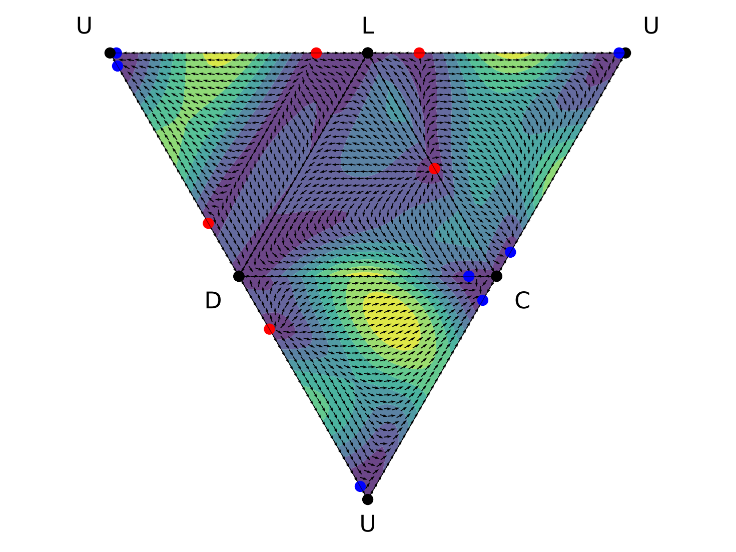
Results 1: fairly nonlinear benefits function

$h = 0.4$

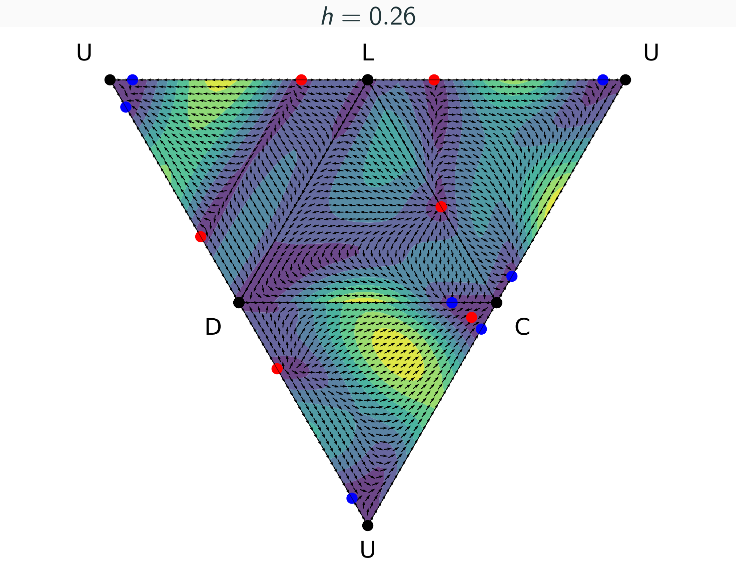


Results 1: fairly nonlinear benefits function

$h = 0.3$

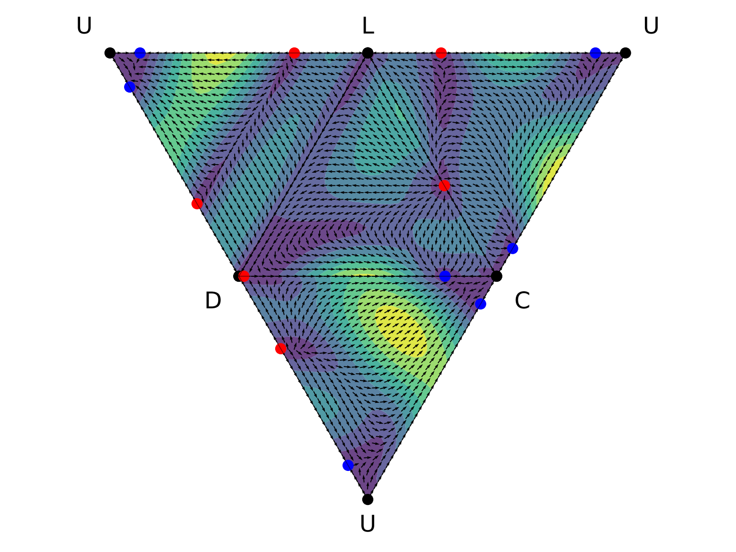


Results 1: fairly nonlinear benefits function

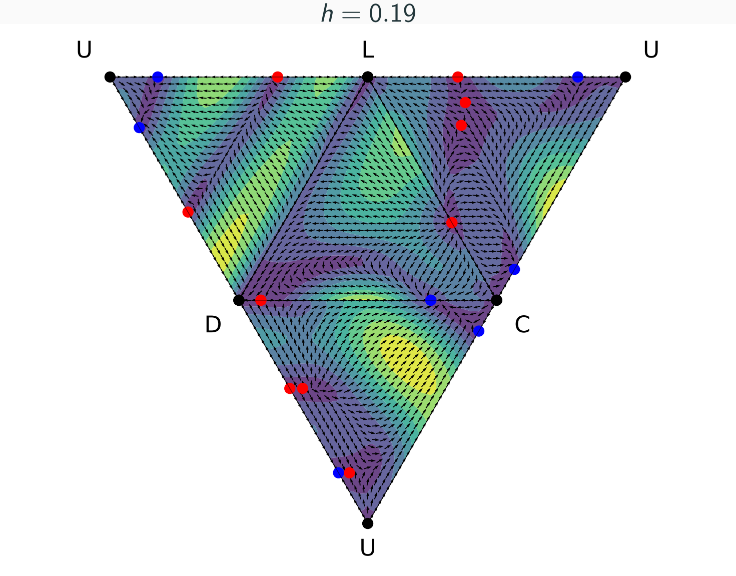


Results 1: fairly nonlinear benefits function

$h = 0.24$

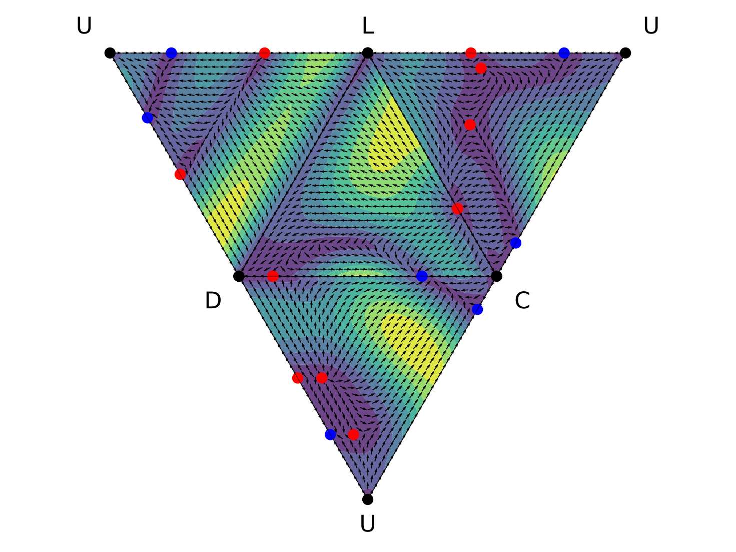


Results 1: fairly nonlinear benefits function



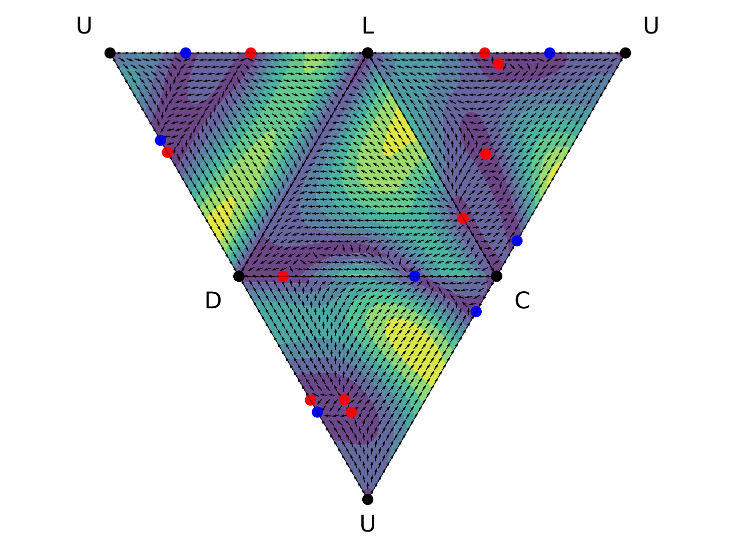
Results 1: fairly nonlinear benefits function

$h = 0.15$



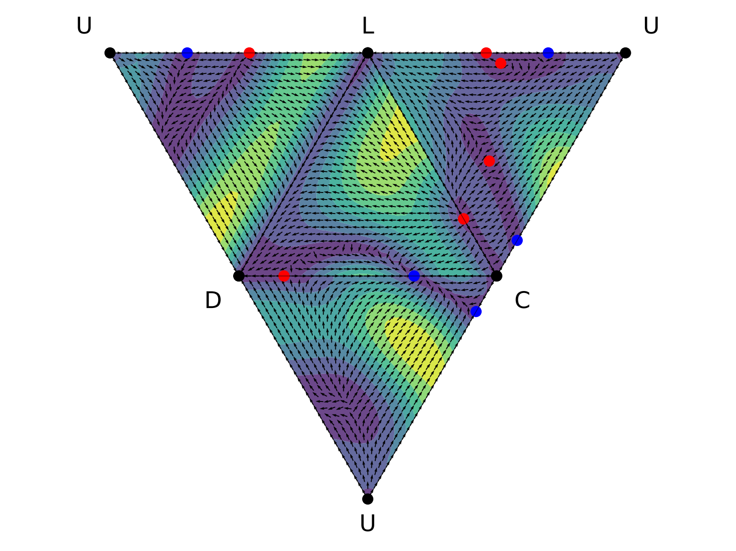
Results 1: fairly nonlinear benefits function

$h = 0.11$



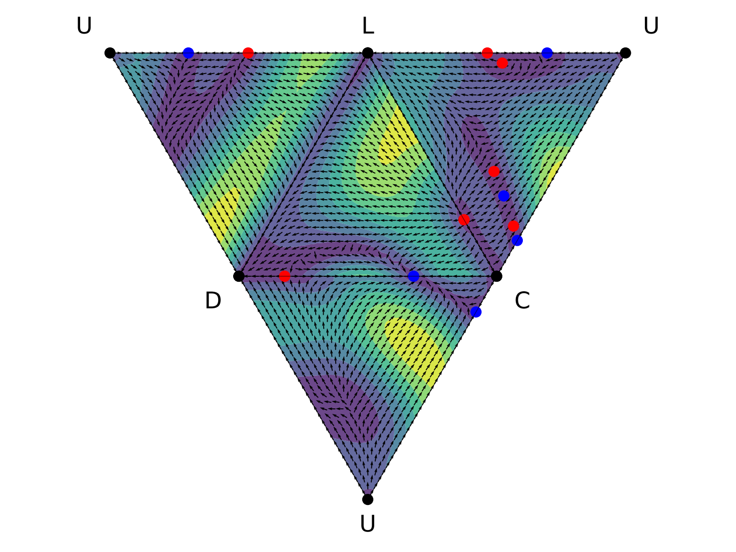
Results 1: fairly nonlinear benefits function

$h = 0.106$

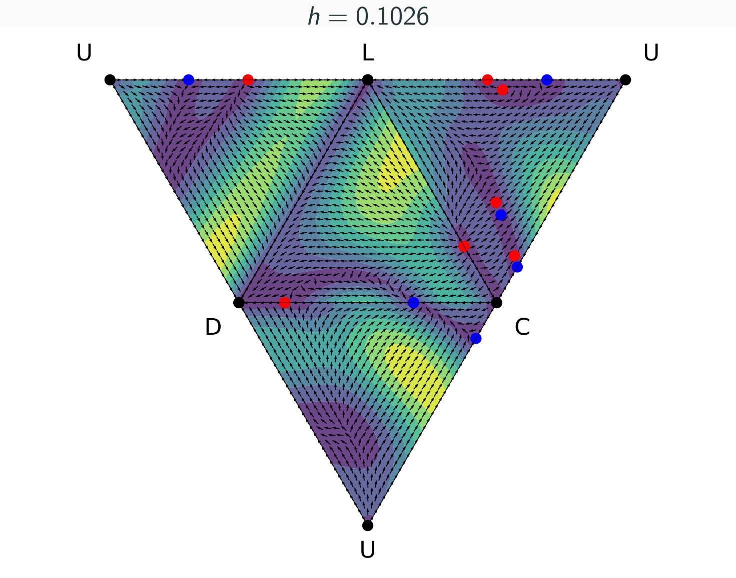


Results 1: fairly nonlinear benefits function

$h = 0.1031$

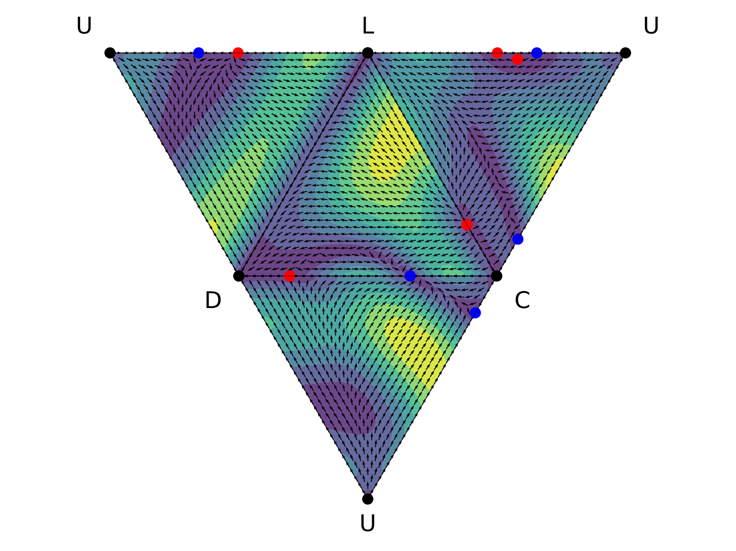


Results 1: fairly nonlinear benefits function

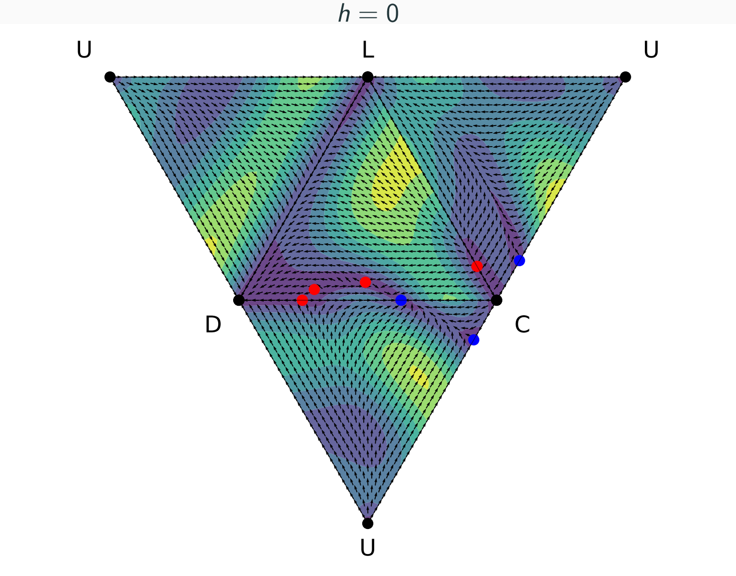


Results 1: fairly nonlinear benefits function

$h = 0.08$

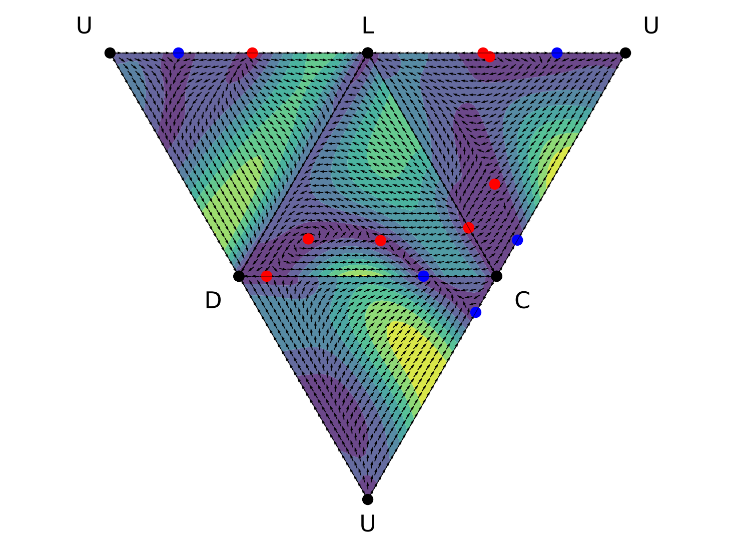


Results 1: fairly nonlinear benefits function



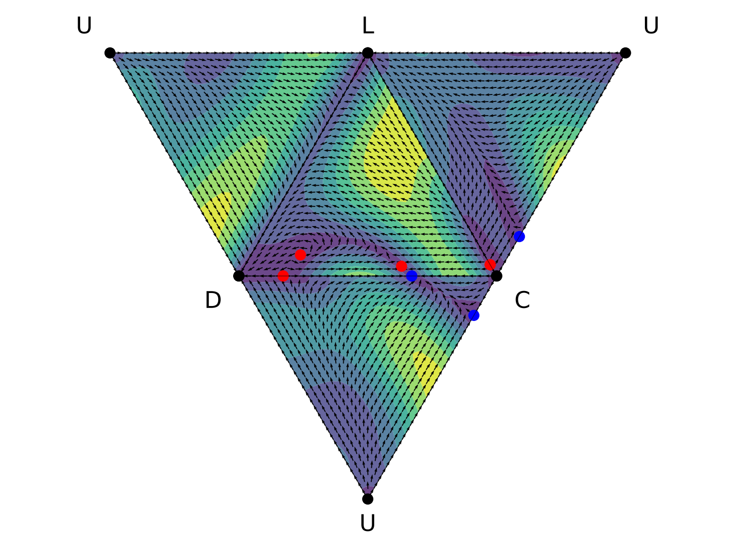
Results 2: more linear benefits function

$h = 0.18$



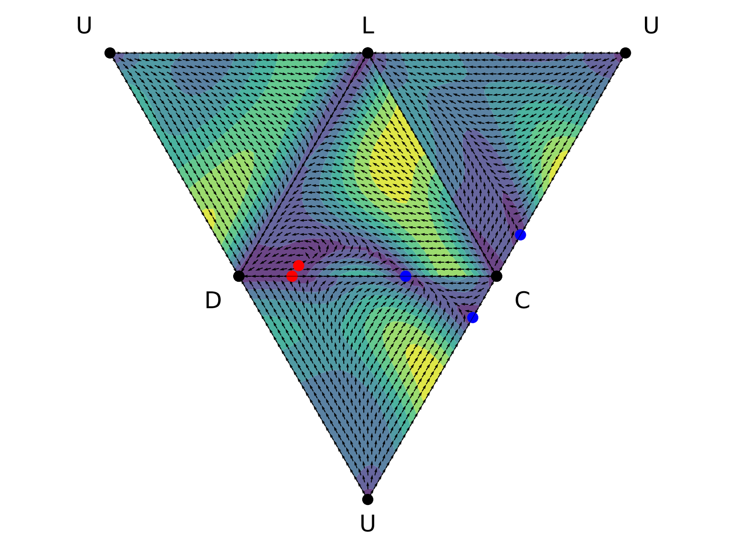
Results 2: more linear benefits function

$h = 0.12$



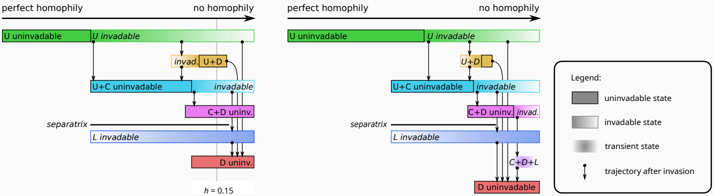
Results 2: more linear benefits function

$h = 0.08$



Results summary

(a) more nonlinear (b) more linear



- Results:
  1. Coordination allows cooperation where it cannot otherwise persist
  2. First arose through kin selection
  3. To persist in modern scenario, either:
     - Keep some degree of homophily in modern interactions
     - Payoff function non-linear enough

Talk summary

- Mathematical framework combines discrete-strategy group games with kin selection (or ‘matching rules’)
- Investigate: how cooperation first arose, and how it can persist

github.com/nadiahpk

nadiah.org