The evolution of human cooperation: homophily, non-additive benefits, and higher-order relatedness

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Acknowledgements

Country:

I acknowledge the Turrbal and Yugara people and as the owners of this land. I pay respect to their Elders, past and present, and recognise this land has always been a place of teaching, learning, and research.

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Humans are cooperative

- Introspection — ‘I am a moral being’
- Humans are a highly cooperative speces
- Eusocial insects– relatives
- Humans– non-relatives, strangers

Why cooperate?

- Help others at a cost to yourself?
    - Violates evolutionary logic
    - Though tricky to think about costs and benefits
- Problem in its pure form – Game Theory

Prisoner's Dilemma

		player 2	
		cooperate	defect
player 1	cooperate	3, 3	0, 4
	defect	4, 0	1, 1

- Implications:
    - You should never cooperate
    - A cooperative type will never evolve
- Yet people do
    - Anecdotally
    - Experimentally

Scope of this talk

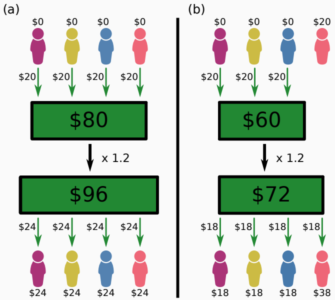
		player 2	
		cooperate	defect
player 1	cooperate	3, 3	0, 4
	defect	4, 0	1, 1

- Two big ideas I won't be covering today:
  1. Repeated interaction
  2. Reputation
- I will assume interactions are:
  1. One-shot
  2. Anonymous
- What I'll be talking about instead:
  1. Kin selection
  2. Nonlinear payoffs (or economies of scale)

Public goods game example

- Group-game version of Prisoner's Dilemma

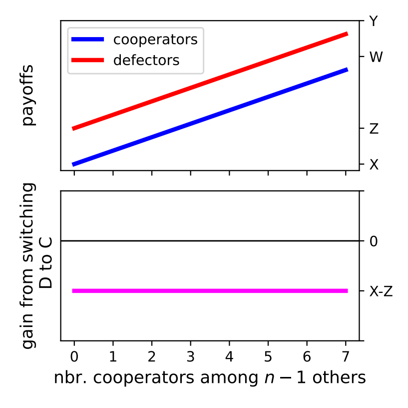
- Example:
    1. Public good that multiplies contributions by 1.2
    2. Everyone contributes $\rightarrow$ maximise total payoffs
    3. However, not contributing maximises individual payoff



- Never makes sense to contribute
    - Returns are split equally
    - Marginal per-capita return $= 1.2/4 = 0.3 < 1$
        - 30c return for every $1 contributed

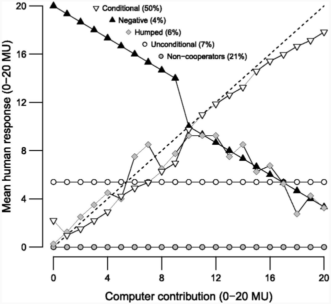
Linear public goods game

- Example was a linear public goods game
  - Benefit increases at constant rate with nbr. cooperators
  - No matter how many cooperators in the group, always lose by switching C to D
- *n*-player generalisation of PD



How do people really behave in linear PGGs?

- Contribute + depends on contribution of others; why?
- Computer opponent, so not for fairness reasons
- Seem to sincerely believe this is payoff maximising
    - Iterated game → learn not to contribute
    - But it's interesting that's their default



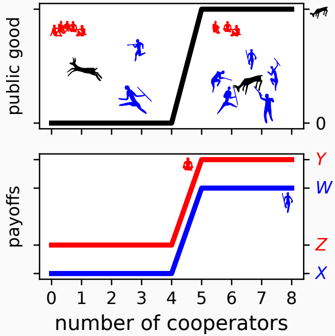
Burton-Chellew *et al.* (2016, PNAS)

- Deeply unnatural scenario: one-shot interaction with strangers
- Previous work focuses on two 'mistakes': iterated game, reputation concerns
- I want to focus on another 'mistake': mistaking the linear game for nonlinear

Nonlinear public goods game

- Claim: sigmoid-shaped benefit functions particularly relevant to our early history

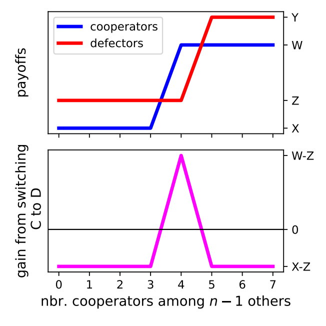
- Defectors always get higher payoffs than Cooperators



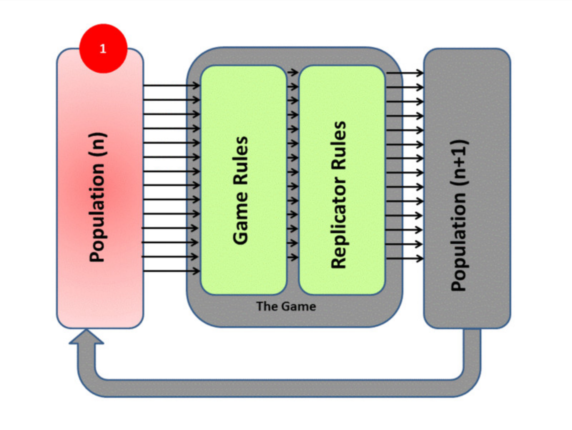
Nonlinear public goods game

- Claim: sigmoid-shaped benefit functions particularly relevant to our early history

- Defectors always get higher payoffs than Cooperators
- However, if you are the pivotal player, you should cooperate
- Evolutionary perspective:
  - if cooperates rare, don't cooperate
  - if cooperate common, might get higher payoffs if you also cooperate



Replicator dynamics overview



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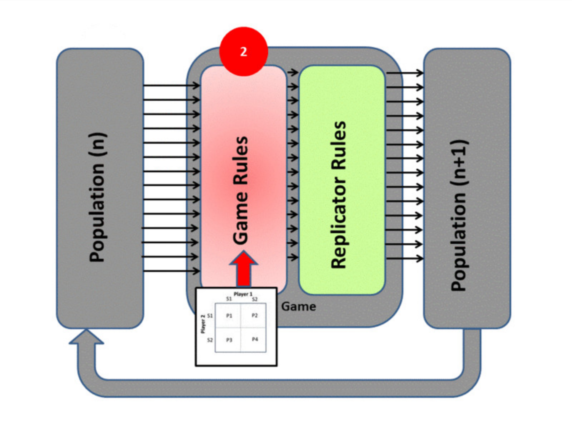
Replicator dynamics overview



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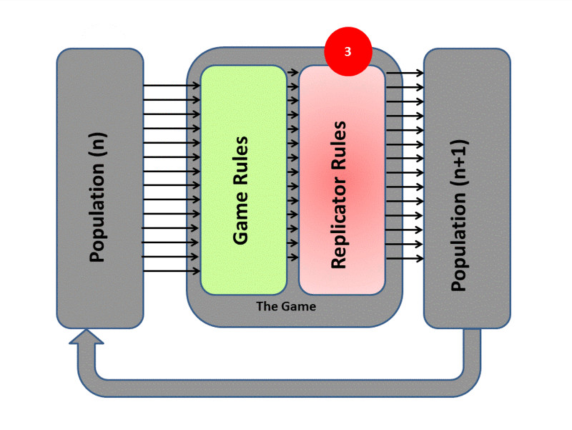
Replicator dynamics overview



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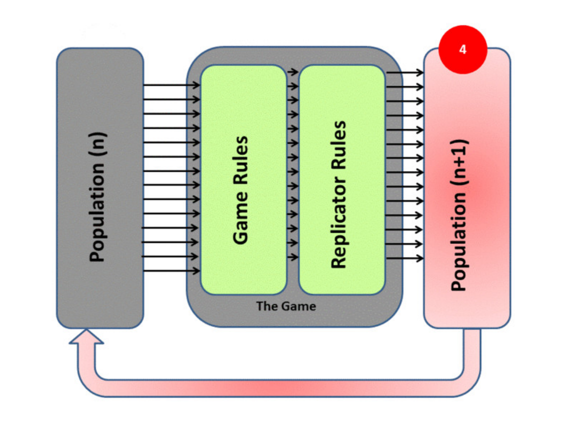
Replicator dynamics overview



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Replicator dynamics maths (1)

Change in number of x-strategists in population:

$$\dot{N}_x = \underbrace{N_x}_{\text{nbr. } x\text{-strategists}} \left(\overbrace{\beta}^{\text{background effects on fitness}} + \underbrace{\bar{\pi}_x}_{\text{expected payoff to } x\text{-strategists}} \right)$$

Change in proportion of x-strategists in population:

$$\dot{p}_x = \underbrace{p_x}_{\text{proportion of } x\text{-strategists}} \left(\overbrace{\bar{\pi}_x}^{\text{expected payoff to } x\text{-strategists}} - \overbrace{\underbrace{\sum_{i=1}^{m} p_i \bar{\pi}_i}_{\text{expected payoff in population}}}^{m \text{ is nbr. strategies}} \right)$$

Replicator dynamics maths (2)

$$\dot{p}_x = \underbrace{p_x}_{\text{proportion of } x\text{-strategists}} \left(\overbrace{\bar{\pi}_x}^{\text{expected payoff to } x\text{-strategists}} - \underbrace{\overbrace{\sum_{i=1}^{m} p_i \bar{\pi}_i}^{m \text{ is nbr. strategies}}}_{\text{expected payoff in population}} \right)$$

New notation: strategy indicator vector $\boldsymbol{e}_x = (0, \ldots, 0, \underbrace{1}_{x-\text{th}}, 0, \ldots, 0)$

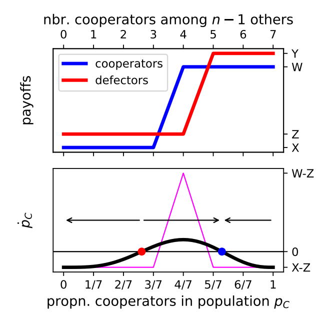
For 2-strategy games:

$$\begin{aligned} \bar{\pi}_x &= \sum_{g=0}^{n-1} \overbrace{\pi(e_x, g_{\text{nf}})}^{\text{payoff}} \overbrace{\mathbb{P}[G_{\text{nf}} = g_{\text{nf}}]}^{\text{probability } g_{\text{nf}} \text{ non-focals are cooperators}}, \\ &= \sum_{g_{\text{nf}}=0}^{n-1} \pi(e_x, g_{\text{nf}}) \underbrace{\binom{n-1}{g_{\text{nf}}} p_x^{g_{\text{nf}}} (1-p_x)^{n-1-g_{\text{nf}}}}_{\text{binomial}} \end{aligned}$$

Cooperation can be sustained

Two main known results:

1. Cooperation can be sustained
    - Do people 'mistake' linear games for a nonlinear ones?
2. But cooperation cannot invade
    - Imagine a small nbr. of cooperators invading defectors...



note: $\dot{p}_C$ is a Bezier curve (Jorge Peña's papers)

But what if, instead of randomly formed groups, groups tend to form of family members?

Homophilic group formation

Change in proportion of x-strategists in population:

$$\dot{p}_x = \underbrace{p_x}_{\text{proportion of } x\text{-strategists}} \left(\overbrace{\bar{\pi}_x}^{\text{expected payoff to } x\text{-strategists}} - \underbrace{\sum_{i=1}^{m} p_i \bar{\pi}_i}_{\text{expected payoff in population}} \right)$$

where

$$\bar{\pi}_x = \sum_{g_{\mathrm{nf}}=0}^{n-1} \pi(e_x, g_{\mathrm{nf}}) \underbrace{\mathbb{P}[G_{\mathrm{nf}} = g_{\mathrm{nf}} \mid G_0 = e_x]}_{\text{nonfocal strategy distribution depends on focal's strategy}}$$

Some notation

Let ρ_ℓ be the probability that ℓ players sampled without replacement from the group have strategy 1.

prob. ℓ sampled have m common ancestors

$$\rho_\ell = \sum_{m=1}^{\ell} \boxed{\theta_{\ell \to m}} \; \boxed{p_1}^{m}$$

propn. strategy-1 in populatn

Examples:

- Sample 1 individual: $\rho_1 = p_1$
- Sample 2 individuals:

prob. same ancestor

prob. strategy-1

$$\rho_2 = \boxed{\theta_{2 \to 1}} \; \boxed{p_1} + \boxed{\theta_{2 \to 2}} \; \boxed{p_1^2}$$

prob. two ancestors

prob. both strategy-1

Hisashi's equation

Ohtsuki (2014, Phil Trans R Soc):

$$\dot{p}_1 = \sum_{g_{\mathrm{nf}}=0}^{n-1} \sum_{\ell=g_{\mathrm{nf}}}^{n-1} (-1)^{\ell-g_{\mathrm{nf}}} \binom{\ell}{g_{\mathrm{nf}}} \binom{n-1}{\ell}$$

relatedness terms

$$\Big[(1 - \rho_1) \rho_{\ell+1} \pi(e_1, g_{\mathrm{nf}}) - \rho_1 (\rho_\ell - \rho_{\ell+1}) \pi(e_2, g_{\mathrm{nf}}) \Big]$$

payoff terms

Linear PGG is a function of dyadic relatedness only

- If the PGG is linear, only need dyadic relatedness

$$\dot{p}_1 = f(\underbrace{\overbrace{\theta_{2\to 1}}^{\text{dyadic relatedness}}}_{\text{(Hamilton's } r\text{)}})$$

because:

  - Payoff function in n-player game can be written as a sum of payoffs in 2-player games

$$\overbrace{\pi^{(n)}(e_x, g_{\mathrm{nf}}^{(n)})}^{\text{payoff in } n\text{-player game}} \equiv \sum_{g_{\mathrm{nf}}^{(2)}} \overbrace{\pi^{(2)}(e_x, g_{\mathrm{nf}}^{(2)})}^{\text{payoff in 2-player game}}$$

  - So the n-player linear game is equivalent to the sum of 2-player games
  - So only dyadic relatedness is needed to calculate expected payoff
- But if the payoff function is nonlinear, higher-order relatedness coefficients are needed

How do we calculate the higher-order relatedness terms?

From group family-size distribution. For example:

	partition	$\theta_{2\to1}$	explanation
$F_{[4]}$		1	Any 2 will have a common ancestor.
$F_{[3,1]}$		$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$	Both must be blue (family size 3).
$F_{[2,2]}$		$1 \times \frac{1}{3} = \frac{1}{3}$	Choose any, then its 1 family member.
$F_{[2,1,1]}$		$\frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$	Only possible in the partition of 2.
$F_{[1,1,1,1]}$		0	Not possible.

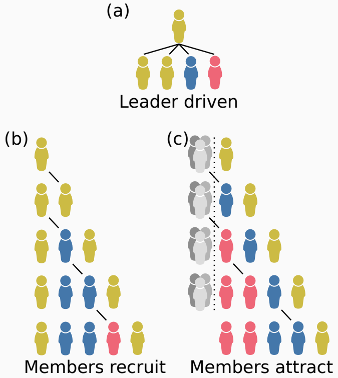
So if we can calculate the $F_{\boldsymbol{q}}$, we can calculate the needed $\theta_{l\to m}$

$h =:$ genetic homophily

(a) Leader driven:

- The leader is chosen at random from the population.
- Leader recruits/attracts kin with probability h and nonkin with probability $1 - h$.
- Group family size distribution

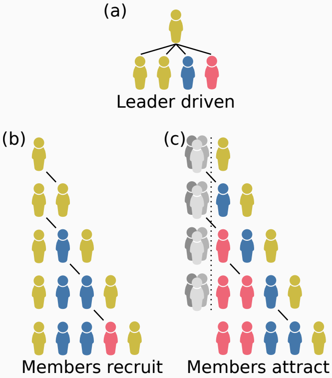
$$F_{[\ell,1,\ldots,1]} = \binom{n-1}{\ell-1} h^{\ell-1}(1-h)^{n-\ell}.$$



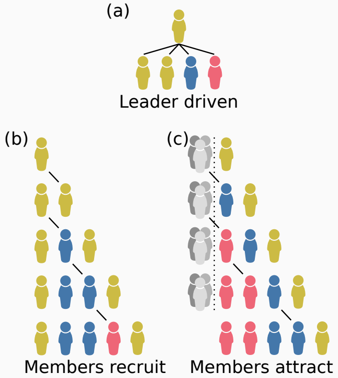
$h =:$ genetic homophily

(b) Members recruit:

- First member is chosen at random.
- Current group members have an equal chance to recruit the next member.
- Member recruits kin with probability h and nonkin with probability $1 - h$.
- Equation in Kristensen *et al.* (2022)



$h =:$ genetic homophily

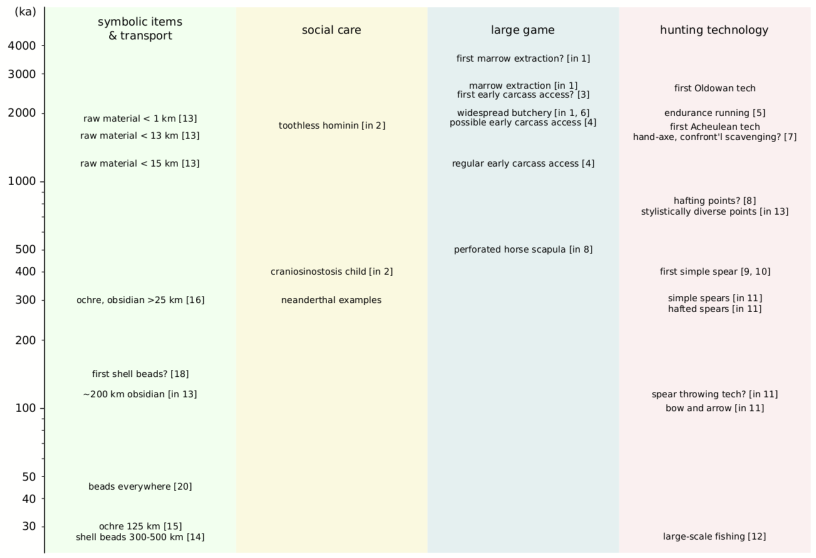


(c) Members attract:

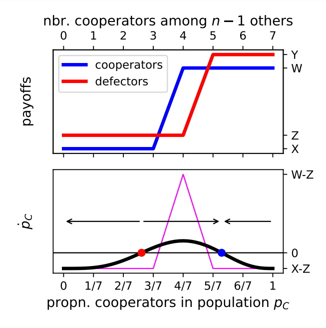
- First member is chosen at random.
- Current group members have equal weighting 1 of attracting a new member who is kin,
- but nonkin members are also attracted to the group itself with collective weighting $\alpha \in [0, \infty)$.
- See Ewens' formula (Ewen 1972) for equation.

NOTE: can be interpreted in terms of 'matching rules', i.e., strategy homophily

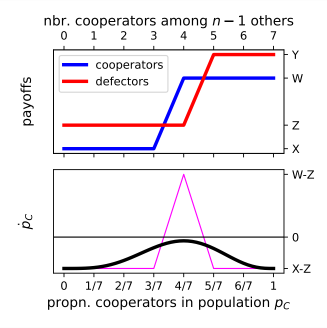
Homophily has decreased over time



Results



nbr. cooperators among $n-1$ others
0 1 2 3 4 5 6 7
cooperators
defectors
payoffs
Y
W
Z
X
$\dot{p}_C$
W-Z
0
X-Z
0 1/7 2/7 3/7 4/7 5/7 6/7 1
propn. cooperators in population p_C



nbr. cooperators among $n-1$ others
0 1 2 3 4 5 6 7
cooperators
defectors
payoffs
Y
W
Z
X
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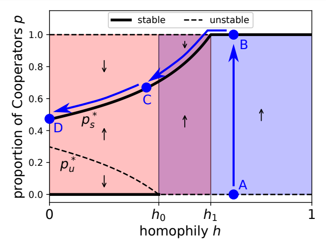
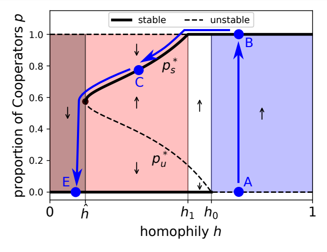
W-Z
0
X-Z
0 1/7 2/7 3/7 4/7 5/7 6/7 1
propn. cooperators in population p_C



stable
unstable
proportion of Cooperators p
1.0
0.8
0.6
0.4
0.2
0.0
B
C
D
A
p_s^*
p_u^*
0
h_0
h_1
1
homophily h



stable
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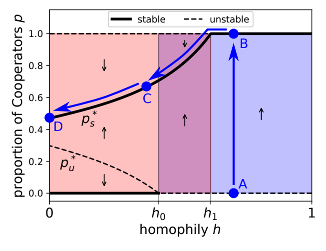
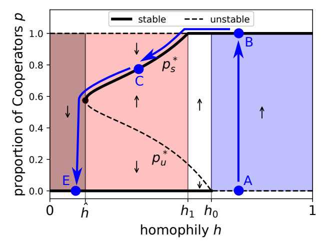
proportion of Cooperators p
1.0
0.8
0.6
0.4
0.2
0.0
B
C
E
A
p_s^*
p_u^*
0
$\hat{h}$
h_1
h_0
1
homophily h

Mini summary





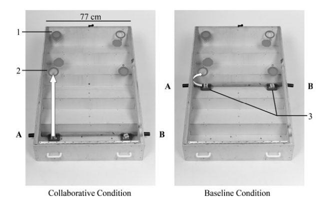
- Cooperation cannot invade a threshold game
    - Also true for sigmoid games (Peña *et al.* (2014) J Theor Biol)
- Can arise through historical homophily
- To persist, either:
    - Parameters such that it can be sustained in a well-mixed population
    - Some degree of homophily maintained

Many discrete strategies

- So far, 2 strategies; natural extension, *m* strategies
- Discrete strategies:
  - I could have modelled cooperate and defect as *degree* of cooperation — one continuous strategy
  - However, some strategies are naturally discrete
    - e.g., conditioning contribution on the existence of a punishment institution (Garcia and De Monte, 2013)
    - e.g., conditioning contribution on the contributions of others (Takezawa and Price, 2010)
    - typically involve some conceptual 'leap'
  - Shared intentionality (Genty et al., 2020; Tomasello, 2020):
    - form a collective 'we' with a jointly optimised goal
    - make a joint commitment (!?) to the goal
    - coordinate our actions towards achieving it

Commitment

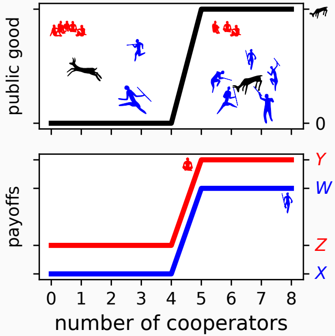
- Commitment is a norm:
  - One should do what one promised
    - Kerr and Kaufman-Gilliland (1994, J Pers Soc Psychol)
- Commitment distinguishes us from other apes
  - In a experimental situation where one individual receives their reward early, 3.5-year-old children will continue contributing until their partner also receives their reward (Hamann et al., 2012), whereas chimpanzees don't distinguish between continuing to help in an existing collaboration versus starting a new one (Greenberg et al., 2010).



Hamann and Warneken (2012, Child Dev)

Commitment and coordination

- In the threshold game, hunters are a bit stupid
  - Cooperator will run off to do the hunt by themselves
- But people don't really behave this way– they coordinate
  - If we were in this situation, we'd have a conversation
  - That's also how people behave experimentally (e.g., Van de Kragt *et al.* (1983, Am Pol Sci Rev))
- Plus, coordination improves the evolutionary prospects for cooperation!

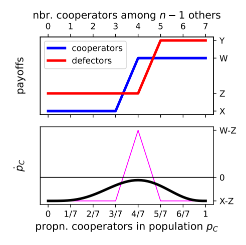


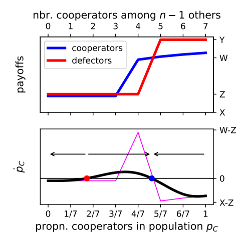
- Newton (2017 Games Econ Behav) ‘shared intentionality’ evolves under fairly general conditions in a public goods game

Jonathan Newton

Coordination in a threshold game example

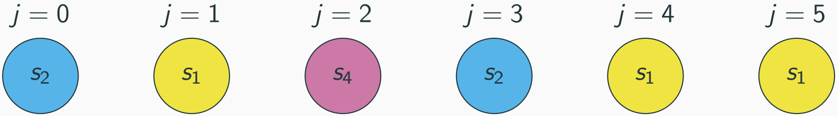
- Threshold game example:
  - Coordinating cooperators draw straws to decide who will contribute
  - The ability to coordinate entails a small cognitive cost ε
- Result:
  - Sustain cooperation where it could not otherwise be sustain
  - Can't invade, but we already know we can overcome this with homophily
- Even works in a linear game!
  - But remember, we know it never makes sense to contribute in the linear game
  - It's true the Defectors can't invade, but what about a type who participates in the lottery but doesn't follow through?
- New strategy: Liars





New notation

- Strategy indicator for player j of $\boldsymbol{g}_j = \boldsymbol{e}_x$ means j plays strategy s_x
- Focal player has subscript '0', nonfocals 'nf', and all players 'a'
- Nonfocal strategy indicator is the sum of nonfocal strategies $\boldsymbol{g}_{\mathrm{nf}} = \boldsymbol{g}_1 + \ldots + \boldsymbol{g}_{n-1}$



The strategy indicators for each individual are

$$\boldsymbol{g}_1 = \boldsymbol{g}_4 = \boldsymbol{g}_5 = (1, 0, 0, 0), \quad \boldsymbol{g}_0 = \boldsymbol{g}_3 = (0, 1, 0, 0), \text{ and } \boldsymbol{g}_2 = (0, 0, 0, 1).$$

The whole-group strategy composition is: $\boldsymbol{g}_{\mathrm{a}} = (3, 2, 0, 1)$

The nonfocal strategy composition is: $\boldsymbol{g}_{\mathrm{nf}} = (3, 1, 0, 1)$

Many strategies

How does a trait change frequency over time?

George Robert Price

$$\underbrace{\Delta p_x}_{\text{dynamics of propn. of } s_x} = \mathbb{C}\text{ov}[\underbrace{G_{0,x}}_{\text{focal's strategy indicator}}, \underbrace{W_0}_{\text{fitness of focal}}],$$

$$G_{0,x} = \begin{cases} 1 & \text{if focal strategy } s_x, \\ 0 & \text{otherwise.} \end{cases}$$

Many strategies

dynamics of propn. of s_x

$$\Delta p_x = \mathbb{C}\mathrm{ov}[\, G_{0,x} \,,\, W_0 \,],$$

1 if focal plays s_x; 0 otherwise

fitness of focal

selection strength

focal payoff random variable

$$W_0 = s + \overbrace{(1 - s)}^{\text{vacancies}} \frac{1 + \delta \, \Pi_0}{1 + \delta \, \overline{\pi}}$$

survival probability

avg. payoff in population

Useful identity: $\mathbb{C}\mathrm{ov}[X, aY + b] = a\, \mathbb{C}\mathrm{ov}[X, Y]$

Substituting and rearranging:

strategy indicator

focal payoff

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[\, G_{0,x} \,,\, \Pi_0 \,\right]$$

Other member accounting

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[\overbrace{G_{0,x}}^{\text{focal's strategy indicator}}, \overbrace{\Pi_0}^{\text{focal payoff}}\right]$$

Payoff to the focal individual:

$$\Pi_0 = \sum_{i=1}^{m} \overbrace{G_{0,i}}^{\text{1 if focal plays } s_i\text{; 0 otherwise}} \; \overbrace{\pi(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}})}^{\text{payoff to } s_i\text{-player}}$$

Useful identity: $\mathbb{C}\mathrm{ov}[X, Y] = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$

$$\Delta p_x = \mathbb{E}\left[G_{0,x}\,\pi(\boldsymbol{e}_x, \boldsymbol{G}_{\mathrm{nf}})\right] - p_x \sum_{i=1}^{m} \mathbb{E}\left[G_{0,i}\,\pi(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}})\right]$$

nonfocal strategy composition

nonfocal strategy composition

$$\Delta p_x = \mathbb{E}\left[G_{0,x}\,\pi(\boldsymbol{e}_x,\ \boxed{\boldsymbol{G}_{\mathrm{nf}}}\ )\right] - p_x \sum_{i=1}^{m} \mathbb{E}\left[G_{0,i}\,\pi(\boldsymbol{e}_i,\ \boxed{\boldsymbol{G}_{\mathrm{nf}}}\ )\right]$$

The possible values of $\boldsymbol{G}_{\mathrm{nf}}$ are: $\mathcal{G}_{\mathrm{nf}} = \left\{\boldsymbol{g}_{\mathrm{nf}} \in \mathbb{N}^m_{\geq 0} \,\middle|\, \sum_{i=1}^m g_{\mathrm{nf},i} = n-1\right\}$

So the expectations are:

$$\begin{aligned}
\mathbb{E}\left[G_{0,i}\pi(\boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}})\right] &= \sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \mathbb{P}[\boldsymbol{G}_0 = \boldsymbol{e}_i, \boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}}] \\
&= \sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \underbrace{\mathbb{P}[\boldsymbol{G}_0 = \boldsymbol{e}_i]}_{p_i}\, \mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i] \\
&= p_i \underbrace{\sum_{\boldsymbol{g}_{\mathrm{nf}} \in \mathcal{G}_{\mathrm{nf}}} \pi(\boldsymbol{e}_i, \boldsymbol{g}_{\mathrm{nf}})\, \mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]}_{\overline{\pi}_i}
\end{aligned}$$

Recover replicator eqn: $\Delta p_x \propto p_x \left(\overline{\pi}_x - \sum_{i=1}^m p_i \overline{\pi}_i\right) = p_x \left(\overline{\pi}_x - \overline{\pi}\right)$.

But $\mathbb{P}[\boldsymbol{G}_{\mathrm{nf}} = \boldsymbol{g}_{\mathrm{nf}} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]$ is not obvious.

$$\Delta p_x \propto \mathbb{C}\mathrm{ov}\left[\underbrace{G_{0,x}}_{\text{strategy indicator}}, \underbrace{\Pi_0}_{\text{focal payoff}}\right]$$

But this time, let's focus on the whole-group distribution. Idea: draw a group at random, then draw a focal individual.

$$\Pi_0 = \sum_{i=1}^{m} G_{0,i} \; \underbrace{\hat{\pi}(\boldsymbol{e}_i, \boldsymbol{G}_\mathrm{a})}_{\text{new payoff fnc wrt whole-group strategy composition}}$$

Using a similar method to before involving covariance identities and re-arranging, we obtain

$$\Delta p_x = \sum_{\boldsymbol{g}_\mathrm{a} \in \mathcal{G}_\mathrm{a}} \left(\frac{g_{\mathrm{a},x}}{n} \hat{\pi}(\boldsymbol{e}_x, \boldsymbol{g}_\mathrm{a}) - p_x \sum_{i=1}^{m} \frac{g_{\mathrm{a},i}}{n} \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{g}_\mathrm{a}) \right) \underbrace{\mathbb{P}[\boldsymbol{G}_\mathrm{a} = \boldsymbol{g}_\mathrm{a}]}_{\text{prob. of whole-group strategy composition}}$$

Whole-group accounting

$$\Delta p_x = \sum_{\boldsymbol{g}_\mathrm{a} \in \mathcal{G}_\mathrm{a}} \left(\frac{g_{\mathrm{a},x}}{n} \hat{\pi}(\boldsymbol{e}_x, \boldsymbol{g}_\mathrm{a}) - p_x \sum_{i=1}^{m} \frac{g_{\mathrm{a},i}}{n} \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{g}_\mathrm{a}) \right) \boxed{\mathbb{P}[\boldsymbol{G}_\mathrm{a} = \boldsymbol{g}_\mathrm{a}]}$$

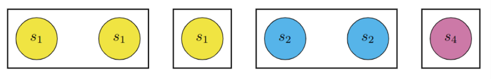
prob. of whole-group strategy composition

- The probability $\mathbb{P}[\boldsymbol{G}_\mathrm{a} = \boldsymbol{g}_\mathrm{a}]$ depends on the family structure of the group
- Two rules for allocating individuals to families:
  - Individuals with different strategies $\rightarrow$ different families
  - Individuals with the same strategy $\rightarrow$ same or different families
- Introduce two new objects:
  1. $\boldsymbol{z}$: strategywise family-size distribution
  2. $\boldsymbol{y}$: group's family-size distribution

Strategywise family-size distribution

Example:

Consider the group illustrated below:



Each strategy's family-size distribution is

$$\boldsymbol{z}_1 = (1, 1, 0, 0, 0, 0), \quad \boldsymbol{z}_2 = (0, 1, 0, 0, 0, 0),$$
$$\boldsymbol{z}_3 = (0, 0, 0, 0, 0, 0), \quad \boldsymbol{z}_4 = (1, 0, 0, 0, 0, 0).$$

So the group's strategywise family-size distribution is

$$\boldsymbol{z} = \big((1, 1, 0, 0, 0, 0),\ (0, 1, 0, 0, 0, 0),\ (0, 0, 0, 0, 0, 0),\ (1, 0, 0, 0, 0, 0)\big),$$

The group's family-size distribution is

$$\boldsymbol{y} = \mathrm{sum}(\boldsymbol{z}) = (2, 2, 0, 0, 0, 0).$$

Probability of whole-group strategy composition

Let $\mathscr{Z}_{\boldsymbol{g}_\mathrm{a}}$ be the set of all possible strategywise family-size distributions $\boldsymbol{z}$ consistent with the strategy distribution $\boldsymbol{g}_\mathrm{a}$. Then

$$\mathbb{P}[\boldsymbol{G}_\mathrm{a} = \boldsymbol{g}_\mathrm{a}] = \sum_{\boldsymbol{z} \in \mathscr{Z}_{\boldsymbol{g}_\mathrm{a}}} \mathbb{P}[\boldsymbol{Z} = \boldsymbol{z}] = \sum_{\boldsymbol{z} \in \mathscr{Z}_{\boldsymbol{g}_\mathrm{a}}} \mathbb{P}[\boldsymbol{Y} = \boldsymbol{y}]\, \mathbb{P}[\boldsymbol{Z} = \boldsymbol{z} \mid \boldsymbol{Y} = \boldsymbol{y}],$$

$$= \sum_{\boldsymbol{z} \in \mathscr{Z}_{\boldsymbol{g}_\mathrm{a}}} \boxed{F_{\boldsymbol{y}}}\, \mathbb{P}[\boldsymbol{Z} = \boldsymbol{z} \mid \boldsymbol{Y} = \boldsymbol{y}],$$

get from homophilic group-formation model

Probability of strategywise family-size distribution:

nbr. ways to order strategies across families

$$\mathbb{P}[\boldsymbol{Z} = \boldsymbol{z} \mid \boldsymbol{Y} = \boldsymbol{y}] = \boxed{C(\boldsymbol{z})}\ \boxed{A(\boldsymbol{z}, \boldsymbol{p})}$$

count of multiset permutations

prob. families' strategies

nbr. families pursuing strategy s_i

$$A(\boldsymbol{z}, \boldsymbol{p}) = \prod_{i=1}^{m} p_i^{\boxed{\|\boldsymbol{z}_i\|}}$$

Whole-group accounting

Bringing it all together:

$$\Delta p_x \propto \underbrace{\sum_{\boldsymbol{g}_\mathrm{a} \in \mathcal{G}_\mathrm{a}}}_{\text{sum over group strategy compositions}} \left(\overbrace{\frac{g_{\mathrm{a},x}}{n}}^{\text{prob. focal pursues } s_x} \hat{\pi}(\boldsymbol{e}_x, \boldsymbol{g}_\mathrm{a}) - p_x \sum_{i=1}^{m} \frac{g_{\mathrm{a},i}}{n} \hat{\pi}(\boldsymbol{e}_i, \boldsymbol{g}_\mathrm{a}) \right) \left(\overbrace{\sum_{\boldsymbol{z} \in \mathcal{Z}_{\boldsymbol{g}_\mathrm{a}}}}^{\text{over strategywise family-sizes}} C(\boldsymbol{z})\, A(\boldsymbol{z}, \boldsymbol{p})\, F_{\mathrm{sum}(\boldsymbol{z})} \right)$$

- Not as intuitive as the traditional replicator equation
  - $\Delta p_x \propto p_x \left(\overline{\pi}_x - \overline{\pi}\right)$
- Useful from computational perspective because we've split homophily calculations off from strategy identity
  - Components of $C(\boldsymbol{z})$ and $A(\boldsymbol{z}, \boldsymbol{p})$ can be precalculated and stored
  - But computational explosion as group size and number of strategies increase
- Now it's clearer how to calculate $\mathbb{P}[\boldsymbol{G}_\mathrm{nf} = \boldsymbol{g}_\mathrm{nf} \mid \boldsymbol{G}_0 = \boldsymbol{e}_i]$

Payoff transformation 2 players

- Payoff transformation
  - Transform payoff matrix by writing as a function of relatedness
  - Solve the well-mixed dynamics
- Example: 2 players many strategies
  - Define matrix with elements equal to payoffs $a_{i,j} = \pi(\boldsymbol{e}_i, \boldsymbol{e}_j)$
  - Replicator dynamics in matrix form:
    $\dot{p}_i = p_i(\overline{\pi}_i - \overline{\pi}) = p_i((A\boldsymbol{p})_i - \boldsymbol{p}^T A\boldsymbol{p})$
  - For example, we can write out $\overline{\boldsymbol{\pi}} = A\boldsymbol{p}$ in detail

$$\begin{pmatrix}\overline{\pi}_1\\ \vdots \\ \overline{\pi}_i \\ \vdots \\ \overline{\pi}_m\end{pmatrix} = \begin{array}{cc} & \xrightarrow{\text{nonfocal's strategy}} \\ \text{focal's strategy}\downarrow & \begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & & \vdots \\ a_{i,1} & \dots & a_{i,m} \\ \vdots & & \vdots \\ a_{m,1} & \dots & a_{m,m}\end{pmatrix}\end{array} \begin{pmatrix}p_1\\ \vdots \\ p_i \\ \vdots \\ p_m\end{pmatrix} = \begin{pmatrix} a_{1,1}p_1 + \ldots + a_{1,m}p_m \\ \vdots \\ a_{i,1}p_1 + \ldots + a_{i,m}p_m \\ \vdots \\ a_{m,1}p_1 + \ldots + a_{m,m}p_m\end{pmatrix}$$

Payoff transformation 2 players

- Example: 2 players many strategies continued
  - We are up to here; $\overline{\boldsymbol{\pi}} = A\boldsymbol{p}$:

$$\begin{pmatrix} \overline{\pi}_1 \\ \vdots \\ \overline{\pi}_i \\ \vdots \\ \overline{\pi}_m \end{pmatrix} = \begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & & \vdots \\ a_{i,1} & \dots & a_{i,m} \\ \vdots & & \vdots \\ a_{m,1} & \dots & a_{m,m} \end{pmatrix} \begin{pmatrix} p_1 \\ \vdots \\ p_i \\ \vdots \\ p_m \end{pmatrix} = \begin{pmatrix} a_{1,1}p_1 + \dots + a_{1,m}p_m \\ \vdots \\ a_{i,1}p_1 + \dots + a_{i,m}p_m \\ \vdots \\ a_{m,1}p_1 + \dots + a_{m,m}p_m \end{pmatrix}$$

  - Now consider the game under dyadic relatedness $\theta_{2\to 1}$

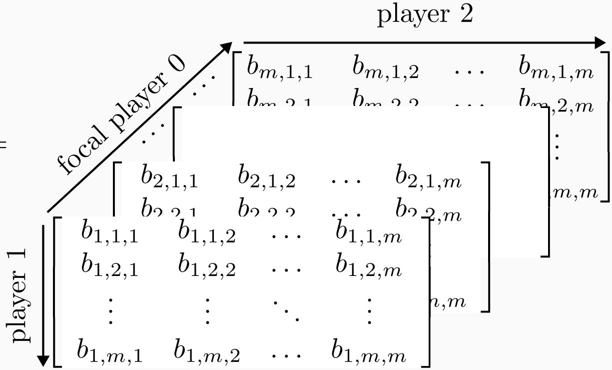
$$B = \underbrace{\theta_{2\to 1} \begin{pmatrix} a_{1,1} & \dots & a_{1,1} \\ \vdots & & \vdots \\ a_{i,i} & \dots & a_{i,i} \\ \vdots & & \vdots \\ a_{m,m} & \dots & a_{m,m} \end{pmatrix}}_{i \text{ matched with } i \text{ with prob. } \theta_{2\to 1}} \underbrace{+ (1 - \theta_{2\to 1}) \begin{pmatrix} a_{1,1} & \dots & a_{1,m} \\ \vdots & & \vdots \\ a_{i,1} & \dots & a_{i,m} \\ \vdots & & \vdots \\ a_{m,1} & \dots & a_{m,m} \end{pmatrix}}_{i \text{ matched with random with prob. } 1 - \theta_{2\to 1}}$$

  - Dynamics of A with homophily $\equiv$ dynamics of B well-mixed

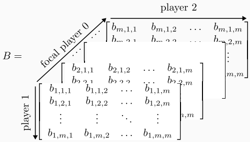
$$\dot{p}_i = p_i((B\boldsymbol{p})_i - \boldsymbol{p}^T B\boldsymbol{p})$$

Seeking a solution to:

$B =$



Payoff transformation n players



$$b_{\boldsymbol{u}} = \sum_{\boldsymbol{q} \vdash n} F_{\boldsymbol{q}} \left(\sum_{q_0 \in \boldsymbol{q}} \frac{q_0}{n|\mathcal{J}_{q_0,\boldsymbol{q}}|} \left(\sum_{\boldsymbol{j} \in \mathcal{J}_{q_0,\boldsymbol{q}}} a_{\boldsymbol{u}_{\boldsymbol{j}}} \right) \right)$$

get from group-formation model

Code to calculate it on Github:

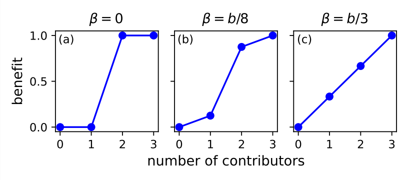
1. Numerically: `TransmatBase` class `functions/transmat_base.py`.
2. Symbolically: `functions/symbolic_transformed.py`.

But why would you want to do this?

- B is expensive to calculate, but matrix multiplication is optimised, can be worth the trade-off when finding steady states
- Use maths from well-mixed case, e.g., Jorge Peña's analysis techniques (example in appendix)

Coordinated cooperation

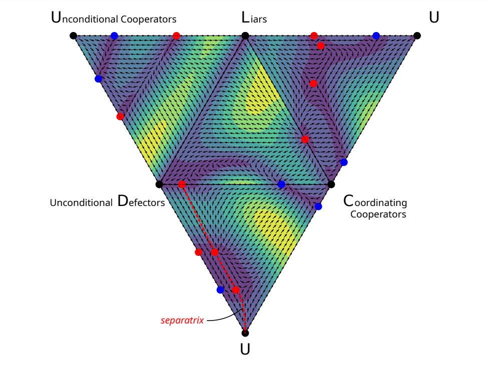
- Game with 4 strategies:
    1. D: unconditional Defector, never contributes
    2. C: Coordinating cooperator, hold lottery, follow through if chosen
        - Nbr. contributors $\tau =$ threshold, or inflection point if sigmoid
    3. L: Liar, participate in lottery, never contributes
    4. U: Unconditional cooperator, always contributes
- C and L pay cognitive cost ε regardless of game outcome
- U and C pay contribution cost c if contributing
- Explore the range from linear to threshold game



Example 3-player - symbolic analysis

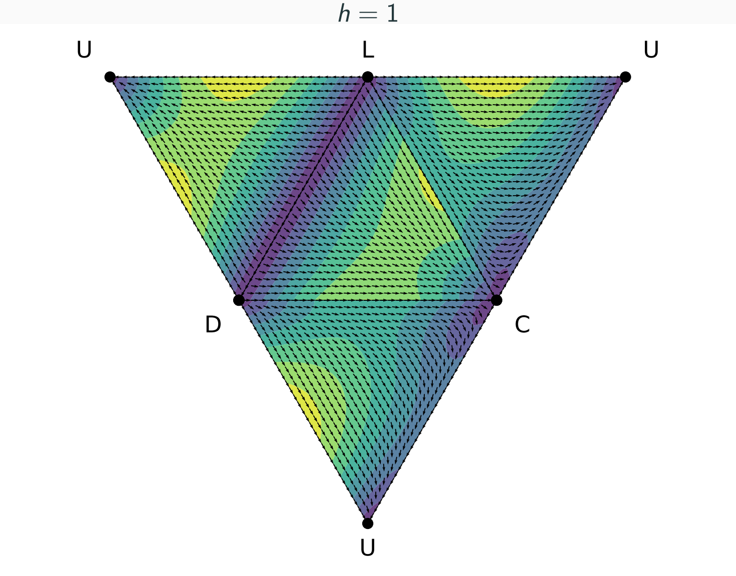
Example 8-player - numerical analysis

How to read results

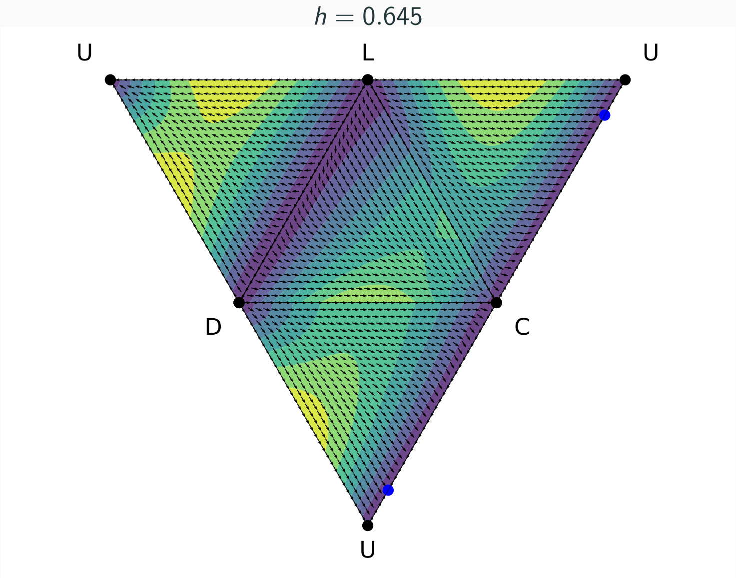


- Evolutionary dynamics for a given homophily level h
  - Dynamics inside a triangular pyramid
  - The points represent a population with just one strategy, lines 2 strategies, triangles 3
  - Blue points are stable in that dimension, red points unstable

Results 1: fairly nonlinear benefits function

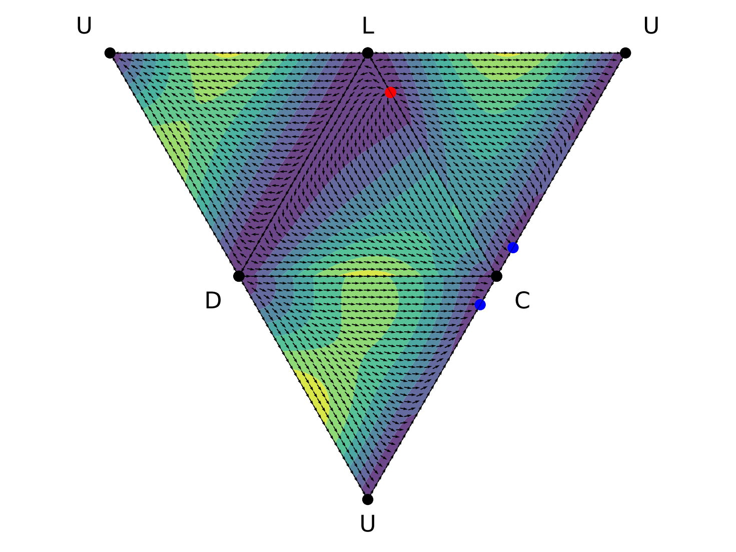


Results 1: fairly nonlinear benefits function

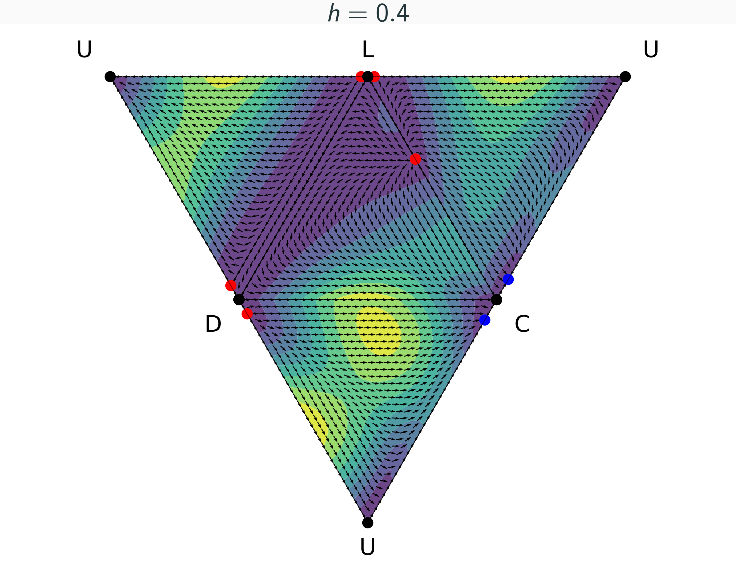


Results 1: fairly nonlinear benefits function

$h = 0.5$

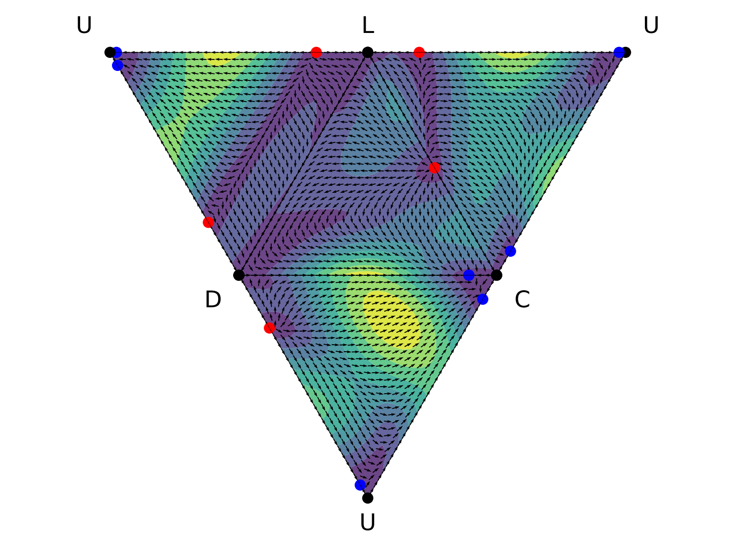


Results 1: fairly nonlinear benefits function



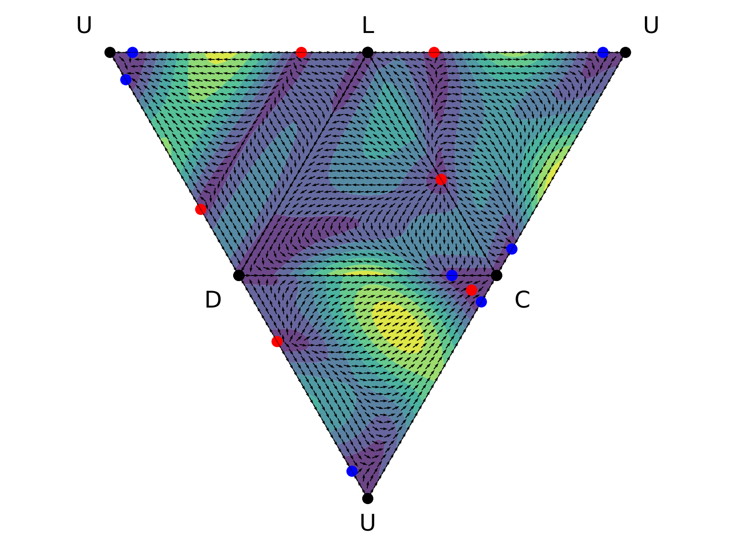
Results 1: fairly nonlinear benefits function

$h = 0.3$



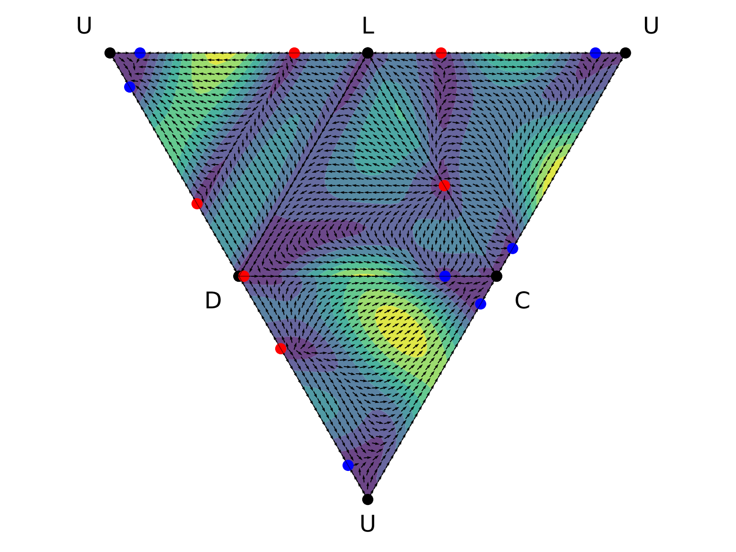
Results 1: fairly nonlinear benefits function

$h = 0.26$



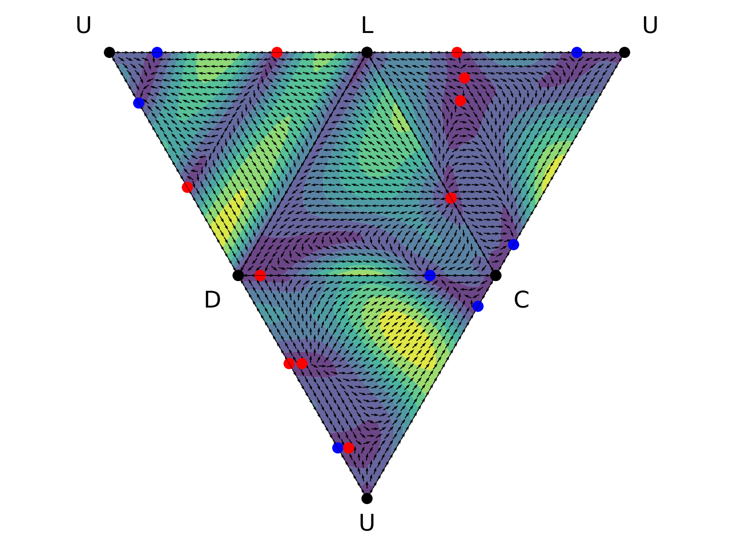
Results 1: fairly nonlinear benefits function

$h = 0.24$



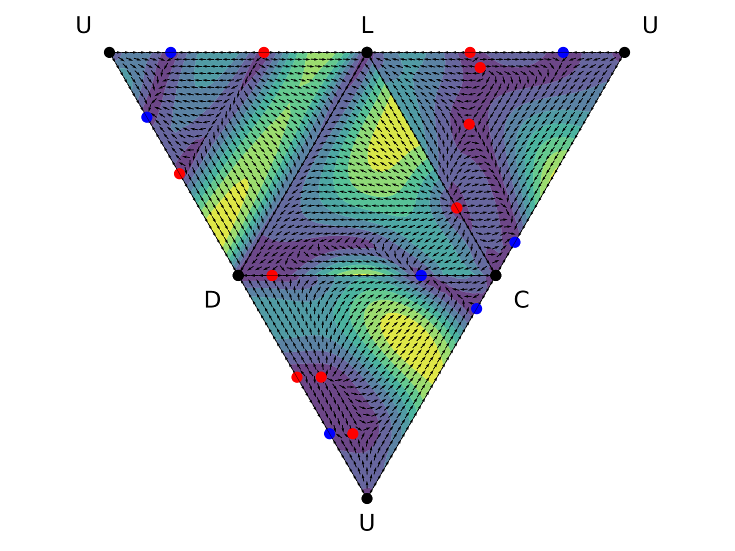
Results 1: fairly nonlinear benefits function

$h = 0.19$



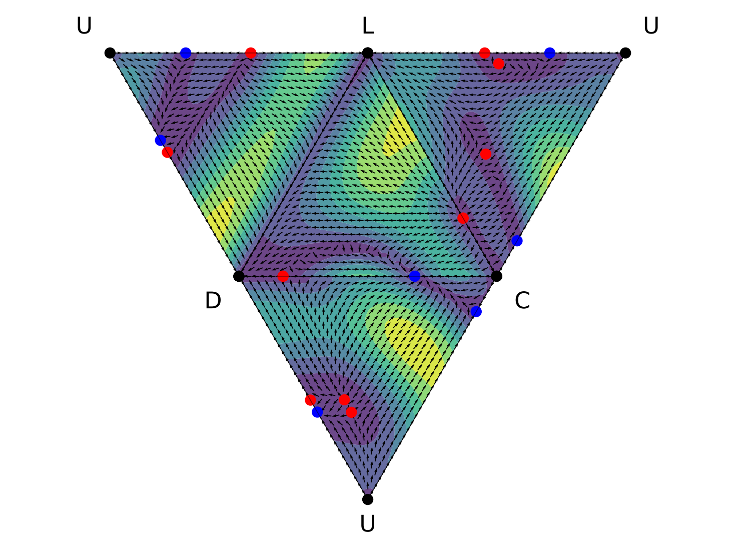
Results 1: fairly nonlinear benefits function

$h = 0.15$



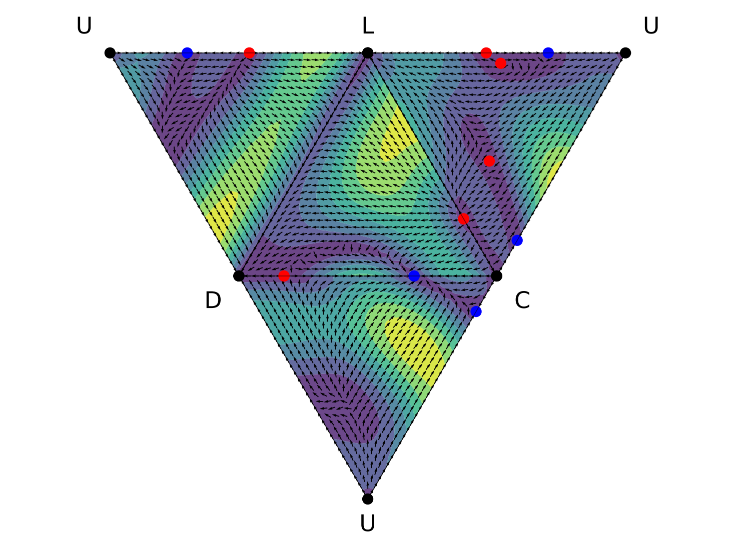
Results 1: fairly nonlinear benefits function

$h = 0.11$



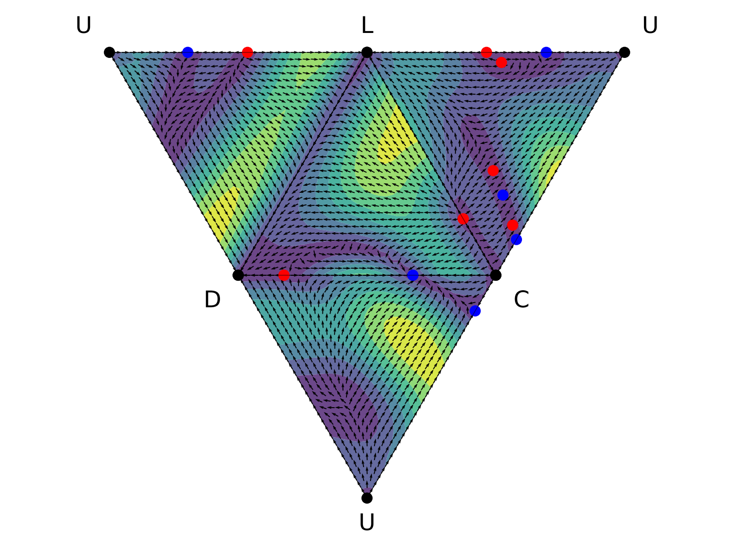
Results 1: fairly nonlinear benefits function

$h = 0.106$

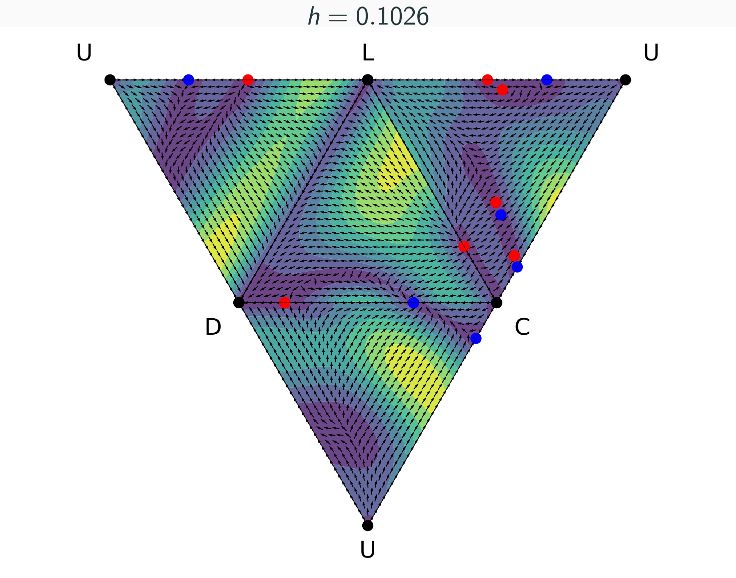


Results 1: fairly nonlinear benefits function

$h = 0.1031$

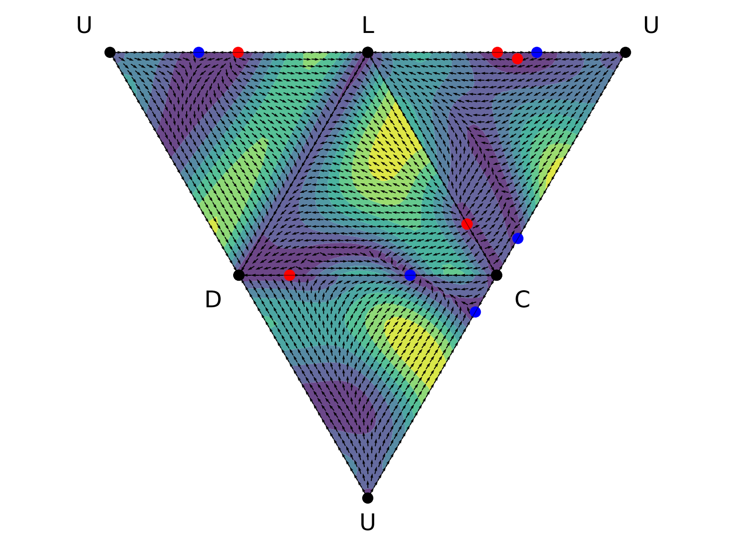


Results 1: fairly nonlinear benefits function

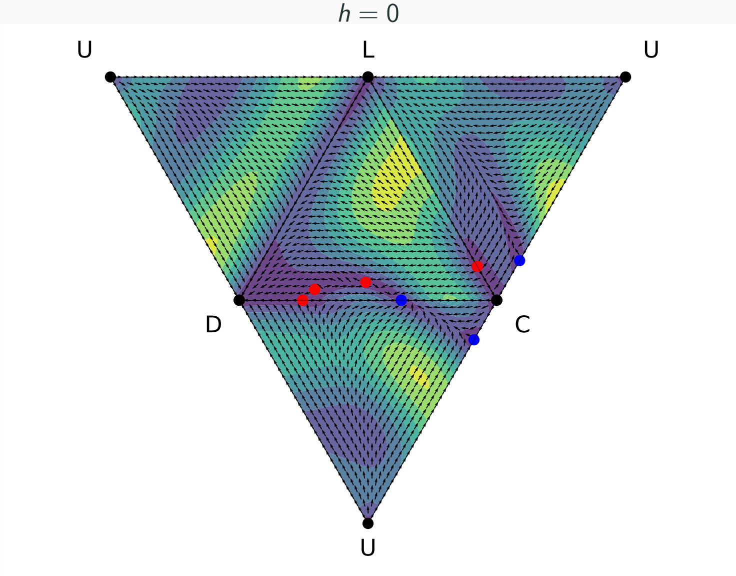


Results 1: fairly nonlinear benefits function

$h = 0.08$

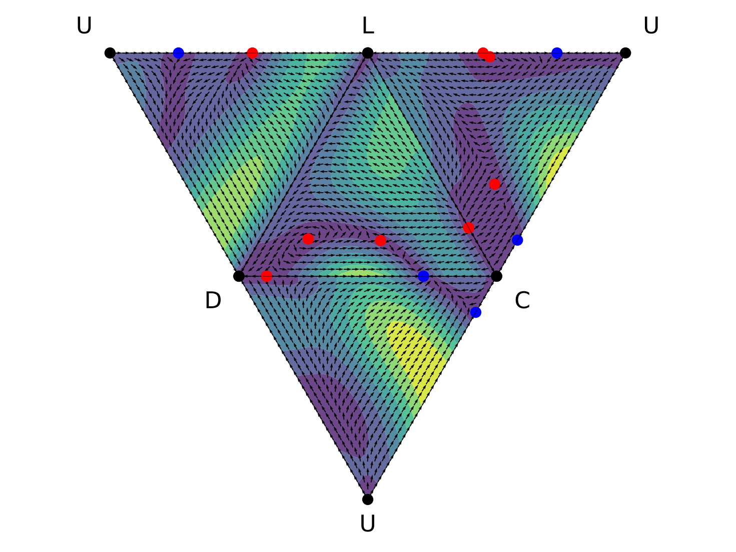


Results 1: fairly nonlinear benefits function

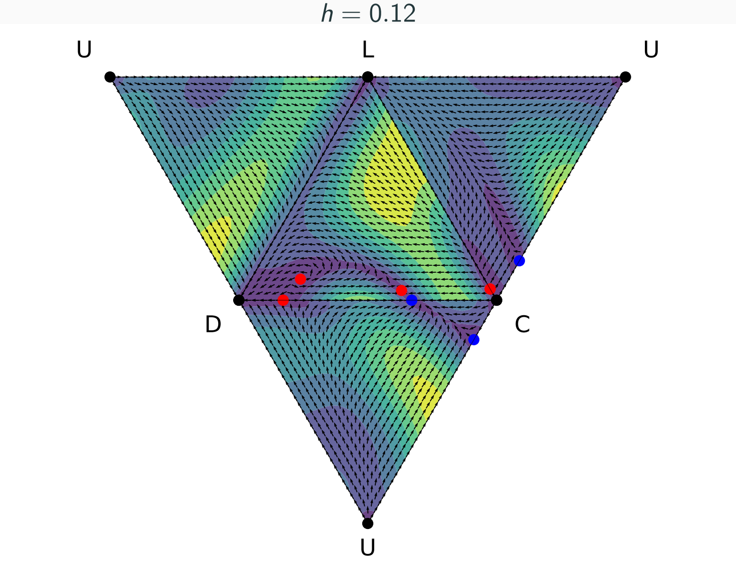


Results 2: more linear benefits function

$h = 0.18$

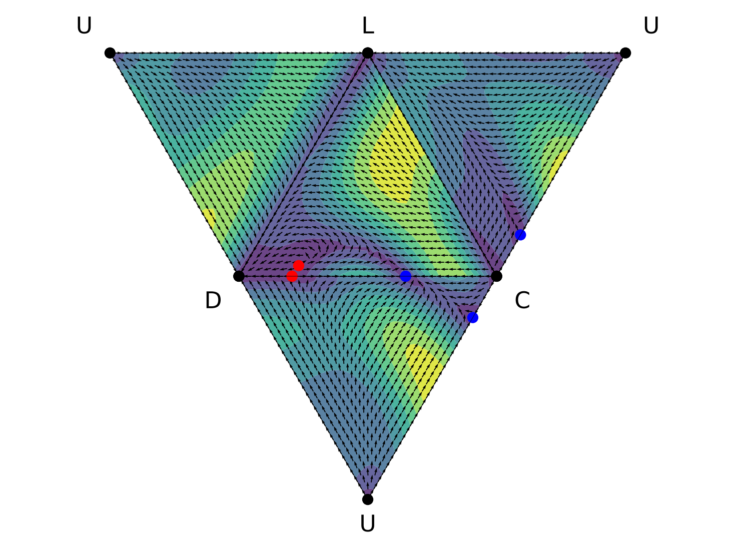


Results 2: more linear benefits function

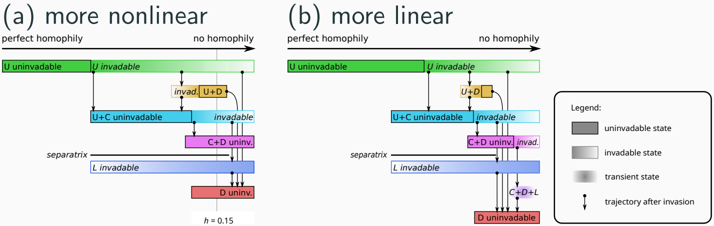


Results 2: more linear benefits function

$h = 0.08$



Results summary



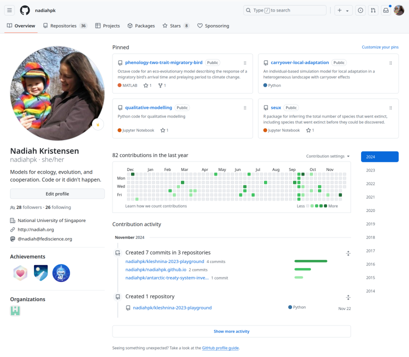
- Commitment beyond self-interest?
    - Interdependence is not enough
        - Enough experimentally, e.g., children's task
        - But evolutionary, must be some kind of 'mistake'
    - Reputation?
        - Players with good reputation only cooperate with others with good reputation
        - A kind of strategy-based homophily
        - Prisoner's Dilemma model by Kellner & Han (2025, arXiv)
        - Good reputation obtained by commitment adherence

Summary

- Built a mathematical framework to combine discrete-strategy group games with kin selection (or ‘matching rules’)
- Past homophily can explain how cooperation first arose
- Conditioned, coordinated cooperation allows cooperation to persist where unconditional cooperation cannot
- Commitment beyond self-interest either requires some level of homophily or some other mechanism

Thank you!

github.com/nadiahpk



nadiah.org