BUSN 33921: Problem Set 1

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Problem 1

Value added is a general function of TFP, labor, and capital

$$Y = f(A, L, K)$$

Assuming perfect competition and constant returns to scale, we first think about the profit maximization problem of the firm

$$\max_{L,K} PY - wL - rK$$

Normalizing prices to 1 we have

$$\max_{L,K} f(A, K, L) - wL - rK$$

$$\implies \frac{\partial f}{\partial L} = w$$
$$\frac{\partial f}{K} = r$$

And then we can totally differentate the production function, divide by Y and plug in the above conditions

$$dY = \frac{\partial f}{\partial A}dA + \frac{\partial f}{\partial L}dL + \frac{\partial f}{\partial K}dK$$
$$\frac{dY}{Y} = \frac{\partial f}{\partial A}\frac{dA}{Y} + w\frac{dL}{Y} + r\frac{dK}{Y}$$

With some more manipulations, multiplying and dividing by L and K

$$\frac{dY}{Y} = \frac{\partial f}{\partial A}\frac{dA}{Y} + w\frac{LdL}{LY} + r\frac{KdK}{KY}$$

Given the definition of α and accounting from the profit max condition, we have

$$\alpha = \frac{wL}{Y}$$
$$1 - \alpha = \frac{rK}{Y}$$

Now we can easily plug in these values to the above expression

$$\frac{dY}{Y} = \frac{\partial f}{\partial A}\frac{AdA}{YA} + \alpha\frac{dL}{L} + (1-\alpha)\frac{dK}{K}$$

Normalizing

$$\frac{A}{Y}\frac{\partial f}{\partial A} = 1$$

And then using the log rul approximation for small changes in levels, we can approximate $dA/A = d = da$ for all of our derivative terms

$$\begin{aligned} dq &= da + \alpha dl + (1-\alpha)dk \\ \implies a &= q - \alpha l - (1-\alpha)dk \end{aligned}$$

Problem 2

a

The firm's maximizes taking K and w as exogenous

$$\begin{aligned} \max_L PY - wL - rK \\ \max_L PAL^{0.7}K^{0.3} - wL - rK \end{aligned}$$

Taking the FOC

$$\begin{aligned} PA(0.7)\left(\frac{K}{L}\right)^{0.3} &= w \\ \implies L &= \left(\frac{P * A * 0.7}{w}\right)^{\frac{1}{0.3}} K \end{aligned}$$

Conceptually normalizing prices to 1 and just having a w term, we can convert to logs and denote the logged variables with notation consistent with the problem setup

$$l_{it} = \frac{1}{0.3}[a_{it} + log(0.7) - w_{it}] + k_{it}$$

b

See attached code

c

The coefficient on labor inputs is biased away from 0.7 in the positive direction (see Table 1 below). This is because TFP is the error term (a_{it}) and as we showed above, labor is co-determined with firm-specific TFP. Thus the positive correlation between the error term and log labor inputs leads to an upward biased estimate from OLS.

d

Now we estimate using fixed and random effects (Table 2). The coefficients are still biased although less so. This is due to the firm fixed effects capturing some of the unknown TFP term through γ_i. Using random effects still demonstrates this bias term is positive but smaller than before.

Table 1: Production function estimation

	(1) OLS
Logged labor inputs	0.860***
	(0.002)
Logged capital inputs	0.213***
	(0.050)
Constant	0.186***
	(0.006)
Observations	5000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Production function estimation using Fixed and Random Effects

	(1) Fixed Effects	(2) Random Effects
Logged labor inputs	0.727***	0.760***
	(0.001)	(0.004)
Logged capital inputs	0.291***	0.273***
	(0.020)	(0.023)
Constant	0.030***	0.069*
	(0.002)	(0.041)
Observations	5000	5000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

e

In the data, we see that two of the determinants of l_{it} are exogenously determined and those are the wage rate, w_{it} and k_{it}. To track with a more realistic view of the world, I've chosen to use w_{it} as an instrument that is a factor determining l_{it} but is not affecting productivity, a_{it}. w_{it} satisfies the exclusion restriction and I do a fixed effects IV to estimate the production function. We see that it's quite precisely estimated right around 0.7 which is much closer to the true value of α.

Table 3: Production function estimation using IV

	(1) IV
Logged labor inputs	0.701***
	(0.001)
Logged capital inputs	0.311***
	(0.023)
Constant	-0.001
	(0.053)
Observations	5000

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Problem 3

Suppose firms in an industry have a variable profit function $\pi(\phi_i, \mu, \sigma)$. Fixed cost of production is f. Firms ex-ante do not know their type, ϕ_i, and can choose to pay a sunk entry cost s to find out. They draw fomr $G(\phi)$ over $[\phi^l, \phi^h]$. No firm produces at a loss and there is free entry.

a

The first equilibrium condition is that no firm produces at a loss (they can always choose not to produce). Given the domain of G we know that for any μ and σ

$$\pi(\phi^l, \mu, \sigma) < f$$
$$\pi(\phi^h, \mu, \sigma) > f$$

Assuming that $G(\phi)$ is continuous, this means that for any σ and μ there must exist some $phi^* > \phi^l$ such that for any σ and μ, $pi(\phi^*, \mu, \sigma) = f$. This implies zero profit so the firm does not operate. Further, for any $\hat{\phi} < \phi^*$

$$\pi(\phi^*, \mu, \sigma) = f \quad \text{and} \quad \pi_\phi(.) > 0$$
$$\implies \pi(\hat{\phi}, \mu, \sigma) < f \quad \forall \hat{\phi} < \phi^*$$

So any firm with type $\phi < \phi^*$ does not operate.

b

Derive the condition under which $d\phi^*/df > 0$. We know that μ is a function of ϕ^*, so the higher ϕ^* is, the higher the average productivity level in the industry ($\partial\mu/\partial\phi^* > 0$). From part a we have that the zero marginal profit condition, or how to set ϕ^* is

$$\pi(\phi^*, \mu(\phi^*), \sigma) - f = 0$$

Now we totally differentiate this zero profit condition with respect to f to get

$$\frac{\partial\pi(\phi^*, \cdot)}{\partial\phi^*}\frac{d\phi^*}{df} + \frac{\partial\pi(\phi^*, \cdot)}{\partial\mu}\frac{\partial\mu}{\partial\phi^*}\frac{d\phi^*}{df} - 1 = 0$$
$$\implies \frac{d\phi^*}{df} = \frac{1}{\pi_\phi(\phi^*, \cdot) + \pi_\mu(\phi^*, \cdot)\frac{\partial\mu}{\partial\phi^*}}$$

Using the notation in the setup of the problem. Since we know that $\partial\mu/\partial\phi^* > 0$ and $\pi_\mu < 0$ then we know that $d\phi^*/df > 0$ if

$$\pi_\phi(\phi^*, \cdot) > -\pi_\mu(\phi^*, \cdot)\frac{\partial\mu}{\partial\phi^*}$$

The intuition behind this result is that there are two opposing forces that happen when raising f, the first is that it raises the firm type needed to be profitable absent any change in μ or σ. However, it also will increase the average firm type in the market, thus increasing μ and making it harder for firms of any type to be competitive in this more competitive market. Thus we could see that ϕ^* could go up or down, depending on which effect dominates.

c

The second free entry equilibrium condition is that there is free entry so in equilibrium, the expected value of entry is equal to s. The probability of successful entry is written as the integral of possible realized profits over the type distribution.

$$\int_{\phi^*}^{\phi^h} (\pi(x, \mu, \sigma) - f)\, g(x)dx = s$$

We now show that $d\phi^*/ds < 0$ by totally differentiating

$$\left[(\pi(\phi^h, \mu, \sigma)g(\phi^h) - f) * 0 - (\pi(\phi^*, \mu, \sigma)g(\phi^*) - f) * 1 + \int_{\phi^*}^{\phi^h} \left(\pi_\mu(x, \mu, \sigma)\frac{\partial\mu}{\partial\phi^*}\right) g(x)dx\right] \frac{d\phi^*}{ds} = 1$$

Since we know that for any ϕ^*, $\pi(\phi^*, \mu, \sigma)g(\phi^*) - f = 0$, the expression reduces to

$$\left[\int_{\phi^*}^{\phi^h} \left(\pi_\mu(x, \mu, \sigma)\frac{\partial\mu}{\partial\phi^*}\right) g(x)dx\right] \frac{d\phi^*}{ds} = 1$$
$$\frac{d\phi^*}{ds} = \frac{1}{\left[\int_{\phi^*}^{\phi^h} \left(\pi_\mu(x, \mu, \sigma)\frac{\partial\mu}{\partial\phi^*}\right) g(x)dx\right]}$$

Similar to part b, we showed that we are given $\pi_\mu(\cdot) < 0$ and we show already that $\partial\mu/\partial\phi^* > 0$. So it must be that the denominator is negative and as desired

$$\frac{d\phi^*}{ds} < 0$$

The intuition here is that increasing the sunk cost in equilibrium raises the level of expected profits realized in order to enter. The only channel this can work through is lowering μ (by lowering ϕ^*) since no individual types are realized yet so this is a barrier to entry to firms of potentially any type. This is opposite to the increase in the lump-sum fixed production cost because increasing fixed costs means that once types are realized, increasing f increases the lowest type which can produce. Lowering competitiveness only by increasing sunk cost means that some high productivity producers may never enter the market so in a sense phi^* is artificially low because there is less competition as it's expensive to enter the market.

d

The condition on the profit function required so $d\phi^*/d\sigma > 0$. Totally differentiating we use the zero profit condition

$$\pi(\phi^*, \mu, \sigma) - f = 0$$

Recall that μ is a direct funtion ϕ^*. Totally differentiating gives

$$\frac{\partial \pi}{\partial \phi^*}\frac{d\phi^*}{d\sigma} + \frac{\partial \pi}{\partial \mu}\frac{\partial \mu}{\phi^*}\frac{d\phi^*}{d\sigma} + \frac{\partial \pi}{\sigma} = 0$$
$$\frac{d\phi^*}{d\sigma} = \frac{-\pi_\sigma(\phi^*, \cdot)}{\pi_\phi(\phi^*, \cdot) + \pi_\mu(\phi^*, \cdot)\frac{\partial \mu}{\partial \phi^*}}$$

We get the same denominator as in part b which we know is positive by the conditions imposed. Thus the condition needed for $d\phi^*/d\sigma > 0$ is $\pi_\sigma(\phi^*, \cdot) < 0$. This would mean that as the output firms had more substitability, profit decreases which makes sense because substitutability leads to greater competitiveness. Thus the result that $\phi^*/d\sigma > 0$ is similarly intuitively plausible because the more substitutable the products are, the less likely firms can mark up a product or retain market power leading to higher type firms surviving and producing in the market.

e

Empirically testing these implications we in essence want to measure the lowest 'type' of a firm that enters a certain market and starts to produce. And how that changes as fixed cost of production varies, how sunk cost of entry varies, and how substitutability, σ, varies.

To see whether ϕ^* increases or decreases, measuring the spread of TFP in an industry would be a good indicator. The tighter the productivity level of firms, the higher ϕ^* would be because higher ϕ^* truncates the distribution on the low end.

Empirically it's difficult to come up with these natural experiments that exogenously move around f, s, or σ. I think for fixed cost of production, f, one could think about a technology shock such as comparing productivity spreads across different powerplants by fuel type. Some fuels have high capital costs to run a plant (wind) and some fuels have lower capital costs (maybe coal). Since they all produce the same output, it could be interesting to study productivity dispersion across fuel types by fixed cost as technology changes differentially across time. Additionally, within an industry, looking at how fixed costs move over time and subsequent productivity distribution would also be empirically viable. If fixed costs decreasing over time lead to higher productivity dispersion (lower ϕ^*) then this theory holds.

This thought experiment similarly holds for sunk costs, for example studying an industry where barriers to entry such as fees or difficulty obtaining permits (perhaps in some countries as a result of bribes) change over time. If sunk costs decreasing over time lead to lower productivity dispersion (higher ϕ^*) then this theory holds.

Substitutability could be thought of perhaps by comparing different industries depending on how differentiated the product is. One could think of comparing the instant coffee industry with the artisan coffee industry and see whether productivity spreads are higher or lower. If they are higher in the less substitutable industry (lower ϕ^*) then part d would hold.