NADIA MAKMAK

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1. A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?

$$\frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15} \cdot \frac{12}{15} \cdot \frac{11}{15} \cdot \frac{10}{15} \cdot \frac{9}{15} \cdot \frac{8}{15} = 10.12\%$$

2. An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all digits are unique. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?

Trying to find how many numbers fit the criteria:

_ _ _ _ _

Has to be 1 3 5 7 9

1 3 5 7 9 but not these

Anything but the previous two

but ous (+0)

0 2 4 6 8 but not previous two

$$\underbrace{5}_{} \cdot \underbrace{4}_{} \cdot \underbrace{8}_{} \cdot \underbrace{7}_{} \cdot \underbrace{\left(\frac{5}{1}+\frac{4}{1}+\frac{3}{1}\right)}_{} = 13440 \text{ possibilities}$$

Has to be 1 3 5 7 9

1 3 5 7 9 but not these

Anything but the previous two (+0)

Anything but the previous three (+0)

0 2 4 6 8 but not previous two

100000 numbers total

$$\overset{1}{\frac{13440}{100000}} \cdot \overset{2}{\frac{13439}{99999}} \cdot \overset{3}{\frac{13438}{99998}} \cdot \overset{4}{\frac{13437}{99997}} \cdot \overset{5}{\frac{13436}{99996}} \cdot \overset{6}{\frac{13435}{99995}} \cdot \overset{7}{\frac{13434}{99994}} \cdot \overset{8}{\frac{13433}{99993}}$$

$= 0.00001\%$

3. You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?

$P(A|B) = P(A)$ and $P(B|A) = P(B)$

$P(A \cap B) = P(A) \cdot P(B)$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$P(A)$: $3/6 \cdot 3/6 \cdot 1$ Doesn't matter what this is?

$P(B)$: $6/6 \cdot 1/6 \cdot 1/6$ Any number but the rest have to match

$P(A \cap B) = 3/6 \cdot 3/6 \cdot 1/6 \cdot 1/6$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/6 \cdot 3/6 \cdot 1/6 \cdot 1/6}{1/6 \cdot 1/6} = 3/6 \cdot 3/6 = P(A)$$

$P(A|B) = P(A)$

A and B are independent

1. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).

COMBINATIONS

$\binom{13}{5} = 1287 \quad 1287 \cdot 4 = 5148$ possible flushes

$\binom{52}{5} = 2\,598\,960$ different 5-card hands

$$P(X=5) = \frac{5148}{2\,598\,960}$$

where X is the number of cards pulled from the same suit

$$\longrightarrow E[X] = \frac{1}{P} = \frac{2\,598\,960}{5148} \approx 504 \text{ hands}$$

2. A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).

Event S is the superstar playing

Event E is the team winning 4/5

Looking for $P(S/E)$

$P(S') = 1 - P(E) = 25\% = .25$

$P(S) = 75\% = .75$

$$P(S|E) = \frac{P(E|S) \cdot P(S)}{P(E|S)P(E) + P(E|S')P(S')} = \frac{x(0.75)}{x(0.75) + y(0.25)} = \frac{0.36 \cdot 0.75}{0.36 \cdot 0.75 + 0.156 \cdot .25} \approx 0.87 \approx 87\%$$

$P(E|S) = \binom{5}{4} \cdot (0.7)^4 \cdot 0.3 = 0.36$

$P(E|S') = \binom{5}{4} \cdot (0.5)^5 = 0.156$