Robotsyn øving 3

$$\underbrace{\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix}}_{\text{homogenous img coords}} = H \underbrace{\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}}_{\text{points on plane (world)}}$$

a)

$$x = \frac{r_{11} X + r_{12} Y + t_x}{r_{31} X + r_{32} Y + t_z} = \frac{\tilde{x}}{\tilde{z}}$$

$$y = \frac{r_{21} X + r_{22} Y + t_y}{r_{31} X + r_{32} Y + t_z} = \frac{\tilde{y}}{\tilde{z}}$$

$$\left.\begin{aligned} \tilde{x} &= r_{11} X + r_{12} Y + t_x \\ \tilde{z} &= r_{31} X + r_{32} Y + t_z \\ \tilde{y} &= r_{21} X + r_{22} Y + t_y \end{aligned}\right| \cdot \lambda$$

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \lambda \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

(1)

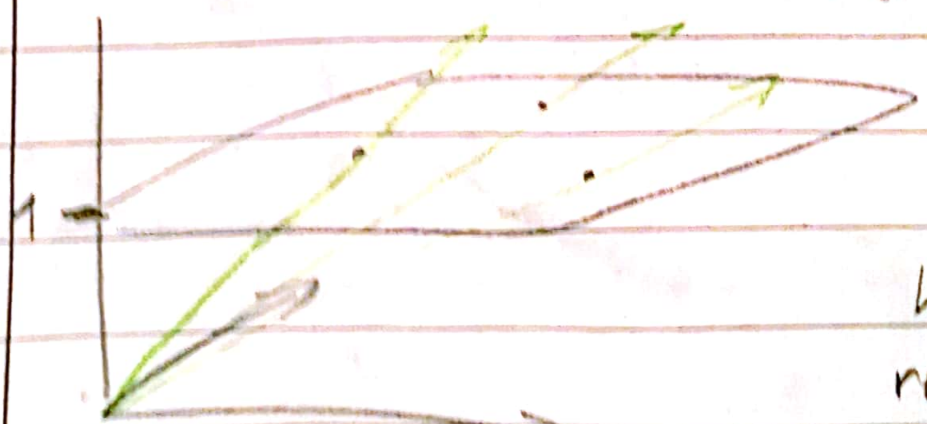
b) H is defined up to scale

So H is true as in $X2 = H X1$
is correct except for the scale.

Homography H is chosen such that
$z1 = 1$ so $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.

Scale is a free variable, we dont have
any information on the scale.

$y = Hx$
$cy = (cH)x$, if $c \neq 0$ H and cH
are the same homography.



* scale of vectors can change and homogenous coord remains the same.

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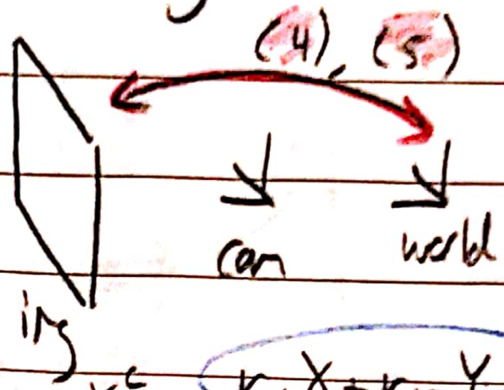
②



$$x = \frac{X^c}{Z^c} = \frac{r_{11}X + r_{12}Y + t_x}{r_{31}X + r_{32}Y + t_z} \quad (4)$$

From equation (4) and (5) we find

$$r_{11}X + r_{12}Y + t_x - (r_{31}X + r_{32}Y + t_z)x = 0$$
$$r_{21}X + r_{22}Y \;\; t_y - (r_{31}X + r_{32}Y + t_z)y = 0$$

$Ah = 0$ $\downarrow$ matrix representation

one point correspondence

$$\left\{ \begin{bmatrix} X & Y & 1 & 0 & 0 & 0 & -Xx & -Yx & -x \\ 0 & 0 & 0 & X & Y & 1 & -Xy & -Yy & -y \end{bmatrix} \right. \begin{bmatrix} r_{11} \\ r_{12} \\ t_x \\ r_{21} \\ r_{22} \\ t_y \\ r_{31} \\ r_{32} \\ t_z \end{bmatrix} = 0$$

rank 8 ish, but noise.

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} h = 0, \quad \arg\min_h \|Ah\| \text{ s.t. } \|h\| = 1$$

least squares.

Robotsyn øving 3

③ Extracting R, t from H

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \lambda \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \quad (6)$$

$$\|H_i\| = 1 \iff \underbrace{\sqrt{h_{11}^2 + h_{21}^2 + h_{31}^2}}_{\text{should be 1}} = \underbrace{\sqrt{h_{12} + h_{22} + h_{32}}}_{\text{should be 1}} = \pm\lambda$$

From (6) we see that r_3 is not present but $r_3 = r_1 \times r_2$

a) $\sqrt{h_{11}^2 + h_{21}^2 + h_{31}^2} = \sqrt{h_{12}^2 + h_{22}^2 + h_{32}^2} = \pm\lambda$

~~Al~~ eqn 1) find λ from

$$\begin{bmatrix} r & \cdots & t_x \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} = \frac{1}{\lambda} H$$