

| | | | | | |
|---|---|---|----|--------------|--|
| X | 3 | 0 | 70 | | |
| | 2 | 1 | 9 | | |
| | 1 | 0 | 1 | | |
| | | | | Y | |
| | | | | 100 | |
| | | | | 7 | |
| | | | | 2 | |

$$X \in \mathbb{R}^{l \times d}$$

$$Y \in \mathbb{R}^C$$

$$l=3 \quad d=3$$

| | | |
|---|---|---|
| 7 | 2 | 9 |
|---|---|---|

?

Инициализация персеп.
 Дано: X, Y

Найти: $a(x) \approx y$

$$a(x) = \underbrace{w_1 x_1 + w_2 x_2 + \dots}$$

Задано ≈ 1 $W = (\underline{0.5}, 1)$

или $1 - 0.5 = 0.5$

Дано: x, y

Модель: $a(\underline{x}) = \sum_{j=1}^d \underbrace{w_j}_{w_1 x_1 + \dots + w_d x_d} \underline{x}_j + \underbrace{b}_b$

Обучение: $b - ?$

$$Q(w, b) = \sum_{i=1}^l L(y_i, a(x_i)) \stackrel{\text{кв. ф. н.}}{=} \sum_{i=1}^l (y_i - a(x_i))^2$$

w_1 $w_d - ?$

$$Q(w, b) = \sum_i (y_i - a(x_i))^2 =$$

$$\sum_i (y_i - (w_1 x_{i1} + w_2 x_{i2} + \dots + w_d x_{id}))^2$$

Задача 2: $a(x) \equiv b$ — константа $\Rightarrow \min_{w_1}$

$$b = ? \quad Q(b) = \sum_{i=1}^l (y_i - b)^2 \rightarrow \min_b$$

$$Q(b) = \sum_{i=1}^p (y_i - b)^2 \rightarrow \min_b$$

$$Q(b) = (y_1 - b)^2 + \dots + (y_p - b)^2$$

$$Q'(b) = \cancel{-2}(y_1 - b) + \dots + \cancel{-2}(y_p - b)$$

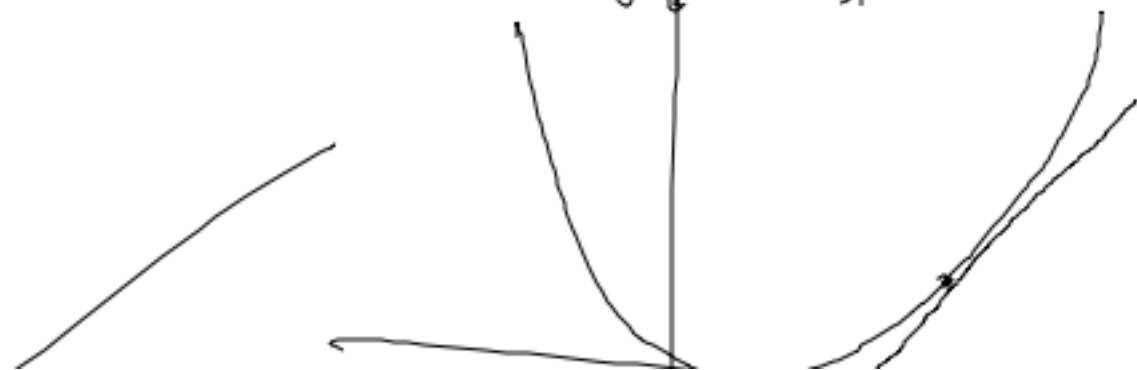
$$y_1 + \dots + y_p - b \cdot p = 0$$

$$\therefore b = \frac{y_1 + \dots + y_p}{p} = \frac{1}{p} \sum_{i=1}^p y_i$$

$$Q(b) = \frac{(b - y_1)^2}{2(b - y_1)} + \dots + \frac{(b - y_n)^2}{2(b - y_n)}$$

$$y^2 =$$

$$y = (b - y_1)^2$$



$$y^2 = 2y$$

$$y = 1$$

$$Q(b) = \sum_i (y_i - b)^2 \rightarrow \min_b$$

mean squared error
MSE

$$b = \text{mean}(y)$$

$$Y = (100, 1, 2)$$

$$b = 34,3$$

$$Q(b) = \sum_i |y_i - b| \rightarrow \min_b$$

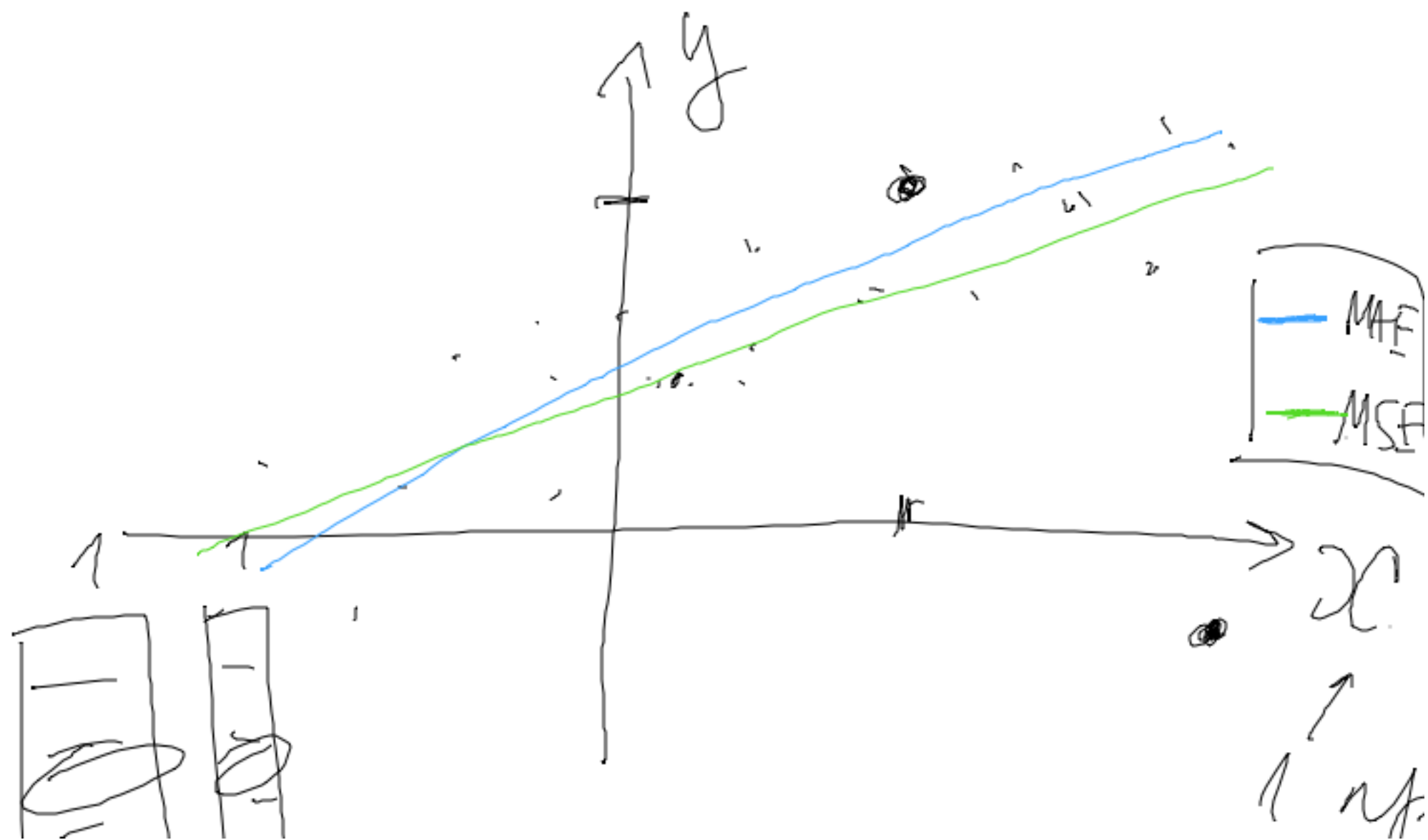
mean absolute error

$$Y = (\underline{100}, \underline{1}, \underline{2})$$

$$\begin{array}{ccc} 1 & 2 & 100 \\ & \uparrow & \end{array}$$

$$Y = (3, 5, 100, 1, 2)$$

$$\begin{aligned} \text{mean}(Y) &= \\ &= \frac{100 + 1 + 2}{3} \\ \text{median}(Y) &= \end{aligned}$$



$$a(x) = w_1 x_1 + \dots + w_d x_d$$

$$Q(w) = \sum_i (y_i - a(x_i))^2 \rightarrow$$

< sup: no $w_j = 0$, per case

$$\widehat{w} = \underbrace{\begin{pmatrix} \cancel{X^T} & \cancel{X} \end{pmatrix}^{-1}}_{d \times 1} \underbrace{\begin{pmatrix} \cancel{X^T} & Y \end{pmatrix}}_{d \times d} \underbrace{\quad}_{1 \times 1}^T$$

| Y | $a(x)$ |
|-----|--------|
| 3 | 4 |
| 5 | 4 |
| 7 | 11 |
| 9 | -1 |

$$\begin{aligned}
 \text{MSE} &= \frac{1}{n} \sum_i (y_i - a(x_i))^2 \\
 &= \frac{1}{4} ((3-4)^2 + (5-4)^2 + (7-11)^2 + (9+1)^2) = \frac{108}{4} = 27 \dots
 \end{aligned}$$

$$\begin{aligned}
 \text{MAE} &= \frac{1}{n} \sum_i |y_i - a(x_i)| \\
 &= \frac{1}{4} (|3-4| + |5-4| + |7-11| + |9+1|) = \frac{14}{4} = 3.5
 \end{aligned}$$

$$w = (X^T X)^{-1} X^T Y$$

hp. lin
matrix