Programming Challenge

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Introduction

This programming challenge is designed to assess you proficiency in formulating and solving a problem as a computer program. The choice of programming language is arbitrary, but Python or Matlab are encouraged. Also, there is no fixed deadline, however, earlier submissions are strongly encouraged; within 2-3 days of receiving this challenge is ideal. If you need more time, please let me know.

Please send back to me a fully functional code that I can run and get a figure with the results. No report is necessary; just the code.

Problem statement

The cart-and-pendulum system in Figure 1 moves along an infinitely-long and frictionless horizontal track. The pendulum's joint is also frictionless. The cart is moved by the input force u. The equations of motion for this system are as follows:

$$(m+M)\ddot{x} = -ml\ddot{\phi}\cos\phi + ml\dot{\phi}^2\sin\phi + u \tag{1}$$

$$l\ddot{\phi} = -g\sin\phi - \ddot{x}\cos\phi \tag{2}$$

where x and ϕ are the cart's position and the pendulum's angle respectively. The cart and pendulum masses are $M = 3\ kg$, and $m = 1\ kg$, and the pendulum length is $l = 1\ m$. Take $g = 9.81\ m/s^2$ to be downward.

The cart and pendulum are initially at rest, with the cart at $x = 0$ and the pendulum vertically downward, $\phi = 0$. The goal is to hit a target located at $\mathcal{T} = (1m, 0.5m)$ with the tip of the pendulum at precisely $t = 2\ s$.

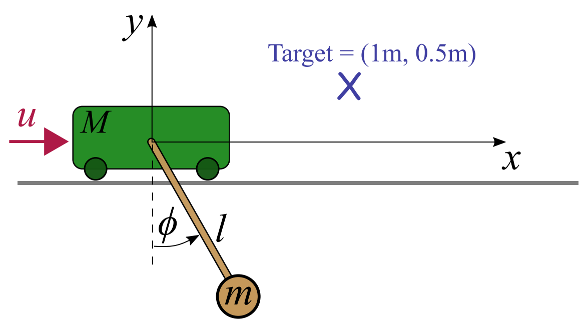


Figure 1: The cart and pendulum system

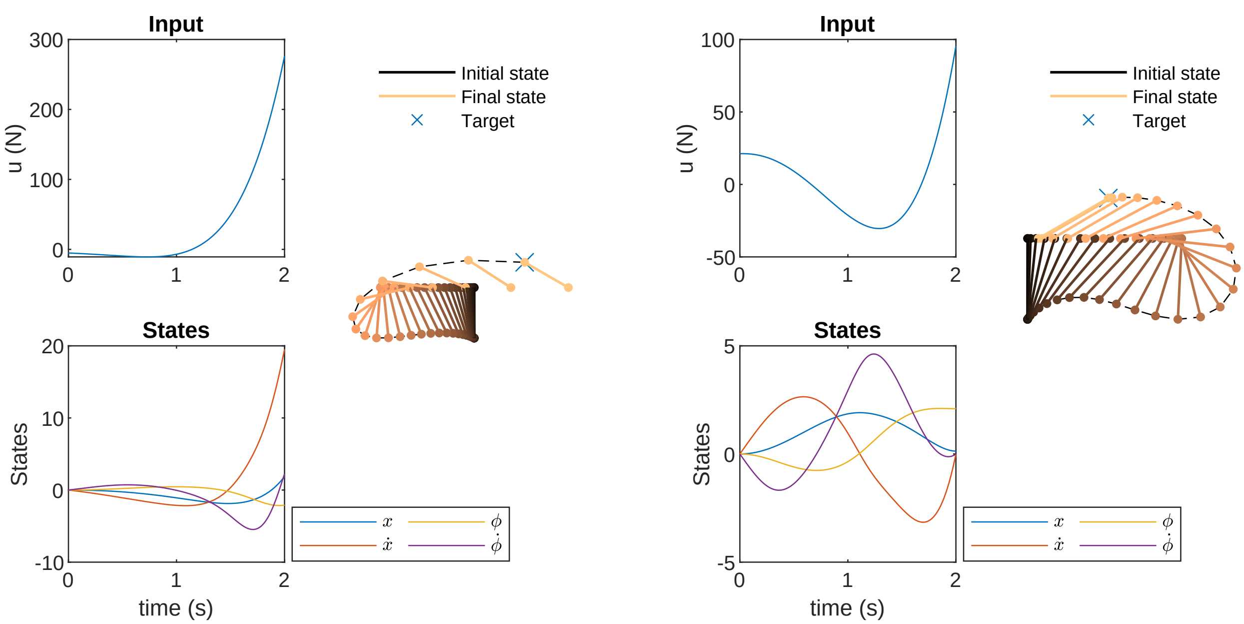
For this control problem, we use trajectory optimization, and specifically the method of "direct transcription", i.e. we solve for the input directly using an optimization. We can parameterize the control input as a fifth-order polynomial function of time ($u(t) = \sum_{i=0}^{5} c_i t^i$), and use optimization to find the polynomial coefficients c_i such that the distance to target at the end of motion is minimized. In other words, we want to minimize the objective function:

$$J = ||\mathbf{P}(t_f) - \mathcal{T}||^2 \tag{3}$$

where $||.||$ is the Euclidean norm. $\mathbf{P}(t_f)$ is the position of the tip of the pendulum at final time $t_f = 2\ s$ and $\mathcal{T}$ is the target position. Any nonlinear optimization algorithm may be used for this problem, e.g, `fmincon` in Matlab and `scipy.optimize` in Python.

Implement this method in your programming language of choice, and generate a figure similar to Figure 2a (showing the input, system states, and the pendulum's path). Note that the final results depend on the optimization algorithm, initial conditions, etc. and the trajectories may look different from those in Figure 2a.

Bonus Modify the method such that the pendulum "lightly touches" the target; i.e., the velocity of the tip of the pendulum should be zero at the time of contact (Figure 2b).



(a) (b)

Figure 2: Examples of the controlled system's response. The stick-figure on the right shows multiple snapshots of the pendulum during the motion. (a) Control without any constraint. (b) Control with the constraint that the velocity of the tip of the pendulum is zero at the target.