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The Algebraic Galaxy of *Simple* Macroeconomic Models: A Hitchhiker's Guide

Prepared by Evan C.Tanner*

***IMF Working Papers* describe research in progress by the author(s) and are published to elicit comments and to encourage debate.** The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

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Institute for Capacity Development

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A Hitchhiker's Guide**

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Authorized for distribution by Laura Kodres

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Abstract

Simple macroeconomic frameworks like the IS/LM have survived because they help us conceptualize complex problems while also providing 'back of the envelope' estimates of macroeconomic outcomes. Herein, a bare-bones New Keynesian extension of the IS/LM model yields solutions for core macro variables (output gap, inflation, interest rate, real exchange rate misvaluation)—expressed in *percent*. We then extend that standard model to also generate a corresponding set of demand-side elements—expressed in *currency units*. A key aim of the paper is to reconcile these two metrics in ways that also aid communication and intuition—including through IS/LM-style graphs.

JEL Classification Numbers: A22, E12, E27

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I. Introduction

Paul Krugman (2000) argued that traditional, static macroeconomic models—the IS/LM or one of its heirs—perform some tasks better than more recent models that are based on dynamic intertemporal optimization precisely *because* they are so simple. More recently, Olivier Blanchard (2009) made a similar point when proposed the "re-legalization of shortcuts and of simple models..." He noted that "Approximating complex relations by simple ones helps intuition and communication." In this vein, Romer (2016) also argued that simpler models in macroeconomics can be superior to more complex ones for many of these same reasons.

Precisely what functions do such models fulfill? As an entry point for novice economists, including undergraduate students, the IS/LM remains the preferred teaching vehicle. For applied professionals, as Krugman noted, such a framework often serves as the default 'mental model' to guide policy discussions.

But, simple static models can do even more. A bare-bones version of the New Keynesian model—direct heir to the IS/LM—can generate equilibrium values of standard 'core' macroeconomic variables: the output gap, the interest rate, the inflation rate, and the real exchange rate (percent deviation from norm). Such an extended 'back-of-the-envelope' calculation can serve as a cross-check against results from more complex methods.

By extension, we can also obtain the corresponding *expenditure components* of output— consumption, investment, government, and net exports—expressed in *currency units*—Dollars, Euros, Pesos, and so on. Doing so helps us answer questions like "To what extent is investment crowded out?" or "Is the net export deficit too high?" But, some algebraic rescaling is required to reconcile these currency unit flows with the core variables which are typically measured in *percent*.

This paper examines whether such a rescaling can be done in a way that aids our intuition— in support of Blanchard's goal. One idea discussed herein is to recast simple linear expenditure equations (i.e. a Keynesian consumption function) in terms of potential output and the real rate of interest relative to some neutral rate. Recasting familiar equations in this way should help illustrate short-run and long-run economic relationships.

This paper does not present any new model *per se*. Instead, it suggests a way that different parts of previously existing (and quite standard) models can correspond with one another.[1]

This paper was written with two key audiences in mind. First, applied macroeconomists should find the framework useful in their day-to-day assessments. The methods are especially useful for comparing alternative scenarios for one or several periods. The open-economy model illustrated herein would be especially useful for a financial programming exercise—scenarios which fully illustrate prospective values for both expenditure and

[1] The paper's title draws upon a literary reference. In Douglas Adams' (1979) novel "The Hitchhiker's Guide to the Galaxy," creatures from different planets can communicate with one another if they implant in their ear an animal that translates different languages – a "Babel fish." Hence the analogy: we seek a way for different units in the macroeconomic galaxy to speak with one another – a macroeconomic "Babel fish."

financing flows. This idea is not new: Khan (1987) argued for the inclusion of a model within the International Monetary Fund's financial programming exercise as a way to assess both economic risks and policies required to eliminate macroeconomic imbalances. [2]

Second, instructors of macroeconomics at the intermediate level should find the methods discussed herein transparent enough to convey to their student—perhaps as a supplement to their current text. For both audiences, the author has developed companion spreadsheets that are available on line.[3]

The model herein does *not* address some fundamental critiques that have been levied against Keynesian macroeconomic models (see for example Chari, Kehoe, and McGrattan. 2009): expectations are exogenous and short-run relationships are not explicitly derived from an optimizing model. However, as discussed in the final section, the model discussed herein has substantial advantages in terms of transparency and communication for policy makers, while still conveying some important microeconomic relationships.

The paper is organized as follows. In Part II, we begin with the case of the closed economy, developing and re-interpreting the standard linear expenditure equations: consumption, investment, and government spending. In Part III, we re-derive the core macroeconomic variables and their equilibrium counterparts on the expenditure side. In Part IV, we extend the model to the open economy. In Part V, we discuss how the long-run expenditures shares in such a model might be determined by appealing to microeconomic reasoning. In Part VI, we extend the model to match up the traditional currency unit equations with those expressed in natural logarithms or percentage changes; such a procedure was used in a module to help teach financial programming and policies (FPP) that was developed by the International Monetary Fund's Institute for Capacity Development (IMF/ICD) (see also the Appendix). Part VII concludes.

II. Expenditures in the Closed Economy

We begin with several linear equations that describe expenditures in a closed economy. Consumption C_t is described by a standard (Keynesian) consumption function:

$$C_t = a_{C0} + a_{CY}(Y_t - T_t) \tag{1a}$$

where Y_t denotes output, T_t denotes tax revenue taken by the government and a_{CY} is the marginal propensity to consume out of disposable income $Y_t - T_t$, $0 < a_{CY} < 1$.

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[2] Ordinarily, the financial programming exercise is conducted over a one-year horizon. For this reason, the model developed in this paper has no lags – but could be extended to include them.

[3] The spreadsheet corresponding to the graphs and numbers in this paper is available online at: http://www.evanctanner.com/simple-models. A full textbook may be found online at: http://www.evanctanner.com/textbook-macro. Other papers that propose extensions of the traditional IS/LM framework for the undergraduate classroom include, Romer (2000), Walsh (2002), Weerapana (2003), and Carlin and Soskice (2005).

We assume that tax revenue itself is a constant fraction of output τ plus a temporary lump sum component "tax policy" component, TP_t:

$$T_t = \tau Y_t + TP_t \tag{1b}$$

Thus the consumption function is now:

$$C_t = a_{C0} + a_{CY}(Y_t(1-\tau) - TP_t) \tag{1c}$$

Next, the level of investment I_t is a function the real interest rate r_t:

$$I_t = a_{I0} + a_{Ir} r_t \tag{2}$$

where $a_{Ir} < 0$.

Finally, government expenditure is some normal level (a constant), namely a_{G0} plus a "government policy" shock, GP_t:

$$G_t = a_{G0} + GP_t \tag{3}$$

In this example we assume government spending to be essentially useless. However, that assumption is not essential and may be modified in an extension of the model.

In the closed economy, output equals the sum of these three components:

$$Y_t\big|_{Closed} = C_t + I_t + G_t \tag{4}$$

A. Consumption: A More Revealing View

Equations (1a-c) summarize the tried-and-true consumption function that has appeared in many textbooks over the years. Unfortunately, such a formulation fails to bring out some of the important ideas that economists have developed to understand the household's consumption decision. For example, a life-cycle (LC) approach stresses that a rational household consumption will use some estimate of its long-run disposable income—not just its current value—when it decides how much to save and consume.

In a similar vein, a strict interpretation of the permanent income hypothesis (PIH, Friedman, 1957) suggests that households will refrain from changing their level of consumption when their income deviates from its permanent value on a transitory basis. However, there may be 'hand-to-mouth' or 'liquidity constrained' households whose consumption expenditures do vary with transitory income.

Fortunately, we can adapt that traditional consumption function to incorporate these ideas. As a first step, *we assume some level of potential output in any period*, Y^P. Second, to obtain proxy measures of permanent and transitory disposable income, we maintain the constant tax rate assumption and we add and subtract τY^P from (1b) to obtain:

$$T_t = \tau Y^P + \tau(Y_t - Y^P) + TP_t \tag{5}$$

Thus, the cyclical component of taxes would be the second term on the right hand side, $\tau(Y_t - Y^P)$.[4]

It will now be useful to introduce some deeper parameters that describe the consumption and savings behavior of households. Specifically, we assume that the economy's long-run and short-run (cyclical) savings rates before taxes are σ and σ_{cyc}, respectively. The long-run savings parameter σ can be clearly traced back to a long-run growth model that includes taxes—either the Solow model or (as discussed below in Part VI) one based on intertemporal optimization. Also, without considering taxes, in the long-run, the average propensity to consume is $(1-\sigma)$. Note also that the pre-tax short-run marginal propensity to consume is $(1-\sigma_{cyc})$.

We may now reinterpret the coefficients a_{C0} and a_{CY} in equation (1a) in terms of these deeper parameters. Looking first at short-run behavior, a_{CY} retains its interpretation from traditional Keynesian models: $a_{CY} = (1-\sigma_{cyc})$. What about a_{C0}? This coefficient is traditionally interpreted superficially as a constant level of consumption. Below, we will show that a_{C0} also has a 'deeper' interpretation, namely:

$$a_{C0} = Y^P * [(1-\sigma) - (1-\sigma_{cyc})](1-\tau) \tag{1d}$$

Why? To see why this must be so, we must add and subtract the term $a_{CY}(1-\tau)Y^P$ from the right-hand side of consumption function (1c) and rearrange to obtain:

$$C_t = \tilde{a}_{C0} + a_{CY}\{[(1-\tau)(Y_t - Y^P)] - TP_t\} \tag{6a}$$

where $\tilde{a}_{C0} = a_{C0} + a_{CY}(1-\tau)Y^P$. We see that a_{C0} and $\tilde{a}_{C0}$ each include the term $(1-\sigma_{cyc})$ (the short-run marginal propensity to consume)—but with opposite signs which cancel one

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[4] This simple version of the model does not include automatic stabilizers. To do so is easy: we just assume a cyclical tax rate τ_{cyc} that is negative. In this case, the second term in equation (5) becomes $\tau_{cyc}(Y_t - Y^P)$ whose positive value when output is below potential tells us the size of government safety-net transfers.

another out. Thus, we may write $\tilde{a}_{C0}$ in an even more compact way, namely: $\tilde{a}_{C0} = Y^P * (1-\sigma)(1-\tau) = Y^P * (1-\tilde{\sigma})$.

Thus, linear equation (1) is now reinterpreted as:

$$C_t = \underbrace{(1-\tilde{\sigma})Y^P}_{\substack{\text{long-run} \\ \text{component}}} + (1-\sigma_{cyc})\underbrace{\{[(1-\tau)(Y_t - Y^P)] - TP_t\}}_{\substack{\text{short-run} \\ \text{component}}} \tag{6b}$$

where $(1-\tilde{\sigma}) = (1-\sigma)*(1-\tau)$. The interpretational advantages of (6b) over (1c) should be readily apparent. In both equations, the first term on the right-hand side is a constant. In the rescaled equation (6b), that constant *explicitly* informs us about household consumption and disposable income in the long run. The parameter $\tilde{\sigma}$ may be thought of as the long-run savings rate but now adjusted for taxes. Such a parameter can be clearly traced back to a long-run growth model that includes taxes—either the Solow model or (as discussed below in Part VI) one based on intertemporal optimization.

The second term on the right-hand side tells us the short-run or cyclical component of consumption—including the effect of one-off tax measures. According to a strict interpretation of the permanent income hypothesis, consumers save the entirety of any temporary windfall: under this interpretation, σ_{cyc} would be unity. However, under a less stringent interpretation, there are some hand-to-mouth (or liquidity constrained consumers). Therefore, under a less strict reading of the PIH, $0 < \sigma_{cyc} < 1$. Put differently $(1-\sigma_{cyc})$ is simply an alternative name for the marginal propensity to consume out of transitory income.

Likewise, equation (6b) can also accommodate a continuum of views on the Ricardian Equivalence Hypothesis (REH). Since TP_t is defined as a one-off tax change, all else equal, a strict reading of the REH would also imply $(1-\sigma_{cyc})$ =0.

B. Rescaling Investment and Government Spending

Consider next a reinterpretation of the investment function (2). We assume that there is a natural (or neutral) real rate of interest $\bar{r}$ which, in the absence of any other shocks to the economy, yields a zero output gap. We may interpret $\bar{r}$ as an interest rate which would hold in a steady state: the marginal product of capital at a steady state minus the depreciation rate. *Also, as we confirm in Part V. A below,* in a model based on intertemporal optimization by a representative consumer, this steady-state interest rate approaches the subjective rate of time preference. We may subtract and add $a_{Ir}\bar{r}$ to the right-hand side of that equation and rearrange to obtain:

$$I_t = \tilde{a}_{I0} + a_{Ir}(r_t - \bar{r}) \tag{7}$$

where $\tilde{a}_{I0} = a_{I0} + a_{Ir}\bar{r}$. Again, it will be convenient to reinterpret $\tilde{a}_{I0}$ in terms of potential output. In a closed economy, investment exactly equals saving, therefore then $\tilde{a}_{I0}$ must equal

$\psi Y^P = \sigma * (1-\tau)Y^P$. Note that ψ, the steady state investment ratio, corresponds to depreciation on the steady state capital stock; see section V. A, equation (41) for further details. Likewise, the national income identity requires that the constant in the government expenditure equation be: $a_{G0} = \tau Y^P$.

C. Deriving the IS Curve—A Rescaled Approach.

In most textbooks, the IS curve is derived in levels by substituting equations (1c), (2) and (3) into identity (4) and solving for the equilibrium level of output (in currency units) as a function of the interest rate (negative) and fiscal policy shocks. In the rescaled approach suggested here, we may instead obtain the IS curve whose output and the interest rate are respectively measured as percentage gaps from potential output and the neutral interest rate, and the fiscal shocks are measured as a percent of potential.

That is, by substituting (6b), (7) and (3) into (4) and the rearranging, we obtain:

$$Y_t = Y^P * [1 + (1-\sigma_{cyc})\{[(1-\tau)(gap_t)] - tp_t\} + \varphi_{Ir}(r_t - \bar{r}) + gp_t] \qquad (8)$$

where $gap_t = Y_t / Y^P - 1$, $\varphi_{Ir} = a_{Ir} / Y^P$ is a response parameter that is scaled to potential output and both fiscal shocks are also expressed as a percent of potential: $tp_t = TP_t / Y^P$ and $gp_t = GP_t / Y^P$. Note that the first term inside the brackets must be unity—the sum of the long run ratios to GDP of household consumption, investment, and government spending: namely [$(1-\sigma)(1-\tau) + \sigma(1-\tau) + \tau$ =1]. The framework also conveniently brings out the one-to-one correspondence of the output gap on the expenditure side. That is, ratios of consumption, investment, and government spending to GDP *minus* their long-run values are equal to $(1-\sigma_{cyc})\{[(1-\tau)(gap_t)] - tp_t\}$, $\varphi_{Ir}(r_t - \bar{r})$ and gp_t respectively—the right most terms on the right-hand side of (8). *These terms sum up to the output gap*.

We then subtract and divide both sides of the equation by potential output Y^P and fully solve to obtain an expression for the *output gap* IS curve:

$$gap_t \Big|_{IS,\text{Closed}} = \frac{\varphi_{Ir}(r_t - \bar{r}) + gp_t - (1-\sigma_{cyc})tp_t}{\tilde{\sigma}_{cyc}} \qquad (9)$$

where $\tilde{\sigma}_{cyc} = 1 - (1-\sigma_{cyc})(1-\tau)$; that is $1/\tilde{\sigma}_{cyc}$, is the familiar Keynesian multiplier for a closed economy. Note that $\varphi_{Ir} / \tilde{\sigma}_{cyc} < 0$. This ensures that the IS curve will have its familiar negative slope. The equation also confirms that the IS curve will shift to the right when there is a *fiscal expansion*, that is when $gp_t - (1-\sigma_{cyc})tp_t > 0$.

III. Equilibrium in a Closed Economy: Core and Expenditure Components

A New Keynesian model for a closed economy consists of three core equations: the IS curve, a monetary policy reaction function, and an aggregate supply or Phillips curve relationship. These three variables yield three equilibrium core values—the output gap, the interest rate (real and nominal), and the inflation rate.

The monetary policy reaction function is typically phrased along the lines of John Taylor's (1993) interest rate rule for central banks, which may be written as:

$$i_t\big|_{Closed} = \overline{r} + \pi^e + \beta_\pi(\pi_t - \pi^*) + \beta_{gap} gap_t + r_t^{DISC} \qquad (10)$$

where i_t is the nominal (policy) interest rate, π^e is the market's expectation of inflation, π_t is the realized rate of inflation, π^* is the central bank's target rate of inflation, and r_t^{DISC} captures any discretionary deviation of the central bank from the previous elements of the equation. The interpretation of this equation is an appealing one: the central bank has a *dual mandate*—to stabilize both prices and output.

In a similar vein, the inflation rate is determined by the following simple Phillips-curve relationship:

$$\pi_t = \pi^e + \frac{1}{\eta}(gap_t - ss_t) \qquad (11)$$

where $\eta > 0$ is the elasticity of aggregate supply in the short run (assuming some nominal rigidity in prices) and ss_t is a supply shock—a perturbation of the aggregate supply curve, measured in percent of potential output. The term inside the parenthesis on the right hand side may be thought of as a marginal cost term. [5]

[5] To see this interpretation, consider the following expression for the quantity of goods and services supplied in the short run: $Y_t^S = Y^P[1+\eta(P_t - P_t^e) + ss_t]$, where Y_t^S = the quantity of output supplied at time t, P_t is the price level at time t, P_t^e is the expected price level at time t (as of t-1), $\eta \geq 0$ is the short-run elasticity of supply with respect to the price level, and ss_t is a supply shock (in percent of potential output). That is, $ss_t > 0$ may be thought of as a reduction in the marginal cost of production—a level shift. Next, normalize the price level in the previous period P_{t-1} to unity and add and subtract one from the term inside brackets to obtain: $Y_t^S = Y^P[1+\eta(\pi_t - \pi_t^e) + ss_t]$. Finally, after dividing both sides through by potential output and subtracting one from both sides, we note that $gap_t = Y_t^S / Y^P - 1$. By inverting that expression, we obtain the Phillips Curve in the text. That expression contains some key ideas from standard literature in macroeconomics. The term $gap_t - ss_t$ on the right hand side of the Phillips curve tells us that prices rise when *marginal costs* rise—an idea that is central in the New Keynesian Phillips Curve literature (see for example Walsh, 2010, p. 381). By itself, the term ss_t corresponds to the 'cost push shock' discussed by Clarida, Galí, and Gertler

(continued…)

To keep the model as simple as possible, inflation expectations π^e are assumed to be determined exogenously. This assumption permits us to calculate the output gap in a way that uses elementary algebra. First, note that the IS curve (9) may be flipped over to show the real interest rate rather than the output gap on the left-hand side:

$$r_t\Big|_{IS,Closed} = \bar{r} + \frac{gap_t * \tilde{\sigma}_{cyc} - gp_t + (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} \tag{12}$$

We then obtain the equilibrium real interest rate from monetary policy and the supply side by combining equations (10) and (11). The first step is to substitute in Phillips curve equation (11) into Taylor rule curve equation (10) to obtain a reduced-form expression for the nominal interest rate. This substitution has an appealing interpretation: the Phillips Curve (11) poses a *constraint* for the central bank whose goal is to stabilize prices and output. Then, subtract off (11) from this term to obtain an expression for the real interest rate consistent with both the central bank reaction function and the Phillips curve—the "RR" schedule:

$$r_t\Big|_{RR,Closed} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t - b_{RRss}ss_t + r_t^{DISC} \tag{13}$$

where $b_{RR\pi} = \beta_\pi$, $b_{RRgap} = \dfrac{(\beta_\pi - 1)}{\eta} + \beta_{gap}$, and $b_{RRss} = \dfrac{(\beta_\pi - 1)}{\eta}$.

Note that IS and RR curves are both in output gap/interest rate space. A standard graphical depiction is shown in Figure 2 (supplementary material is shown in Figure 1).

The equilibrium output gap is now obtained by equating combining the IS and RR curves:

$$gap_t^{eq}\Big|_{Closed} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + \dfrac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\dfrac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]} \tag{14}$$

That is, equation (14) is the intersection of the (red) IS and (blue) RR curves in Figure 2 (top portion). We then obtain the two remaining core variables—the equilibrium real interest rate and inflation rate, r_t^{eq} and π_t^{eq} —by substituting in gap_t^{eq} along with the other exogenous variables, including π^e, ss_t, gp_t, tp_t, and r_t^{DISC} into equations (13) and (11) respectively.

(1998). Deviations of π_t from π^e may be interpreted in (at least) two ways. Agents may face a signal extraction problem (Lucas, 1973) or may exhibit bounded rationality (Gali, Lopez-Salido, and Valles, 2004). Alternatively, price adjustment may sticky, along the lines of Calvo (1983). In this case, we reinterpret π^e as $\kappa\bar{\pi}$, where $\bar{\pi}$ is the long-run rate of inflation consistent with monetary neutrality.

(continued…)

The equilibrium nominal interest rate is computed as $i_t^{eq} = r_t^{eq} + \pi_t^{eq}$ —consistent with other equations in the model. Coefficient values shown in the example may be said to be *coherent* in the sense that movements in r_t^{DISC}, the nominal interest rate, and the real interest rate are all in the same direction. [6] That is, an exogenous rise in r_t^{DISC} will be only partially offset by reductions in the interest rate—those that are induced by changes in the output gap and inflation. Likewise, to obtain the expenditure side components in real currency units (Dollars), results for gap_t^{eq} and r_t^{eq} are inserted into equations (6b), (7) and (3) alongside the assumptions regarding tax and expenditure policy, gp_t and tp_t respectively.

Examples of all calculations are presented in Figure 1 (corresponding charts in Figure 2). (The spreadsheet that generated the table and graphs is available online at http://www.evanctanner.com/simple-models). The top section of Figure 1 shows assumed shocks for three scenarios. Under the baseline, there are no shocks. Under alternative scenario (i) government spending expands by 1 percent of potential output but the monetary authority remains on its initial Taylor rule. Alternative scenario (ii) retains that 1 percent fiscal expansion but assumes in addition that the monetary authority accommodates by setting an interest rate that is 1 percent lower than the Taylor rule: r_t^{DISC} = -1.0 percent. The impacts of shocks on the output gap are shown in the lower 2/3rd of the figure. Under alt(i), we see that the government spending multiplier which takes into account the endogenous response of monetary policy is 0.7 (the fiscal component of output gap is 0.7 percent in response to the initial 1 percent shock). Under alt(ii), the fiscal component of the shock is equal to alt(i), but the effect of the monetary loosening—a movement *along* the IS curve—is an additional 0.4 percent. Thus, the output gap under is 0.7 percent under alt(i) but 1.1 percent under alt(ii).

[6] The necessary and sufficient conditions for such coherence are: $[\frac{b_{RRgap}}{den} + 1] > 0$ for the real interest rate and $\{\frac{b_{\pi}}{\eta * den} + \frac{b_{gap}}{den} + 1\} > 0$ for the nominal interest rate, where $den = \left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]$ is the denominator term in equation (14).

Figure 1: Closed Economy Model: Shocks and Output Gap

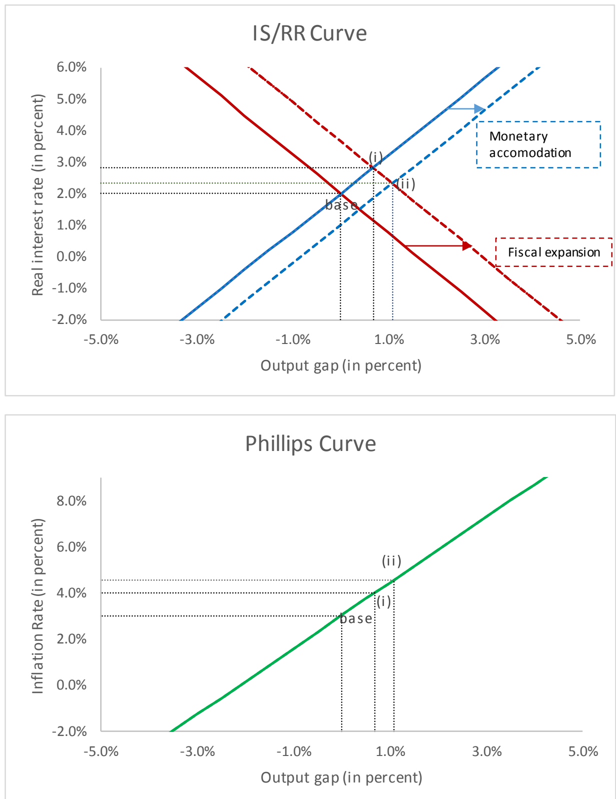
		Shocks -- Expenditure		
In percent of potential output		base	alt(i)	alt(ii)
Gov't Spending	$\boxed{gp_t}$	0.0%	1.0%	1.0%
Tax Measures (one-off)	$\boxed{tp_t}$	0.0%	0.0%	0.0%
In percent		Shocks - Supply / Expected inflation		
Supply shock (% of Y^P)	$\boxed{ss_t}$	0.0%	0.0%	0.0%
Inflation expectations (gap w.r.t. target)	$\boxed{\pi^e - \pi^*}$	0.0%	0.0%	0.0%
In percent		Shocks - Discretionary monetary policy		
Deviation from Taylor Rule (shift)	$\boxed{r_t^{DISC}}$	0.0%	0.0%	-1.0%

Calculation of equlibrium output gap -- component by component

	base	alt(i)	alt(ii)
(a) Inflation expectations component $\boxed{\text{inflation expectations component} = \frac{b_{RR\pi}(\pi^e - \pi^*)}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}}$	0.0%	0.0%	0.0%
(b) Supply shock component $\boxed{\text{supply shock component} = \frac{b_{RRss} ss_t}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}}$	0.0%	0.0%	0.0%
(c) Fiscal component $\boxed{\text{fiscal component} = \frac{\frac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}}$	0.0%	0.7%	0.7%
(d) Discretionary monetary component $\boxed{\text{discretionary monetary policy} = \frac{r_t^{DISC}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}}$	0.0%	0.0%	0.4%
Output gap (a)-(b)+(c)+(d) $\boxed{gap_t^{eq}\big\vert_{Closed} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss} ss_t + \frac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\frac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}}$	0.0%	0.7%	1.1%

Source: Author's calculations.

Figure 2: Closed Economy Model: Graphics, Core Variables and Expenditure Components



alt(i) – fiscal expansion, central bank remains on Taylor Rule

alt(ii) – fiscal expansion, monetary accommodation discretionary loosening off Taylor Rule.

	base	alt(i)	alt(ii)
Output gap (a)-(b)+(c)+(d)	0.0%	0.7%	1.1%
$gap_t^{eq}\Big\rvert_{Closed} = \dfrac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss}ss_t + \dfrac{gp_t - (1-\sigma_{cyc})tp_t}{\varphi_{Ir}} + r_t^{DISC}}{\left[\dfrac{\tilde{\sigma}_{cyc}}{\varphi_{Ir}} - b_{RRgap}\right]}$			
Equilibrium inflation rate			
$\pi_t = \pi^e + \dfrac{1}{\eta}(gap_t^{eq} - ss_t)$	3.0%	4.0%	4.6%
Equilibrium real interest rate			
$r_t^{eq}\Big\rvert_{RR,Closed} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap}gap_t^{eq} - b_{RRss}ss_t + r_t^{DISC}$	2.0%	2.8%	2.3%
Currency-unit results			
Gross Domestic Product	16.79	16.90	16.97
Consumption	11.21	11.24	11.26
Investment	3.06	2.97	3.03
Government Spending	2.52	2.69	2.69
Demand side decomposition of output gap (percent of potential)			
Output gap	0.0%	0.7%	1.1%
Consumption	0.0%	0.2%	0.3%
Investment	0.0%	-0.5%	-0.2%
Government Spending	0.0%	1.0%	1.0%

Source: Author’s calculations.

Figure 2 shows the graphical representation of these scenarios. The upper diagram shows the IS and RR schedules in output gap/real interest rate space. Under alt (i) the downward sloping red line shifts to the right, from the solid to the dotted/dashed line, along the RR curve, which remains in its original position. Thus, we see the joint increase of output and the real interest rate under this scenario. Under alt(ii), we see no further shift of the IS curve but the RR curve now shifts to the right. Accordingly, output under alt(ii) is higher but the interest rate is lower than under alt(i). Directly below, we see the Phillips Curve outcome: inflation rises under alt(i) and higher still under alt(ii)—as reflected in movements along the green PC schedule.

Under the graphs, a table shows the results for both the core variables and the expenditure components. The bottommost table shows the contribution to the output gap by expenditure component. For example, under alt(i), we see that when government spending rises by 1 percent, there is an induced increase of consumption of 0.2 percent but higher interest rates help crowd-out investment by 0.5 percent. Under alt(ii), since interest rates are lower, the investment crowding-out component drops to 0.2 percent while consumption rises even further—an additional induced increase of 0.1 percent above alt(i).

IV. The Small Open Economy: Exports, Imports, and External Financial Pressures

The framework may be extended to show the impacts that internal and external shocks will have on a small open economy—including core variables, the real exchange rate, and net exports. The first step is to expand the goods market to include both exports and imports. Thus, the output identity in an open economy rewritten:

$$Y_t\big|_{Open} = C_t + I_t + G_t + X_t - IM_t \tag{15}$$

where X_t and IM_t denote exports and imports of goods and services, respectively.

However, there will also be trade in assets. Here we assume the economy to be small relative to the rest of the world. It faces an external rate of interest (including a risk premium) that it has no influence over. Importantly, the model can help us understand in a straightforward way how externally-based shocks, in addition to domestic shocks, will affect the economy's short-run equilibrium. As illustrated in Figure 3, such effects will be transmitted through to the markets for exports and imports.

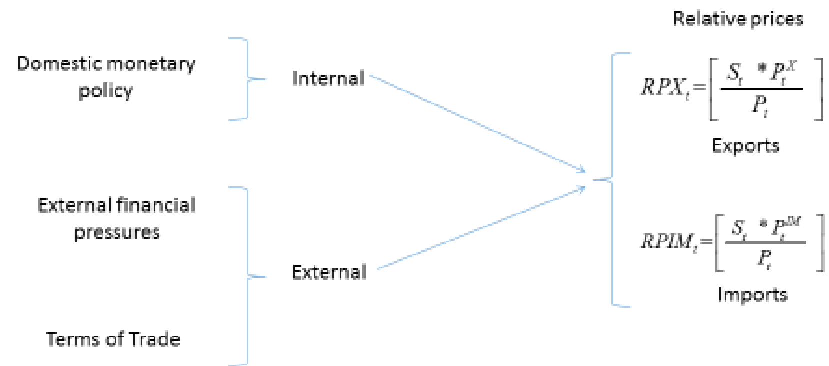
A. Exports, Imports, and the Trade Balance

The level of exports is assumed to comprise two elements: a long-run component, which is expressed as a constant fraction of potential output, and a short-run component which is linked to deviations of the relative price of exports from its long run norm:

$$X_t = Y^P[x + \eta_x * rpx_t] \tag{16}$$

where the share term x is determined by long-run external prices and productivity levels in the export and non-traded goods sectors, $\eta_x > 0$ is a response function and rpx_t is the percent deviation of the relative price of exports RPX_t from some long run norm which is defined below.

Figure 3: Internal and External Factors that *Directly* Impact the Relative Prices of Exports and Imports



That is, the relative price of exports is defined as the world currency price of exports P_t^X converted to domestic currency by the nominal exchange rate S_t (domestic currency units per unit of foreign currency—appreciation minus), and divided by domestic price level, P_t:

$$RPX_t = \left[\frac{S_t \; * P_t^X}{P_t} \right] \tag{17}$$

We may thus think of RPX_t as a real exchange rate that applies specifically to exporters. We define a baseline value of RPX_t namely $\overline{RPX}$ which we normalize to unity. Thus, $rpx_t \approx \ln(RPX_t)$. The conditions required for the baseline value to obtain are discussed below.

The corresponding equation for imports is structured much like that of exports but also includes a term for the output gap:

$$IM_t = Y_t^P [im + im_{cyc} * gap_t + \eta_{im} * rpim_t] \tag{18}$$

where the share term im is determined by long-run external prices and productive capacity of the economy, im_{cyc} is a short-run marginal propensity to import ($im_{cyc} > 0$), $\eta_{im} < 0$ is a response function and $rpim_t$ is the percent deviation of the relative price of imports $RPIM_t$ from some long run norm. Symmetric with exports, that price ratio is defined as:

$$RPIM_t = \left[\frac{S_t \; * P_t^{IM}}{P_t} \right] \tag{19}$$

Here, we may think of $RPIM_t$ as a real exchange rate that applies specifically to import markets. Again, the baseline value $\overline{RPIM}$, which is discussed below, is normalized to unity. Thus, $rpim_t \approx \ln(RPIM_t)$.

B. The Open Economy IS Curve

An open economy is vulnerable to shocks that originate externally. Such shocks will have first order impacts on exports and imports. The model highlights the fact that shocks—both domestic and external—are transmitted to the economy through their impacts on the relative prices of imports and exports, as summarized in Figure 3. It will be shown that the real prices of exports and imports attain their reference or baseline values, $\overline{RPX}$ and $\overline{RPIM}$ respectively, when the domestic and foreign real interest rates and the terms of trade are at *their* baseline values.

To help model such relationships in an intuitive way, it will be useful first to recall that the traditional definition of the real exchange—a key price in any open economy model is:

$$Q_t = \left[\frac{S_t \; * P_t^{EXT}}{P_t} \right] \tag{20}$$

where P_t^{EXT} is the level of *external* prices. As Hinkle and Nsengiyumva (1999), Dabos and Juan-Ramón (2000) and others have pointed out, that price level may be written as a (geometric) weighted average of export and import prices, namely: [7]

$$P_t^{EXT} \equiv (P_t^X)^{\nu} (P_t^{IM})^{(1-\nu)} \tag{21}$$

[7] Note that this is appropriate for an open economy model with three goods: exports, imports, and non-tradables. For further details, see Hinkle and Nsengiyumva (1999).

(continued…)

where ν and $(1-\nu)$ are the relative importance of a country's exports and imports, respectively, the overall external price level. [8]

As an intermediate step, the natural logarithm of the real exchange rate may be written as $q_t = \ln(S_t P_t^{EXT} / P_t) = \ln[S_t (P_t^X)^{\nu} (P_t^{IM})^{(1-\nu)} / P_t]$. Then, we may manipulate this expression to show that that the relative prices of exports and imports are compound functions of the real exchange rate and the (appropriately scaled) terms of trade, $TT_t = P_t^X / P_t^{IM}$, namely:

$$\underset{\substack{\text{Relative price}\\ \text{of exports}}}{rpx_t} = \underset{\text{Real exchange rate}}{q_t} + \underset{\substack{\text{Scaled External}\\ \text{Terms of Trade}}}{(1-\nu)\ln(TT_t)} \tag{22}$$

$$\underset{\substack{\text{Relative price}\\ \text{of imports}}}{rpim_t} = \underset{\text{Real exchange rate}}{q_t} - \underset{\substack{\text{Scaled External}\\ \text{Terms of Trade}}}{\nu\ln(TT_t)} \tag{23}$$

That is, the right hand sides of (22) and (23) show that the relative prices of exports and imports each include two components: the real exchange rate and a scaled terms-of-trade component. Below, we assume that there is a long-run or baseline value for the external terms of trade, namely $\overline{TT}$ which is normalized to unity (thus $\ln(\overline{TT}) = 0$. (For further discussion of this decomposition, see Dabos and Juan-Ramón, 2000; see also Clarida, 2009, for a discussion of terms-of-trade impacts.)

Next, we may extend this decomposition to distinguish between domestic and externally based components—the decomposition shown in Figure 3. We do this element-by-element. First, as discussed below, the real exchange rate is determined by a real interest rate parity condition that is adjusted for a country-specific risk premium. Movements in the real exchange rate correspond one-to-one with movements in the risk-adjusted real interest differential. Such movements come from two sources: domestic monetary policy and external financial conditions (foreign monetary policy/risk premium). Second, changes in the external terms of trade are determined entirely in world markets. [9]

We are also now in a position to establish the baseline values for the relative prices of exports and imports as discussed above. A key to this baseline lies in the definition of the natural rate of interest, which is now assumed to be the sum of the natural external rate of interest $\overline{r}^{EXT}$ plus a baseline risk premium $\overline{rp}$. Importantly, the natural rate of interest is a steady-state construct. That is, the natural rate of interest is written $\overline{r} = \overline{r}^{EXT} + \overline{rp}$. *Hence the*

[8] Of course, the widely used measures of the real exchange rate may use familiar proxies for the external price level such as trading partner CPI or WPI. Such indices may also reflect movements in goods / services that are, in the strict sense, irrelevant for the economy in question. This model may be modified to incorporate such 'noise' elements with no change in interpretation.

[9] We ignore changes in export taxes and import tariffs. For a discussion of this issue see Connolly and Devereux (1992).

baseline corresponds to a steady state. In any economy, open or closed, the steady state natural interest rate converges to the steady state marginal product of capital net of depreciation, namely $mpk^{SS} - \delta$. In an open economy, $mpk^{SS} - \delta$ converges to an exogenous value, namely $\bar{r}^{EXT} + \overline{rp}$ (thorough net capital accumulation). *This issue is discussed further below, in Part V.b.*

We may discuss changes in foreign monetary and financial conditions as deviations from that steady state. Tighter foreign monetary policy implies that $r_t^{EXT} > \bar{r}^{EXT}$. An idiosyncratic revision to investor perceptions of a country will be reflected its risk premium: a capital-flight scenario would imply that $rp_t > \overline{rp}$. Jointly, external financial pressures efp_t reflects the divergence between the external interest rate plus risk premium from their baseline values: $efp_t = [r_t^{EXT} + rp_t] - [\bar{r}^{EXT} + \overline{rp}]$.

We next introduce the real interest parity condition that tells us the short run deviation of the real exchange rate from its long-run baseline value: $q_t = r_t^{EXT} + rp_t - r_t + \bar{q}$, $\bar{q} = 0$. (That is, $\exp(\bar{q}) = 1$.) This condition implies impacts of domestic monetary policy and external financial pressures on the real exchange rate that are symmetric: a domestic monetary tightening causes the real exchange rate to appreciate; this reduces relative prices of both exports and imports. By contrast, an increase in external financial pressures will bring about a depreciation of the real exchange rate; this increases relative prices of both exports and imports.

Thus, relative prices of exports and imports are determined by both domestic and external factors is shown in an exact decomposition *based solely on an identity* (no behavioral parameters) in the rightmost terms of equations (22') and (23'):

$$\underbrace{rpx_t}_{\text{Relative price of exports}} = \underbrace{q_t}_{\text{Real exchange rate}} + \underbrace{(1-\nu)\ln(TT_t)}_{\text{Scaled External Terms of Trade}} = \boxed{\underbrace{[\bar{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t + (1-\nu)\ln(TT_t)}_{\text{External shocks}}} \tag{22'}$$

$$\underbrace{rpim_t}_{\text{Relative price of imports}} = \underbrace{q_t}_{\text{Real exchange rate}} - \underbrace{\nu\ln(TT_t)}_{\text{Scaled External Terms of Trade}} = \boxed{\underbrace{[\bar{r} - r_t]}_{\text{Domestic monetary policy}} + \underbrace{efp_t - \nu\ln(TT_t)}_{\text{External shocks}}} \tag{23'}$$

Domestic monetary policy affects these relative prices through the real exchange rates. Externally, both external financial pressures and changes in the external terms-of-trade have impacts on these key relative prices.

Accordingly, export (supply) and import (demand) functions may be re-written as:

$$X_t = Y^P[x - \underbrace{\eta_x * (r_t - \bar{r})}_{\text{endogenous}} + \underbrace{\eta_x efp_t + \eta_x(1-\nu)\ln(TT_t)}_{\text{exogenous}}] \tag{16'}$$

$$IM_t = Y_t^P[im + \underbrace{im_{cyc} * gap_t - \eta_{im} * (r_t - \bar{r})}_{\text{endogenous}} + \underbrace{\eta_{im} efp_t - \eta_{im} \nu \ln(TT_t)}_{\text{exogenous}}] \qquad (18')$$

Both exports and imports will move either when domestic monetary policy changes (an endogenous factor in the model), when there are externally-based financial pressures (exogenous) or when the external terms of trade change (exogenous).

We may now develop the open economy IS curve by substituting domestic expenditure functions (6b), (7) and (3) and external sector equations (16 ') and (18 ') into the open economy GDP identity (15) and rearranging:

$$gap_t \Big|_{IS,\text{Open}} = \frac{(\varphi_{Ir} - \eta_{nx})(r_t - \bar{r}) + fp_t + \eta_{nx} efp_t + \tilde{\eta}_{nx} \ln(TT_t)}{\{1 - [(1 - \sigma_{cyc})(1 - \tau) + im_{cyc}]\}} \qquad (24)$$

where the compressed fiscal policy component is $fp_t = gp_t - (1 - \sigma_{cyc}) tp_t$ while compound parameters are $\eta_{nx} = \eta_x - \eta_{im}$ and $\tilde{\eta}_{nx} = \eta_x (1 - \nu) + \eta_{im} \nu$. Note that the terms-of-trade TT_t is shift term for the IS curve that appears only for the open economy. As might be expected, the IS curve will shift to the right if the terms of trade improve. To the extent that a country's export prices rise when foreign demand rises, TT_t thus reflects changes in foreign *demand*. However, TT_t is more comprehensive. It also captures third-party effects on the *supply* side such as a surge in production by competitor countries.

C. Monetary Policy and Inflation in the Open Economy

Regarding monetary policy in an open economy, since $\bar{r} = \bar{r}^{EXT} + \overline{rp}$, we add external financial pressures to the monetary reaction function:

$$i_t \Big|_{Open} = \bar{r} + \pi^e + b_\eta (\pi_t - \pi^*) + b_{gap} gap_t + efp_t + r_t^{DISC} \qquad (25)$$

The presence of efp_t in this equation implies that, all else equal, the interest rate set by the central bank will track developments in world financial markets. Assuming (as a special case) that nominal exchange rate depreciation equals the inflation rate, if the inflation expectations gap, the output gap, and discretionary policy are all zero, equation (25) would be interpreted as a traditional "International Fisher" equation.

As an alternative interpretation, by including efp_t in equation (25), the mandate of the central bank in an open economy is expanded compared to a closed economy. For example, $efp_t > 0$ may be thought of as a capital outflow scenario. In this case, the central bank will take measures to defend the exchange rate by raising the interest rate. Such a reaction suggests that the central bank in an open economy is concerned about balance sheet effects of exchange rate movements—in addition to price and output stabilization. Such an idea is frequently discussed in the literature on emerging market macroeconomic policy; for example, see Calvo and Reinhart's (2002) discussion of 'fear of floating'.

Note that, in *equilibrium* the interest rate will not move one-to-one with efp_t. Instead, an initial interest rate hike will be partially offset by an *induced* interest rate decline which reflects the central banks goal to stabilize both prices and output on the downside. (As shown below, this will be reflected in a leftward shift in the RR curve that is partially offset by a movement *along* that same curve).

Finally, inflation in the open economy differs from the closed economy expression insofar as a fraction θ of external financial pressures, which have impacts on the real exchange rate, are also passed through to the domestic economy:

$$\pi_t \Big|_{Open} = \pi^e + \frac{1}{\eta}(gap_t - ss_t) + \theta efp_t \tag{26}$$

That is, the term θefp_t may be thought of as exchange rate pass-through that reflects external financial market developments.

By combining the open economy monetary policy rule (25) and the Phillips curve (26) we obtain the equilibrium real rate of interest in the open economy:

$$r_t \Big|_{RR,Open} = \bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap} gap_t - b_{RRss} ss_t + b_{RRefp} efp_t + r_t^{DISC} \tag{27}$$

where the reduced form coefficient for external financial pressures is defined as $b_{RRefp} = [b_\pi - 1]\theta + 1$. This term will exceed unity: the central bank is assumed to raise interest rates both to defend the currency and to restrain exchange rate pass through to inflation.

D. Equilibrium in the Open Economy—Core Variables and the Trade Balance

To obtain equilibrium output in the small open economy, again flip IS curve (24) to obtain an expression for the real interest rate:

$$r_t \Big|_{IS,\text{Open}} = \bar{r} + \frac{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}]\} gap_t - [\, fp_t + \eta_{nx} efp_t + \tilde{\eta}_{nx} \ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} \tag{28}$$

We then solve for the gap by combining IS (28) and RR (27) curves for the open economy:

$$gap_t^{eq} \Big|_{Open} = \frac{b_{RR\pi}(\pi^e - \pi^*) - b_{RRss} ss_t + efp_t [b_{RRefp} + \dfrac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}] + \dfrac{[\, fp_t + \tilde{\eta}_{nx} \ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} + r_t^{DISC}}{\left[\dfrac{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}]\}}{(\varphi_{Ir} - \eta_{nx})} - b_{RRgap} \right]} \tag{29}$$

The term $(\varphi_{Ir} - \eta_{nx})$ has critical importance for adverse financial pressures efp_t—an increase in external interest rates or other external tightening—including 'sudden stops' of capital inflows discussed by Calvo (1998) and Blanchard, Ostry, Ghosh and Chamon (BOGC, 2015). By itself, φ_{Ir} <0 captures the expenditure *reducing* effects of an external financial tightening that is transmitted to domestic financial markets; this means lower output, consistent with both the Calvo and BOGC papers.[10] By contrast, η_{nx} >0 conveys an impact in the opposite direction; since the exchange rate depreciates, net exports increase; such an expenditure *switching* effect raises output. However, the assumption that $(\varphi_{Ir} - \eta_{nx})$ <0 ensures that the *net* effect of an external financial tightening will be to reduce (not increase) output.

Next, note that the denominator on the right-hand side is negative. This must be so since: $\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}$ >0, $(\varphi_{Ir} - \eta_{nx})$ <0 and b_{RRgap} >0. We can therefore say that an increase in inflationary expectations relative to the target $(\pi^e - \pi^*)$ >0 will reduce output, a favorable supply shock ss_t will increase output, while increases in domestic and foreign demand, fp_t and $\tilde{\eta}_{nx} \ln(TT_t)$ respectively, will each increase output. Importantly, an increase in external financial pressures will work through two channels that have opposite effects on output. The central bank's response to such pressures which are included in its reaction function will squeeze expenditures and reduce output. In the other direction, the currency depreciation effect, reflected in the term $[\eta_{nx}/(\varphi_{Ir} - \eta_{nx})] * efp_t$, will have a positive impact on output (a rightward shift of the IS curve). However, since the absolute value of $[\eta_{nx}/(\varphi_{Ir} - \eta_{nx})]$ must be less than one, the former effect dominates the latter. Finally, a discretionary interest rate hike will reduce output and cause the real exchange rate to appreciate.

As before, we obtain the two remaining core variables by substituting in gap_t^{eq} along with the other exogenous variables into equations (27) and (26)—the equilibrium real interest rate and inflation rate, r_t^{eq} and π_t^{eq} respectively. The equilibrium nominal interest rate is $i_t^{eq} = r_t^{eq} + \pi_t^{eq}$ —consistent with equation (25). As in the case of the closed economy, we assume coherence between movements in the discretionary component of monetary policy and those of equilibrium values for interest rates (real and nominal): they are all in the same direction. To obtain the real exchange rate, we incorporate the equilibrium real interest rate and assumptions regarding external financial pressures into the parity equation $q_t = r_t^{EXT} + rp_t - r_t^{eq} + \bar{q}$.

And, as before, to obtain the expenditure side components in real currency units (Dollars), results for gap_t^{eq} and r_t^{eq} are inserted into equations (6b), (7), (3), (16) and (18), alongside the assumptions regarding tax and expenditure policy, gp_t and tp_t respectively, external financial pressures, efp_t, and foreign *ne*t demand through the terms-of-trade effect, TT_t.

[10] Their model includes two interest rates: a policy rate and a market rate. It would be straightforward to extend this model to include a market interest rate as well.

The equilibrium trade balance, in percent of potential, is a function of the equilibrium output gap and real exchange rate (conditional on terms-of-trade shocks):

$$nx_t^{eq} = x - im + (\eta_x - \eta_{im})q_t^{eq} + [\eta_x * (1-\nu) + \eta_{im}\nu] * \ln(TT_t)] - im_{cyc} * gap_t^{eq} \qquad (30)$$

where the equilibrium real exchange rate is: $q_t^{eq} = r_t^{EXT} + rp_t - r_t^{eq} + \bar{q}$.

Examples of open economy calculations and their corresponding graphical exposition are presented in Figures 4-6. (The spreadsheet that generated the table and graphs is available online at http://www.evanctanner.com/simple-models).

The top section of Figure 4 shows assumed shocks for three scenarios. Under the baseline, there are no shocks. Under alternative scenario (i) there is an adverse shock to external demand—export prices fall by 4 percent from their baseline values. Alternative scenario (ii) retains that 4 percent export price decline but adds in a 400 basis point increase in the risk premium: efp_t =4 percent. For evidence that terms-of-trade and financial shocks are not orthogonal, consistent with scenario (ii), see Kuralbayeva and Vines (2008).

The impacts of these shocks on the output gap are shown in the lower portion of Figure 6 on a component-by-component basis. Under both alt(i) and alt(ii), the terms of trade shock reduce output by 0.55 percent. Under alt(ii), adverse external financial pressures further reduce output by another 0.98 percent.

The graphs in Figure 5 illustrate the channels of transmission for each of these shocks. By itself, the terms-of-trade shock in alt(i) shifts the IS curve (red line, uppermost chart) to the left, from solid to dotted. The central bank, consistent with its mandate and keeping on its Taylor Rule, reduces the interest rate—a movement along the RR curve downward and to the left. Thus, the initial impact of the adverse demand shock is softened by lower interest rates.

Unsurprisingly, there is lower inflation under scenario (i)—as reflected in a leftward movement along the solid green Phillips Curve line in the middle chart. The corresponding numerical solution, as shown in Figure 6 (upper portion) implies a fall in the equilibrium inflation rate from its initial target of 3 percent to 2.2 percent.

It is also not surprising that the net exports deteriorate but the real exchange rate depreciates under scenario (i). This is shown graphically in the bottommost chart of Figure 5. Here, net exports as a ratio to potential output are shown on the horizontal axis (leftward movement means higher deficit) while the real exchange rate is on the vertical axis (upward movement means currency depreciation).

Thus, the tan line reflects the relationship between the real exchange rate and net exports for given values of the output gap and the terms of trade; this is equation (30). That line slopes upward: when the currency depreciates, exports are encouraged, imports are discouraged, and the trade balance improves. An increase in the output gap means more imports (an income effect); this means that the curve will shift to the left. A reduction in the terms of trade means lower external demand—again a leftward shift of the tan line. Of course, a reduction of the terms of trade also reduces the output gap: demand decreases. This is reflected as second order effect—a partially offsetting shift of the tan line the right. However, the initial impact typically dominates: a fall in the terms of trade will mean a net shift of the tan line to the left.

Such an impact is shown for alternative (i). The terms-of-trade impact, which dominates the output impact, shifts the net export line to the left from the solid tan NX line to the dotted line: any given level of the real exchange rate, the net export deficit will now be higher. As an offsetting factor, the depreciation of the real exchange rate—the outcome of a monetary loosening—brings about an upward movement *along* the dotted tan line.

What happens to inflation under alternative (ii)? Adverse external financial pressures cause the currency to depreciate in real term—higher import prices. To some extent, these higher costs will be passed on to domestic purchasers—as the rightmost term in equation (26) (the open economy Phillips Curve) reveals. For this reason, the green line shifts up and to the left, from the solid to the dashed line. At the same time, the equilibrium output gap is even lower under alt(ii) than alt(i)—as reflected in a shift *along* that green dashed line. The calculation in Figure 5 confirms this: equilibrium inflation is now 1.2 percent (compared to 2.2 percent under alt(i)).

What happens to net exports under alternative (ii)? Because higher real interest rates at home further squeeze demand, equilibrium output is even lower under alt(ii) than alt(i). Import compression under alt(ii) means that the tan NX line now shifts to the right—from the dotted to the dashed line. At the same time, the real exchange rate depreciates more under alt(ii) than alt(i). (Without the central bank's defensive interest rate hike under alt(ii), the depreciation would have been even more severe.) This means a further shift along the dashed tan NX line.

Together, income and price effects bring about an improvement in the net export balance under alt(ii) relative to alt(i). But the net export improvement under alt(ii) should be interpreted as a *forced adjustment*—a capital outflow scenario that is accompanied by an even sharper reduction of output under alt(ii) than under alt(i).

The numbers in the bottom section of Figure 6 confirm the graphical analysis. In addition, the demand side contributions at the bottom of that figure show that, under a capital outflow scenario, the burden of adjustment falls mainly in investment (minus 1.4 percent of potential output) and also to some extent on consumption (minus 0.4 percent of potential output). This is consistent with the analyses of *capital outflows* found in both Calvo (1998) and Blanchard, Ostry, Ghosh, and Chamon (2015): the current account balance will improve but domestic expenditures will fall—enough so to bring down overall output.

Figure 4: Small Open Economy Model: Shocks and Output Gap Components

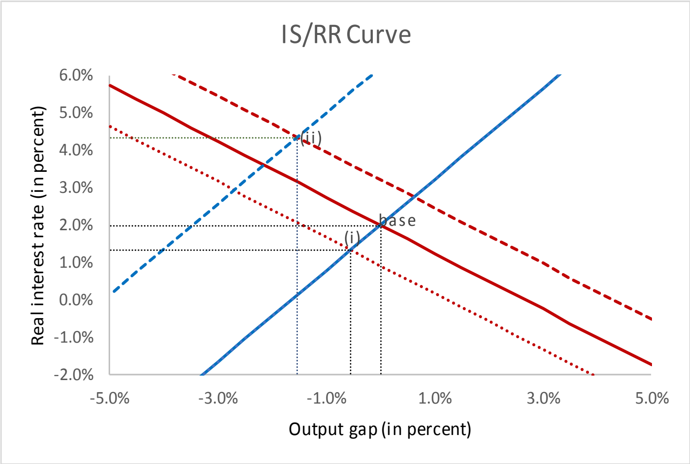
		Shocks -- Expenditure		
In percent of potential output		base	alt(i)	alt(ii)
Gov't Spending	gp_t	0.0%	0.0%	0.0%
Tax Measures (one-off)	tp_t	0.0%	0.0%	0.0%
In percent		Shocks - Supply / Expected inflation		
Supply shock (% of Y^P)	ss_t	0.0%	0.0%	0.0%
Inflation expectations (gap w.r.t. target)	$\pi^e - \pi^*$	0.0%	0.0%	0.0%
In percent		Shocks - Discretionary monetary policy		
Deviation from Taylor Rule (shift)	r_t^{DISC}	0.0%	0.0%	0.0%
In percent		Shocks - External financial pressures (efp)		
Total external financial pressures		0.0%	0.0%	4.0%
Real interest rate -- dev.from baseline	$r_t^{EXT} - \bar{r}^{EXT}$	0.0%	0.0%	0.0%
Risk premium -- dev.from baseline	$rp_t - \overline{rp}$	0.0%	0.0%	4.0%
In percent		Shocks - External Terms of Trade (ln(TT))		
TT log deviation from baseline		0.0%	-4.0%	-4.0%
Export price log dev		0.0%	-4.0%	-4.0%
Import price log dev		0.0%	0.0%	0.0%

Calculation of equlibrium output gap -- component by component

	base	alt(i)	alt(ii)
(a) Inflation expectations component			
$\text{Inflation expectations component} = \frac{b_{RR\pi}(\pi^e - \pi^*)}{den}$	0.0%	0.0%	0.0%
(b) Supply shock component			
$\text{Supply shock component} = \frac{b_{RRss} ss_t}{den}$	0.0%	0.0%	0.0%
(c) Fiscal component			
$\text{Fiscal component} = \frac{\frac{fp_t}{(\varphi_{Ir} - \eta_{nx})}}{den}$	0.0%	0.0%	0.0%
(d) Discretionary monetary component			
$\text{Discretionary monetary component} = \frac{r_t^{DISC}}{den}$	0.0%	0.0%	0.0%
(e) External financial pressure component			
$\text{External financial pressures component} = \frac{efp_t[b_{RRefp} + \frac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}]}{den}$	0.00%	0.00%	-0.98%
(f) External terms of trade component			
$\text{External terms of trade component} = \frac{\frac{\tilde{\eta}_{nx} \ln(TT_t)}{(\varphi_{Ir} - \eta_{nx})}}{den}$	0.00%	-0.55%	-0.55%

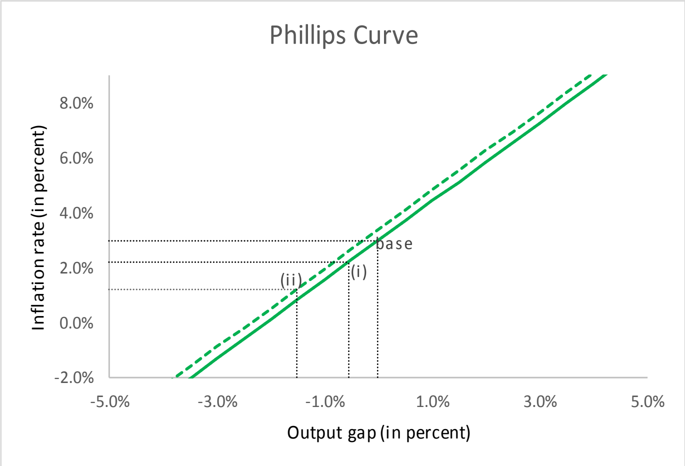
Source: Author's calculations.

Figure 5: Small Open Economy Model: Graphical Exposition: Core Variables, Net Exports, and Real Exchange Rate



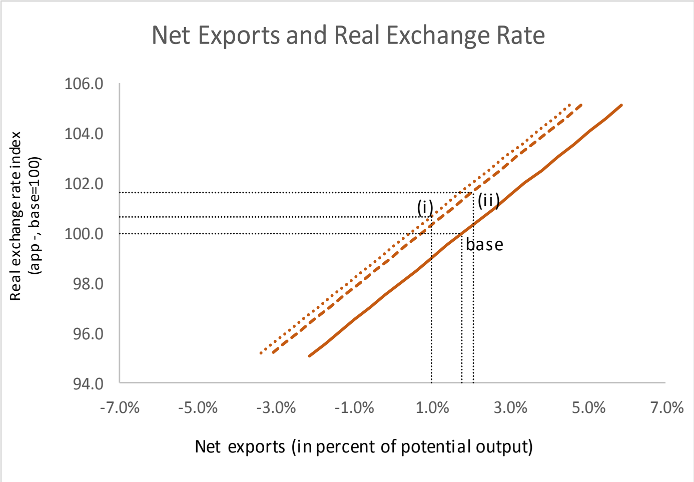
alt(i) – external terms of trade deterioration shifts IS curve to left (from solid to dotted) along solid RR curve.

alt(ii) – external financial pressures shift IS curve to right (dotted to dashed) but RR curve to the left (solid to dashed, exchange rate depreciation, interest rate defense)



alt(i) – external terms of trade deterioration reduces output gap and inflation – shift along original PC line.

alt(ii) – external financial pressures shift PC up and to the left (solid to dashed exchange rate depreciation passed through to inflation) but also further along dashed PC (output gap even lower).



alt(i) – external terms of trade deterioration dominates income effect, shifting NX curve to the left (solid to dotted, higher deficit); exchange rate depreciation implies upward shift *along* dotted tan NX line.

alt(ii) – external financial pressures imply lower output gap, rightward shift of tan NX line (from dotted to dashed, lower imports); further exchange rate depreciation means a further upward shift along dashed NX line.

Source: Author's calculations.

Figure 6: Small Open Economy Model: Solutions for Core variables, External Sector, and Expenditure Components

	base	alt(i)	alt(ii)
Output gap (a)-(b)+(c)+(d)+(e)+(f)	0.0%	-0.5%	-1.5%

$$\boxed{gap_t^{eq}\Big|_{Open} = \frac{b_{RR\pi}(\pi^e-\pi^*) - b_{RRss}ss_t + efp_t[b_{RRefp} + \dfrac{\eta_{nx}}{(\varphi_{Ir}-\eta_{nx})}] + \dfrac{[fp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir}-\eta_{nx})} + r_t^{DISC}}{\left[\dfrac{\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}}{(\varphi_{Ir}-\eta_{nx})} - b_{RRgap}\right]}}$$

	base	alt(i)	alt(ii)
Equilibrium inflation rate	3.0%	2.2%	1.2%

$$\boxed{\pi_t\Big|_{Open} = \pi^e + \frac{1}{\eta}(gap_t^{eq} - ss_t) + \theta efp_t}$$

	base	alt(i)	alt(ii)
Equilibrium real interest rate	2.0%	1.3%	4.3%

$$\boxed{r_t\Big|_{RR,Open} = \overline{r} + b_{RR\pi}(\pi^e-\pi^*) + b_{RRgap}gap_t^{eq} - b_{RRss}ss_t + b_{RRefp}efp_t + r_t^{DISC}}$$

	base	alt(i)	alt(ii)
Real price of exports (deviation from norm)	0.0%	-3.2%	-2.2%

$$\boxed{\underset{\substack{\textit{Relative price}\\ \textit{of exports}}}{rpx_t} = \underset{\textit{Real exchange rate}}{q_t} + \underset{\substack{\textit{Scaled External}\\ \textit{Terms of Trade}}}{(1-\nu)\ln(TT_t)} = \underbrace{[\overline{r}-r_t]}_{\textit{Domestic monetary policy}} + \underbrace{efp_t + (1-\nu)\ln(TT_t)}_{\textit{External shocks}}}$$

	base	alt(i)	alt(ii)
Real price of imports (deviation from norm)	0.0%	0.8%	1.8%

$$\boxed{\underset{\substack{\textit{Relative price}\\ \textit{of imports}}}{rpim_t} = \underset{\textit{Real exchange rate}}{q_t} - \underset{\substack{\textit{Scaled External}\\ \textit{Terms of Trade}}}{\nu\ln(TT_t)} = \underbrace{[\overline{r}-r_t]}_{\textit{Domestic monetary policy}} + \underbrace{efp_t - \nu\ln(TT_t)}_{\textit{External shocks}}}$$

	base	alt(i)	alt(ii)
Real exchange rate (deviation from norm) (appreciaton -)	0.0%	0.7%	1.7%
Real exchange rate index (Base = 100, app -)	100.0	100.7	101.7
Currency-unit results			
Gross Domestic Product	18.33	18.23	18.05
Consumption	11.92	11.89	11.85
Investment	3.34	3.41	3.08
Government Spending	2.75	2.75	2.75
Net Exports	0.33	0.18	0.38
Exports	1.45	1.21	1.29
Imports	1.12	1.04	0.91
Net Exports/Y^P	1.8%	1.0%	2.1%
Net Exports/Y^P Baseline	1.8%	1.8%	1.8%
NX gap	0.0%	0.8%	-0.3%
Demand side decomposition of output gap (percent of potential)			
Output gap	0.0%	-0.5%	-1.5%
Consumption	0.0%	-0.1%	-0.4%
Investment	0.0%	0.4%	-1.4%
Government Spending	0.0%	0.0%	0.0%
Net Exports	0.0%	-0.8%	0.3%
Exports	0.0%	-1.3%	-0.9%
Imports	0.0%	-0.5%	-1.2%

Source: Author’s calculations.

As a related scenario, but one not reported here, would be to jointly examine the impact of an increase in government spending with adverse financial pressures. Note that, if those pressures are large enough to offset the endogenous monetary tightening, the real exchange rate would *depreciate*, not appreciate, consistent with recent evidence by Kollman (2010).

E. The Trade Balance and Monetary Policy: In the Spirit of Marshall and Lerner

A question arises: what is the impact of a discretionary shift in monetary policy on the trade balance? Since the movement of the discretionary component of monetary policy and the equilibrium real interest rate itself (including induced movements) are assumed to be in the same direction, and there are no other shocks, the real exchange rate must depreciate when there is a discretionary monetary loosening $r_t^{DISC}<0$. That is, assuming no other shocks, the impact of discretionary monetary policy on the equilibrium real interest rate is:

$$r_t^{eq}\Big|_{RR,Open} = \overline{r} + [\frac{b_{RRgap}}{den} + 1] * r_t^{DISC} \qquad (31)$$

where den is the denominator in equation (29):

$$den = \left[\frac{\{1-[(1-\sigma_{cyc})(1-\tau)+im_{cyc}]\}}{(\varphi_{Ir}-\eta_{nx})} - b_{RRgap} \right] < 0$$

That is, the full impact of discretionary monetary policy on the equilibrium real interest rate includes a second-order (or induced) change (sign opposite to r_t^{DISC}). Recall that coherence between discretionary policy and the equilibrium real interest rate implies that $[\frac{b_{RRgap}}{den}+1]>0$. This ensures that the equilibrium real interest rate will move in the same direction as the discretionary element.

Correspondingly, the full impact of discretionary monetary policy on the real exchange rate, assuming *no other shocks* and that the future real exchange rate $\overline{q}$ equals zero, can be seen as the sum of the direct impact plus the second-order (or induced) change:

$$q_t^{eq} = \overline{r} - r_t^{eq} = -[\frac{b_{RRgap}}{den} + 1] * r_t^{DISC} \qquad (32)$$

We may now sign the effect of monetary policy shocks to the trade balance. Since we have assumed that the real exchange rate is domestic currency per unit of foreign currency (a depreciation means a rise in q), $(\eta_x - \eta_{im}) > 0$. We may think of this as a Marshall-Lerner condition, but in a narrow sense: a depreciation of the currency, ignoring all other effects, causes the trade balance to improve.

Alone, the coherence property discussed above ensures that monetary loosening will cause the real exchange rate to depreciate. However, coherence alone is not sufficient to ensure that the trade balance improves when there is a monetary loosening. Rather, the output gap effect on imports must be considered. Using equation (29) this impact is written:

$$im_{cyc} * gap_t^{eq} = im_{cyc} * \frac{r_t^{DISC}}{den} \tag{33}$$

Thus, assuming no other shocks, the full impact of discretionary monetary policy on the trade balance is obtained by substituting the direct price and output effects of a discretionary change to monetary policy into export and import equations (16 ') and (18 '):

$$(x_t^{eq} - im_t^{eq}) - (x - im) = r_t^{DISC} * [(\eta_x - \eta_{im}) * \{-[\frac{b_{RRgap}}{den} + 1]\} - \frac{im_{cyc}}{den}] \tag{34}$$

This permits us to consider an expanded Marshall-Lerner (EML) condition—one that also includes the impact of changes in the output gap on imports.[11] A monetary loosening will cause the trade balance to improve if:

$$[(\eta_x - \eta_{im}) * \{-[\frac{b_{RRgap}}{den} + 1]\} - \frac{im_{cyc}}{den}] > 0 \tag{35}$$

That is, the EML condition implies that the improvement in the trade balance that takes place purely through price changes (the real exchange rate) exceeds any deterioration thereof that reflects the impact of the output gap effect on imports.

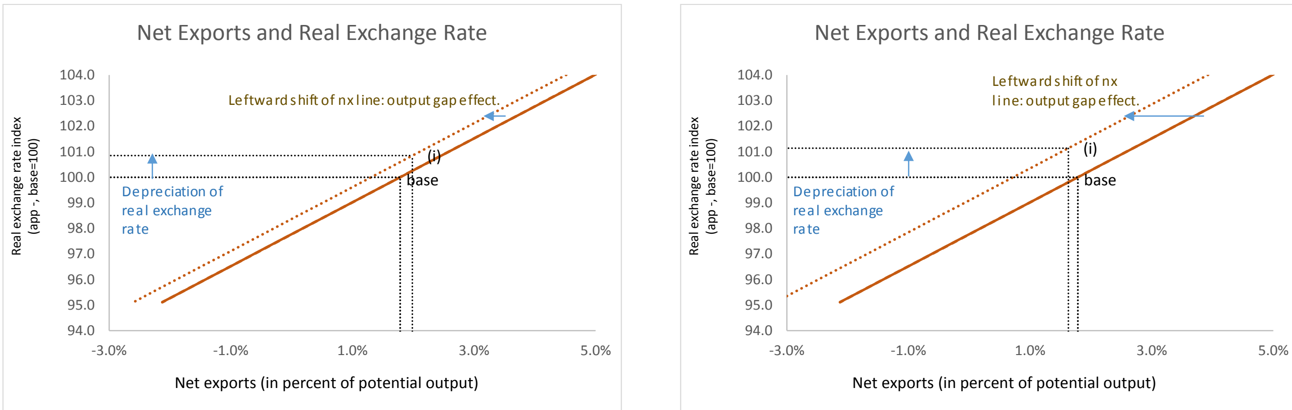
Figure 7 presents a graphical interpretation of the EML condition. In the left-hand panel, alternative scenario (i) shows the effect of a discretionary monetary loosening (r^{DISC} <0) compared to the baseline when the Expanded Marshall-Lerner (EML) condition holds.

The depreciation of the real exchange rate, which is reflected as an upward movement along the tan lines dominates the output gap effect which is reflected in a leftward shift of the tan line, from solid to dotted.

Thus, since EML is satisfied, a monetary loosening causes net exports to increase. In the right-hand panel, the alternative scenario shows the effect of a that same monetary loosening when the Expanded Marshall-Lerner (EML) condition fails to hold. The depreciation of the real exchange rate, which is reflected as an upward movement along the tan lines is dominated by the output gap effect which is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is not satisfied, a monetary loosening causes net exports to decrease.

[11] The idea of an expanded Marshall-Lerner condition is not new. See for example Hostland and Schembri (2005).

Figure 7: The Expanded Marshall-Lerner (EML) Condition



Alt (i) shows the effect of a discretionary monetary loosening r^{DISC} =-2% compared to the baseline when the Expanded Marshall-Lerner (EML) condition holds. The depreciation of the real exchange rate, which is reflected as an upward movement along the tan lines dominates the output gap effect which is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is satisfied, a monetary loosening causes net exports to increase.

Alt (i) shows the effect of a discretionary monetary loosening r^{DISC} =-2% compared to the baseline when the Expanded Marshall-Lerner (EML) condition fails to hold. The depreciation of the real exchange rate, which is reflected as an upward movement along the tan lines is dominated by the output gap effect which +is reflected in a leftward shift of the tan line, from solid to dotted. Thus, since EML is not satisfied, a monetary loosening causes net exports to decrease.

Source: Author's calculations.

V. Long-run Expenditure Shares: Some Links to Microeconomics

In order to keep the preceding models simple, little effort has been made to derive the parameters from the explicit maximization of an objective function. However, it is not necessary to entirely jettison links to microeconomic models. There are instances where we might draw informal or heuristic linkages with such models.

One such instance regards the values of the long-run expenditure shares—steady-state ratios of consumption, investment, government, and net exports to potential output. In this case, it is both possible and helpful to our intuition to appeal to some microeconomics when deriving these values. Since the reasoning for the closed and open economy differ from one another somewhat, we present them as distinct analytical cases. *However, it can be seen that, in both cases, the long-run expenditure shares can be linked to the underlying 'deep parameters' of preferences and the production function.*

A. The Closed Economy

Output is comprised of a private (after tax) component plus government expenditures:

$$Y = AK^{\alpha}L^{1-\alpha} = \underbrace{(1-\tau)AK^{\alpha}L^{1-\alpha}}_{\text{Private disposable income}} + \underbrace{G}_{\substack{\text{Government}\\ \text{Consumption}}} \tag{36}$$

where $G \equiv \tau * AK^{\alpha}L^{1-\alpha}$, reflecting a balanced budget assumption. There is one representative consumer / worker in the private sector: L is thus normalized to unity. Lifetime (logarithmic) utility for that individual is:

$$U(C) = \ln(C_t) + \frac{\ln(C_{t+1})}{(1+\rho)} + \frac{\ln(C_{t+2})}{(1+\rho)^2} + ... \tag{37}$$

where $1+\rho$ is the gross rate of time preference. In this Robinson Crusoe setting, the opportunity cost of current private consumption must be the gross after-tax marginal product of capital net of depreciation, $1+(1-\tau)\alpha AK^{\alpha-1} - \delta$. Thus, to satisfy the familiar Euler equation in steady state ($C_t = C_{t+1} = \overline{C}$), that marginal productivity term must equal the subjective rate of time preference:

$$(1-\tau)\alpha AK^{\alpha-1} - \delta = \rho \tag{38}$$

Rearranging this identity permits us to solve for the steady state stock of capital:

$$K^{SS} = \left[\frac{\rho+\delta}{(1-\tau)\alpha A}\right]^{\frac{1}{\alpha-1}} \tag{39}$$

In steady state, output is thus:

$$Y^{SS} = A\left[\frac{\rho+\delta}{(1-\tau)\alpha A}\right]^{\frac{\alpha}{\alpha-1}} \tag{40}$$

The steady state gross investment share must be the depreciation rate multiplied times the steady state capital/output ratio:

$$\psi = \delta\left[\frac{K}{Y}\right]^{SS} = \frac{\delta}{A}\left[\frac{\rho+\delta}{(1-\tau)\alpha A}\right]^{-1} \tag{41}$$

Finally, using the adding up constraint in (8) (i.e. Y=C+I+G, G=T) the net consumption ratio must be:

$$(1-\tilde{\sigma}) = 1-\psi-\tau \tag{42}$$

That is, private savings provides financing for private investment plus the government expenditures (output equals consumption plus savings plus tax revenue).

B. The Small Open Economy

In extending the model to a small open economy, there are certain analytical issues that need to be revisited. Importantly, the natural real interest rate is exogenously determined in world markets. It is assumed to be the sum of baseline values for a global external real rate of interest and a risk premium. We need to show conditions under which this interest rate is consistent with output at the steady state level—a zero output gap—assuming that all other variables are at their baseline values (i.e. no shocks).

To generate such a result, two conditions much be satisfied: (i) the capital stock is at its steady state level and (ii) the allocation of employment across export and non-tradable sectors is consistent with equalized wages. Below, we develop a model of a small open economy that produces two goods: non-tradables (*N*) and exports (*X*). In this model, we show how both conditions will be satisfied. Such a model is similar in spirit to one developed by Hinkle and Nsengiyumva (1999).

The economy-wide production function is assumed to be:

$$\tilde{Y} = Y_N + P_X Y_X = A_N K^{\alpha} L_N^{1-\alpha} + P_X A_X K^{\alpha} L_X^{1-\alpha} \tag{43}$$

where A_N and A_X are respectively, total factor productivities in non-traded and export sectors, K is the total capital stocks deployed to these two sectors, and L_N and L_X are the employment levels in each sector, and α is capital's share of total output. The labor force is assumed to be fixed: $L = L_N + L_X$. The price of non-tradables P_N is normalized to unity. To ensure a tractable solution, we assume that capital services are non-rival across sectors—for example, generic infrastructure that is essentially shared between sectors.

As noted above, in a steady state, the level and composition of output must satisfy two conditions: (i) the capital stock must be at its steady-state levels; (ii) the allocation of employment across sectors is consistent with equalized wages.

To satisfy condition (i), recall first that in the open economy, the natural rate of interest is exogenous: $\overline{r} = \overline{r}^{EXT} + \overline{rp}$. In the steady state, this interest rate must equal the after tax marginal product of capital net of depreciation. Thus, the steady-state first order condition for capital accumulation is:

$$\alpha(1-\tau)(K^{\alpha-1}) * [A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}] - \delta = \overline{r} \tag{44}$$

Then, the steady-state capital stock is:

$$K^{SS} = \left[\frac{\overline{r} + \delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]} \right]^{\frac{1}{\alpha-1}} \tag{45}$$

Accordingly, steady-state output must be:

$$Y^{SS} = \left[\frac{\overline{r} + \delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]} \right]^{\frac{\alpha}{\alpha-1}} * \left[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha} \right] \tag{46}$$

To satisfy condition (ii) note that wage equality means that the value of marginal products of labor must be equalized across sectors:

$$\underbrace{(1-\alpha)A_N K_N^{\alpha} L_N^{-\alpha}}_{\text{Marginal product of labor, nontradables}} = \underbrace{(1-\alpha)P_X A_X K_X^{\alpha} L_X^{-\alpha}}_{\text{Marginal product of labor, exports}} \tag{47a}$$

The ratio of labor devoted to non-tradeables relative to exports is a function of relative productivities and output prices in the sectors, namely:

$$\frac{L_N}{L_X} = \left[\frac{P_X A_X}{A_N}\right]^{\frac{-1}{\alpha}} \tag{47b}$$

Recall that the economy wide labor constraint implies that the total labor supply is the sum of L_X and L_X . Thus the share of labor that is allocated non-traded goods (in percent of total labor supply) must be:

$$\frac{L_N}{L_X + L_N} = \left\{1 + \left[\frac{P_X A_X}{A_N}\right]^{\frac{1}{\alpha}}\right\}^{-1} = \omega \tag{47c}$$

That is, if wages are equalized, a fraction of the total labor force ω will be deployed in the non-traded goods sector while $(1-\omega)$ will be deployed in the export sector. To find out steady state export production, we substitute the expression for steady state capital into the export production function:

$$\left[P_X X\right]^{SS} = \underbrace{\left[\frac{\bar{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]}\right]^{\frac{\alpha}{\alpha-1}}}_{\text{Steady-state capital stock}} * \left[P_X A_X L_X^{1-\alpha}\right] \tag{48}$$

Then, divide steady state exports (48) by steady state output (48) and then substitute in the wage equalization condition (47c), the steady-state share of exports (ratio to potential output) must be:

$$x = \frac{P_X A_X \left([1-\omega]L\right)^{1-\alpha}}{\left[A_N \left(\omega L\right)^{1-\alpha} + P_X A_X \left([1-\omega]L\right)^{1-\alpha}\right]} \tag{49}$$

We assume that in the steady state, the country receives constant external transfer Z^{SS} which is minus one times net exports:

$$Z^{SS} \equiv zY^{SS} \equiv im - x \equiv -nx \tag{50}$$

The volume of gross investment expenditures corresponds entirely to maintenance (constant capital stock):

$$\psi = \delta\left[\frac{K}{Y}\right]^{SS} = \delta\left[\frac{\bar{r}+\delta}{\alpha(1-\tau)[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}]}\right]^{\frac{1}{\alpha}} \left[A_N L_N^{1-\alpha} + P_X A_X L_X^{1-\alpha}\right] \tag{51}$$

The share of government expenditures must match the tax rate: $\gamma = \tau$. Finally, the steady-state share of private consumption may be computed as a residual:

$$c = 1 - \psi - \gamma + z \tag{52}$$

VI. EXTENSION: LOGARITHMS, GROWTH RATES, AND ELASTICITIES

Macroeconomists often write behavioral relationships in terms of growth rates rather than levels. Parameters in their models take the familiar from of an elasticity rather than level response parameters (such as those introduced above).

As an example, an econometrician may estimate a consumption function as:

$$\ln(C_t) = \eta_{C0} + \eta_{CY} * \ln(Y_t^d) \tag{53a}$$

Using the approximation that $\Delta \ln(X_t) \approx \% \Delta X_t$ we may obtain an expression for the growth rate of consumption, namely:

$$\% \Delta C_t = \eta_{CY} \% \Delta Y_t^d \tag{53b}$$

Consistent with equation (53b), consumption in the current period is:

$$C_t = C_{t-1} + \eta_{CY} \frac{C_{t-1}}{Y_{t-1}} \Delta Y_t^d$$

That is, we simply re-write the short-run marginal propensity to consume from equation (1a) as $a_{CY} \equiv \eta_{CY} C_{t-1} / Y_{t-1}$.

Correspondingly, an econometric estimate for the investment function might be:

$$\ln(I_t) = \eta_{I0} + \eta_{Ir} * r_t \tag{54a}$$

where η_{Ir} is the familiar semi-elasticity of investment with respect to the real interest rate. Again using the approximation for growth rates, we have:

$$\% \Delta I_t = \eta_{Ir} * \Delta r_t \tag{54b}$$

Thus,

$$I_t = I_{t-1}[1 + \eta_{Ir} \Delta r_t] \tag{54c}$$

Here, we have simply re-written the interest rate response parameter in equation (2) as: $a_{Ir} \equiv \eta_{Ir} I_{t-1}$.

If we assume that that baseline values potential output and the natural rate of interest remain constant, we can use these expressions to generate scenarios based on alternative assumptions for short-run fiscal, monetary, and external shocks for a given period.

VII. STATIC MACROECONOMIC MODELS: SOMETHING'S LOST, BUT SOMETHING'S GAINED

In recent years, macroeconomic models have become increasingly complex. Dynamic stochastic general equilibrium (DSGE) models do an important job: they help us understand the behavior of macroeconomic aggregates in a way that reflects rational, optimizing behavior by economic agents. The dominance of DSGE models in the economics profession is *prima facie* evidence of their rigor and potential. And many have rejected simpler Keynesian-style models (including more recent New Keynesian heirs) for their lack microeconomic rigor (see for example Chari, Kehoe, and McGrattan, 2009).

However, this current in macroeconomic modeling is not without its critics. Several prominent authors, including Krugman (2000), Blanchard (2009) and, in a more strident tone, Romer (2016) have recently raised doubts about the merits of these more recent models precisely because they are so complex and difficult to understand—*even for seasoned professionals*.

And, even as more complex models dominate the profession, models that are simpler—the traditional IS/LM and its New Keynesian heirs—have not vanished from use. But, why? If we are dropping the microeconomic rigor of DSGE and other recent modeling techniques, *what do we get in return?*

Rather than presenting a new model per se, this paper has attempted to restate what might be thought of as models that are traditional and familiar. In exchange for the limitations, elements of the material presented herein should provide a useful toolbox for applied economists, both seasoned and novices. Below, find several of these advantages. [12]

Traditional graphical exposition. As with other traditional comparative static models, this one can easily be displayed graphically. Many important insights can be derived by shifting the familiar IS and monetary policy (LM or RR) curve.

Impact multipliers and illustrative calculations: This paper contains easy ways to obtain impact multipliers and other convenient calculations for key macroeconomic variables that are consistent with the graphs. We might think of such calculations as an extended 'back-of-

[12] To see how such ideas might be presented to undergraduates, see the author's textbook manuscript (2014) currently available online at: http://www.evanctanner.com/textbook-macro. For example, the issue of household consumption is addressed in Chapter 7.

the-envelope' exercise that might be used as a cross-check for more complex ones—with the important caveat that the results should treated as illustrative and qualitative.

Expenditure-side equations that illustrate deeper macroeconomic issues. More than previous static models, the algebraic scaling for expenditure equations in this one help illustrate some deeper economic issues. For example, the constant term in consumption function (6b) links directly back to a model of long-run growth—for example the Solow model—which textbooks often feature as an antecedent to the short-run model. Likewise, an equation such as (6b) can help illustrate other key ideas in macroeconomics, such as the permanent income/life cycle approach and the Ricardian equivalence hypothesis, more readily than its traditional counterpart.

Succinct portrayal of key open economy relationships. Impacts of domestic monetary policy and external shocks in both goods and financial markets are summarized in a transparent and sucking way. Specifically, equations (22′) and (23′) show, *as identities*, exact decompositions of the relative prices of exports and imports into domestic monetary policy effects, external financial tightening, and changes in the external terms of trade. The model is thus especially helpful to understand the macroeconomic implications of shocks to the terms of trade and to external financial flows (i.e. 'sudden stops'). An application of the open economy model to such issues was shown in Figures 4-6. As an additional exercise, the model clearly shows how fiscal policy may affect the real exchange rate. Whether a fiscal expansion causes the real exchange rate to appreciate or depreciate depends the relative strengths of domestic monetary policy and risk premium responses—arguably a more transparent linkage than is found in extant DSGE models (see for example Kollman, 2010).

Cross-checks of parameter values for mutual consistency. Romer (2016) notes that many models rely upon parameters whose value can never be estimated. While we cannot avoid this criticism, we can show how certain *combinations* of parameter values may make sense whether they are mutually consistent. There are three main examples of such cross-checks. *First,* the interest rate response (IS) and short run aggregate supply (Phillips Curve), in combination with one another, should yield a *coherent* result for discretionary monetary policy: movements in the nominal and real interest rates should be in the same direction (See part III for details). *Second*, the model shows the conditions required to ensure that an external financial outflow (i.e. a 'sudden stop' to external capital) will result in a fall of domestic expenditures and output but a rise in net exports—as reflected in equation (29). This is consistent with evidence in Calvo (1998) and Blanchard, Ostry, Ghosh, and Chamon (BOGC,2015). *Third*, the expanded Marshall Lerner condition (see Part IV.E for details) shows the parameter combinations required to yield a result that a domestic monetary tightening will both appreciate the exchange rate and worsen the trade balance.

Theoretical foundations for steady-state expenditure shares. There are elements of the model whose microeconomic foundations are more transparent and robust than in other models of at this level of complexity. For example, in Section V, long run shares are linked to technology and tastes in a way that is consistent with some real business cycle or 'fresh water' models. Also, the treatment of external prices—relative prices of exports and imports, real exchange rate, and terms of trade—equations (22') and (23')—also represents an improvement over most extant models.

Extensions of the model: At the same time, an effort to extend a model like the one in this paper to include forward looking inflation and exchange rate expectations is found in Tanner (2017, forthcoming). Also, the author is currently developing extensions of the model, such as including exchange rate management and sterilized intervention, along the lines of Benes, Berg, Portillo, and Vavra (2015).

In sum, 'simple' models can serve as supplements to (and cross-checks on) their more sophisticated counterparts. By paying some attention to algebra that links up different metrics we may add considerable insight to simple models.

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IX. APPENDIX

The 'Macronia' Model—A Financial Programming Exercise

An online course on Financial Programming and Policies (FPP2x) that was recently developed by the International Monetary Fund's Institute for Capacity Development (ICD) includes a model for a hypothetical case study ("Macronia") that is similar to the open economy model presented in the main body of the paper. For the real sector, participants are given equations in the more traditional 'elasticities' like those shown in Section VI. Using these equations, they build a real sector—element by element. The model, whose parameters are calibrated to match each sector's equations in section VI, is used to both assess the economy's vulnerabilities and to design an adjustment program.

However, note that several additional features have been included into that model. First, output appears as a determinant of investment—an accelerator function. Thus, the investment equation is now written:

$$I_t = \tilde{a}_{I0} + a_{IY} Y_t + a_{Ir} r_t \tag{A1}$$

The corresponding expression written in terms of potential output and the output gap is:

$$I_t = Y^P[\psi + \varphi_{Icyc} gap_t + \varphi_{Ir}(r_t - \bar{r})] \tag{A2}$$

Thus, in equation (A2), the constant term would now be interpreted as:

$$\tilde{a}_{I0} = Y^P[\psi - \varphi_{Icyc} - \varphi_{Ir}\bar{r}]$$

Thus, the IS equation (extension to 24) is now written as:

$$gap_t \Big|_{IS,\text{Open}} = \frac{(\varphi_{Ir} + \eta_{nx})(r_t - \bar{r}) + fp_t + \eta_{nx} efp_t + \tilde{\eta}_{nx} \ln(TT_t)}{\{1 - [(1 - \sigma_{cyc})(1 - \tau) + im_{cyc}] - \varphi_{Icyc}\}} \tag{A3}$$

Second, interest rate smoothing has been added to the central bank's interest rate rule. In nominal terms, we have:

$$i_t \Big|_{Open} = \rho i_{t-1} + (1-\rho)[\bar{r} + \pi^e + b_\eta(\pi_t - \pi^*) + b_{gap} gap_t + efp_t] + r_t^{DISC} \tag{A4}$$

Where the continuous smoothing parameter is $0 < \rho < 1$ ($\rho = 0$ implies no smoothing). The equilibrium real interest rate (Phillips curve substituted into Taylor rule) yields:

$$\begin{aligned} r_t \Big|_{RR,Open} = {} & \rho i_{t-1} + (1-\rho)[\bar{r} + b_{RR\pi}(\pi^e - \pi^*) + b_{RRgap} gap_t - b_{RRss} ss_t + b_{RRefp} efp_t] \\ & - [\pi^e + \frac{1}{\eta}(gap_t - ss_t) + \theta efp_t] + r_t^{DISC} \end{aligned} \tag{A5a}$$

or more compactly:

$$r_t \Big|_{RR,Open} = \bar{r} + \tilde{b}_{RR\pi}(\pi^e - \pi^*) + \tilde{b}_{RRgap} gap_t - \tilde{b}_{RRss} ss_t + \tilde{b}_{RRefp} efp_t + \tilde{r}_t^{DISC} \tag{A5b}$$

where $\tilde{b}_{RR\pi} = (1-\rho)b_{RR\pi}$, $\tilde{b}_{RRgap} = (1-\rho)b_{RRgap} - \dfrac{\rho}{\eta}$, $\tilde{b}_{RRss} = (1-\rho)b_{RRss} - \dfrac{\rho}{\eta}$,

$\tilde{b}_{RRefp} = (1-\rho)b_{RRefp} - \dfrac{\rho}{\theta}$ are compound coefficients and $\tilde{r}_t^{DISC} = r_t^{DISC} + \rho(i_{t-1} - \pi^e - \overline{r})$ is a compound error term that includes both the contemporaneous discretionary element and the smoothing component $\rho(i_{t-1} - \pi^e - \overline{r})$.

Thus, adding these two elements, the equilibrium output gap is now:

$$gap_t^{eq}\Big|_{Open} = \frac{\tilde{b}_{RR\pi}(\pi^e - \pi^*) - \tilde{b}_{RRss}ss_t + efp_t[\tilde{b}_{RRefp} + \dfrac{\eta_{nx}}{(\varphi_{Ir} - \eta_{nx})}] + \dfrac{[\, fp_t + \tilde{\eta}_{nx}\ln(TT_t)]}{(\varphi_{Ir} - \eta_{nx})} + \tilde{r}_t^{DISC}}{\left[\dfrac{\{1 - [(1-\sigma_{cyc})(1-\tau) + im_{cyc}] - \varphi_{Icyc}\}}{(\varphi_{Ir} - \eta_{nx})} - \tilde{b}_{RRgap}\right]} \quad \text{(A6)}$$

Examples of this solution are found in the ICD's online financial programming course FPP2x, applied to the case of "Macronia." An explanation of the model, in videos, may be found online at: http://www.evanctanner.com/imf-fpp2x-model-videos.