NA-DPVI AISTATS 2025 Rebuttal

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November 2024

1 NA-DPVI algorithm blocks

Global variables

$(\boldsymbol{x}_i)_{i=1}^N$: data;
(ϵ, δ) : DP privacy level;
C : clipping threshold;
T : training iterations;
λ : learning rate;
κ : Poisson sampling rate;
$\boldsymbol{\phi}_0$: initial values for $\boldsymbol{\phi}$;
σ_{DP} : PRVAccountant$(\epsilon, \delta, T, \kappa)$;

Algorithm 1 DPVI

```
for t = 1, ..., T − 1 do
    g_{t+1} ← Σ_{i∈B_{t+1}} clip(∇_φ ℓ(φ_t; x_i), C);
    Sample η_{t+1} ~ N(0, I_d);
    g̃_{t+1} ← g_{t+1} + σ_DP C η_{t+1}
    φ_{t+1} ← φ_t − λ g̃;
end for
return T = (φ_t)_{t=0}^T, g̃ = (g̃_t)_{t=1}^T;
```

for $t = 1, \ldots, T-1$ **do**
  $\boldsymbol{g}_{t+1} \leftarrow \sum_{i \in \mathcal{B}_{t+1}} \mathrm{clip}\left(\nabla_{\boldsymbol{\phi}} \ell(\boldsymbol{\phi}_t; \boldsymbol{x}_i), C\right)$;
  Sample $\boldsymbol{\eta}_{t+1} \sim \mathcal{N}(0, \mathbf{I}_d)$;
  $\widetilde{\boldsymbol{g}}_{t+1} \leftarrow \boldsymbol{g}_{t+1} + \sigma_{\mathrm{DP}} C \boldsymbol{\eta}_{t+1}$
  $\boldsymbol{\phi}_{t+1} \leftarrow \boldsymbol{\phi}_t - \lambda \widetilde{\boldsymbol{g}}$;
end for
return $\mathcal{T} = (\boldsymbol{\phi}_t)_{t=0}^T, \widetilde{\boldsymbol{g}} = (\widetilde{\boldsymbol{g}}_t)_{t=1}^T$;

Algorithm 2 NA-DPVI

Input:
  M : number of samples;
  $\boldsymbol{\mu}_{\boldsymbol{\phi}^*}$: $\boldsymbol{\phi}^*$ prior distribution mean;
  $\boldsymbol{\Sigma}_{\boldsymbol{\phi}^*}$: $\boldsymbol{\phi}^*$ prior distribution covariance matrix;
  $\boldsymbol{\mu}_{\mathbf{A}}$: $\mathbf{A}$ entries prior distribution mean;
  $\boldsymbol{\Sigma}_{\mathbf{A}}$: $\mathbf{A}$ entries prior distribution covariance matrix;
  $\boldsymbol{\mu}_{\Sigma_{\mathrm{sub}}}$: Σ_{sub} entries prior distribution mean;
  $\boldsymbol{\Sigma}_{\Sigma_{\mathrm{sub}}}$: Σ_{sub} entries prior distribution covariance matrix;
  $\mathcal{T}, \widetilde{\boldsymbol{g}}$: DPVI output;
model-definition $\widetilde{\boldsymbol{g}} \mid \mathcal{T}, \boldsymbol{\phi}^*, \mathbf{A}, \Sigma_{\mathrm{sub}}$
  $\boldsymbol{\phi}^* \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\phi}^*}, \boldsymbol{\Sigma}_{\boldsymbol{\phi}^*})$;
  $\mathbf{A} \sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{A}}, \boldsymbol{\Sigma}_{\mathbf{A}})$;
  $\Sigma_{\mathrm{sub}} \sim \mathcal{N}(\boldsymbol{\mu}_{\Sigma_{\mathrm{sub}}}, \boldsymbol{\Sigma}_{\Sigma_{\mathrm{sub}}})$;
  $\widetilde{\boldsymbol{g}}_{t+1} \mid \boldsymbol{\phi}_t, \mathbf{A}, \boldsymbol{\phi}^*, \Sigma_{\mathrm{sub}} \sim \mathcal{N}\left(\kappa \mathbf{A}\left(\boldsymbol{\phi}_t - \boldsymbol{\phi}^*\right), \sigma_{\mathrm{DP}}^2 C^2 \mathbf{I}_d + \Sigma_{\mathrm{sub}}\right)$;
end model-definition
Sample $(\boldsymbol{\phi}_i^*, \mathbf{A}_i, \Sigma_{\mathrm{sub},i})_{i=1}^M \sim p\left(\boldsymbol{\phi}^*, \mathbf{A}, \Sigma_{\mathrm{sub}} \mid \widetilde{\boldsymbol{g}}, \mathcal{T}\right)$; ▷ sampling using any approximate inference method.
return $\widetilde{p}(\boldsymbol{\theta} \mid \mathcal{T}) = \frac{1}{M} \sum_{i=1}^M q_{\mathrm{VI}}(\boldsymbol{\theta}; \boldsymbol{\phi}_i^*)$; ▷ approximate noise-aware posterior mixture model.

2 Exponential families experiment

See the benchmarks in Figure 1 and Table 1 which include the TARP coverages and the RMSE of the coverage errors for NA-DPVI (NUTS), Bernstein & Sheldon's (2018) method, last iterate DPVI, and DPVIm (Jälkö et al., 2023).

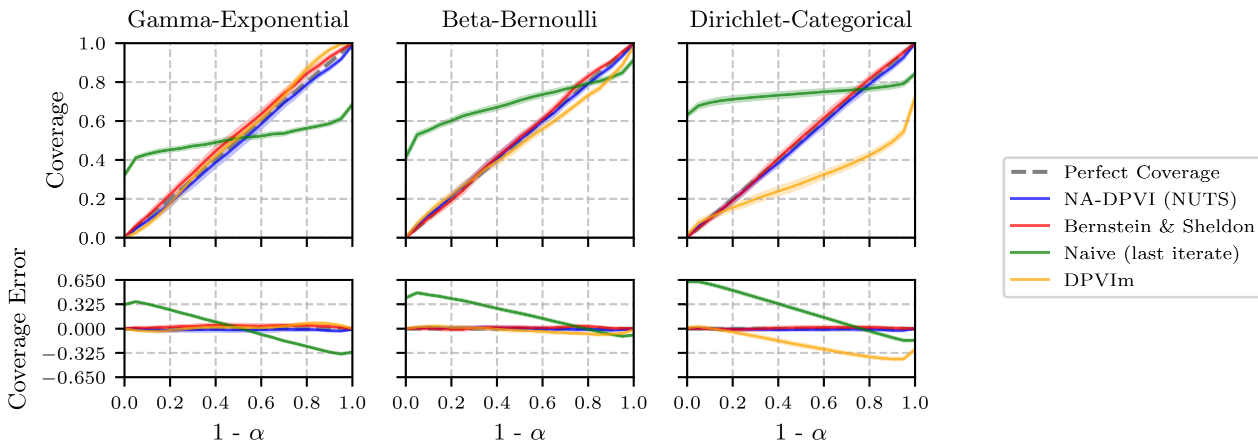

Figure 1: The first row in the figure shows the TARP coverages for the exponential families experiment for NA-DPVI (NUTS), Bernstein & Sheldon's (2018) method, last iterate DPVI, and DPVIm (Jälkö et al., 2023). The second row shows the error for the coverages ($C(\alpha) - (1 - \alpha)$). The solid lines show the average performance over 20 independent TARP repetitions and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $N = 5000$, $\kappa = 0.1$ and $T = 10^4$.

Table 1: The mean RMSE errors ± std corresponding to the coverages in Figure 1.

Method	**Gamma-Exponential**	**Beta-Bernoulli**	**Dirichlet-Categorical**
NA-DPVI (NUTS)	**0.023 ± 0.008**	**0.016 ± 0.006**	0.020 ± 0.004
Bernstein & Sheldon	0.034 ± 0.011	0.018 ± 0.006	**0.017 ± 0.007**
Naive (last iterate)	0.232 ± 0.003	0.273 ± 0.007	0.355 ± 0.009
DPVIm	0.038 ± 0.005	0.044 ± 0.004	0.251 ± 0.011

3 10D Bayesian linear regression experiment

See the benchmarks in Figure 2 and Table 2 which include the TARP coverages and the RMSE of the coverage errors for NA-DPVI (NUTS), last iterate DPVI, and Bernstein & Sheldon's (2019) Gibbs-SS-Noisy method.

Table 2: The mean RMSE errors ± std corresponding to the coverages in Figure 2.

Method	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 1.0$
NA-DPVI (NUTS)	**0.036 ± 0.011**	**0.035 ± 0.011**	**0.027 ± 0.009**
NA-DPVI (Laplace)	0.078 ± 0.011	0.098 ± 0.006	0.06 ± 0.007
Naive (last iterate)	0.512 ± 0.003	0.307 ± 0.003	0.36 ± 0.003
Bernstein & Sheldon	0.301 ± 0.001	0.299 ± 0.001	0.323 ± 0.003
DPVIm	0.584 ± 0.0	0.584 ± 0.0	0.584 ± 0.0

4 UCI Adult Bayesian logistic regression experiment

See the benchmarks in Figure 3 and Table 3 which include the predictive calibrations and the RMSE of the calibration errors for NA-DPVI (NUTS), NA-DPVI (Laplace), last iterate DPVI, and DPVIm (Jälkö et al., 2023).

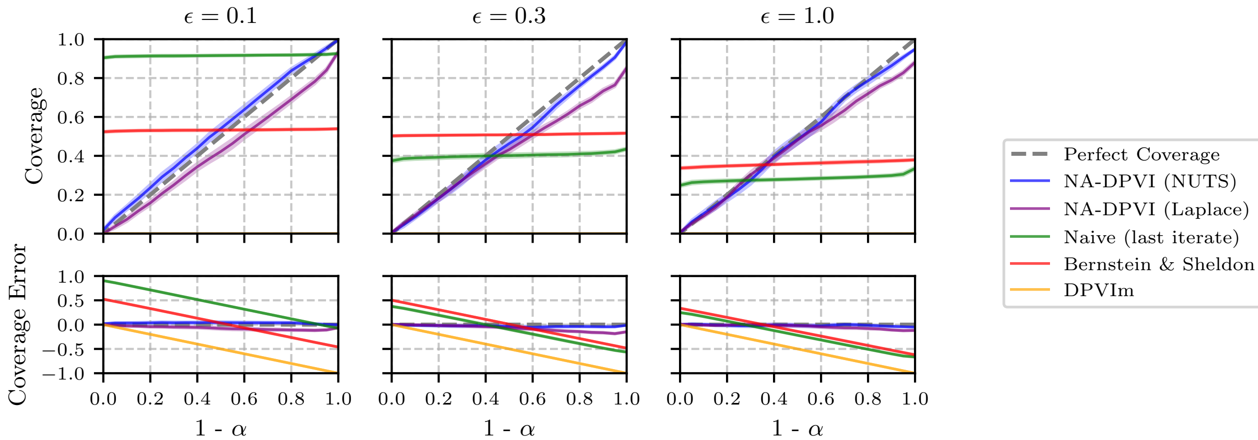

Figure 2: The first row in the figure shows the TARP coverages for the 10D Bayesian linear regression experiment for NA-DPVI (NUTS), last iterate DPVI, and Bernstein & Sheldon's (2019) Gibbs-SS-Noisy method. The second row shows the error for the coverages ($C(\alpha) - (1 - \alpha)$). The solid lines show the average performance over 20 independent TARP repetitions and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $N = 5000$, $\kappa = 0.1$ and $T = 10^4$.

Table 3: The mean RMSE errors ± std corresponding to the calibrations in Figure 3.

Method	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 1.0$
NA-DPVI (NUTS)	**0.061 ± 0.031**	**0.026 ± 0.005**	**0.024 ± 0.007**
NA-DPVI (Laplace)	0.084 ± 0.043	0.044 ± 0.011	0.046 ± 0.014
Naive (last iterate)	0.12 ± 0.065	0.067 ± 0.024	0.054 ± 0.017
DPVIm	0.101 ± 0.052	0.065 ± 0.025	0.038 ± 0.014

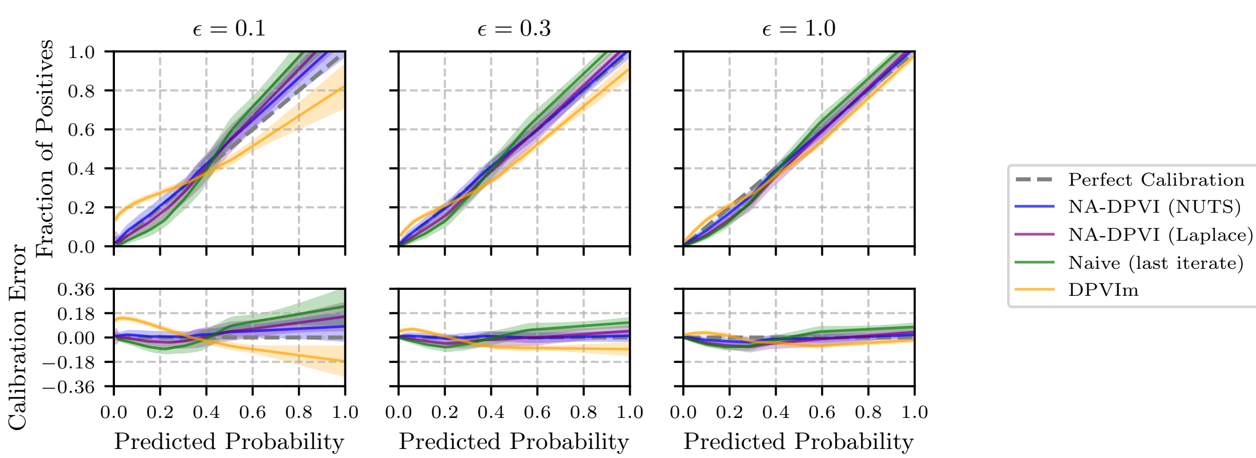

Figure 3: The first row in the figure shows the predictive calibration for the UCI Adult logistic regression experiment for NA-DPVI (NUTS), NA-DPVI (Laplace), last iterate DPVI, and DPVIm (Jälkö et al., 2023). The second row shows the calibration error (Fraction of Positives - Predicted Probability). The solid lines show the average performance over 20 independent repetitions, and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $\kappa = 0.1$ and $T = 10^4$.