

NA-DPVI AISTATS 2025 Rebuttal

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1 NA-DPVI algorithm blocks

Global variables

$(\mathbf{x}_i)_{i=1}^N \leftarrow \text{data};$
 $(\epsilon, \delta) \leftarrow \text{DP privacy level};$
 $C \leftarrow \text{clipping threshold};$
 $T \leftarrow \text{training iterations};$
 $\lambda \leftarrow \text{learning rate};$
 $\kappa \leftarrow \text{Poisson sampling rate};$
 $\boldsymbol{\phi}_0 \leftarrow \text{initial values for } \boldsymbol{\phi};$
 $d \leftarrow \dim(\boldsymbol{\phi}_0)$ $\triangleright \boldsymbol{\phi}_0$ dimensions.
 $\sigma_{\text{DP}} \leftarrow \text{PRVAccountant}(\epsilon, \delta, T, \kappa);$

Algorithm 1 DPVI

1: **for** $t = 1, \dots, T - 1$ **do**
2: $\mathbf{g}_{t+1} \leftarrow \sum_{i \in \mathcal{B}_{t+1}} \text{clip}(\nabla_{\boldsymbol{\phi}} \ell(\boldsymbol{\phi}_t; \mathbf{x}_i), C);$
3: Sample $\boldsymbol{\eta}_{t+1} \sim \mathcal{N}(0, \mathbf{I}_d);$
4: $\tilde{\mathbf{g}}_{t+1} \leftarrow \mathbf{g}_{t+1} + \sigma_{\text{DP}} C \boldsymbol{\eta}_{t+1}$
5: $\boldsymbol{\phi}_{t+1} \leftarrow \boldsymbol{\phi}_t - \lambda \tilde{\mathbf{g}};$
6: **end for**
7: **return** $\mathcal{T} = (\boldsymbol{\phi}_t)_{t=0}^T, \tilde{\mathbf{g}} = (\tilde{\mathbf{g}}_t)_{t=1}^T;$

Algorithm 2 NA-DPVI

1: $M \leftarrow \text{number of samples};$
2: $\boldsymbol{\mu}_{\boldsymbol{\phi}^*} \leftarrow \boldsymbol{\phi}^*$ prior distribution mean;
3: $\Sigma_{\boldsymbol{\phi}^*} \leftarrow \boldsymbol{\phi}^*$ prior distribution covariance matrix;
4: $\boldsymbol{\mu}_{\mathbf{A}} \leftarrow \mathbf{A}$ entries prior distribution mean;
5: $\Sigma_{\mathbf{A}} \leftarrow \mathbf{A}$ entries prior distribution covariance matrix;
6: $\boldsymbol{\mu}_{\Sigma_{\text{sub}}} \leftarrow \Sigma_{\text{sub}}$ entries prior distribution mean;
7: $\Sigma_{\Sigma_{\text{sub}}} \leftarrow \Sigma_{\text{sub}}$ entries prior distribution covariance matrix;
8: $\mathcal{T}, \tilde{\mathbf{g}} \leftarrow \text{DPVI};$
9: **model-definition** $\tilde{\mathbf{g}} \mid \mathcal{T}, \boldsymbol{\phi}^*, \mathbf{A}, \Sigma_{\text{sub}}$
10: $\boldsymbol{\phi}^* \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{\phi}^*}, \Sigma_{\boldsymbol{\phi}^*});$
11: $\mathbf{A} \sim \mathcal{N}(\boldsymbol{\mu}_{\mathbf{A}}, \Sigma_{\mathbf{A}});$
12: $\Sigma_{\text{sub}} \sim \mathcal{N}(\boldsymbol{\mu}_{\Sigma_{\text{sub}}}, \Sigma_{\Sigma_{\text{sub}}});$
13: $\tilde{\mathbf{g}}_{t+1} \mid \boldsymbol{\phi}_t, \mathbf{A}, \boldsymbol{\phi}^*, \Sigma_{\text{sub}} \sim \mathcal{N}(\kappa \mathbf{A}(\boldsymbol{\phi}_t - \boldsymbol{\phi}^*), \sigma_{\text{DP}}^2 C^2 \mathbf{I}_d + \Sigma_{\text{sub}});$
14: **end model-definition**
15: Sample $(\boldsymbol{\phi}_i^*, \mathbf{A}_i, \Sigma_{\text{sub}, i})_{i=1}^M \sim \boldsymbol{\phi}^*, \mathbf{A}, \Sigma_{\text{sub}} \mid \tilde{\mathbf{g}}, \mathcal{T};$ \triangleright using any approximate inference method.
16: **return** $\tilde{p}(\boldsymbol{\theta} \mid \mathcal{T}) = \frac{1}{M} \sum_{i=1}^M q_{\text{VI}}(\boldsymbol{\theta}; \boldsymbol{\phi}_i^*);$ \triangleright approximate noise-aware posterior mixture model.

2 Exponential families experiment

See the benchmarks in Figure 1 and Table 1 which include the TARP coverages and the RMSE of the coverage errors for NA-DPVI (NUTS), Bernstein & Sheldon’s (2018) method, last iterate DPVI, and DPVIm (Jätkö et al., 2023).

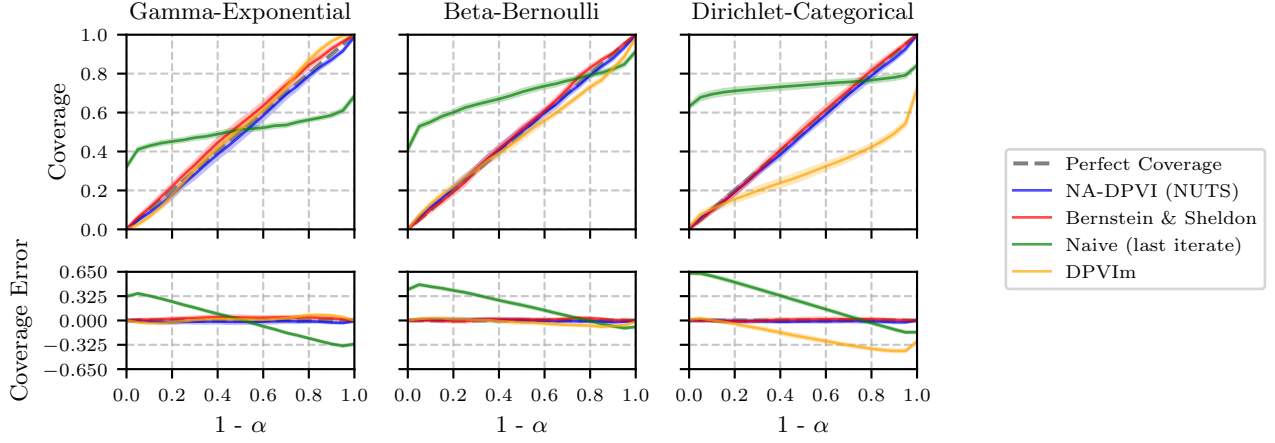


Figure 1: The first row in the figure shows the TARP coverages for the exponential families experiment for NA-DPVI (NUTS), Bernstein & Sheldon’s (2018) method, last iterate DPVI, and DPVIm (Jätkö et al., 2023). The second row shows the error for the coverages ($C(\alpha) - (1 - \alpha)$). The solid lines show the average performance over 20 independent TARP repetitions and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $N = 5000$, $\kappa = 0.1$ and $T = 10^4$.

Table 1: The mean RMSE errors \pm std corresponding to the coverages in Figure 1.

Method	Gamma-Exponential	Beta-Bernoulli	Dirichlet-Categorical
NA-DPVI (NUTS)	0.023 \pm 0.008	0.016 \pm 0.006	0.020 \pm 0.004
Bernstein & Sheldon	0.034 \pm 0.011	0.018 \pm 0.006	0.017 \pm 0.007
Naive (last iterate)	0.232 \pm 0.003	0.273 \pm 0.007	0.355 \pm 0.009
DPVIm	0.038 \pm 0.005	0.044 \pm 0.004	0.251 \pm 0.011

3 10D Bayesian linear regression experiment

See the benchmarks in Figure 2 and Table 2 which include the TARP coverages and the RMSE of the coverage errors for NA-DPVI (NUTS), last iterate DPVI, and Bernstein & Sheldon’s (2019) Gibbs-SS-Noisy method.

Table 2: The mean RMSE errors \pm std corresponding to the coverages in Figure 2.

Method	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 1.0$
NA-DPVI (NUTS)	0.036 \pm 0.011	0.035 \pm 0.011	0.027 \pm 0.009
NA-DPVI (Laplace)	0.078 \pm 0.011	0.098 \pm 0.006	0.06 \pm 0.007
Naive (last iterate)	0.512 \pm 0.003	0.307 \pm 0.003	0.36 \pm 0.003
Bernstein & Sheldon	0.301 \pm 0.001	0.299 \pm 0.001	0.323 \pm 0.003

4 UCI Adult Bayesian logistic regression experiment

See the benchmarks in Figure 3 and Table 3 which include the predictive calibrations and the RMSE of the calibration errors for NA-DPVI (NUTS), NA-DPVI (Laplace), last iterate DPVI, and DPVIm (Jätkö et al., 2023).

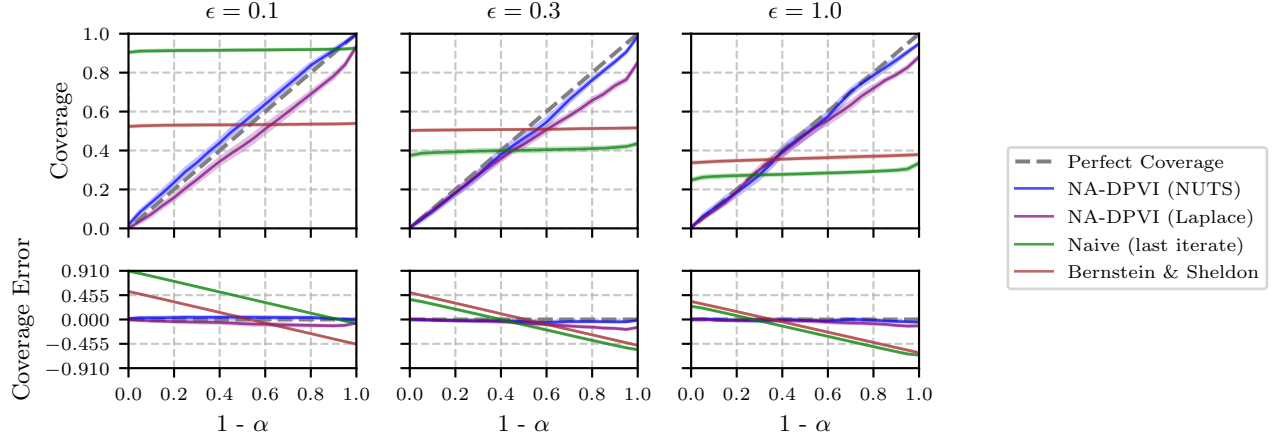


Figure 2: The first row in the figure shows the TARP coverages for the 10D Bayesian linear regression experiment for NA-DPVI (NUTS), last iterate DPVI, and Bernstein & Sheldon’s (2019) Gibbs-SS-Noisy method. The second row shows the error for the coverages ($C(\alpha) - (1 - \alpha)$). The solid lines show the average performance over 20 independent TARP repetitions and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $N = 5000$, $\kappa = 0.1$ and $T = 10^4$.

Table 3: The mean RMSE errors \pm std corresponding to the calibrations in Figure 3. Fig. 3.

Method	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 1.0$
NA-DPVI (NUTS)	0.061 ± 0.031	0.026 ± 0.005	0.024 ± 0.007
NA-DPVI (Laplace)	0.084 ± 0.043	0.044 ± 0.011	0.046 ± 0.014
Naive (last iterate)	0.12 ± 0.065	0.067 ± 0.024	0.054 ± 0.017
DPVIm	0.101 ± 0.052	0.065 ± 0.025	0.038 ± 0.014

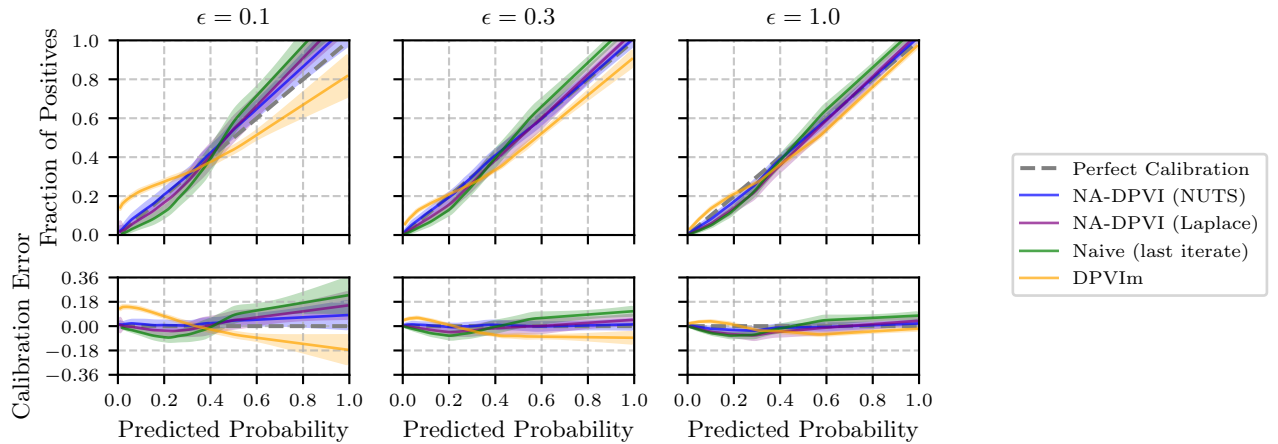


Figure 3: The first row in the figure shows the predictive calibration for the UCI Adult logistic regression experiment for NA-DPVI (NUTS), NA-DPVI (Laplace), last iterate DPVI, and DPVIm (Jälkö et al., 2023). The second row shows the calibration error (Fraction of Positives - Predicted Probability). The solid lines show the average performance over 20 independent repetitions, and the error bars show the corresponding std. The parameters for NA-DPVI are $\delta = 10^{-5}$, $\kappa = 0.1$ and $T = 10^4$.