Department of Electronic and Telecommunications Engineering

EN3150 - Pattern Recognition

Assignment 01: Learning from data and related challenges and linear models for regression

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1. Data Pre-processing

1.1 Feature Scaling

- **Feature 1**: Max-Abs Scaling
  Feature 1 has a sparse data distribution. Max-abs scaling is chosen for this feature because it scales the data to lie within the range of -1 to 1. This method preserves the structure of Feature 1 after scaling, making it an appropriate choice.

- **Feature 2**: Standard Scaling
  Feature 2 has a relatively high variability within the data distribution. Standard scaling is chosen since it normalizes the data effectively and preserves the structure.

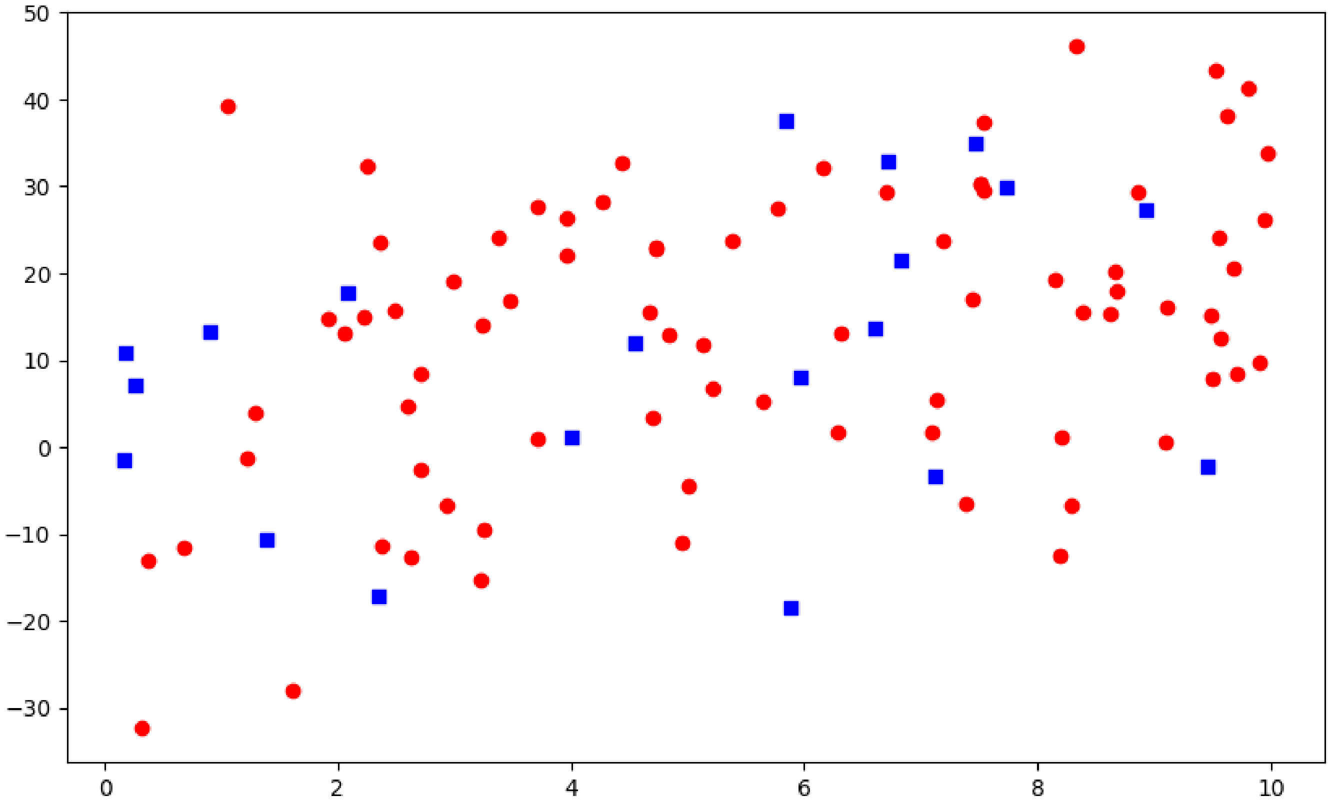
2. Learning from data

2.1 Data generation

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression

# Generate 100 samples
n_samples = 100

# Generate X values (uniformly distributed between 0 and 10)
X = 10 * np.random.rand(n_samples, 1)

# Generate epsilon values (normally distributed with mean 0 and standard deviation 15)
epsilon = np.random.normal(0, 15, n_samples)

# Generate Y values using the model Y = 3 + 3X + epsilon
Y = 3 + 2 * X + epsilon[:, np.newaxis]
```

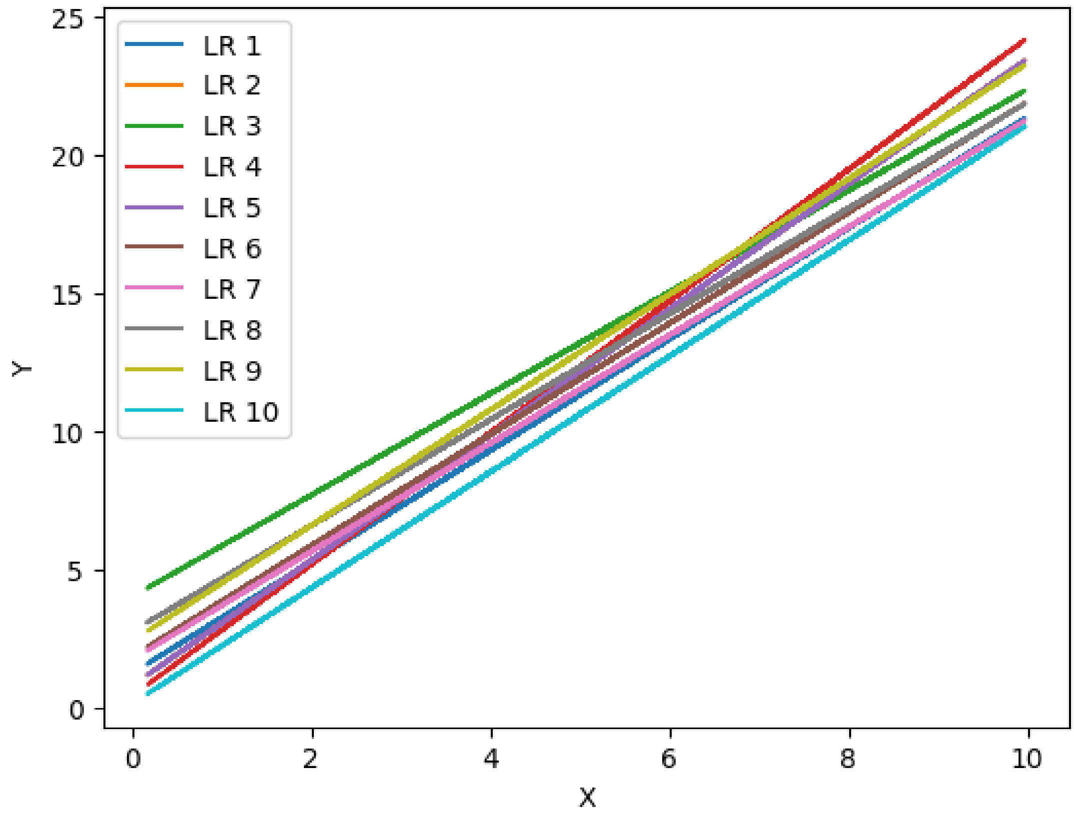
2.2 Data Visualization

In [2]:
```
r=np.random.randint(104)

# Split the data into training and test sets (80% train,20% test)
X_train, X_test, Y_train, Y_test = train_test_split(X, Y,test_size=0.2, random_state=r)

# Plot the data points
plt.figure(figsize=(10, 6))
plt.scatter(X_train, Y_train, alpha=1, marker='o',color='red',label='Training Data')
plt.scatter(X_test, Y_test, alpha=1, marker='s',color='blue',label='Testing Data')
plt.show()
```



2.3 Linear Regression

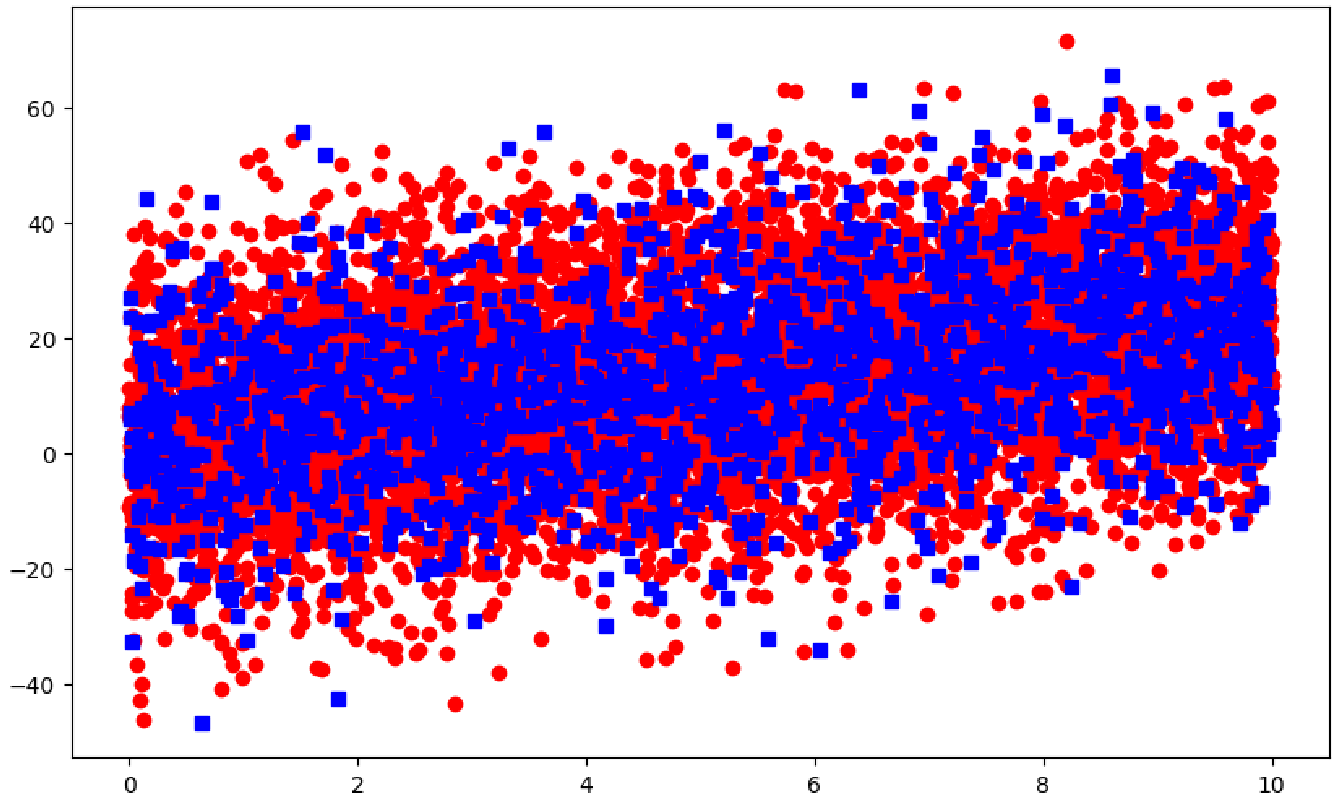
In [3]:
```
for i in range(10): # Plotting 10 different instances
    X_train, X_test, Y_train, Y_test = train_test_split(X,Y, test_size=0.2, random_state
    model = LinearRegression()
    model.fit(X_train, Y_train)
    Y_pred_train = model.predict(X_train)
    plt.plot(X_train, Y_pred_train, label=f'LR {i+1}')
plt.xlabel('X')
plt.ylabel('Y')
plt.legend()
plt.show()
```



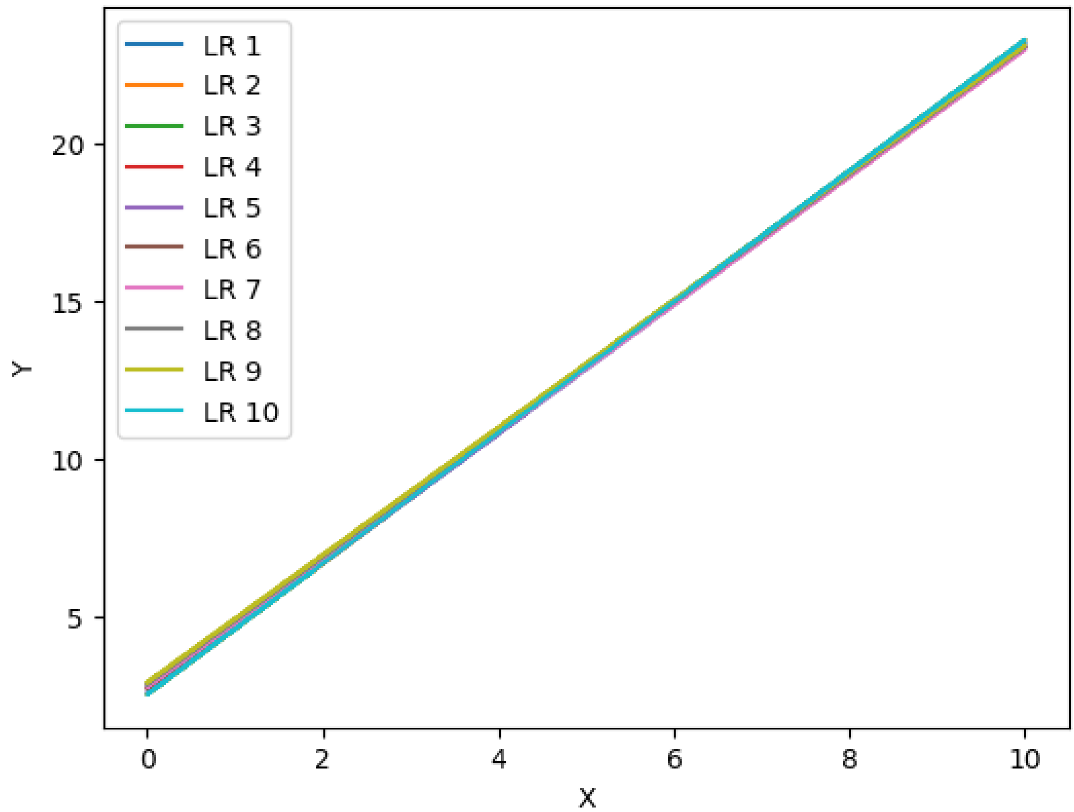
- Because, the 100 data points are generated randomly each time, resulting in a different dataset for the model to fit on each instance. Therefore, the linear regression model varies from one instance to another.

2.4 Above tasks for 10,000 data samples,

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression

# Generate 100 samples
n_samples = 10000

# Generate X values (uniformly distributed between 0 and 10)
X = 10 * np.random.rand(n_samples, 1)

# Generate epsilon values (normally distributed with mean 0 and standard deviation 15)
epsilon = np.random.normal(0, 15, n_samples)

# Generate Y values using the model Y = 3 + 3X + epsilon
Y = 3 + 2 * X + epsilon[:, np.newaxis]

r=np.random.randint(104)

# Split the data into training and test sets (80% train,20% test)
X_train, X_test, Y_train, Y_test = train_test_split(X, Y,test_size=0.2, random_state=r)

# Plot the data points
plt.figure(figsize=(10, 6))
plt.scatter(X_train, Y_train, alpha=1, marker='o',color='red',label='Training Data')
plt.scatter(X_test, Y_test, alpha=1, marker='s',color='blue',label='Testing Data')
plt.show()
```



In [5]:
```
for i in range(10): # Plotting 10 different instances
    X_train, X_test, Y_train, Y_test = train_test_split(X,Y, test_size=0.2, random_state
    model = LinearRegression()
    model.fit(X_train, Y_train)
    Y_pred_train = model.predict(X_train)
    plt.plot(X_train, Y_pred_train, label=f'LR {i+1}')
plt.xlabel('X')
plt.ylabel('Y')
plt.legend()
plt.show()
```



- This data generation process creates a uniformly distributed sample between 0 and 10. Since the number of data samples has increased from 100 to 100,000, the dataset has become more generalizable. As a result, in different instances, the datasets are almost identical because they are sampled from the same range with a large number of 100,000 data points. Therefore, the linear regression model remains almost the same across different instances.

3. Linear regression on real world data

3.1 Load the dataset

In [6]:

```
# If package not installed, install it using pip install ucimlrepo
from ucimlrepo import fetch_ucirepo

# fetch dataset
infrared_thermography_temperature = fetch_ucirepo(id=925)

# data (as pandas dataframes)
X = infrared_thermography_temperature.data.features
y = infrared_thermography_temperature.data.targets

# metadata
print(infrared_thermography_temperature.metadata)

# variable information
print(infrared_thermography_temperature.variables)
```


```
{'uci_id': 925, 'name': 'Infrared Thermography Temperature', 'repository_url': 'https://
archive.ics.uci.edu/dataset/925/infrared+thermography+temperature+dataset', 'data_url':
'https://archive.ics.uci.edu/static/public/925/data.csv', 'abstract': 'The Infrared Ther
mography Temperature Dataset contains temperatures read from various locations of inferr
ed images about patients, with the addition of oral temperatures measured for each indiv
idual. The 33 features consist of gender, age, ethnicity, ambiant temperature, humidity,
distance, and other temperature readings from the thermal images. The dataset is intende
d to be used in a regression task to predict the oral temperature using the environment
information as well as the thermal image readings. ', 'area': 'Health and Medicine', 'ta
sks': ['Regression'], 'characteristics': ['Tabular'], 'num_instances': 1020, 'num_featur
es': 33, 'feature_types': ['Real', 'Categorical'], 'demographics': ['Gender', 'Age', 'Et
hnicity'], 'target_col': ['aveOralF', 'aveOralM'], 'index_col': ['SubjectID'], 'has_miss
ing_values': 'no', 'missing_values_symbol': None, 'year_of_dataset_creation': 2021, 'las
t_updated': 'Tue Dec 12 2023', 'dataset_doi': '10.13026/9ay4-2c37', 'creators': ['Quanze
ng Wang', 'Yangling Zhou', 'Pejman Ghassemi', 'David McBride', 'J. Casamento', 'T. Pfefe
r', 'Quanzeng Wang', 'Yangling Zhou', 'Pejman Ghassemi', 'David McBride', 'J. Casament
o', 'T. Pfefer'], 'intro_paper': {'title': 'Infrared Thermography for Measuring Elevated
Body Temperature: Clinical Accuracy, Calibration, and Evaluation', 'authors': 'Quanzeng
Wang, Yangling Zhou, Pejman Ghassemi, David McBride, J. Casamento, T. Pfefer', 'publishe
d_in': 'Italian National Conference on Sensors', 'year': 2021, 'url': 'https://www.seman
ticscholar.org/paper/443b9932d295ca3a014e7d874b4bd77a33a276bd', 'doi': None}, 'additiona
l_info': {'summary': None, 'purpose': None, 'funded_by': None, 'instances_represent': No
ne, 'recommended_data_splits': None, 'sensitive_data': None, 'preprocessing_descriptio
n': None, 'variable_info': '- gender\n- age\n- ethnicity\n- ambiant temperature\n- humid
ity\n- distance\n- temperature readings from the thermal images', 'citation': None}, 'ex
ternal_url': 'https://physionet.org/content/face-oral-temp-data/1.0.0/'}
```


```
           name     role         type demographic  \
0     SubjectID       ID  Categorical        None
1      aveOralF   Target   Continuous        None
2      aveOralM   Target   Continuous        None
3        Gender  Feature  Categorical      Gender
4           Age  Feature  Categorical         Age
5     Ethnicity  Feature  Categorical   Ethnicity
6         T_atm  Feature   Continuous        None
7      Humidity  Feature   Continuous        None
```

```
8       Distance  Feature   Continuous        None
9      T_offset1  Feature   Continuous        None
10     Max1R13_1  Feature   Continuous        None
11     Max1L13_1  Feature   Continuous        None
12   aveAllR13_1  Feature   Continuous        None
13   aveAllL13_1  Feature   Continuous        None
14         T_RC1  Feature   Continuous        None
15     T_RC_Dry1  Feature   Continuous        None
16     T_RC_Wet1  Feature   Continuous        None
17     T_RC_Max1  Feature   Continuous        None
18         T_LC1  Feature   Continuous        None
19     T_LC_Dry1  Feature   Continuous        None
20     T_LC_Wet1  Feature   Continuous        None
21     T_LC_Max1  Feature   Continuous        None
22          RCC1  Feature   Continuous        None
23          LCC1  Feature   Continuous        None
24    canthiMax1  Feature   Continuous        None
25   canthi4Max1  Feature   Continuous        None
26       T_FHCC1  Feature   Continuous        None
27       T_FHRC1  Feature   Continuous        None
28       T_FHLC1  Feature   Continuous        None
29       T_FHBC1  Feature   Continuous        None
30       T_FHTC1  Feature   Continuous        None
31     T_FH_Max1  Feature   Continuous        None
32    T_FHC_Max1  Feature   Continuous        None
33        T_Max1  Feature   Continuous        None
34         T_OR1  Feature   Continuous        None
35     T_OR_Max1  Feature   Continuous        None

                                          description units missing_values
0                                          Subject ID  None             no
1              Oral temperature measured in fast mode  None             no
2           Oral temperature measured in monitor mode  None             no
3                                      Male or Female  None             no
4                            Age ranges in categories\n  None             no
5   American Indian or Alaska Native, Asian, Black...  None             no
6                                 Ambiant temperature  None             no
7                                   Relative humidity  None             no
8        Distance between the subjects and the IRTs.   None             no
9   Temperature difference between the set and mea...  None             no
10  Max value of a circle with diameter of 13 pixe...  None             no
11  Max value of a circle with diameter of 13 pixe...  None             no
12  Average value of a circle with diameter of 13 ...  None             no
13  Average value of a circle with diameter of 13 ...  None             no
14  Average temperature of the highest four pixels...  None             no
15  Average temperature of the highest four pixels...  None             no
16  Average temperature of the highest four pixels...  None             no
17  Max value of a square of 24x24 pixels around t...  None             no
18  Average temperature of the highest four pixels...  None             no
19  Average temperature of the highest four pixels...  None             no
20  Average temperature of the highest four pixels...  None             no
21  Max value of a circle with diameter of 13 pixe...  None             no
22  Average value of a square of 3x3 pixels center...  None             no
23  Average value of a square of 3x3 pixels center...  None             no
24                Max value in the extended canthi area  None             no
25  Average temperature of the highest four pixels...  None             no
26  Average value in the center point of forehead,...  None             no
27  Average value in the right point of the forehe...  None             no
28  Average value in the left point of the forehea...  None             no
29  Average value in the bottom point of the foreh...  None             no
30  Average value in the top point of the forehead...  None             no
31  Maximum temperature within the extended forehe...  None             no
32  Max value in the center point of forehead, a s...  None             no
33  Maximum temperature within the whole face region.  None             no
34  Average temperature of the highest four pixels...  None             no
35       Maximum temperature within the mouth region.  None             no
```

3.2 Independant and Dependant Variables

In [7]:
```
print(f"Independaent Variables: {X.shape[1]}")
print(f"Dependaent Variables: {y.shape[1]}")
```

```
Independaent Variables: 33
Dependaent Variables: 2
```

3.3 Possibility to apply linear regression

- No, it is not possible because the dataset has categorical type features.
- In order to apply linear regression to this dataset, we need to use labeling or one-hot encoding like encoding method on this categorical features.

3.4 Remove NaN/missing values using,

X = X.dropna()

y = y.dropna()

- This approach provided is incorrect because it drops missing values from X and y separately, which can cause a misalignment between the feature set X and the target variable y. If a missing value is dropped in X but not in the corresponding y, or vice versa, the data will no longer match up correctly.

In [8]:
```
print(f"Number of samples: {X.shape[0]}")
```

```
Number of samples: 1020
```

In [9]:
```
import pandas as pd

#Removing missing/NaN values
data = pd.concat([X, y], axis=1)
data = data.dropna()

X = data.drop(['aveOralF', 'aveOralM'], axis=1)
y = data[['aveOralF', 'aveOralM']]
```

Shape after removing missing/NaN values.

In [10]:
```
print(f"X shape: {X.shape}")
print(f"y shape: {y.shape}")
```

```
X shape: (1018, 33)
y shape: (1018, 2)
```

3.5 Select features

dependent_feature = 'aveOralM'

independant_features = ['T_atm', 'Humidity', 'T_offset', 'T_RC1', 'Age']

Since 'Age' is a categorical type feature,

In [11]:
```
# One-hot encode 'Age' feature
X_encoded = pd.get_dummies(X, columns=['Age'], drop_first=True)

# Print the new columns to confirm one-hot encoding
print(X_encoded.head())
```

```
   Gender                     Ethnicity  T_atm  Humidity  Distance  T_offset1  \
0    Male                         White   24.0      28.0       0.8     0.7025
1  Female  Black or African-American   24.0      26.0       0.8     0.7800
2  Female                         White   24.0      26.0       0.8     0.8625
3  Female  Black or African-American   24.0      27.0       0.8     0.9300
4    Male                         White   24.0      27.0       0.8     0.8950

   Max1R13_1  Max1L13_1  aveAllR13_1  aveAllL13_1  ...  T_Max1   T_OR1  \
0    35.0300    35.3775      34.4000      34.9175  ...  35.6925  35.6350
1    34.5500    34.5200      33.9300      34.2250  ...  35.1750  35.0925
2    35.6525    35.5175      34.2775      34.8000  ...  35.9125  35.8600
3    35.2225    35.6125      34.3850      35.2475  ...  35.7200  34.9650
4    35.5450    35.6650      34.9100      35.3675  ...  35.8950  35.5875

   T_OR_Max1  Age_21-25  Age_21-30  Age_26-30  Age_31-40  Age_41-50  \
0    35.6525      False      False      False      False       True
1    35.1075      False      False      False       True      False
2    35.8850      False       True      False      False      False
3    34.9825      False       True      False      False      False
4    35.6175      False      False      False      False      False

   Age_51-60  Age_>60
0      False    False
1      False    False
2      False    False
3      False    False
4      False    False

[5 rows x 39 columns]
```

In [12]:
```python
dependent_feature = "aveOralM"
independent_features = ['T_atm', 'Humidity', 'T_offset1', 'T_RC1'] + [column for column

X_final = X_encoded[independent_features]
y_final = y[dependent_feature]
```

In [13]:
```python
X_final
```

Out[13]:

	T_atm	Humidity	T_offset1	T_RC1	Age_21-25	Age_21-30	Age_26-30	Age_31-40	Age_41-50	Age_51-60	Age_>60
0	24.0	28.0	0.7025	34.9850	False	False	False	False	True	False	False
1	24.0	26.0	0.7800	34.7100	False	False	False	True	False	False	False
2	24.0	26.0	0.8625	35.6850	False	True	False	False	False	False	False
3	24.0	27.0	0.9300	35.2075	False	True	False	False	False	False	False
4	24.0	27.0	0.8950	35.6025	False	False	False	False	False	False	False
...	...	...	...	...	...	...	...	...	...	...	...
1015	25.7	50.8	1.2225	35.7525	True	False	False	False	False	False	False
1016	25.7	50.8	1.4675	35.9700	True	False	False	False	False	False	False
1017	28.0	24.3	0.1300	36.4100	False	False	False	False	False	False	False
1018	25.0	39.8	1.2450	35.7700	False	False	True	False	False	False	False
1019	23.8	45.6	0.8675	35.7025	False	False	False	False	False	False	False

1018 rows × 11 columns

In [14]:
```python
y_final
```

Out[14]:
```
0        36.59
```

```
1        37.19
2        37.34
3        37.09
4        37.04
         ...
1015     36.99
1016     37.19
1017     37.59
1018     37.29
1019     37.19
Name: aveOralM, Length: 1018, dtype: float64
```

3.6 Data splitting

In [15]:
```
#split the data
X_train, X_test, y_train, y_test = train_test_split(X_final, y_final, test_size=0.2, ran

print(f"X_train shape: {X_train.shape}")
print(f"X_test shape: {X_test.shape}")
print(f"y_train shape: {y_train.shape}")
print(f"y_test shape: {y_test.shape}")
```

```
X_train shape: (814, 11)
X_test shape: (204, 11)
y_train shape: (814,)
y_test shape: (204,)
```

3.7 Train a linear regression model

In [16]:
```
from sklearn.linear_model import LinearRegression

# Fit the model
model = LinearRegression()
model.fit(X_train, y_train)
```

Out[16]:
```
LinearRegression
LinearRegression()
```

- Estimated coefficients corresponding to independent variables

In [17]:
```
#Retrieve the coefficients corresponds to independent features
coefficients = model.coef_
intercept = model.intercept_

for feature, coef in zip(independent_features, coefficients):
    print(f"{feature}: {coef}")
```

```
T_atm: -0.06022755085277936
Humidity: 0.0013094634816224054
T_offset1: 0.04018229206283859
T_RC1: 0.763191169629071
Age_21-25: 0.04908563499575056
Age_21-30: 0.16823450873539156
Age_26-30: 0.05754835526898491
Age_31-40: 0.011791307076780826
Age_41-50: 0.2817743528981443
Age_51-60: 0.11321020120241118
Age_>60: 0.7091377165441106
```

3.8 Highly contributed independent feature

- T_RC1 feature contributes highly for the 'aveOralM' dependant feature from selected feature as it has a highest coefficient of 0.7632

3.9 Linear regression model for 'T_OR1', 'T_OR_Max1', 'T_FHC_Max1', 'T_FH_Max1' as features.

In [18]:
```
#feature selection
dependent_feature = "aveOralM"
independent_features_new = ['T_OR1', 'T_OR_Max1', 'T_FHC_Max1', 'T_FH_Max1']

X_sel = X[independent_features_new]
y_sel = y[dependent_feature]

#data splitting
X_train, X_test, y_train, y_test = train_test_split(X_sel, y_sel, test_size=0.2, random_

#linear regression model
model_2 = LinearRegression()
model_2.fit(X_train, y_train)
```

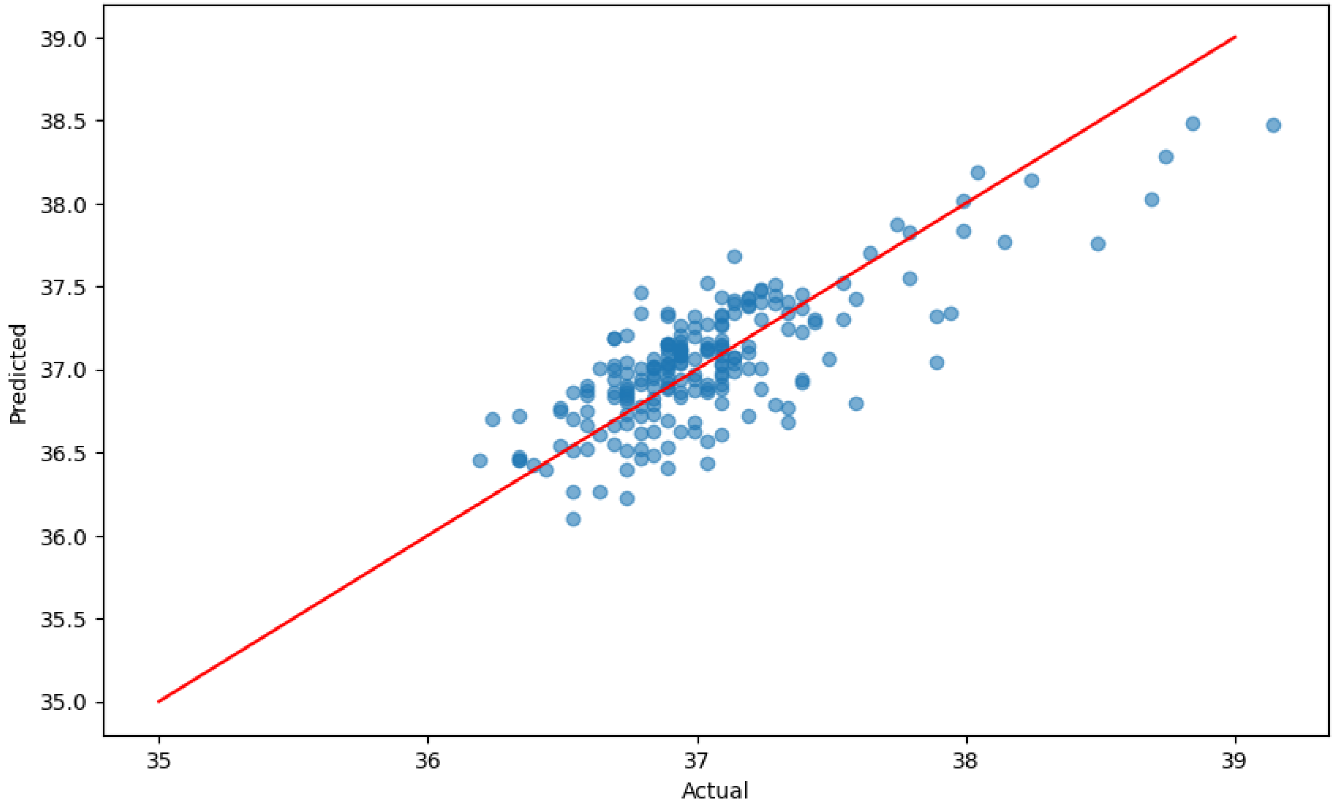
Out[18]:
```
LinearRegression
LinearRegression()
```

- Estimated coefficients corresponding to independent variables

In [19]:
```
#Retrieve the coefficients corresponds to independent features
coefficients_sel = model_2.coef_
intercept_sel = model_2.intercept_

for feature, coef in zip(independent_features_new, coefficients_sel):
    print(f"{feature}: {coef}")
```

```
T_OR1: 0.2054577632399466
T_OR_Max1: 0.3481968431600288
T_FHC_Max1: -0.08371846705362104
T_FH_Max1: 0.3765643420653233
```

In [20]:
```
#visualizing the model and the predictions
y_pred = model_2.predict(X_test)

plt.figure(figsize=(10, 6))
plt.scatter(y_test, y_pred, alpha=0.6)
plt.plot(range(35,40), range(35,40), color='red')
plt.xlabel('Actual')
plt.ylabel('Predicted')
plt.show()
```



3.10 Evaluation metrics

- Calculating evaluation metrics for test data for evaluate the model.

In [21]:
```
import numpy as np
from scipy.stats import t

# Residual sum of squares (RSS)
RSS = np.sum(np.square(y_test - model_2.predict(X_test)))
print(f"Residual sum of squares (RSS): {RSS}")

# Residual standard error (RSE)
N = X_test.shape[0]
d = X_test.shape[1]
RSE = np.sqrt(RSS/(N-d-1))
print(f"Residual standard error (RSE): {RSE}")

# Mean squared error (MSE)
MSE = np.mean(np.square(y_test - model_2.predict(X_test)))
print(f"Mean squared error (MSE): {MSE}")

# R2 statistic
y_mean = np.mean(y_test)
TSS = np.sum(np.square(y_test - y_mean))
R2 = 1 - RSS/TSS
print(f"R2 statistic: {R2}")

# Standard error for each feature
standard_errors = []
for feature in independent_features_new:
    X_test_feature = X_test[feature]
    X_test_feature_mean = np.mean(X_test_feature)
    X_test_feature_std = np.std(X_test_feature)
    SE2 = X_test_feature_std**2 / np.sum(np.square(X_test_feature - X_test_feature_mean)
    standard_errors.append(np.sqrt(SE2))
    print(f"Standard error for {feature}: {np.sqrt(SE2)}")
```

```
# t-statistic for each feature
t_statistics = []
for coef, SE, feature in zip(coefficients_sel, standard_errors, independent_features_new

    t_stat = coef / (SE**2)
    t_statistics.append(t_stat)
    print(f"t-statistic for {feature}: {t_stat}")
```

```
Residual sum of squares (RSS): 15.923399754377385
Residual standard error (RSE): 0.28287291173396434
Mean squared error (MSE): 0.07805588114890875
R2 statistic: 0.6076047374475756
Standard error for T_OR1: 0.07001400420140048
Standard error for T_OR_Max1: 0.07001400420140048
Standard error for T_FHC_Max1: 0.0700140042014005
Standard error for T_FH_Max1: 0.07001400420140048
t-statistic for T_OR1: 41.91338370094872
t-statistic for T_OR_Max1: 71.03215600464588
t-statistic for T_FHC_Max1: -17.07856727893869
t-statistic for T_FH_Max1: 76.81912578132597
```

- Calculating the p-values for the training dataset.

In [22]:
```
import pandas as pd
import statsmodels.api as sm


# Add a constant column to the feature matrix (required by statsmodels)
X_train_with_constant = sm.add_constant(X_train)

# Fit the OLS (Ordinary Least Squares) model
ols_model = sm.OLS(y_train, X_train_with_constant).fit()

# Get summary statistics of the model
summary = ols_model.summary()

# Extract p-values from the summary for all features
p_values = summary.tables[1].data[1:]

# Create a DataFrame to associate p-values with feature names
p_values_df = pd.DataFrame(p_values, columns=['Feature', 'Coefficient', 'Standard Error'
p_values_df['P-Value'] = p_values_df['P-Value'].astype(float)


print(summary)
print(p_values_df)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                aveOralM   R-squared:                       0.651
Model:                             OLS   Adj. R-squared:                  0.650
Method:                  Least Squares   F-statistic:                     377.8
Date:                 Sat, 07 Sep 2024   Prob (F-statistic):          2.11e-183
Time:                         15:32:41   Log-Likelihood:                -200.37
No. Observations:                  814   AIC:                             410.7
Df Residuals:                      809   BIC:                             434.2
Df Model:                            4
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          6.7936      0.801      8.486      0.000       5.222       8.365
T_OR1          0.2055      0.893      0.230      0.818      -1.547       1.958
T_OR_Max1      0.3482      0.890      0.391      0.696      -1.400       2.096
```

```
T_FHC_Max1    -0.0837      0.044     -1.921      0.055      -0.169       0.002
T_FH_Max1      0.3766      0.049      7.756      0.000       0.281       0.472
==============================================================================
Omnibus:                       49.892   Durbin-Watson:                   2.020
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               76.197
Skew:                           0.481   Prob(JB):                     2.84e-17
Kurtosis:                       4.149   Cond. No.                     8.25e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifi
ed.
[2] The condition number is large, 8.25e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
      Feature Coefficient Standard Error    t-value  P-Value   Lower CI  \
0       const      6.7936          0.801      8.486    0.000      5.222
1       T_OR1      0.2055          0.893      0.230    0.818     -1.547
2   T_OR_Max1      0.3482          0.890      0.391    0.696     -1.400
3  T_FHC_Max1     -0.0837          0.044     -1.921    0.055     -0.169
4   T_FH_Max1      0.3766          0.049      7.756    0.000      0.281

    Upper CI
0      8.365
1      1.958
2      2.096
3      0.002
4      0.472
```

3.11 Discard of features

- We can reject the features that have p-values grater the 5% based on p-values. Therefore, we can discard the features 'T_OR1', 'T_OR_Max1', 'T_FHC_Max1'.

4. Performance evaluation of Linear regression

Table 1: SSE and TSS of linear regression models.

	Model A	Model B
SSE= $\sum_{i=1}^{N}(y_i - \boldsymbol{w}^T\boldsymbol{x}_i)^2$	9	2
TSS= $\sum_{i=1}^{N}(y_i - \tilde{y}_i)^2$	90	10
Number of data samples (N)	10000	10000

$$ModelA : y = w_0 + w_1x^1 + w_1x^2$$
$$ModelB : y = w_0 + w_1x^1 + w_1x^2 + w_3x^3 + w_4x^4$$

4.2 RSE Calculation

$$RSE = \sqrt{\frac{RSS}{N-d-1}} = \sqrt{\frac{SSE}{N-d-1}}$$

RSE of Model A:

$$RSE = \sqrt{\frac{9}{10000-2-1}} = 0.0300$$

RSE of Model B:

$$RSE = \sqrt{\frac{2}{10000-4-1}} = 0.01415$$

Conclusion: RSE is lower in Model B. Therefore, based on RSE, Model B performs better.

4.3 R^2 Calculation

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{SSE}{TSS}$$

R^2 of Model A:

$$R^2 = 1 - \frac{9}{90} = 0.9$$

R^2 of Model B:

$$R^2 = 1 - \frac{2}{10} = 0.8$$

Conclusion: R^2 is higher for model A, which is 0.9. Therefore, based on R^2, model A performs better.

4.4 Model evaluation

R^2 (R-squared) metric is more fair because the variance of the data in the test dataset are taking into account by R^2 metric whereas RSE only takes sum of squared errors. Since the variance of the data varies we can't evaluate the performance only by looking at SSE. Therefore, model A performs better.

5. Linear regression impact on outliers.

$$L_1(w) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_i^2}{a^2 + r_i^2}\right) = \frac{1}{N}\sum_{i=1}^{N} L_{1,i}$$

$$L_2(w) = \frac{1}{N}\sum_{i=1}^{N}\left(1 - \exp\left(\frac{-2|r_i|}{a}\right)\right) = \frac{1}{N}\sum_{i=1}^{N} L_{2,i}$$

Where:

$$r_i = \hat{y}_i - y_i, \quad \hat{y}_i = w^T x_i$$

5.2 When $a \to 0$:

$$\lim_{a\to 0} L_1(w) = \lim_{a\to 0}\left\{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_i^2}{a^2 + r_i^2}\right)\right\} = \frac{1}{N}\sum_{i=1}^{N} 1 = 1$$

$$\lim_{a\to 0} L_2(w) = \lim_{a\to 0}\left\{\frac{1}{N}\sum_{i=1}^{N}\left(1 - e^{-\frac{2|r_i|}{a}}\right)\right\} = \frac{1}{N}\sum_{i=1}^{N} 1 = 1$$

Both $L_1(w)$ and $L_2(w)$ become 1 when $a \to 0$. Despite the r_i, the loss remains constant. If we take gradient descent for an instant:

$$w \leftarrow w - \alpha\frac{\partial L(w)}{\partial w} = w - \alpha\frac{\partial(1)}{\partial w} = w$$

Since the loss is 1, parameters cannot be optimized. Therefore, the model cannot be trained when $a \to 0$ because the loss remains 1.

5.3 Choose 'a' and loss functions.

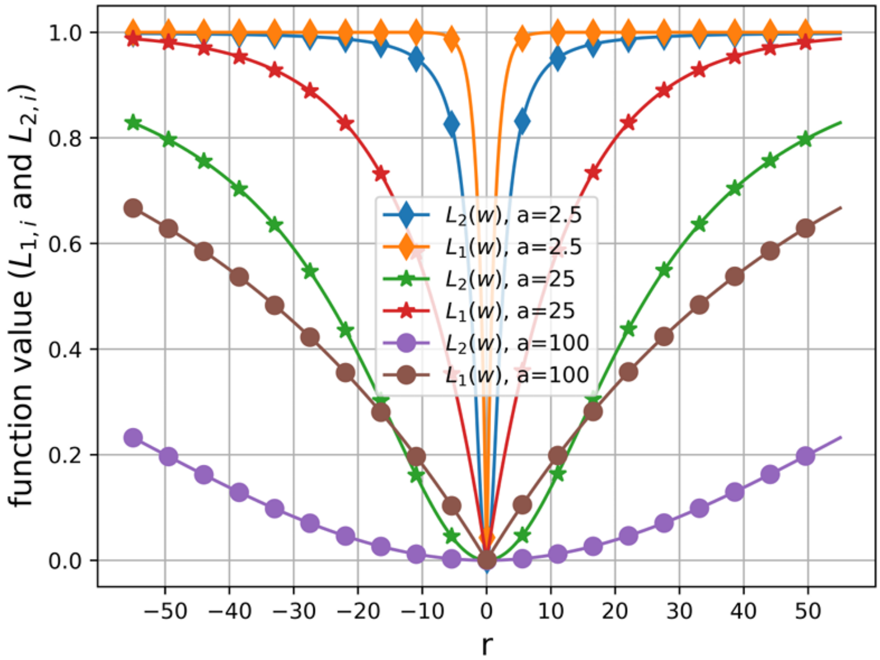


Figure 2: $L_1(\boldsymbol{w})$ and $L_2(\boldsymbol{w})$ with respect to different "a" values.

To minimize the influence of data points where $|r_i| \geq 40$, **choose** $L_2(w)$ **or** $L_1(w)$ **as the loss function** and **set** $a = 25$. From the given graph of $L_1(w)$ and $L_2(w)$ with respect to different values of a shown in the above figure ??, it can be observed that, with these hyperparameters, the gradient of the curve decreases significantly after $|r_i| \geq 40$. Therefore, selecting these parameters will reduce the influence of such data points.