Uncertainty Propagation in Nonlinear Dynamic Systems using Adaptive Gaussian Sum Filter

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Nonlinear Filtering
Methods in use

1. Kalman Filter - Originally for Gaussian Linear Systems
   - Eliminate Gaussian Assumption: Approximate a non-Gaussian pdf with weighted sum of n number of Gaussian pdf

$$
\begin{aligned}
&\hat{p}(t, x(t)) = \sum_{i=1}^{n} w_i \mathcal{N}(x(t) | \mu_i(t), P_i(t)) \\
&\sum_{i=1}^{n} w_i(t) = 1;\ w_i(t) \geq 0, \forall i
\end{aligned} \tag{1}
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2. Nonlinear Systems

Linearize about mean

Assumption: Small Higher Moments

Overhead: Calculation of Jacobian

Apply

Kalman Filter

(a) Extended Kalman Filter(EKF)

Unscented Transformation

Idea: Approximation of pdf is easier than approx of nonlinear function

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(f) Unscented Kalman Filter(UKF)

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Apply Kalman Filter	Apply Kalman Filter
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Nonlinear Filtering
Algorithm/Flowchart

Algorithm

for t = t_0 to t_{end}

  Propagate mean and covariance of each Gaussian component using EKF/UKF.

  If measurement is available

    do measurement update

    do Weight update

end

Flowchart

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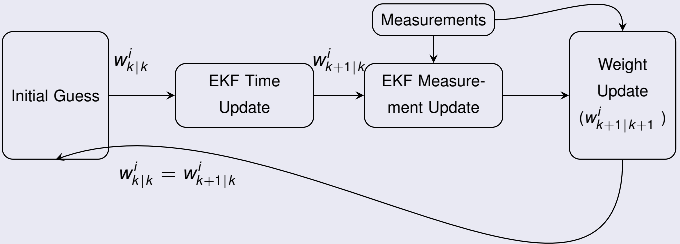

Adaptive gaussian Sum Filter(AGSF)

- Measurements are not available frequently
- Large Uncertainty
- Forecast weight Update: Minimize the error between the pdf using Gaussian sum approximation and the true pdf generated by solving Chapman Kolmogorov equation (CKE). The problem reduces to a convex quadratic optimization which guarantees convergence.

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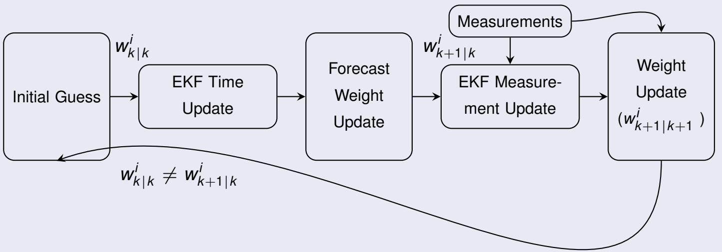


Equation of motion

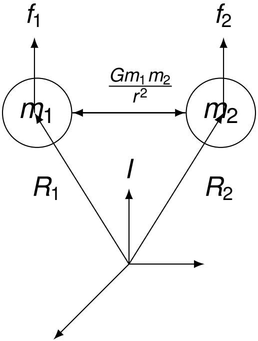
r = R_2 - R_1

$$m_1\ddot{R}_1 = \frac{Gm_1m_2}{r^3}\mathbf{r} + f_1 \qquad m_1\ddot{R}_2 = \frac{-Gm_1m_2}{r^3}\mathbf{r} + f_2$$

$$\ddot{R}_1 = \frac{Gm_2}{r^3}\mathbf{r} + \frac{f_1}{m_1} \qquad \ddot{R}_2 = \frac{-Gm_1}{r^3}\mathbf{r} + \frac{f_2}{m_2}$$

$$\ddot{R}_2 - \ddot{R}_1 = \frac{G(m_1+m_2)}{r^3}\mathbf{r} + a_d$$

$$\ddot{\mathbf{r}} = \frac{-\mu}{r^3}\mathbf{r} + a_d$$



Problem Statement

1. Large Uncertainty exist in the initial states.
    - Use UKF for propagation of mean and covariance.
    - Use 6n+1 Gaussian components for approximation.
    - Use parallel computing to get quick results.
2. Use Monte carlo simulations to validate the results.

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