Uncertainty Propagation in Nonlinear Dynamic Systems using Adaptive Gaussian Sum Filter

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Nonlinear Filtering

Methods in use

1. Kalman Filter - Originally for Gaussian Linear Systems
   - Eliminate Gaussian Assumption: Approximate a non-Gaussian pdf with weighted sum of n number of Gaussian pdf

$$\hat{p}(t, x(t)) = \sum_{i=1}^{n} w_i \mathcal{N}(x(t)|\mu_i(t), P_i(t))$$
$$\sum_{i=1}^{n} w_i(t) = 1; w_i(t) \geq 0, \forall i \quad (1)$$

2. Nonlinear Systems

Nonlinear Filtering

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(e) Extended Kalman Filter(EKF) (f) Unscented Kalman Filter(UKF)

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2. **Nonlinear Systems**

Linearize about mean Assumption: Small Higher Moments Overhead: Calculation of jacobian	Unscented Transformation Idea: Approximation of pdf is easier than approx of nonlinear function
↓	↓
Apply Kalman Filter	Apply Kalman Filter
(i) Extended Kalman Filter(EKF)	(j) Unscented Kalman Filter(UKF)

Nonlinear Filtering
Algorithm/Flowchart

Algorithm

for t = t_0 to t_{end}

Propagate mean and covariance of each Gaussian component using EKF/UKF.

If measurement is available

do measurement update

do Weight update

end

Flowchart

Nonlinear Filtering
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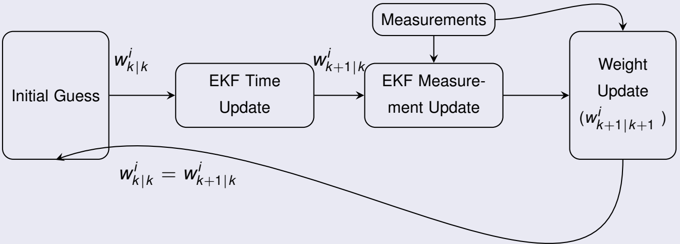

Adaptive gaussian Sum Filter(AGSF)

- Measurements are not available frequently
- Large Uncertainty
- Forecast weight Update:Minimize the error between the pdf using Gaussian sum approximation and the true pdf generated by solving Chapman Kolmogorov equation (CKE). The problem reduces to a convex quadratic optimization which guarantees convergence.

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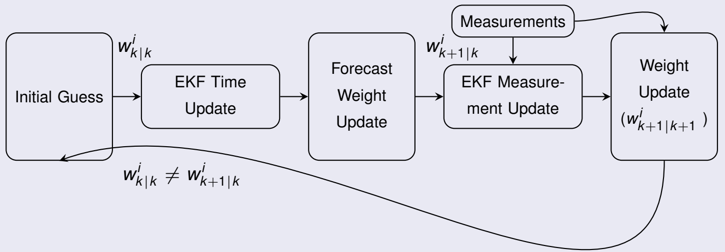

Equation of motion

$\mathbf{r} = R_2 - R_1$

$m_1 \ddot{R}_1 = \frac{Gm_1 m_2}{r^3}\mathbf{r} + f_1$

$\ddot{R}_1 = \frac{Gm_2}{r^3}\mathbf{r} + \frac{f_1}{m_1}$

$\ddot{R}_2 - \ddot{R}_1 = \frac{G(m_1+m_2)}{r^3}\mathbf{r} + a_d$

$\ddot{\mathbf{r}} = \frac{-\mu}{r^3}\mathbf{r} + a_d$

$m_1 \ddot{R}_2 = \frac{-Gm_1 m_2}{r^3}\mathbf{r} + f_2$

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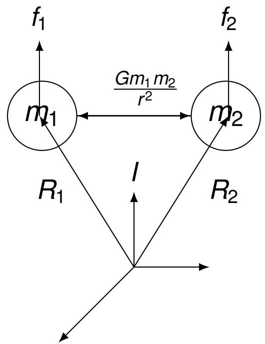

Problem Statement

1. Large Uncertainty exist in the initial states.
   - Use UKF for propagation of mean and covariance.
   - Use 6n+1 Gaussian components for approximation.
   - Use parallel computing to get quick results.
2. Use Monte carlo simulations to validate the results.

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