

# Bending of Bernouilli beams

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# Overview

## 1 Introduction to the problem

- Tacoma Bridge
- Beam equation
- Boundary conditions

## 2 Implementation

## 3 Results

- Static case
- Dynamic case

## 4 Conclusion

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# Tacoma Bridge



# Beam equation

## Dynamic beam equation

$$\begin{cases} \mu \ddot{w}(t, x) + (EIw(t, x)'' )'' = p(t, x) \\ \mu \ddot{v}(t, x) - (EA v(t, x)')' = q(t, x) \end{cases} \quad (1)$$

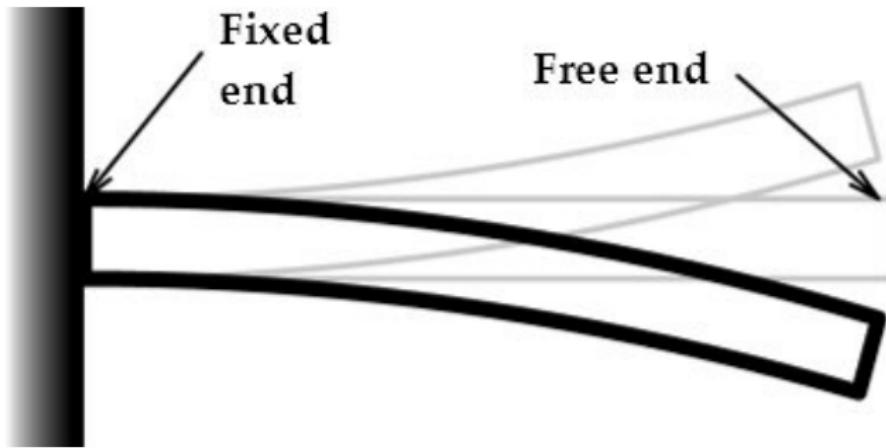
- $w(t, x)$  is the lateral displacement of the beam at  $x$  and time  $t$
- $v(t, x)$  is the axial displacement of the beam at  $x$  and time  $t$
- $E = E(x)$  is Young's modulus
- $I = I(x)$  is the area moment of inertia of the beam
- $A$  is the cross section of the beam
- $q(t, x)$  is the load at  $x$  and time  $t$

# Boundary conditions

## Clamped

No translation nor rotation allowed

Example: Beam bolted to the wall

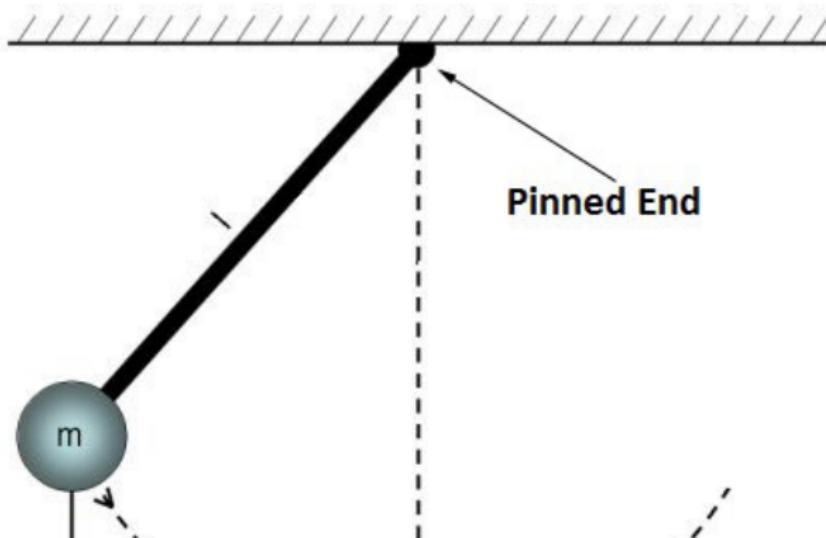


# Boundary conditions

## Pinned

Rotation allowed, no translation

Example: Attached end of a pendulum

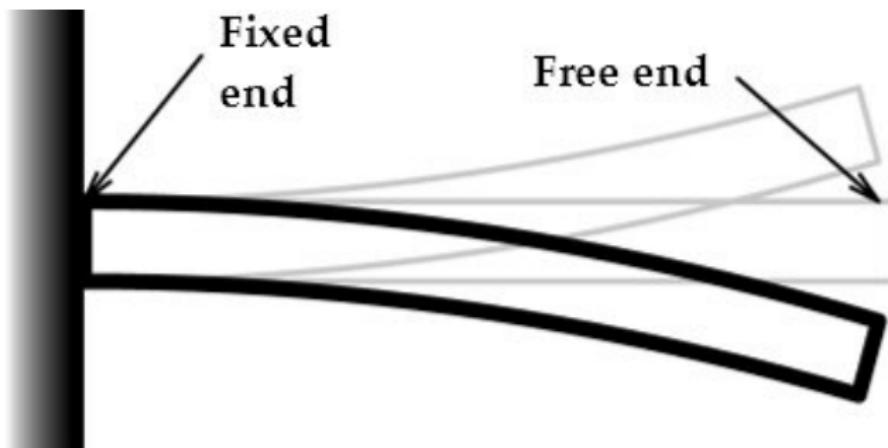


# Boundary conditions

Free

No restrictions

Example: Free end of a pendulum



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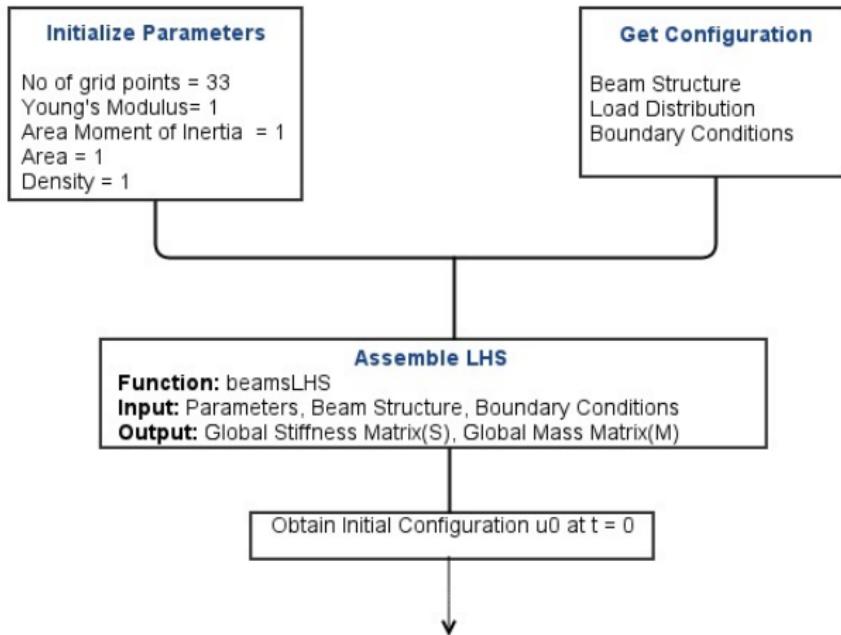
## 2 Implementation

## 3 Results

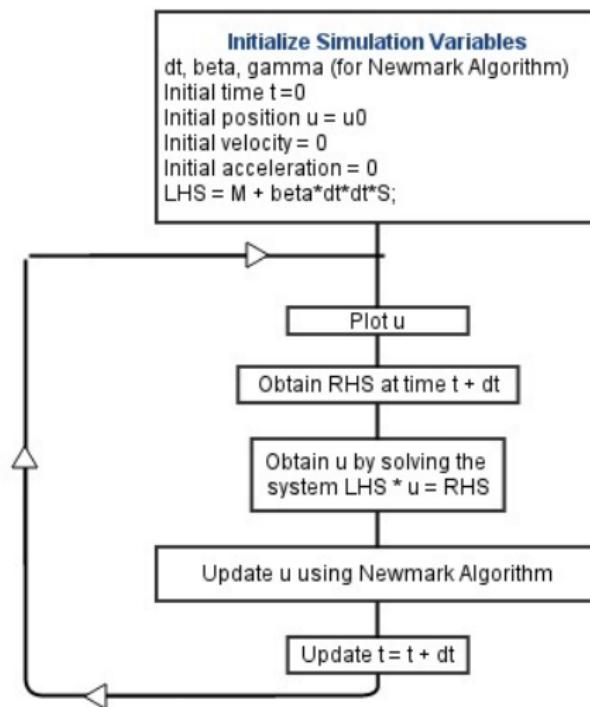
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# Implementation I



## Implementation II



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# Cantilever Beam

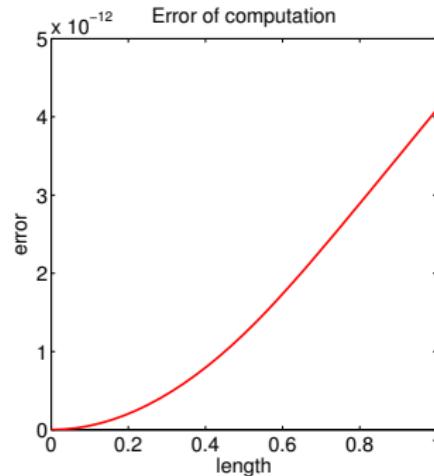
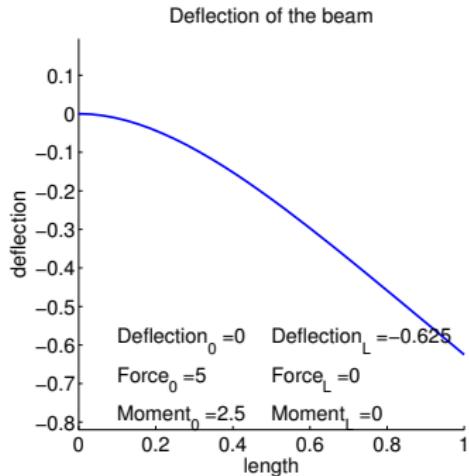


Figure: Results for cantilever beam with uniformly distributed load of 5N

# Beam clamped on both sides

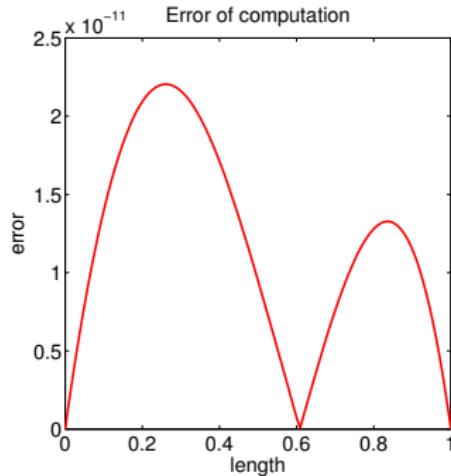
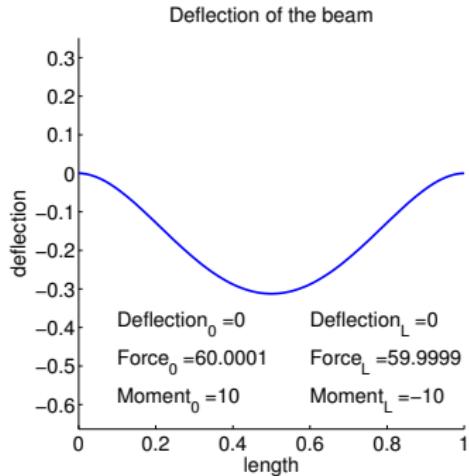


Figure: Results for clamped - clamped beam with uniformly distributed load of 120N

# Beam pinned on both sides

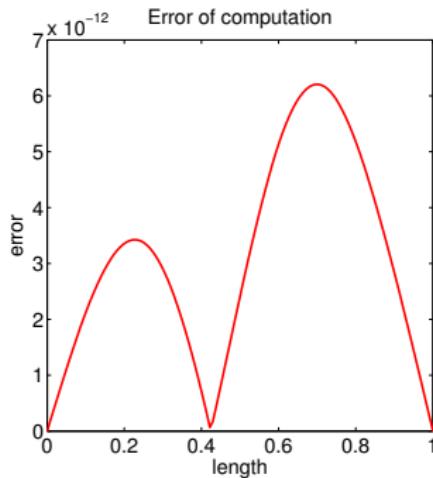
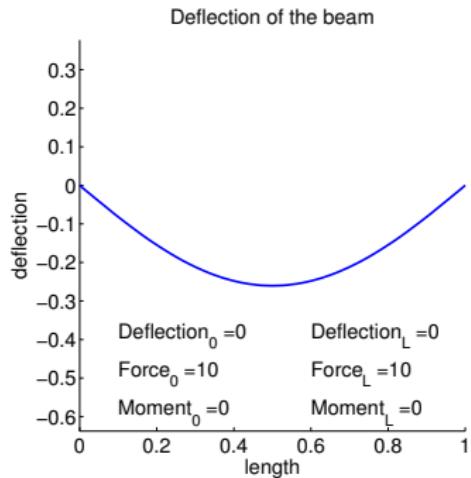


Figure: Results for simply supported beam with uniformly distributed load of 20N

# Unloaded tower

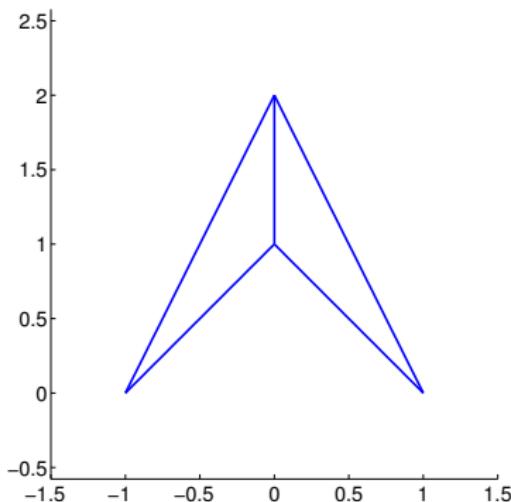
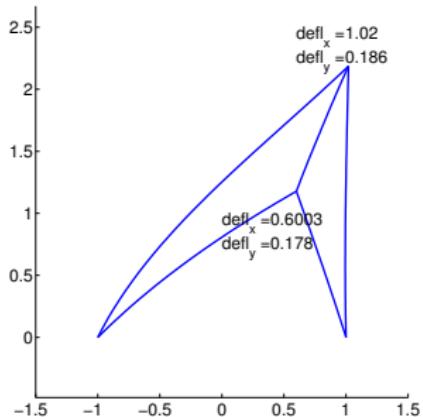
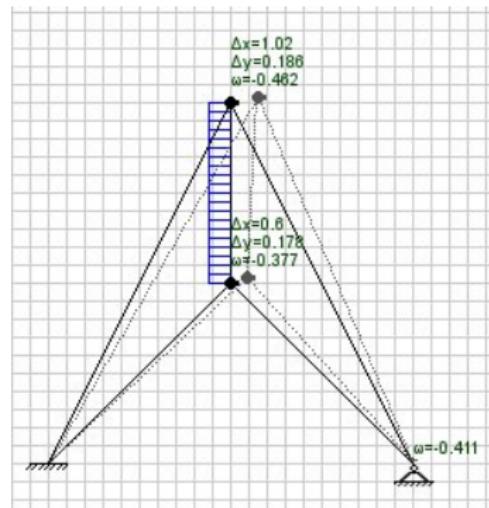


Figure: The unloaded tower structure

# Loaded tower

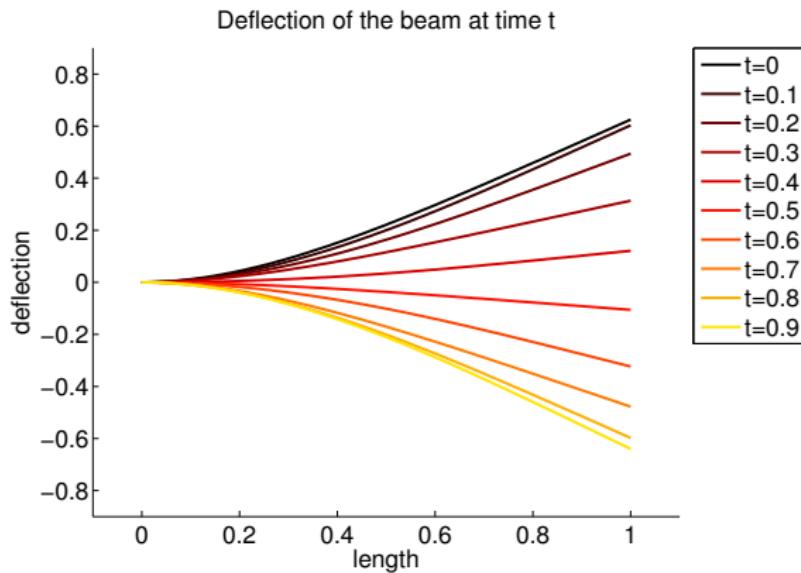


**Figure:** The tower structure loaded with the distributed load of 1N applied on the central beam

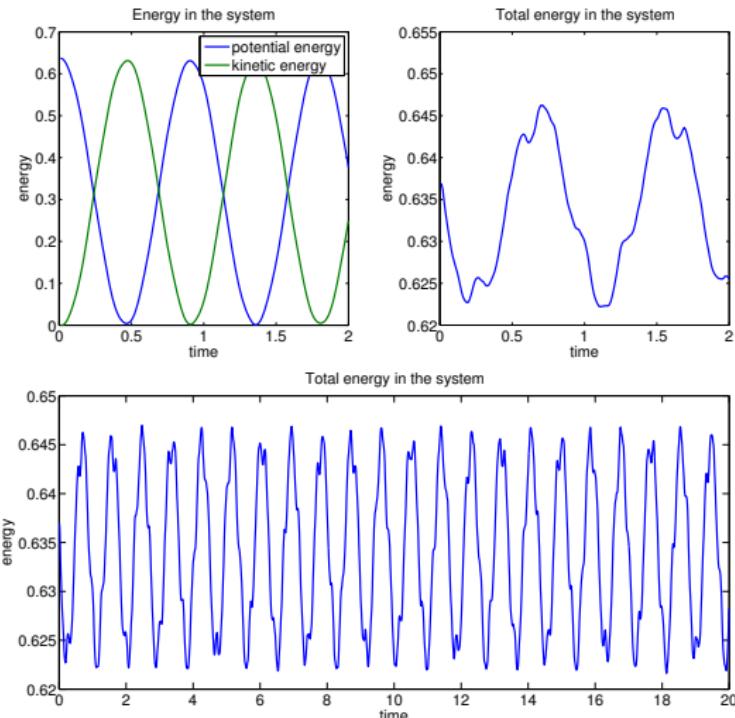


**Figure:** The solution from Sopromat software

# Vibrating cantilever beam



# Energy of vibrating cantilever beam



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# Conclusion

Single static beam

Small error compared to the exact solution ☺

Static framework

Result recovered by another solver ☺

Single dynamic beam

Energy conservation ☺

# Simulated Bridge

