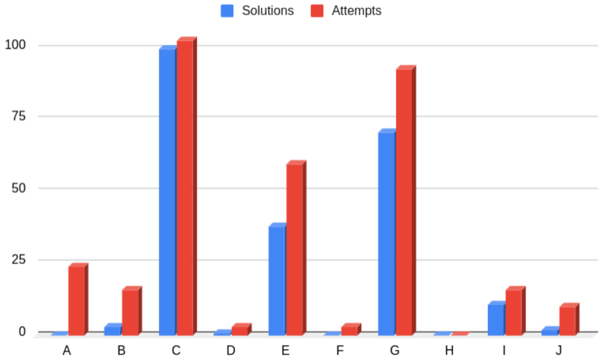
National Girls' Programming Contest 2022

Judge Notes

February 25, 2023

Statistics

- 10 problems.
- 104 teams.



C. Strong Passwords

Author: Rafid Bin Mostofa

Number of Teams Solved: 100
Number of Teams Tried: 103

Abridged Problem Statement
Suggest a standard strong password with certain requirements.

Solution

One such solution is **Mitu1997@qt!**

G. Alligator Sky

Author: Iftekhar Hakim Kaowsar

Number of Teams Solved: 71
Number of Teams Tried: 93

Abridged Problem Statement
Maximum value of $x + y$ such that $x \cdot a + y \cdot b = n$, $a + b \geq k$ and $0 \leq (x - y) \leq 1$.

Solution

Optimal to choose $a,\ b$ such that $a + b = k$.

$$\text{Result} = \begin{cases} \lfloor n/k \rfloor \cdot 2 & \text{if } k \text{ divides } n \\ \lfloor n/k \rfloor \cdot 2 + 1 & \text{otherwise} \end{cases}$$

Solution (contd)

Handle $k = 1$ separately, result is n.

E. Shortest Path, Yet Again

Author: Sabbir Rahman Abir

Number of Teams Solved: 38
Number of Teams Tried: 60

Abridged Problem Statement
Given n, m, k, create a $n \times m$ grid with start cell, end cell and obstacles such that shortest path between start and end cell is k.

Solution (1/2)

Just make a zigzag path with start cell at top left.

```
S......
######.
.......
.######
.......
######.
.......
```

Then traverse this path for k cells and put the end cell!

Solution (2/2)

This works as $2 \leq k \leq \frac{nm}{2}$ and in the contsruction, more than half cells are free.

The traversal can be done via a BFS or just a while loop with some if else.

I. Farewell Gift for Messi

Author: Md. Shariful Islam

Number of Teams Solved: 11
Number of Teams Tried: 16

Abridged Problem Statement
Given n, p, x, y, find number of possible arrays with length n with integer values in $[0, 2^p)$ such that bitwise OR of the array is x and bitwise AND of the array is y.

Solution

Consider the number of ways for each of the p bits separately.

For a particular bit,

- If both OR and AND have same value, we must assign that value for every index in the array.
- If AND has 1 but OR has 0, there is no such array, hence answer will be 0.
- If AND has 0 but OR has 1, the number of ways is $(2^n - 2)$ (excluding all 0's and all 1's).

Solution (contd.)

So, the answer is 0 if there is some bit where AND is 1 but OR is 0.

Otherwise the answer is $(2^n - 2)^x$, where x is the number of bits where AND is 1 but OR is 0.

B. Angels and Demons

Author: Tariq Bin Salam

Number of Teams Solved: 3
Number of Teams Tried: 16

Abridged Problem Statement
Given Q range addition/subtraction updates, what is the initial minimum value, such that applying these updates sequentially does not make the running sum 0 or lower at any point?

Solution

- **Observation:** It is possible to do a binary search on the result.
- How to check if you can survive with initial health H?
- Sort all intervals according to S and simulate the updates from checkpoint 1 to N with health H and see if you survived
- **Complexity:** $O(Q\log(H))$ where H is the maximum possible initial health.

J. Wizard Duel

Author: Rafid Bin Mostofa

Number of Teams Solved: 2
Number of Teams Tried: 10

Abridged Problem Statement
Given a few ratios of unknown variables, compute ratio between two query variables.

Solution

Fractions can be chained and multiplied together to get the values of a new fraction.

$$\frac{a}{b} = \frac{a}{c} \times \frac{c}{d} \times \cdots \times \frac{z}{b}$$

Solution (contd.)

Consider the wizards as vertices.

For each info $\frac{P_i}{P_j} = \frac{x}{y}$, we add two **directed** edges:

- i to j with weight $\frac{x}{y}$
- j to i with weight $\frac{y}{x}$

Solution (contd.)

The value of $\frac{P_a}{P_b}$ is the multiplication of weights of the edges in the path from a to b.

If there are no such paths, the outcome is `unknown`.

If the value equals 1, it is a `draw`. Otherwise, if it's greater than 1, it is a `win` and `lose` otherwise.

We can use DFS, BFS or DSU to traverse the graph.

D. Interesting Parenthesis

Author: Mahdi Hasnat Siyam

Number of Teams Solved: 1
Number of Teams Tried: 3

Abridged Problem Statement
Count the number of parentheses sequence of length $2n$, that can be balanced by at most one swap.

Solution

Consider all opening parentheses as $+1$, and all closing parentheses as -1.

To check if a parentheses sequence is balanced:

- Summation of the $+1$, -1 values equals to 0.
- None of the prefix sum is less than 0.

A parenthesis sequence is **interesting** if all of the prefix sums are greater than or equal to -2 and the total sum is 0.

Solution (contd.)

Let $dp(n, c)$ be the number of parenthesis sequence we can generate such that we have remaining n position left and current prefix sum is c.

$$dp(0, 0) = 1$$

$$dp(0, c \neq 0) = 0$$

We can compute $dp(n, c)$ by the following:

$$dp(n, c) = \begin{cases} dp(n-1, c+1) + dp(n-1, c-1) & \text{if } c - 1 \geq -2 \\ dp(n-1, c+1) & \text{otherwise} \end{cases}$$

Solution (contd.)

Complexity: $O(N^2)$.

There exists a faster approach using the idea of Catalan Numbers. See OEIS series A026012.

H. Plantik

Author: Kazi Md Irshad

Number of Teams Solved: 0
Number of Teams Tried: 0

Abridged Problem Statement
Calculate the probability of a team winning a penalty shootout, given scoring probabilities of each player.

Solution

The optimal shooting order is sorted in non-increasing order. Which means the best shooter will go first and the worst will go last. 5 best shooters already took shots.

Remaining shooting order is from 6th to 11th, then 1st to 5th.

Let us relabel the shooters (from 6th to 11th, then 1st to 5th) as 1st to 11th.

Solution (contd.)

Let f_i be the probability that Argentina wins from the situation when the next shooters of each team is i. Let X_i be the probability that Argentina win after each team's i shoots. And Y_i be the probability that the shootout continues.

If Argentina score and France miss then Argentina win.

$$X_i = A_i \cdot (1 - F_i)$$

If both players score or miss, the shootout continues.

$$Y_i = A_i \cdot F_i + (1 - A_i) \cdot (1 - F_i)$$

Solution (contd.)

Now we can write for each i in range $[1, 10]$

$$f_i = X_i + Y_i \cdot f_{i+1}$$

And additionally

$$f_{11} = X_{11} + Y_{11} \cdot f_1$$

We need to find f_1.

Solution (contd.)

We can solve the equations by iteratively merging the linear equations.

We can also solve it by algebraic manipulation.

$$f_1 = \frac{\sum_{i=1}^{11} (X_i \cdot \prod_{j=1}^{j<i} Y_i)}{1 - \prod_{i=1}^{11} Y_i}$$

We can also solve it by gaussian elimination.

A. Half Measures

Author: Pritom Kundu

Number of Teams Solved: 0
Number of Teams Tried: 24

Abridged Problem Statement
The numbers $1, 2, \cdots n$ are written on a board. In one operation, you can halve any number. Find minimum sum after k operations.

Solution

- **Observation:** Its always optimal to choose the largest number and halve it.
- O(k) solution by simulation. Too slow.

Solution (contd.)

- Multiple ways to optimize. Instead of choosing one number at a time, we can choose an interval $[l, r]$ with $l > r/2$, and perform all moves at once.
- Keep intervals of the form (l, r, x) meaning numbers from l to r exists exactly x times. make sure that $l > r/2$. Then take the last interval, halve it and merge with existing intervals.
- It can be proven that there will be at most $O(log(n))$ intervals leading to a $O(log^2(n))$ sol.
- $O(log(n))$ solutions exist, but even harder to implement.

F. Predict The Frequency

Author: Iftekhar Hakim Kaowsar

Number of Teams Solved: 0
Number of Teams Tried: 3

Abridged Problem Statement
Given N, M, X, construct all possible array with length N with values in $[1, M]$ such that none of their frequency exceeds X. Score of an array is the maximum frequency of its values. Find the sum of score of all possible arrays.

Solution

- Let us define $dp(n, m, x)$ as the number of arrays with length n, constructed by values $[1, m]$ and maximum frequency of its values **does not exceed** x.
- In the transition part, we will iterate over how many values will get frequency x. This count will surely not exceed $\lfloor \frac{n}{x} \rfloor$. Otherwise, final array size will exceed n. So, count $c = 0 \cdots \lfloor \frac{n}{x} \rfloor$.

Solution (contd.)

- If we fix a value of c, we can choose c distinct values in $\binom{m}{c}$ ways.
- We can choose $c \cdot x$ positions to place them in $\binom{n}{c \cdot x}$ ways.
- Placing those values in those position has $\frac{(c \cdot x)!}{(x!)^c}$ ways.

Solution (contd.)

$$dp(n, m, x) = \sum_{c=0}^{\lfloor \frac{n}{x} \rfloor} \binom{m}{c} \binom{n}{c \cdot x} \frac{(c \cdot x)!}{(x!)^c} dp(n - c \cdot x, m - c, x - 1)$$

- This may look like $O(n^4)$. But the number of total transition is limited by $O(nlgn)$. To proof this, fix n and m. For those fixed n and m, the total transition will be $\lfloor \frac{n}{1} \rfloor + \lfloor \frac{n}{2} \rfloor + ... + \lfloor \frac{n}{n} \rfloor$. This is shaped by sum of harmonic series which is $nlogn$.
- Hence, total complexity will $O(n^3 logn)$.

Contributors

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