Problem 2.

a)
Attribute set closure –
(classid, id, gender)+ :
  Result = classid, id, gender
  Result = classid, id, gender, name (id->name && id ⊂ Result)
  Result = classid, id, gender, name, age (name -> age, id)
  Result = classid, id, gender, name, age, salary, manager (classid, id, gender -> salary, manager)

Thus, (classid, id, gender) is a superkey of the relational schema.
No combination of attributes (or individual attributes) in this superkey is a superkey itself. Thus, (classid, id, gender) is a candidate key.

Considering manager+ :
  Result = manager
  Result = manager, gender, age, classid, id (manager ⊂ manager && manager -> gender, age, classid, id)
  Result = manager, gender, age, classid, id, salary (classid, id, gender -> salary, manager && classid, id, gender ⊂ Result)
  Result = manager, gender, age, classid, id, salary, name

Thus, manager is a superkey of the relational schema.
Also, since it is a single attribute, it is a candidate key.

b)
1NF:
Since all attributes are atomic, 1NF is satisfied by all FDs.

2NF:
The non prime attributes are: name, age & salary
For name:
Id -> name means that name is functionally dependent on a subset of the candidate key (classid, id, gender). Thus, 2NF second property is violated here.

For age:
Name -> age, id
Thus, by decomposition rule:
Name -> age
It is not a trivial FD. Also, it is functionally dependent on a non primary attributed. Thus, 2NF is violated here as well.

For Salary:
Classid, id, gender -> salary, manager
By decomposition rule:
Classid, id, gender -> salary

It is not a trivial FD. Also, salary is fully functionally dependent on a candidate key. Thus, 2NF is not violated by this FD.

3NF:
Considering FD classid, id, gender -> salary, manager
Classid, id, gender is a superkey of the relational schema, thus, 3NF is obeyed.

Considering FD name -> age, id
It is not a trivial FD.
Name is not a super key of the schema.
Id is a prime attribute, but age is not, so
Name -> age violates 3NF.

Considering FD id -> name
It is not a trivial FD
Id is not a super key of the schema (it is a prime attribute)
Name is not a prime attribute of the schema. Thus, 3NF is violated by this FD

Considering the FD manager -> gender, age, classid, id
Manager is a superkey of the schema (candidate key).
Thus, this FD obeys 3NF

4NF:

Considering FD classid, id, gender -> salary, manager
Classid, id, gender is a superkey of the relational schema, thus, 3NF is obeyed.

Considering FD name -> age, id
It is not a trivial FD.
Name is not a super key of the schema.
Thus, BCNF is violated

Considering FD id -> name
It is not a trivial FD
Id is not a super key of the schema (it is a prime attribute)
Thus, BCNF is violated.

Considering the FD manager -> gender, age, classid, id
Manager is a superkey of the schema (candidate key).
Thus, BCNF is obeyed by this FD.

c)
The highest Normal Form that the relation schema does not satisfy is 2NF.
Thus, functional decomposition to remove the FD's violating 2NF:
Id -> name
Name -> age

Thus, R1 (id, classid, gender, manager, salary)
R2(id, name)
R3(name, age)

For R1:
F+:
classid, id, gender -> salary, manager,
Manager -> gender, classid, id

Candidate keys for R1:

Considering manager+ :
        Result = manager, gender, classid, id (manager -> gender, classid, id && manager ⊂ manager)
        Result = manager, gender, classid, id, salary (classid, id, gender -> salary, manager && classid, id, gender ⊂ Result)

Thus, manager is a candidate key (a single attribute)

Considering (classid, id, gender)+
        Result = classid, id, gender, salary, manager (classid, id, gender -> salary manager and classid, id, gender ⊂ Result)
Thus, classid, id, gender is a super key.
It is also a candidate key as none of the constituent attributes is a super key according to the F+ for this schema.
Thus, prime attributes : manager, classid, id, gender
Non prime attributes: salary
For salary: classid, id, gender -> manager, salary
By the decomposition rule : classid, id, gender -> salary.
Thus, salary is fully dependent on the candidate key classid, id, gender

Also, manager -> gender, classid, id
We know that gender, classid, id -> manager, salary
Thus, manager -> manager, salary
And so, manager -> salary
Thus, salary is fully dependent on the second candidate key, manager.
Thus, R1 is in 2NF.

For R2:

F+: id -> name
Candidate key: id
Non prime attribute: name
Name is fully dependent on the candidate key in R2. Thus, R2 obeys 2NF.

For R3:

F+: name -> age
Candidate key: name
Non prime attribute: age
Name is fully dependent on the candidate key in R3. Thus, R3 obeys 2NF.