# Rendering techniques

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# Contents

# 1 Global Illumination

## 1.1 What is Global illumination

**Definition 1.1.1** (Global Illumination). *Global illumination* or *indirect illumination* is a set of algorithms that take into account not only direct light (from light sources) but also indirect lights (reflections, diffusion, ...)

# 2 Physics

## 2.1 The physics behind the images

### 2.1.1 Electronic waves, light transport equation

**Definition 2.1.1** (Electromagnetic radiation)**.** It consists of waves of EM which propagate through space It's behaviour is determined by the Maxwell equations:

- $\operatorname{div} \vec{E} = \frac{\rho}{\epsilon_0}$,
- $\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$,
- $\operatorname{div} \vec{B} = 0$,
- $\operatorname{rot} \vec{B} = \mu_0 \left( \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$.

Where $\vec{j}$ is the current density, $\rho$ is the charge density.

**Theorem 2.1.1** (Light transport equation)**.** *Every solution to Maxwell equation respect the following transport equation*

- $\Delta \vec{E} = \frac{1}{c_0^2} \frac{\partial^2 \vec{E}}{\partial t^2}$
- $\Delta \vec{B} = \frac{1}{c_0^2} \frac{\partial^2 \vec{B}}{\partial t^2}$

*where $c_0^2 = \mu_0 \epsilon_0$ and $\Delta$ is the Laplacian.*

### 2.1.2 Poynting vector, Radiance, Radiant energy...

**Definition 2.1.2** (Poynting vector)**.** The Poynting vector is the vector defined by:

$$\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}.$$

**Theorem 2.1.2** (Poynting theorem)**.**

$$-\frac{\partial u}{\partial t} = \operatorname{div} \vec{\Pi} + \vec{j} \cdot \vec{E}$$

*Where $u = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right)$ is the local electromagnetic energy, $j$ is the current density.*

**Definition 2.1.3** (Radiant Energy)**.** It is the energy of an electromagnetic radiation, given by The radiant energy flux is given by

$$\Phi_e = \frac{\mathrm{d}Q_e}{\mathrm{d}t}$$

where

$$Q_e = \int_\Sigma \vec{\Pi} \cdot \vec{n} \, \mathrm{d}A$$

and $\Sigma$ is a closed surface.

**Theorem 2.1.3.**

$$\Phi_e \approx \int_\Sigma \langle|\Pi|\rangle \cos\alpha \, \mathrm{d}A$$

*Where $\alpha$ is the angle between $\vec{\Pi}$ and $\vec{n}$.*

**Definition 2.1.4** (Radiance)**.** The radiance is the radiant flux emitted, reflected, transmitted or received by a given surface, per unit solid angle per unit projected area

$$L_{e,\Omega} = \frac{\partial^2 \Phi_e}{\partial\Omega \, \partial(A\cos\theta)},$$

where $\Omega$ is the solid angle, $A\cos\theta$ the projected area.

### 2.1.3 The rendering equation

**Definition 2.1.5.** The rendering equation is an equation that gives the relation between the outgoing radiance along an output direction $\omega_o$ at a given point, and the incoming radiance:

$$L_o(\vec{x}, \omega_o, t) = L_e(\vec{x}, \omega_o, t) + \int_\Omega f_r(\vec{x}, \omega_i, \omega_o, t) L_i(\vec{x}, \omega_i, \omega_o, t)(\omega_i \cdot \vec{n}) \, \mathrm{d}\omega_i.$$

where $L_o$ is the total outgoind radiance along $\omega_0$, $\vec{x}$ the position, $t$ the time, $L_e$ the emitted radiance, $L_i$ the incoming radiance, $f_r$ is the BRDF function.

The equation can also be modified to take into account the wavelength

The previous equation has some limitations: it can't model non-linear effects, interference, polarization...

See transient rendering (Definition **??**) and volume rendering (Definition **??**).

**Definition 2.1.6** (Bidirectional reflectance distribution function — BRDF)**.**

# 3 Rendering techniques

## 3.1 Whitted Rendering

In his 1980 paper[Whi80], J. Turner Whitted introduced the notion of recursive raytracing.

### 3.1.1 The illumination model

The light from an object to the viewer consist of a specular reflection $S$, a transmission reflection $T$ and a diffusive component $D$. Thus the Whitted model is given by the equation

$$I = I_a + \sum_j D(\vec{L}_j) + k_s S + k_t T, \tag{3.1}$$

where

- $I$ is the reflected intensity,
- $I_a$ is the reflection due to ambient lighting,
- $k_t$ the transmission coefficient,
- $k_s$ is the specular reflection constant,
- $\overrightarrow{N}$ is the unit surface normal,
- $\overrightarrow{R}$ is the reflected direction,
- $\overrightarrow{P}$ is the refracted direction,
- $\overrightarrow{L}_j$ is the vector in the direction of the $j$th light source,
- $S$ is the intensity of light incident from the direction $\overrightarrow{R}$,
- $T$ is the intensity of light incident from the direction $\overrightarrow{P}$,
- Finally, $D(\overrightarrow{a})$ gives the diffuse contribution comming from direction $\overrightarrow{a}$

Note that both $\overrightarrow{R}$ and $\overrightarrow{P}$ are computed thanks to the Snell-Descartes formulas.

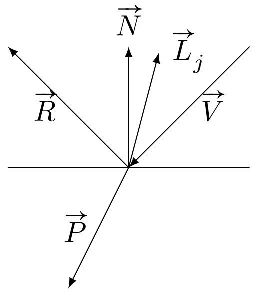


Figure 3.1: The vectors useful in the Whitted illumination model

To compute $S$ and $T$, one need to cast rays more rays until a maximum depth has been reached when the ray does not intersect any object.

### 3.1.2 Variations and subtilities

The function $D$ should check whether path between the object and the light is not obstructed. If so, the light cannot contribute to the illumination. To check whether the contribution of a light should be taken into consideration, a *shadow ray* is cast from the light up to the surface to check intersection with objects.

# Bibliography

[Whi80] Turner Whitted. “An improved illumination model for shaded display.” In: *Communications of the ACM* 23.6 (June 1, 1980), pp. 343–349. ISSN: 0001-0782. DOI: `10.1145/358876.358882`. URL: `https://doi.org/10.1145/358876.358882` (visited on 11/08/2022).