

Rendering techniques

Tristan Gouge

Contents

1	Global Illumination	1
1.1	What is Global illumination	1
2	Physics	3
2.1	The physics behind the images	3
2.1.1	Electronic waves, light transport equation	3
2.1.2	Poynting vector, Radiance, Radiant energy...	3
2.1.3	The rendering equation	4
3	Rendering techniques	5
3.1	Whitted Rendering	5
3.1.1	The illumination model	5
3.1.2	Variations and subtilities	6

1 Global Illumination

1.1 What is Global illumination

Definition 1.1.1 (Global Illumination). *Global illumination* or *indirect illumination* is a set of algorithms that take into account not only direct light (from light sources) but also indirect lights (reflections, diffusion, ...)

2 Physics

2.1 The physics behind the images

2.1.1 Electronic waves, light transport equation

Definition 2.1.1 (Electromagnetic radiation). It consists of waves of EM which propagate through space. Its behaviour is determined by the Maxwell equations:

- $\text{div } \vec{E} = \frac{\rho}{\epsilon_0},$
- $\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t},$
- $\text{div } \vec{B} = 0,$
- $\text{rot } \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right).$

Where \vec{j} is the current density, ρ is the charge density.

Theorem 2.1.1 (Light transport equation). *Every solution to Maxwell equation respects the following transport equation*

- $\Delta \vec{E} = \frac{1}{c_0^2} \frac{\partial^2 \vec{E}}{\partial t^2}$
- $\Delta \vec{B} = \frac{1}{c_0^2} \frac{\partial^2 \vec{B}}{\partial t^2}$

where $c_0^2 = \mu_0 \epsilon_0$ and Δ is the Laplacian.

2.1.2 Poynting vector, Radiance, Radiant energy...

Definition 2.1.2 (Poynting vector). The Poynting vector is the vector defined by:

$$\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}.$$

Theorem 2.1.2 (Poynting theorem).

$$-\frac{\partial u}{\partial t} = \text{div } \vec{\Pi} + \vec{j} \cdot \vec{E}$$

Where $u = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right)$ is the local electromagnetic energy, j is the current density.

Definition 2.1.3 (Radiant Energy). It is the energy of an electromagnetic radiation, given by The radiant energy flux is given by

$$\Phi_e = \frac{dQ_e}{dt}$$

where

$$Q_e = \int_{\Sigma} \vec{\Pi} \cdot \vec{n} \, dA$$

and Σ is a closed surface.

Theorem 2.1.3.

$$\Phi_e \approx \int_{\Sigma} \langle |\Pi| \rangle \cos \alpha \, dA$$

Where α is the angle between $\vec{\Pi}$ and \vec{n} .

Definition 2.1.4 (Radiance). The radiance is the radiant flux emitted, reflected, transmitted or received by a given surface, per unit solid angle per unit projected area

$$L_{e,\Omega} = \frac{\partial^2 \Phi_e}{\partial \Omega \partial (A \cos \theta)},$$

where Ω is the solid angle, $A \cos \theta$ the projected area.

2.1.3 The rendering equation

Definition 2.1.5. The rendering equation is an equation that gives the relation between the outgoing radiance along an output direction ω_o at a given point, and the incoming radiance:

$$L_o(\vec{x}, \omega_o, t) = L_e(\vec{x}, \omega_o, t) + \int_{\Omega} f_r(\vec{x}, \omega_i, \omega_o, t) L_i(\vec{x}, \omega_i, \omega_o, t) (\omega_i \cdot \vec{n}) \, d\omega_i.$$

where L_o is the total outgoing radiance along ω_o , \vec{x} the position, t the time, L_e the emitted radiance, L_i the incoming radiance, f_r is the BRDF function.

The equation can also be modified to take into account the wavelength

The previous equation has some limitations: it can't model non-linear effects, interference, polarization...

See transient rendering (Definition ??) and volume rendering (Definition ??).

Definition 2.1.6 (Bidirectional reflectance distribution function — BRDF).

3 Rendering techniques

3.1 Whitted Rendering

In his 1980 paper[Whi80], J. Turner Whitted introduced the notion of recursive raytracing.

3.1.1 The illumination model

The light from an object to the viewer consist of a specular reflection S , a transmission reflection T and a diffusive component D . Thus the Whitted model is given by the equation

$$I = I_a + \sum_j D(\vec{L}_j) + k_s S + k_t T, \quad (3.1)$$

where

- I is the reflected intensity,
- I_a is the reflection due to ambient lighting,
- k_t the transmission coefficient,
- k_s is the specular reflection constant,
- \vec{N} is the unit surface normal,
- \vec{R} is the reflected direction,
- \vec{P} is the refracted direction,
- \vec{L}_j is the vector in the direction of the j th light source,
- S is the intensity of light incident from the direction \vec{R} ,
- T is the intensity of light incident from the direction \vec{P} ,
- Finally, $D(\vec{a})$ gives the diffuse contribution coming from direction \vec{a}

Note that both \vec{R} and \vec{P} are computed thanks to the Snell-Descartes formulas.

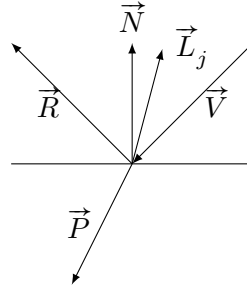


Figure 3.1: The vectors useful in the Whitted illumination model

To compute S and T , one needs to cast more rays until a maximum depth has been reached when the ray does not intersect any object.

3.1.2 Variations and subtleties

The function D should check whether the path between the object and the light is not obstructed. If so, the light cannot contribute to the illumination. To check whether the contribution of a light should be taken into consideration, a *shadow ray* is cast from the light up to the surface to check intersection with objects.

Due to how the Whitted illumination works, emissive materials can't work out of the box because emissive lighting requires diffuse rays to be cast, grabbing light from all incoming directions and not only from light. A way to circumvent this limitation would be to detect emissive materials and consider them as area light in the diffuse calculation.

Bibliography

- [Whi80] Turner Whitted. “An improved illumination model for shaded display.” In: *Communications of the ACM* 23.6 (June 1, 1980), pp. 343–349. ISSN: 0001-0782. DOI: 10.1145/358876.358882. URL: <https://doi.org/10.1145/358876.358882> (visited on 11/08/2022).