Rendering techniques

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Contents

1 Global Illumination

1.1 What is Global illumination

Definition 1.1.1 (Global Illumination) *Global illumination or indirect illumination is a set of algorithms that take into account not only direct light (from light sources) but also indirect lights (reflections, diffusion, ...)*

2 Physics

2.1 The physics behind the images

2.1.1 Electronic waves, light transport equation

Definition 2.1.1 (Electromagnetic radiation) *It consists of waves of EM which propagate through space It's behaviour is determined by the Maxwell equations:*

- $\operatorname{div} \vec{E} = \frac{\rho}{\epsilon_0}$,
- $\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$,
- $\operatorname{div} \vec{B} = 0$,
- $\operatorname{rot} \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$.

Where $\vec{j}$ is the current density, ρ is the charge density.

Theorem 2.1.1 (Light transport equation) *Every solution to Maxwell equation respect the following transport equation*

- $\Delta \vec{E} = \frac{1}{c_0^2} \frac{\partial^2 \vec{E}}{\partial t^2}$
- $\Delta \vec{B} = \frac{1}{c_0^2} \frac{\partial^2 \vec{B}}{\partial t^2}$

where $c_0^2 = \mu_0 \epsilon_0$ and Δ is the Laplacian.

2.1.2 Poynting vector, Radiance, Radiant energy...

Definition 2.1.2 (Poynting vector) *The Poynting vector is the vector defined by:*

$$\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}.$$

Theorem 2.1.2 (Poynting theorem)

$$-\frac{\partial u}{\partial t} = \operatorname{div} \vec{\Pi} + \vec{j} \cdot \vec{E}$$

Where $u = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right)$ is the local electromagnetic energy, j is the current density.

Definition 2.1.3 (Radiant Energy) *It is the energy of an electromagnetic radiation, given by The radiant energy flux is given by*

$$\Phi_e = \frac{\mathrm{d}Q_e}{\mathrm{d}t}$$

where

$$Q_e = \int_\Sigma \vec{\Pi} \cdot \vec{n} \ \mathrm{d}A$$

and Σ is a closed surface.

Theorem 2.1.3

$$\Phi_e \approx \int_\Sigma \langle|\Pi|\rangle \cos\alpha \ \mathrm{d}A$$

Where α is the angle between $\vec{\Pi}$ and $\vec{n}$.

Definition 2.1.4 (Radiance) *The radiance is the radiant flux emitted, reflected, transmitted or received by a given surface, per unit solid angle per unit projected area*

$$L_{e,\Omega} = \frac{\partial^2 \Phi_e}{\partial\Omega \, \partial(A\cos\theta)},$$

where Ω is the solid angle, $A\cos\theta$ the projected area.

2.1.3 The rendering equation

Definition 2.1.5 *The rendering equation is an equation that gives the relation between the outgoing radiance along an output direction ω_o at a given point, and the incoming radiance:*

$$L_o(\vec{x},\omega_o,t) = L_e(\vec{x},\omega_o,t) + \int_\Omega f_r(\vec{x},\omega_i,\omega_o,t) L_i(\vec{x},\omega_i,\omega_o,t)(\omega_i \cdot \vec{n}) \, \mathrm{d}\omega_i.$$

where L_o is the total outgoind radiance along ω_0, $\vec{x}$ the position, t the time, L_e the emitted radiance, L_i the incoming radiance, f_r is the BRDF function.

The equation can also be modified to take into account the wavelength

The previous equation has some limitations: it can't model non-linear effects, interference, polarization...

See transient rendering and volume rendering.

Definition 2.1.6 (Bidirectional reflectance distribution function — BRDF)