__The detailed mathematical model (Excel Sheets) is attached with the PDF of the report.__

Question 1

In this question, we are evaluating the impacts of economic policies on the production profile and trade between countries A, B and C. Each country has different resource availability (Ore, Oil, Land) to produce intermediary products (Metals, Plastics, Crops) and final products (Machines and Food). Also, each country has an exogenous demand for final products and limitations on the export capacity for some products. The only products that can be exported are Ore, Oil, Metal, Crop, Machines, and Food. We will try to figure out the optimal structure for national production, imports, and exports by minimizing the total cost. We used the provided tables and information to come up with this **__notation__**:

O_A	Ore produced in A
O_B	Ore produced in B
O_C	Ore produced in C
L_A	Land available in A
L_B	Land available in B
L_c	Land available in C
G_A	Oil produced in A
G_B	Oil produced in B
G_C	Oil produced in C
M_A	Metal produced in A
M_B	Metal produced in B
M_C	Metal produced in C
C_A	Crop produced in A
C_B	Crop produced in B
C_C	Crop produced in C
P_A	Plastic produced in A
P_B	Plastic produced in B
P_C	Plastic produced in C
R_A	Machines produced in A
R_B	Machines produced in B
R_C	Machines produced in C
F_A	Food produced in A
F_B	Food produced in B
F_C	Food produced in C
O_{A-B}	Ore trade A →B
O_{B-A}	Ore trade B→A
O_{A-C}	Ore trade A→C
O_{C-A}	Ore trade C→A
O_{B-C}	Ore Trade B→C
O_{C-B}	Ore Trade C→B
G_{A-B}	Oil trade A →B
G_{B-A}	Oil trade B→A
G_{A-C}	Oil trade A→C
G_{C-A}	Oil trade C→A
G_{B-C}	Oil Trade B→C
G_{C-B}	Oil Trade C→B
M_{A-B}	Metal trade A →B
M_{B-A}	Metal trade B→A
M_{A-C}	Metal trade A→C
M_{C-A}	Metal trade C→A
M_{B-C}	Metal Trade B→C
M_{C-B}	Metal Trade C→B
C_{A-B}	Crop trade A →B
C_{B-A}	Crop trade B→A
C_{A-C}	Crop trade A→C
C_{C-A}	Crop trade C→A
C_{B-C}	Crop Trade B→C
C_{C-B}	Crop Trade C→B
R_{A-B}	Machine trade A →B
R_{B-A}	Machine trade B→A
R_{A-C}	Machine trade A→C
R_{C-A}	Machine trade C→A
R_{B-C}	Machine Trade B→C
R_{C-B}	Machine Trade C→B
F_{A-B}	Food trade A →B
F_{B-A}	Food trade B→A
F_{A-C}	Food trade A→C
F_{C-A}	Food trade C→A
F_{B-C}	Food Trade B→C
F_{C-B}	Food Trade C→B
sO_A	Total supply of Ore in A
sO_B	Total supply of Ore in B
sO_C	Total supply of Ore in C
sG_A	Total supply of Oil in A
sG_B	Total supply of Oil in B
sG_C	Total supply of Oil in C
sM_A	Total supply of Metal in A
sM_B	Total supply of Metal in B
sM_C	Total supply of Metal in C
sC_A	Total supply of Crop in A
sC_B	Total supply of Crop in B
sC_C	Total supply of Crop in C
sR_A	Total supply of Machines in A
sR_B	Total supply of Machines in B
sR_C	Total supply of Machines in C
sF_A	Total supply of Food in A
sF_B	Total supply of Food in B
sF_C	Total supply of Food in C

Fig 1.1

The Objective Function: (From Tables 6 and 7)

Decision Variables	O_A	O_B	O_C	L_A	L_B	L_C	G_A	G_B	G_C
Objective function Coefficients	180	100	120	100	125	180	160	140	100

Decision Variables	O_{A-B}	O_{B-A}	O_{A-C}	O_{C-A}	O_{B-C}	O_{C-B}	G_{A-B}	G_{B-A}	G_{A-C}
Objective function Coefficients	40	40	45	45	48	48	45	45	50

Decision Variables	G_{C-A}	G_{B-C}	G_{C-B}	M_{A-B}	M_{B-A}	M_{A-C}	M_{C-A}	M_{B-C}	M_{C-B}
Objective function Coefficients	50	45	45	40	40	45	45	48	48

Decision Variables	C_{A-B}	C_{B-A}	C_{A-C}	C_{C-A}	C_{B-C}	C_{C-B}	R_{A-B}	R_{B-A}	R_{A-C}
Objective function Coefficients	50	50	55	55	60	60	55	55	60

Decision Variables	R_{C-A}	R_{B-C}	R_{C-B}	F_{A-B}	F_{B-A}	F_{A-C}	F_{C-A}	F_{B-C}	F_{C-B}
Objective function Coefficients	60	50	50	55	55	60	60	50	50

Fig 1.2

For the parts below, we are assuming that the information provided in the table is given in terms of yearly production/cost.

(a) What is the optimal production profile of each country? How many resources, intermediary products, and final products are made yearly in each country?

The optimal production profile includes the amount of resources, intermediaries, and final products each country makes:

Decision Variables	O_A	O_B	O_C	L_A	L_B	L_C	G_A	G_B	G_C
Optimal Solution	0	2905.96	1444.432	3788.2	2800	1080	1295.84	3200	6385.568

Decision Variables	M_A	M_B	M_C	C_A	C_B	C_C	P_A	P_B	P_C
Optimal Solution	500	3101.2	1641.4	4856.667	3733.333	1500	2150	3148.8	2268.6

Decision Variables	R_A	R_B	R_C	F_A	F_B	F_C
Optimal Solution	1250	4320	2830	5500	6000	3000

Fig 1.a-1

We can see that Country A produces no Ore. Moreover, a couple of countries maxed out on their resources, such as L_B, G_B, M_A, C_c, F_B, and F_C. This will be visible in the supply interpretation below, since the excess resources or products will be exported and any lack will be imported (the case of Country A and Ore)

For easier comprehension of the numbers, we made Supply variables that take into account the exports and imports in each country. However, we didn't provide a supply of Land resources in the countries because there was no export or import of Land, so the "supply" of Land was just the Land in each country. The same goes for Plastic; there are no trading costs provided for plastic, indicating the lack of trade. The Excel model provides this information:

Decision Variables	L_A	L_B	L_C	P_A	P_B	P_C	sO_A	sO_B	sO_C
Optimal Solution	3788.2	2800	1080	2150	3148.8	2268.6	425	2480.96	1444.432

Decision Variables	sG_A	sG_B	sG_C	sM_A	sM_B	sM_C	sC_A	sC_B	sC_C
Optimal Solution	3293.467	4702.373	2885.568	750	2851.2	1641.4	3850	4320	1920

Decision Variables	sR_A	sR_B	sR_C	sF_A	sF_B	sF_C
Optimal Solution	1250	4320	2830	4500	6050	3950

Fig 1.a-2

We can note that country B is the only one of the three countries to use up all of the Land resources it has. And none of the three countries used up all of their plastic resources. Moreover, the three countries satisfy exactly the minimum demand for the final products as given in Table 5 from the Question. This fits the cost minimization, since the countries need to make the least they can.

(b) Please show the optimal trade profile between countries. How many products are traded yearly and what are the origins/destinations?

The optimal trade profile between countries is the amount of each tradable resource, intermediary, or final product. Which means the Land and Plastic resources will not be included for the same reason we mentioned in Part a.

Decision Variables	O_{A-B}	O_{B-A}	O_{A-C}	O_{C-A}	O_{B-C}	O_{C-B}	G_{A-B}	G_{B-A}	G_{A-C}	G_{C-A}	G_{B-C}	G_{C-B}
Optimal Solution	0	425	0	0	0	0	0	0	0	1997.627	0	1502.373

Decision Variables	M_{A-B}	M_{B-A}	M_{A-C}	M_{C-A}	M_{B-C}	M_{C-B}	C_{A-B}	C_{B-A}	C_{A-C}	C_{C-A}	C_{B-C}	C_{C-B}
Optimal Solution	0	250	0	0	0	0	586.6667	0	420	0	0	0

Decision Variables	R_{A-B}	R_{B-A}	R_{A-C}	R_{C-A}	R_{B-C}	R_{C-B}	F_{A-B}	F_{B-A}	F_{A-C}	F_{C-A}	F_{B-C}	F_{C-B}
Optimal Solution	0	0	0	0	0	0	50	0	950	0	0	0

Fig 1.b-1

We can see that most resources aren't traded. However, some are.
It is expected that Country A imports Ore since its production limitations clearly don't satisfy its demand for machinery.
We also notice that Country C is the only exporter of Oil, since it exports to both of the other countries. Moreover, Country A is the only exporter of crops. This is because C and A have the highest limitations for Oil and crops, respectively, given by Table 3. The same argument holds for Food from A and Machines from B.

(c) What is the total cost? What is the cost for each individual country?

Our objective function included all the costs provided, and it is modeled by these variables and their coefficients:

Decision Variables	O_A	O_B	O_C	L_A	L_B	L_C	G_A	G_B	G_C
Objective function Coefficients	180	100	120	100	125	180	160	140	100
Optimal Solution	0	2905.96	1444.432	3788.2	2800	1080	1295.84	3200	6385.568

Decision Variables	O_{A-B}	O_{B-A}	O_{A-C}	O_{C-A}	O_{B-C}	O_{C-B}	G_{A-B}	G_{B-A}	G_{A-C}
Objective function Coefficients	40	40	45	45	48	48	45	45	50
Optimal Solution	0	425	0	0	0	0	0	0	0

Decision Variables	G_{C-A}	G_{B-C}	G_{C-B}	M_{A-B}	M_{B-A}	M_{A-C}	M_{C-A}	M_{B-C}	M_{C-B}
Objective function Coefficients	50	45	45	40	40	45	45	48	48
Optimal Solution	1997.627	0	1502.373	0	250	0	0	0	0

Decision Variables	C_{A-B}	C_{B-A}	C_{A-C}	C_{C-A}	C_{B-C}	C_{C-B}	R_{A-B}	R_{B-A}	R_{A-C}
Objective function Coefficients	50	50	55	55	60	60	55	55	60
Optimal Solution	586.6667	0	420	0	0	0	0	0	0

Decision Variables	R_{C-A}	R_{B-C}	R_{C-B}	F_{A-B}	F_{B-A}	F_{A-C}	F_{C-A}	F_{B-C}	F_{C-B}
Objective function Coefficients	60	50	50	55	55	60	60	50	50
Optimal Solution	0	0	0	50	0	950	0	0	0

Fig 1.c-1

Using this model and the constraints provided, we got the minimum cost of (Z=):
$ 2,987,710.50

We can divide this per country by separating the decision variables for each country and multiplying the coefficient by the optimal production and trade profile for each.

→ *Country A*

Decision Variables	O_A	L_A	G_A	O_{A-B}	O_{A-C}	G_{A-B}	G_{A-C}	M_{A-B}	M_{A-C}	C_{A-B}	C_{A-C}	R_{A-B}	R_{A-C}	F_{A-B}	F_{A-C}
Objective function Coefficients	180	100	160	40	45	45	50	40	45	50	55	55	60	55	60
Optimal Solution	0	3788.2	1295.84	0	0	0	0	0	0	586.6667	420	0	0	50	950

Fig 1.c-2

Cost for Country A= 100*3788.2+160*1295.84+50*586.6667+55*420+55*50+60*950=

$ 698,337.73

→ *Country B*

Decision Variables	O_B	L_B	G_B	O_{B-A}	O_{B-C}	G_{B-A}	G_{B-C}	M_{B-A}	M_{B-C}	C_{B-A}	C_{B-C}	R_{B-A}	R_{B-C}	F_{B-A}	F_{B-C}
Objective function Coefficients	100	125	140	40	48	45	45	40	48	50	60	55	50	55	50
Optimal Solution	2905.96	2800	3200	425	0	0	0	250	0	0	0	0	0	0	0

Fig 1.c-3

Cost for Country B= 100*2905.96+125*2800+140*3200+40*425+40*250=

$ 1,115,596

→ *Country C*

Decision Variables	O_C	L_C	G_C	O_{C-A}	O_{C-B}	G_{C-A}	G_{C-B}	M_{C-A}	M_{C-B}	C_{C-A}	C_{C-B}	R_{C-A}	R_{C-B}	F_{C-A}	F_{C-B}
Objective function Coefficients	120	180	100	45	48	50	45	45	48	55	60	60	50	60	50
Optimal Solution	1444.432	1080	6385.568	0	0	1997.627	1502.373	0	0	0	0	0	0	0	0

Fig 1.c-4

Cost for Country C= 120*1444.432+180*1080+100*6385.568+50*1997.627+45*1502.373=

$ 1,173,776.77

The last three highlighted numbers should add up to the first highlighted number, and that's true:
698337.73+1115596.00+1173776.77=$ 29,877,710.50

(d) There are ongoing talks of conflicts between countries A and B, with country B threatening to double import tariffs from country A, to which country A guaranteed equal tariff levels from country B. If the trade costs between A-B doubled, how would this affect the production profile, the trade, and the total costs?

We will begin by assessing the effects this trade war has on **(i)** the production profile and **(ii)** the trade, linking the two to generate meaningful relations and insights in **(iii)**, and then comparing the total and individual costs in **(iv)**.

(i) If both countries A and B doubled their trade costs between one another, the new optimal production profile (resources, intermediary products, and final products, respectively) of each country would be as follows:

Decision Variables	O_A	O_B	O_C	L_A	L_B	L_C	G_A	G_B	G_C
Optimal Solution	425	2290.394	1650	3788.2	2800	1080	1156.061	3200	6510.345

Decision Variables	M_A	M_B	M_C	C_A	C_B	C_C	P_A	P_B	P_C
Optimal Solution	500	2862.993	1875	4856.667	3733.333	1500	2057.862	3148.8	2365.345

Decision Variables	R_A	R_B	R_C	F_A	F_B	F_C
Optimal Solution	1019.655	4320	3060.345	5500	6000	3000

Fig 1.d-1

For country A's resources, this increase in tariff levels resulted in an increase in national Ore production levels (from 0 to 425 units), no change in national Land production levels, and a decrease in national Oil production levels (from 1295.84 to 1156.061 units). As for country A's intermediary products, the outcome was no change in national metal production levels, no change in national crop production levels, and a decrease in national plastic production levels (from 2150 to 2057.862 units). Lastly, for country A's final products, the result was a decrease in national machinery production levels (from 1250 to 1019.655 units), and no change in national food production levels.

For country B's resources, this increase in tariff levels resulted in a decrease in national Ore production levels (from 2905.96 to 2290.394 units), no change in national Land production levels, and no change in national Oil production levels. As for country B's intermediary products, the outcome was a decrease in national metal production levels (from 3101.2 to 2862.993 units), no change in national crop production levels, and no change in national plastic production levels. Lastly, for country B's final products, the result was no change in national machinery production levels, and no change in national food production levels.

For country C's resources, this increase in tariff levels resulted in an increase in national Ore production levels (from 1444.432 to 1650 units), no change in national Land production levels, and an increase in national Oil production levels (from 6385.568 to 6510.345 units). As for country C's intermediary products, the outcome was an increase in national metal production levels (from 1641.4 to 1875 units), no change in national crop production levels, and an increase in national plastic production levels (from 2268.6 to 2365.345 units). Lastly, for country

C’s final products, the result was an increase in national machinery production levels (from 2830 to 3060.345 units), and no change in national food production levels.

(ii) As for the trade between the three countries,

Decision Variables	O_{A-B}	O_{B-A}	O_{A-C}	O_{C-A}	O_{B-C}	O_{C-B}	G_{A-B}	G_{B-A}	G_{A-C}	G_{C-A}	G_{B-C}	G_{C-B}
Optimal Solution	0	0	0	0	0	0	0	0	0	2045.268	0	1454.732

Decision Variables	M_{A-B}	M_{B-A}	M_{A-C}	M_{C-A}	M_{B-C}	M_{C-B}	C_{A-B}	C_{B-A}	C_{A-C}	C_{C-A}	C_{B-C}	C_{C-B}
Optimal Solution	0	11.7931	0	100	0	0	586.6667	0	420	0	0	0

Decision Variables	R_{A-B}	R_{B-A}	R_{A-C}	R_{C-A}	R_{B-C}	R_{C-B}	F_{A-B}	F_{B-A}	F_{A-C}	F_{C-A}	F_{B-C}	F_{C-B}
Optimal Solution	0	0	0	230.3448	0	0	50	0	950	0	0	0

Fig 1.d-2

All Ore trades have stopped after the newly imposed tariffs. This means all trades have remained unchanged at 0, except B to A, which decreased from its initial value of 425 units, almost certainly due to the higher cost of trade/tariff. All Oil trades except C to A and C to B remained unchanged at 0 units, while the Oil trade from C to A has increased from 1997.627 to 2045.268 units, and the trade from C to B has decreased from 1502.373 to 1454.732 units. Here, the increase in trade from C to A can be attributed to the fact that since the trade from B to A has decreased following the tariff changes, the demand for Ore in A had to be met only by its own national production (which is quite near its limit of 500 units), as well as imports from C. There are no Land trades before nor after the new tariffs.

After the new tariff laws, all metal trades have stopped, except the trade of metal from B to A, which decreased from 250 to 11.7931 units, and C to A, which increased from 0 to 100 units. Once again, it is extremely likely that the decrease in trade from B to A is due to the increased tariffs. And the increase in trade from C to A is a direct result of that, as A is trying to meet its demand for metal from what is now a cheaper option to trade with. All the crop trades have remained unchanged. Plastic remained completely un-traded between countries.

All machinery trades remained unchanged at 0 except the trade from C to A, which increased from 0 to 230.3448 units. All the trades of food also remained unchanged.

Now that we have information on this tariff war’s impact on trade, the reasoning behind the changes in the aforementioned production profiles becomes more apparent. Country A drastically increased its Ore production levels to make up for the new lack of trade of Ore from B to A. And due to the increase in trade of Oil from C to A, country A was able to decrease its national production of Oil. Similarly, the decrease in country A’s national machinery production can be attributed to the increase in the trade of machinery from C to A. The decrease in Ore production in B is likely a consequence of the decrease in Ore trades from B to A. Likewise, the decrease in metal production in B is due to the decrease in metal trades from B to A. As for the increases in country C’s national production of Oil, metal, and machinery result from C’s increasing exports to A. Moreover, the increase in C’s national production of Ore and plastic is presumably to keep up with its increased export trade levels, since Ore and plastic are used in the making of the aforementioned intermediary and final (exported) products. These relationships tell a lot about how each country is coping with this trade war, and so, we take a deeper look into them in the following section, country by country.

(iii) For country A, after the trade war, nothing changed for the trades from A to B and A to C. This is probably because A was not exporting as much as B and C before the increase in tariffs, either. Since the cost of production for Country A's primary products, Ore and Oil, are higher than the rest it leads to Country A not producing its maximum amount of those products, and that leads to Country A only producing intermediary and final products (that uses Ore and Oil) in the amount needed for itself. This means that A would not be able to export these products to any country. However, the production cost of Land is the cheapest for Country A, so it would make sense that the intermediary product and final product that need Land to make (Crop and Food) would be produced more in Country A, so it can produce for itself and for Countries B and C. For Country B, it was either paying the expensive cost to produce Land to make the intermediary and final products or paying the doubled tariffs, and clearly, paying them was the better option. Everything else that Country A makes is not exported.

Similarly, we will now analyse the changes in country B's production and trade costs. After the trade war, Country B stopped exporting any Ore to Country A, which meant that Country B produced slightly less Ore. This forced Country A to produce its own Ore to satisfy its demand. Moreover, Country B was exporting 250 units of metal to Country A, which makes sense because the cost of producing Ore (which makes Metal) in Country B was the cheapest, so Country B would have produced Ore and therefore Metal enough for itself and for other countries. Now, because of the doubled tariffs, Country A is only exporting 11.79 units of metal from Country B, enough metal to help meet demands. Before the trade war, Country B was not exporting to C at all, and the doubled tariffs did not change that.

As for country C, after the trade war, it started producing a little bit more of all its resources and products except for Land, Crop, and Food. This is likely because it started exporting Machines, which are made by the other resources, to Country A so that A could meet its annual demand. Country C also slightly increased its exports to Country A for Oil because Country A's Oil production slightly decreased. Moreover, Country C was initially exporting Oil to Country B, but after the trade war, the amount Country C was exporting has slightly decreased. This is probably due to Country B suddenly stopping exporting Metal (made with Oil) to Country A following the doubled tariffs.

(iv) Lastly, the total costs are detailed below. This includes the total minimum cost (optimal Z value), as well as its breakdown into each country's individual cost in dollars ($).

Total Minimum Cost (Objective Function Value)	3042020.786

Fig 1.d-3

Similarly to the procedure followed in the previous part, to obtain each country's individual total cost, the costs are multiplied by their respective production/trade levels:

→ *Country A:*

Decision Variables	O_A	L_A	G_A	$O_{A\text{-}B}$	$O_{A\text{-}C}$	$G_{A\text{-}B}$	$G_{A\text{-}C}$	$M_{A\text{-}B}$	$M_{A\text{-}C}$	$C_{A\text{-}B}$	$C_{A\text{-}C}$	$R_{A\text{-}B}$	$R_{A\text{-}C}$	$F_{A\text{-}B}$	$F_{A\text{-}C}$		Z
Objective function Coefficients	180	100	160	80	45	90	50	80	45	100	55	110	60	110	60		
Optimal Solution	425	3788.2	1156.061	0	0	0	0	0	0	586.6667	420	0	0	50	950	=	784556.38

Fig 1.d-4

Total Cost for Country A = 180*425 +100*3788.2 + 160*1156.061 + 100*586.6667 + 55*420 + 110*60 + 60*950 = $ 784556.38

→ *Country B:*

Decision Variables	O_B	L_B	G_B	$O_{B\text{-}A}$	$O_{B\text{-}C}$	$G_{B\text{-}A}$	$G_{B\text{-}C}$	$M_{B\text{-}A}$	$M_{B\text{-}C}$	$C_{B\text{-}A}$	$C_{B\text{-}C}$	$R_{B\text{-}A}$	$R_{B\text{-}C}$	$F_{B\text{-}A}$	$F_{B\text{-}C}$		Z
Objective function Coefficients	100	125	140	80	48	90	45	80	48	100	60	110	50	110	50		
Optimal Solution	2290.394	2800	3200	0	0	0	0	11.7931	0	0	0	0	0	0	0	=	1027982.9

Fig 1.d-5

Total Cost for Country B = 100*2290.394 + 125*2800 + 140*3200 + 80*11.7931 = $ 1027982.848

→ *Country C:*

Decision Variables	O_C	L_C	G_C	$O_{C\text{-}A}$	$O_{C\text{-}B}$	$G_{C\text{-}A}$	$G_{C\text{-}B}$	$M_{C\text{-}A}$	$M_{C\text{-}B}$	$C_{C\text{-}A}$	$C_{C\text{-}B}$	$R_{C\text{-}A}$	$R_{C\text{-}B}$	$F_{C\text{-}A}$	$F_{C\text{-}B}$		Z
Objective function Coefficients	120	180	100	45	48	50	45	45	48	55	60	60	50	60	50		
Optimal Solution	1650	1080	6510.345	0	0	2045.268	1454.732	100	0	0	0	230.3448	0	0	0	=	1229481.51

Fig 1.d-6

Total Cost for Country C = 120*1650 + 180*1080 + 100*6510.345 + 50*2045.268 + 45*1454.732 + 45*100 + 60*230.3448 = $ 1,229,481.528

This adds up to $ 3,042,020.76, which is exactly the total minimum cost – taking into account a <0.1 computational error.

(e) Please calculate the average resource cost for the resources supplied in each country for this case and compare it to the national production cost.

First, to compute the average resource cost for the resources supplied in each country, we will find the total cost of the optimal volume of production and imports, and divide it by the total optimal volume of production and imports:

- *Average Resource Cost for Ore Supplied in A:*

$$[(180*425) + (80*0) +(45*0)] / (425 + 0 + 0) = 180 \text{ \$/unit}$$

It is apparent here that since no trade of Ore to A occurred, the average resource cost for Ore supplied in A is equal to the national production cost of Ore in A. In a similar fashion, since in this case there are no trades of Ore nor Land between any of the three countries, it follows that the average Ore and Land costs for the Ore and Land supplied in each country will be equal to their respective national production costs. Thus,

- *Average Resource Cost for Ore Supplied in B* = 100 $/unit
- *Average Resource Cost for Ore Supplied in C* = 120 $/unit
- *Average Resource Cost for Land Supplied in A* = 100 $/unit
- *Average Resource Cost for Land Supplied in B* = 125 $/unit
- *Average Resource Cost for Land Supplied in C* = 180 $/unit

Now, for the resource that did not unanimously have no trade, Oil, we may expect a difference in their average resource costs for resources supplied in each country, compared to the national production costs.

- *Average Resource Cost for Oil Supplied in A:*

$$[(160*1156.01) + (90*0) + (50*2045.268)] / (1156.01 + 0 + 2045.268)] = 89.7 \text{ \$/unit}$$

For country A, the average Oil unit cost for the Oil supplied (both produced and imported) is cheaper than the national unit production cost of 160 $/unit. This makes having a mixture of national production and imports make up the A's supply a more cost-efficient choice.

- *Average Resource Cost for Oil Supplied in B:*

$$[(140*3200) + (90*0) + (45*1454.73)] / (3200 + 0 + 1454.73)] = 110.3 \text{ \$/unit}$$

For country B, the average Oil unit cost for the Oil supplied (both produced and imported) is also cheaper than its national unit production cost of 140 $/unit. Once again, this makes having a mixture of national production and imports make up the B's supply a more cost-efficient choice.

- *Average Resource Cost for Oil Supplied in C:*

[(100*6510.34) + (50*0) + (45*0)] / (6510.34 + 0 + 0) = 100 $/unit

For country C, the average Oil unit cost for the Oil supplied (both produced and imported) is equal to its national unit production cost of 100 $/unit due to C having no Oil imported from A nor B.

(f) Which country would benefit the most and suffer the most from this trade war?

While observing changes in the production profiles and trade levels may provide valuable insight into how this trade war has affected each country, the individual country costs essentially summarize and reflect these effects. For this reason, we may base our conclusions solely on the changes in individual country costs.

Based on the individual country costs before and after the tariff changes, the following can be observed:

Change in Total Costs in Country A = 784556.38 - 698337.73 = $ 86,218.65
Change in Total Costs in Country B = 1027982.848 - 1115596 = $ - 87,613.152
Change in Total Costs in Country C = 1,229,481.528 - 1173776.77= $ 55,704.758

Both countries A and C have suffered from this trade war, since both of their total individual costs have experienced increases. However, Country A's costs have increased more than Country B's; therefore, Country A has suffered the most. Country C is the only country that benefited from the trade war, having had its total individual costs decreased. Therefore, Country C benefited the most.

(g) In this situation, how much idle capacity does each country have for each exported product?

Idle Capacity = Maximum Export Limit - Actual export

→ *Country A:*

- *Ore: 50 Units (0% Used)*
- *Oil: 700 Units (0% Used)*
- *Metal: 50 Units (0% Used)*
- *Crop: 193.33 Units (83.89% Used)*
- *Machines: 300 Units (0% Used)*
- *Food: 500 Units (66.67% Used)*

Country A did not export Ore, Oil, Metal, and Machines at all before and after the trade war. This is probably because the cost of production for the resources mentioned is the highest in Country A. So to minimize costs, Country A would have to produce a small amount of those resources (enough for itself) and import from other countries. So the idle Capacity for the exported products mentioned is at its max. However, the resource used to make Crop and Food, which is Land, is the cheapest to produce in Country A out of the three countries, which explains why A's idle capacity for exporting Crop and Food is reduced to 193.33 Units and 500 Units respectively.

→ *Country B:*

- *Ore: 700 Units (0% Used)*
- *Oil: 1200 Units (0% Used)*
- *Metal: 388.21 Units (2.95% Used)*
- *Crop: 600 Units (0% Used)*
- *Machines: 600 Units (0% Used)*
- *Food: 500 Units (0% Used)*

Country B used to export 425 Units of Ore to only Country A, and now, because of the doubled tariffs, Country B has stopped exporting Ore because it has become too expensive for Country A. So the idle capacity for exporting Ore from Country B is now 700 units, which is the maximum it can export Ore. Moreover, Country B used to export Metal at 62.5% of its export limitation to Country A, and after the tariffs, it drastically decreased to 2.95%. This is probably because Country A started making its own Ore and thus its own Metal. So now the idle capacity for Country B exporting Metal is 388.21 Units. The rest of Country B's idle capacity for exported products is at its maximum limits. (Note that Country B never exported anything to Country C, even after the doubled tariffs.)

→ *Country C:*

- *Ore: 200 Units (0% Used)*
- *Oil: 0 Units (100% Used)*
- *Metal: 100 Units (0% Used)*
- *Crop: 300 Units (0% Used)*
- *Machines: 269.66 Units (46.07% Used)*
- *Food: 400 Units (0% Used)*

Country C exported Oil to its limit to both Country A and Country B, so the idle capacity for Oil exported from C is 0 Units. This is because producing Oil in C was the cheapest option, and so Country C exported Oil to Country A and B as much as it could. Moreover, the doubled tariffs caused Country C to start exporting Machines to Country A. This increased its idle capacity from 500 units to 269.66 Units, meaning it started exporting 46.07% of its Machines export limitations. Ore, Metal, Crop, and Food for Country C have idle export capacities at their Max.

(h) What would be your recommendation for the countries to minimize costs and, therefore, prices for their national markets?

It is recommended to avoid any tariff escalation, as the trade war will only negatively affect the total costs. The total costs went from $2,987,710.51 to $3042020.79 just from doubling the tariffs between Country A and Country B. Moreover, this trade war is causing Country A to make its own Ore. If this goes on, then at some point, each country will make all its own resources and products, even if production costs are expensive, because those costs are cheaper than the tariffs. If the tariffs keep increasing, the costs will increase as well for no reason.

Another recommendation would be for each country to focus on producing what resource it can produce the cheapest. For example, since the production cost of Ore for Country B is the cheapest, it should max out the amount of Ore it makes, which is 3600 units, and export it to A and C as much as it can. The same should be done with Oil for Country C. Although it already exports to its max, an increase in its export capacity limitation would help minimize costs.

Another thing the countries can do is increase trade. Country B does not export at all to Country C. This is creating a burden on Country A as it is the primary source of Crop and Food because they use Land, and Land is the cheapest in Country A. Increasing trade between all countries helps them meet their needs more efficiently and cost-effectively. This would reduce the pressure on Country A. While increasing trade seems unappealing due to the increased tariffs, if the first recommendation is taken into consideration, the countries can reach a more cost-effective resolution.

Question 2

There are 3 different Products that need a specific amount of 3 different parts that are sold by 3 different Suppliers, each with a different cost for parts. In this question, we are required to maximise profit of a company given the information about the products, parts, and the suppliers supplying those parts. We used this **notation** to model the problem:

P1 Product 1	S11 Supplier 1 Part 1	S12 Supplier 1 Part 2	S13 Supplier 1 Part 3
P2 Product 2	S21 Supplier 2 Part 1	S22 Supplier 2 Part 2	S23 Supplier 2 Part 3
P3 Product 3	S31 Supplier 3 Part 1	S32 Supplier 3 Part 2	S33 Supplier 3 Part 3

Fig 2.1

The Objective Function: (from tables 9 and 10)

Decision Variables	P1	P2	P3	S11	S21	S31	S12	S22	S32	S13	S23	S33
Objective Function Coefficients	675	450	500	-7	-9	-5	-14	-12	-18	-8	-9	-10

Fig 2.2

(a) In order to maximize their profit, what is the best production level for each product?

The best production level of each product to maximise profit is:

Decision Variables	P1	P2	P3
Optimal Solution	40	112.5	130

Fig 2.a-1

We're making 40 units of Product 1, around 112 of Product 2, and 130 of Product 3.

(b) What is the optimal profit level?

The Optimal Profit level can be calculated using the objective function and the product and supply profiles:

Decision Variables	P1	P2	P3	S11	S21	S31	S12	S22	S32	S13	S23	S33		Z
Objective Function Coefficients	675	450	500	-7	-9	-5	-14	-12	-18	-8	-9	-10		
Optimal Solution	40	112.5	130	1000	1000	1000	707.5	1000	0	1000	1000	125	=	81470

Fig 2.b-1

Therefore, the optimal profit level given the constraints provided to us is $ 81,470

(c) How much of each part is being bought from each supplier?

The supply profile for each of the Suppliers/Parts is:

Decision Variables	S11	S21	S31	S12	S22	S32	S13	S23	S33
Optimal Solution	1000	1000	1000	707.5	1000	0	1000	1000	125

Fig 2.c-1

We maxed out all the Suppliers for Part 1, where we bought 1000 from each. We didn't need to buy Part 2 from Supplier 3 since buying from Suppliers 2 and 1 was sufficient. And we didn't max out buying Part 3 from supplier 3, but from the other suppliers.

(d) The company is facing pressure from the regulators to promote competition in the market, which is introducing a limitation on how much the Company can spend on a single supplier. As such, the company asked you to evaluate a scenario where a maximum value of 25,000 $ per month can be paid for each supplier. How would this change the production profile and the supply profile of each part?

Neither Supplier 1 and 3 exceeded the new limitation set, however, Supplier 2 did.
As shown below, only supplier 2 reached the upper limit of 25000:

S11	S21	S31	S12	S22	S32	S13	S23	S33			
7			14			8			24905	<=	25000
	9			12			9		25000	<=	25000
		5			18			10	11805.55556	<=	25000

Fig 2.d-1

The new rule didn't change the production profile, but it slightly changed the supply profile:

*<u>Production Profile:</u>

Decision Variables	P1	P2	P3
Optimal Solution	40	112.5	130

Fig 2.d-2

*<u>Supply Profile:</u>

Decision Variables	S11	S21	S31	S12	S22	S32	S13	S23	S33
Optimal Solution	1000	1000	1000	707.5	1000	0	1000	444.4444	680.5556

Fig 2.d-3

The new rule prevented us from maxing out on the limit of Part 3 from Supplier 2, since by the 444th unit, we reached the 25000 limit. Therefore, we bought more of Part 3 from Supplier 3 compared to what was previously shown. The increase in S33 from before and after the limit should equal the decrease in S23, which is exactly the case.
1000-444= $ 555.556 680.555-125= $ 555.56

Now the new profiles with the existing objective function is:

Decision Variables	P1	P2	P3	S11	S21	S31	S12	S22	S32	S13	S23	S33		Z
Objective Function Coefficients	675	450	500	-7	-9	-5	-14	-12	-18	-8	-9	-10		
Optimal Solution	40	112.5	130	1000	1000	1000	707.5	1000	0	1000	444.4444	680.5556	=	80914.44

Fig 2.d-4

Therefore, the optimal profit level given the constraints added is $ 80,914.44. Which is lower than the 81470 profit we got before. This means the new constraint is limiting and affecting our profit.

(e) To negotiate with the regulator, the Company wants to have more detailed information about the impact of this limitation on supplier expenditure. What should the Regulator set minimum cost to prevent the company from not obeying the expenditure limit on the suppliers?

In order to prevent the company from not obeying the expenditure limit, the minimum cost set must be disadvantageous enough to the company to make the benefit gained from not obeying the expenditure limit not worth the risk of paying the fine/cost. In other words, the set minimum cost must be greater than the profit the company would make for every dollar they disobey the expenditure limit by (the shadow price).

As seen below from the Answer Report of this LP problem, the constraints created by the expenditure limits at Supplier 1 and 3 ($O$24 and $O$26) are not binding. Hence, both their shadow prices are 0, i.e., the company would currently not make any profit for each dollar they disobey these expenditure limits by – as they still have yet to reach the limits (still have slack). On the other hand, the constraint created by the expenditure limit at Supplier 2 ($O$25) is binding, meaning the company has reached the expenditure limit and would profit from purchasing more from this supplier (positive shadow price).

Cell	Name	Cell Value	Formula	Status	Slack
$O$24	LHS	24905	$O$24<=$Q$24	Not Binding	95
$O$25	LHS	25000	$O$25<=$Q$25	Binding	0
$O$26	LHS	11805.55556	$O$26<=$Q$26	Not Binding	13194.44444

Fig 2.e-1

This is exactly what can be seen below in the Sensitivity Report. The shadow prices of both constraints for Supplier 1 and Supplier 3 are equal to 0. While the shadow price of the constraint on Supplier 2 is equal to $ 0.111. The interpretation of this value would then be that for every one additional dollar spent over the limit at Supplier, the company increases its profit by $ 0.111.

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$O$24	LHS	24905	0	25000	1E+30	95
$O$25	LHS	25000	0.111111111	25000	5000	2875
$O$26	LHS	11805.55556	0	25000	1E+30	13194.44444

Fig 2.e-2

Therefore, to ensure that going over the expenditure limit brings a decrease in profit to the company, the set minimum cost per dollar over the limit should be anything greater than $ 0.111. So, we may fix the set minimum cost to be $ 0.2 per dollar spent over the expenditure limit. That way, the company makes approximately a $ 0.089 loss for each dollar spent over the limit, making it unappealing and unfavourable for them to disobey the expenditure limit.

(f) To increase competition, the Company is starting a negotiation with an international partner, which could supply roughly 500 units of each part for the price of 6, 13, and 9 $/unit (respectively for Part 1, Part 2, and Part 3). Once again, how would this change the original solution (production, supply, profit)?

(i) Production Profile:

Decision Variables	P1	P2	P3
Optimal Solution	40	154.167	130

Fig 2.f-1

The production of Products 1 and 3 remains the same, while the production of Product 2 increases by around 41 Units after the company started purchasing from the international supplier.

(ii) Supply Profile:

Decision Variables	S11	S21	S31	S41	S12	S22	S32	S42	S13	S23	S33	S43
Optimal Solution	1000	1000	1000	500	499.167	1000	0	500	1000	708.333	0	500

Fig 2.f-2

The new supplier sells 2 of the parts (Parts 1 and 3) at a price lower than the pre-existing “lowest price” given by the national suppliers. It also provides Part 2 at a lower price compared to ⅔ of the suppliers of Part 2. Therefore, the company maxed out on the new supplier's supplies for all three parts, which led to it taking fewer units of Part 2 from Supplier 1 (S12) and Part 3 from Supplier 2 (S23), and completely stopping the purchase of Part 3 from Supplier 3 (S33). This is because the prices of the respective parts from the new international supplier are cheaper than the already existing ones.

- The price of Part 2 from the new supplier is $1 cheaper than the price from Supplier 1.
- The price of Part 3 is $1 cheaper from the new supplier compared to Supplier 3. But it is equivalent to the price from Supplier 2.

Moreover, Product 2 requires more units of Part 2, so it would make sense that it's purchasing a lot more of it, causing S41 to max out.

(iii) Profit:

Decision Variables	P1	P2	P3	S11	S21	S31	S41	S12	S22	S32	S42	S13	S23	S33	S43		Z
Optimal Solution	40	154.1667	130	1000	1000	1000	500	499.1667	1000	0	500	1000	444.4444	263.8889	500	=	92747.78

Fig 2.f-3

After purchasing from the new supplier, the profit went from $81,470 to $93,011.70, an increase of $11,541.70. This shows that introducing the new supplier improved profitability. Although the number of products did not change much, the supplier helped increase the difference between production cost and revenue significantly with their slightly cheaper parts.

Question 3

There is international and national pressure on a company's steel sector to reduce its impact on water consumption, increase energy efficiency, and reduce local pollutants, such as particulate matter, in the air. The company has to meet the demand of 150kt/year in 2025 and 200kt/year in 2035 by effectively using the 3 commercially available processes for iron production, each with a different amount of consumption and emission of Water, Fuel, Electricity, and Pollutant. We need to use the parameters of the processes and their operation costs to minimize cost while meeting demand. We used the **notation**:

x_{i1} Capacity for process 1 in 2025
x_{i2} Capacity for process 2 in 2025
x_{i3} Capacity for process 3 in 2025
x_{f1} Capacity for process 1 in 2035
x_{f2} Capacity for process 2 in 2035
x_{f3} Capacity for process 3 in 2035
e_{f1} Capacity extension for process 1 in 2035
e_{f2} Capacity extension for process 2 in 2035
e_{f3} Capacity extension for process 3 in 2035

W_i Consumption emission of Water in 2025
F_i Consumption emission of Fuel in 2025
E_i Consumption emission of Electricity in 2025
P_i Consumption emission of Pollutants in 2025
W_f Consumption emission of Water in 2035
F_f Consumption emission of Fuel in 2035
E_f Consumption emission of Electricity in 2035
P_f Consumption emission of Pollutants in 2035

Fig 3.1

The Objective Function: (From Table 11)

Decision Variable	x_{i1}	x_{i2}	x_{i3}	x_{f1}	x_{f2}	x_{f3}	e_{f1}	e_{f2}	e_{f3}
Objective function coeffcients	300	400	200	300	400	200	3000	4000	5000

Fig 3.2

(a) What is the least cost option for meeting demand in 2025 and 2035?

The demand in 2025 was 150 kt/year. And for the demand to be met the company would have to use Process 1 with a capacity of 120 kt/year, Process 2 with a capacity of 30 kt/year, and Process 3 with a capacity of 0 kt/year. The capacity for Process 2 is originally 50 kt/year the company meets demands with only 30 kt/year for Process 2, this is because there is a surplus of installed capacity relative to demand. These capacities allowed the company to meet its demand with the least cost. The processes were allowed to have an expansion of capacities to meet the demand in 2035 which is 200 kt/year. The expansions are as follows: Process 1 with a 30 kt/year capacity expansion, making it 150 kt/year, Process 2 with a 20 kt/year capacity expansion, making it 50 kt/year, and no expansion for Process 3. This is probably because increasing capacity for Process 3 is the most expensive and it would increase costs. So instead the company increased the cheapest to expand more than the rest of the processes, which is Process 1. So the company should not install a unit for Process 3.

Decision Variable	x_{i1}	x_{i2}	x_{i3}	x_{f1}	x_{f2}	x_{f3}	e_{f1}	e_{f2}	e_{f3}
Optimal Solution	120	30	0	150	50	0	30	0	0

Fig 3.a-1

(b) Current discussions claim that future policies might request that water consumption levels, energy consumption (fuel and electricity), and pollutant emissions in 2035 be kept at 80% of the levels in 2025 (20% reduction, based on 2025 levels). What would be the impact on the production and capacity in 2035 for each change individually? In other words, please answer individually for the reduction target for (i) water, (ii) fuel, (iii) electricity, and (iv) pollutants.

In light of the possible future policies mandating a reduction of our consumption of water, fuel and electricity, as well as our pollutant emissions by 20% in 2035, we must consider that each of these reductions would impact our production and capacity differently, as these parameters all have unique relations to the production and capacity profiles. Therefore, it would be more meaningful to evaluate these changes independently of one another.

(i) If we set our 2035 water consumption to be kept at 80% of the water consumption levels of 2025 – i.e., fix a 20% decrease in water consumption – the optimal production profile for minimizing our overall yearly consumption changes as follows.

Since the water consumption level in 2025 was 690 units, the water consumption level in 2035 must decrease to 80% of that which is 552 units. However, since we still require meeting the 200 kt/year demand while minimizing our costs, we also experience changes in the allocation of capacity for the processes and expansions of the products. More specifically, in 2035, we'll see a decrease in the capacity for Process 1 from 150 to 56 kt/year. To cancel out this change, an increase in the capacity of Process 2 from 50 to 64 kt/year, and an increase in the capacity of Process 3 from 0 to 80 kt/year can be observed. As for expansion, the expansion capacity for Process 1 decreased from 30 all the way to 0 – while the expansion capacities for Process 2 and 3 increased from 0 to 14 and 80 respectively.

While the new distribution of process capacities results in an overall decrease in costs incurred from the processes ($58400 as opposed to the original $65000), there is also a drastic increase of costs incurred from the expansions due to the new allocation of expansion capacities ($456000 compared to the original $90000). This leaves us with an overall $359000 increase in costs, bringing our total costs up to $562400 from its original value of $203000.

(ii) If we set our 2035 fuel consumption to be kept at 80% of the fuel consumption levels of 2025 – i.e., fix a 20% decrease in fuel consumption – the optimal production profile for minimizing our overall yearly consumption changes as follows.

In an attempt to decrease fuel consumption to 384 units in 2035, the expansion capacity of process 1 decreases from 30 to 8 kt/year, and more importantly, the process capacities, that were originally met by Process 1 and 2, change to being 128 kt/year for Process 1 and 0 for the other two. While this would seemingly give us a lower total cost of $110400, it only appears that way since it is infeasible as it does not meet the new demand of 200 kt/year.

(iii) If we set our 2035 electricity consumption to be kept at 80% of the electricity consumption levels of 2025 – i.e., fix a 20% decrease in electricity consumption – the optimal production profile for minimizing our overall yearly consumption changes as follows.

Once again, in an attempt to decrease electricity consumption to 768 units in 2035, the expansion capacity changes from being solely 30 kt/year of process 1 expansion to solely 80 kt/year of process 3 expansion. Similar to the case of fuel, the process capacities that were originally met by Process 1 and 2, change to being 88 kt/year for Process 1, 80 kt/year forProcess 3, and 0 for Process 2. Again, this change would be infeasible since it does not meet the new demand of 200 kt/year.

(iv) Lastly, if we set our 2035 pollutant emissions to be kept at 80% of the pollutant emissions levels of 2025 – i.e., fix a 20% decrease in pollutant emissions – the optimal production profile for minimizing our overall yearly consumption changes as follows.

Since the pollutant emissions level in 2025 was 1260 units, the pollutant emissions level in 2035 must decrease to 80% of that which is 1008 units.

Because we still need to meet the 200 kt/year demand while minimizing our total costs, we'll also see changes in the allocation of capacity for the processes and expansions of the products. In 2035, we'll see a decrease in the capacity for Process 1 from 150 to 76 kt/year. To balance out this change, an increase in the capacity of Process 2 from 50 to 124 kt/year, while the capacity of Process 3 remained at 0. The expansion capacity for Process 1 decreased from 30 to 0, while the expansion capacities for Process 2 increased from 0 to 76 kt/year, and the expansion capacities for Process 3 remained at 0.

While the new distribution of process capacities results in an overall increase in costs incurred from the processes ($72400 as opposed to the original $65000), there is also a sharp increase of costs incurred from the expansions due to the new allocation of expansion capacities ($296000 compared to the original $90000). This leaves us with an overall $213,400 increase in costs, bringing our total costs up to $416400 from its original value of $203000.

(c) What would be the estimated cost per water ($/unit) and cost per pollutant ($/unit) to the company? (Please answer them individually)?

(i) To find the estimated cost per water for each year, we will first find the amount of operational cost spent on water used by finding the units of water used as a ratio of the total water, fuel, electricity, and pollutants used/emitted, and then multiplying that ratio by the total operational costs for the year. Once we have the amount of operational cost spent on water used, we will divide it by the water used that year to obtain the estimated cost per water ($/unit).

Following this procedure for the year 2025, we obtain a water used to total water, fuel, electricity, and pollutants used/emitted ratio of 0.2035 and a total operational cost of $48000. Multiplying the two values gives us $9769.9 as the amount of operational cost spent on water used. To get the estimated cost per water, we simply divide the amount by the water used for that year, which leaves us with an estimated cost per water of 14.12 $/unit in 2025.

As for 2035, we repeat the same procedure using the values from 2035 instead. This gives us a ratio of 0.2022, a total operational cost of $65000, and hence, the amount of operational cost spent on water used as $13146.1. Finally, dividing this amount by the water used, we conclude that the estimated cost per water in 2035 is 14.61 $/unit.

(ii) We can follow a similar approach for finding the estimated cost per pollutant for each year. That is, we will begin by finding the amount of operational cost spent on pollutants emitted, and divide it by the amount of pollutants emitted that year.

For 2025, we get a pollutant emitted to total water, fuel, electricity, and pollutants used/emitted ratio of 0.3717 and a total operational cost of $48000. Hence, the amount of operational cost spent on pollutants emitted is $17840.7. Now, dividing this value by the amount of pollutants emitted in 2025, we find the estimated cost per pollutant for 2025 to be 14.16 $/unit.

Similarly, for 2035, we get a pollutant emitted to total water, fuel, electricity, and pollutants used/emitted ratio of 0.3596, a total operational cost of $65000. As a result, the amount of operational cost spent on pollutants emitted is $23370.8. Again, dividing this value by the amount of pollutants emitted in 2035, we find the estimated cost per pollutant for 2035 to be 14.61 $/unit.