Mini Project 4
Nafeesa Nawar
Student ID: 33577735

Table of Contents

Part A - An Active Filter

1)

Below we have shown the calculation of C and A_M that will turn the filter into a 2nd order Butterworth filter and give the filter a 3dB frequency of 10kHz.

$$C = \frac{1}{2\pi f_{3dB} R} = \frac{1}{(2\pi)(10kHz)(10k\Omega)}$$
$$= 1.5915\ nF$$

The transfer function given is:

$$H(s) = A_m \frac{1/(RC)^2}{s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}}$$

We know, $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}$

Thus, $\zeta = \frac{3 - A_m}{2}$ $\omega_n = \frac{1}{RC}$

The normalized characteristic equation of a 2nd order butterworth filter is: $s^2 + \sqrt{2}s + 1$. This is because it has poles 45° from the real axis.

Now, $A_m = 3 - \sqrt{2} = 1.5858\ V/V$

Solving for R_1 and R_2 we get.

$A_m = 1 + \frac{R_2}{R_1}$ $R_1 + R_2 = 10k\Omega$

$R_1 = 6306.0194\ \Omega$ $R_2 = 3693.9806\ \Omega$

Using the values obtained above, the following circuit was simulated to achieve its bode plots.

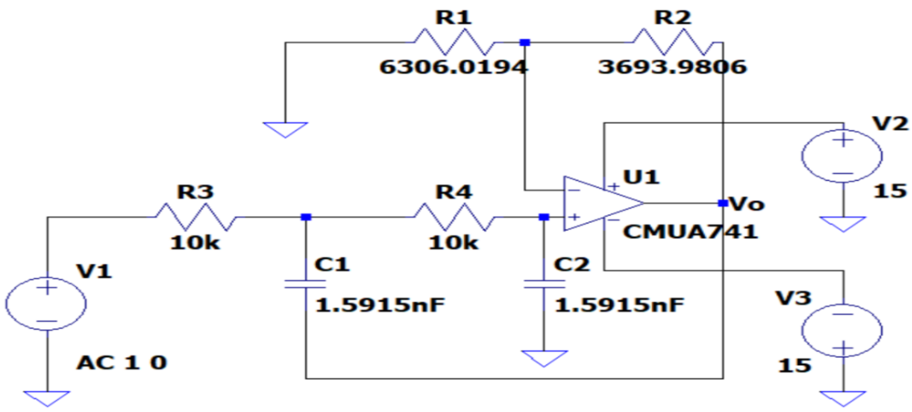

Figure 1.1: Simulated Circuit for bode plot

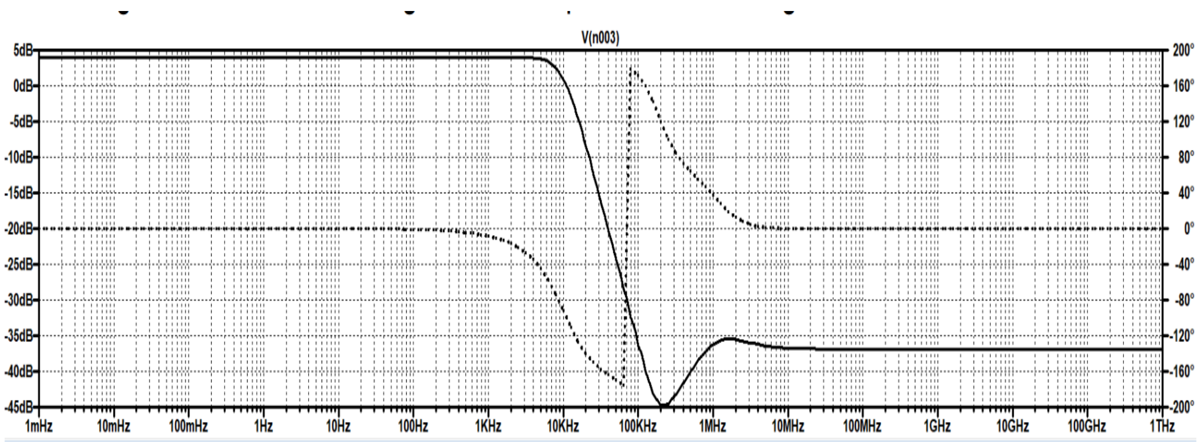

Figure 1.2: Magnitude and phase plot

From the simulation we have found the f_{3dB} to equal to 9.9776554 kHz. This is very close to the given of 10 kHz, proving the accuracy of our calculation.

Using the obtained values, below, we have shown where the poles in the S-plane. The simulation was done using matlab and two complex conjugate poles(-44.5k + 44.5j and -44.5k - 44.5j) was obtained.

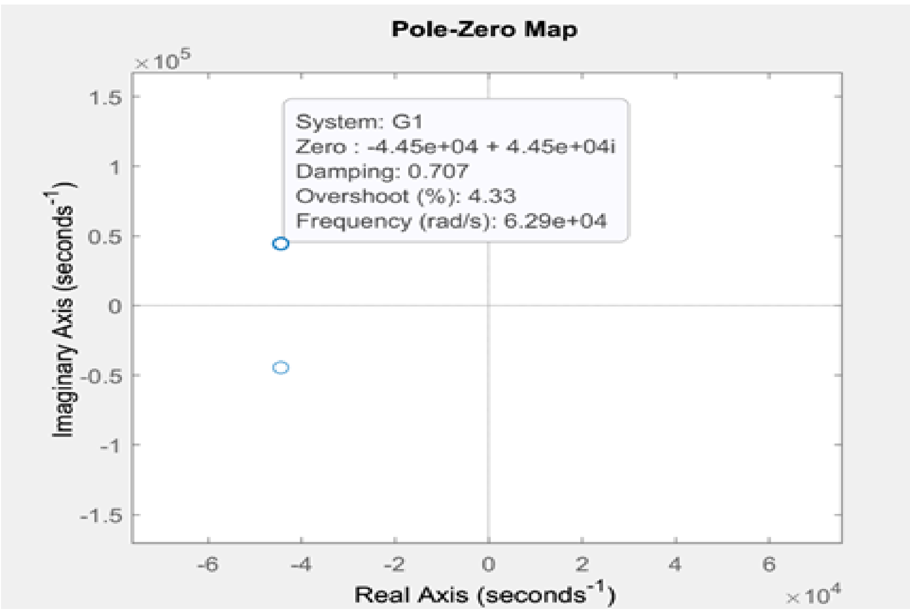

Figure 1.3: Pole-zero Map

2)

In figure 1.4, 1.5 and 1.6, we have depicted the root locus graph for $A_M < 3$, $A_M = 3$ and $A_M > 3$. Note that when $A_M = 3$, the s term of the transfer function disappears and our pole lies on the imaginary axis, meaning $s=\pm j(1/RC)$. In figure 1.5 and 1.6 A_M goes from greater than 3 to 3. When that happens the poles travel from the RHS of the s plane to the imaginary axis. Then again, when A_M goes from 3 to less than 3, the poles travel from imaginary axis to LHS of the s plane.

Therefore, in order for our system to oscilate, we need to ensure that A_M is greater than or equal 3. This will ensure our poles to be on RHS of the complex plane such that there is no exponential decay that can get rid of the oscillation. When A_M is less than 3, we observe stable root locus plot. When A_M is greater than 3 the root locus plot is unstable and the characteristic equation is negative which causes instability and allow the system to oscillate.

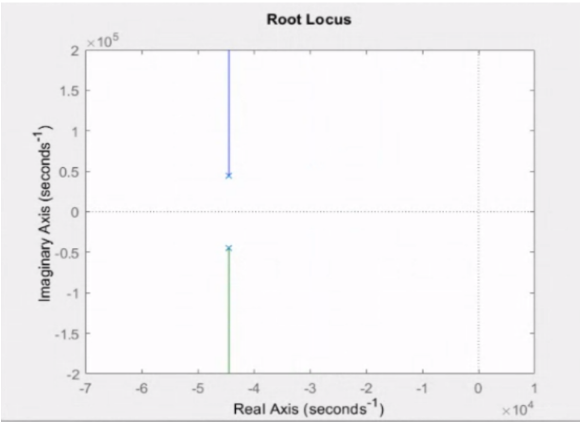

Figure 1.4: $A_M < 3$

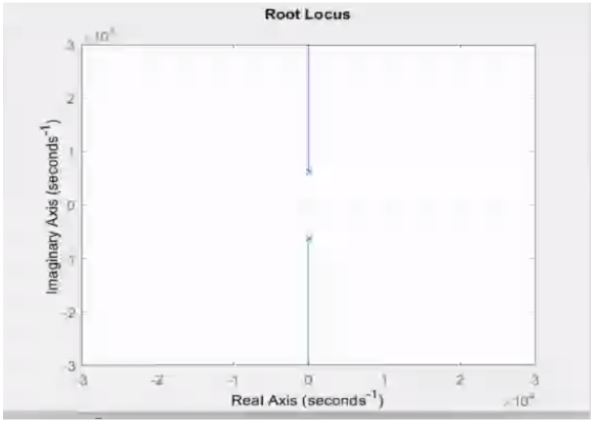

Figure 1.5: $A_M = 3$

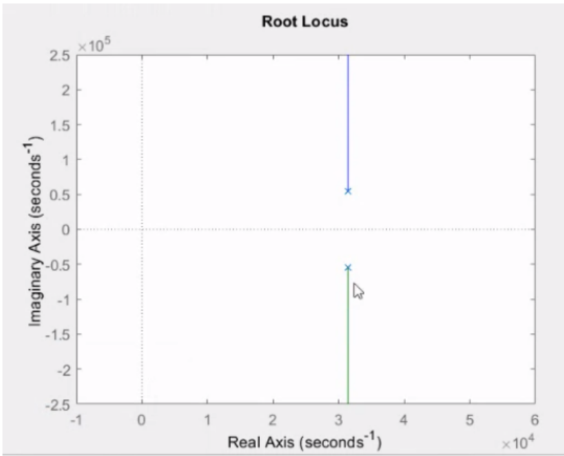

Figure 1.6: $A_M > 3$

As mentioned above oscillation start to occur when $A_M = 3$. Thus, let us choose $A_M = 3$ and observe the output. When $A_M = 3$, $R_1 = 3.3333$ kΩ and $R_2 = 6.6667$ kΩ. In this way, we kept the value of $R_1 + R_2 = 10$ kΩ and changed the value of R_1 and R_2 in order to increase A_M. Now we can observe the output by subbing in the new R_1 and R_2 values.

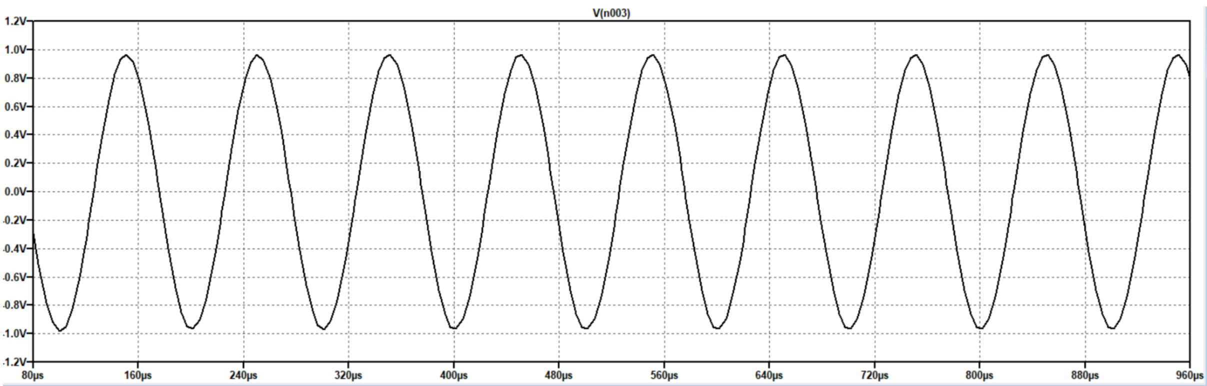

Figure 1.7: Oscilated output via grounding input

Via measuring the differences between the crest of the plot, the oscillating frequency was found to be be $f_o = 9.01$ kHz.

Discussion:

In this segment, we crafted a second-order Butterworth filter in accordance with the provided specifications. Upon revisiting the denominator of our transfer function, it becomes evident that when the term involving 's' raised to the power of 1 is multiplied by zero, the function's poles assume a purely imaginary nature. This signifies an anticipated response characterized by continuous oscillation without decay. Consequently, we foresaw oscillations in the filter when A_M equals 3 V/V, with resistors R1 set at 3.3333kΩ and R2 at 6.6667kΩ. To delve deeper into this oscillatory behavior, an examination of the root locus reveals that an increase in A_M drives the poles towards the jw-axis, resulting in purely imaginary poles and, consequently, inducing oscillations.

Part B - A Phase Shift Oscillator

For this part of the project, we will analyse a phase shift oscillator depicted in figure 2.1. The oscillator comprises a feedback network with capacitors and resistors that induce a shift in the output. It also employs a feedback network with poles positioned on the jω-axis and generates a phase shift proportional to its frequency. The transfer function of the feedback circuit is the following:

$$A_f = \frac{A}{1+A_\beta}$$

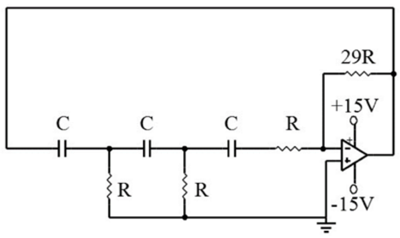

Figure 2.1: Phase Shift Oscillator

In order for the circuit to oscillate with a specific frequency of w_o without requiring an input, A_f would need to equal to infinity and Aβ need to equal to -1. In such a case, the complex conjugate poles would be located on the jw-axis.

As mentioned in the question description, the values of R were chosen to be 1 kΩ and the values of C were 1μF. As mentioned in the question, the values of 29R had to be slightly higher, thus, a value of 29.3 kΩ was chosen.

The following equations below predicts the oscillating frequency of the circuit shown in figure 2.1 and then compared to the equations given in the class handouts.

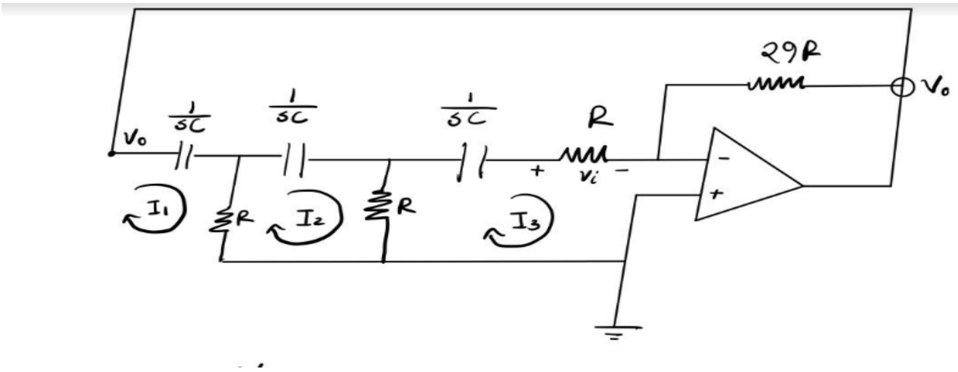

Figure 2.2: Circuit diagram with marked current to predict the equation for oscillating frequency

$$k = \text{gain} = \frac{V_o}{V_i}$$

$$kV_i = \frac{1}{sC} I_1 + R(I_1 - I_2) \longrightarrow (1)$$

$$R(I_2 - I_1) + \frac{1}{sC} I_2 + R(I_2 - I_3) = 0 \longrightarrow (2)$$

$$R(I_3 - I_2) + \left(\frac{1}{sC} + R\right) I_3 = 0 \longrightarrow (3)$$

Solving for I_1, I_2, I_3 we get:

$$I_3 = \frac{V_i}{R}$$

$$I_2 = \frac{V_i}{R}\left(\frac{1 + 2(sRC)}{sRC}\right)$$

$$I_1 = \frac{1}{R}\left(V_i\left(\frac{1 + 2(sRC)^2}{(sRC)^2} - 1\right)\right)$$

Using I_1, I_2 and I_3, we solve for k.

$$kV_i = \frac{1+sRC}{sRC}\left(\frac{(1+2(sRC)^2}{(sRC)^2} - 1\right)V_i - R\left(\frac{V_i}{R}\left(\frac{1+2(sRC)}{(sRC)}\right)\right)$$

thus,

$$\boxed{k = \frac{[(wRC)^3 - 5(\omega RC)] + j(1 - 6(\omega RC)^2)}{(wRC)^3}}$$

By substituting gain to unity and the phase to 180^0, the following equations were obtained which are same as class handouts.

$$\boxed{f = \frac{1}{2\pi\sqrt{6}RC}} \quad \text{and} \quad \boxed{k = -29}$$

Using the obtained equation for f , the oscillation frequency is calculated and listed below in the table. Using LTSpice, the output frequency was measured. The process was then repeated by doubling the value of R and C and halving the value of R and C. All the results obtained are summarised in table 2.1.

R and C	**Calculated f (Hz)**	**Measured f (Hz)**
1 kΩ, 1µF	64.97	64.29
2 kΩ, 2 µF	16.24	16.16
0.5 kΩ, 0.5 µF	259.90	257.89

Table 2.1: measured and calculated output frequencies

As seen from table 2.1 , the calculated and measured frequencies are fairly close to one another which further proves the accuracies of the formulas used to predict the oscillating frequencies.

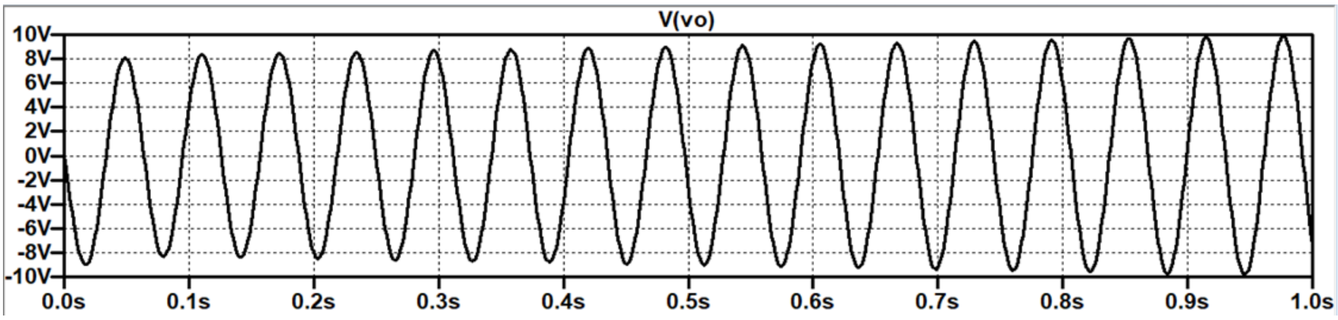

Figure 2.3: Oscillation for Doubled R and C

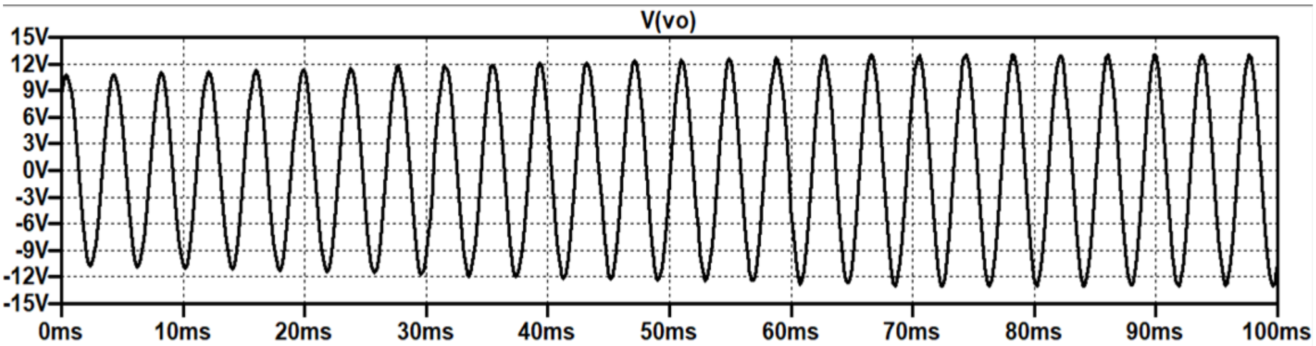

Figure 2.3: Oscillation for Halved R and C

How a phase-shift oscillator works and why we had to increase the 29R resistor:
A Phase Shift Oscillator operates by leveraging feedback through an RC network to generate a sinusoidal waveform. It involves the cascading of three passive high-pass filter stages. Each of these filter stages contributes a phase shift of 60°, resulting in a cumulative phase shift of 180°. This contribution is then augmented by the -180° introduced by the negative feedback from the amplifier. The net effect is a total phase shift of 0° in the transfer function, aligning with the Barkhausen criterion necessary for sustained oscillations.

Our calculations indicated that the operational amplifier necessitates a minimum gain of -29, aligning with the initial design of 29R and R for the inverting operational amplifier. Nevertheless, to regulate the oscillation amplitude, we had to make a slight adjustment to the 29R value to ensure that $|A\beta|$ is slightly greater than 1, causing the poles to shift to the right-hand plane for exponential growth and initiating oscillations. This condition was achieved by increasing the value from 29k to 29.3k.

Discussion:
In this segment, we elaborate on the effective emulation of a phase-shift oscillator employing three distinct combinations of resistor and capacitor parameters. Previously, we delved into the fundamental principles governing oscillators and their mathematical application to phase-shift oscillators. Finally, our formula aligns with the lecture notes, yielding precise results. As depicted in Table 2.1, the computed and simulated values closely correspond. Additionally, we note an inverse relationship between the resistor and capacitor values and the oscillation frequency, in accordance with the deduced equation.

Part C - A Feedback Circuit

In this section, we will analyse a feedback circuit comprising a common emitter stage succeeded by a common collector stage serving as our amplifier. The feedback network employs a resistor. Given that we measure voltage at the output and feeding back a current to the input, we can utilise y-parameters, as this configuration aligns with a shunt-shunt topology.

Initiating the amplifier biassing process is our first step. To create the largest open-loop gain at 1kHz, the crucial task is selecting the appropriate value for R_{B2}. By disconnecting R_f in figure 3.1 and observing the magnitude bode plot at 1kHz, we can explore various R_{B2} values to identify the one that yields the highest gain. Below, the observed gains and the values of R_{B2} is recorded in table 3.1.

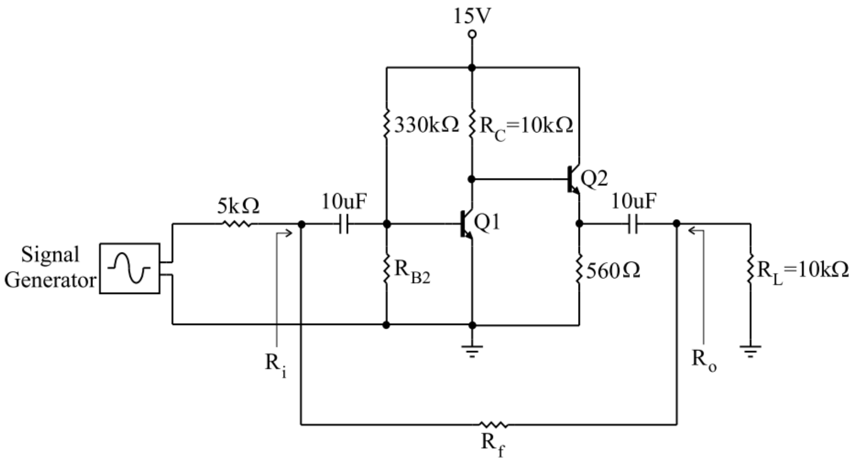

Figure 3.1: The Feedback circuit

R_{B2} Value (kΩ)	1	10	15	20	25	30	35
Gain (dB)	-98.8	-23.7	33.5	42.1	-95.9	-10e1.3	-104.2

Table 3.1: Different Values of R_{B2} at 1 kHZ and Open Loop Gain

From table 3.1, we can notice that the gain is when R_{B2} is around 20 kΩ. Thus, R_{B2} = 20 kΩ.

1)

The DC operating point feature in LTspice is used to obtain the DC operating values for both the transistors and listed in table 3.2

	I_C (mA)	I_B (μA)	I_E (mA)	V_C (V)	V_B (V)	V_E (V)
Q1	1.29	10.81	1.31	1.90	0.65	0
Q2	2.19	15.432	2.22	15	1.90	1.23

Table 3.2: DC Operating Points

Using the obtained values in 3.1, the values of h_{FE}, g_m and r_Π is calculated below.

$(V_T = 25mV)$

$g_{m1} = \frac{I_{C1}}{V_T}$

$= 51.6\ mS$

$g_{m2} = \frac{I_{C2}}{V_T}$

$= 87.6\ mS$

$r_{\pi 1} = \frac{V_T}{I_{B1}}$

$= 2312.67\ \Omega$

$r_{\pi 2} = \frac{V_T}{I_{B2}}$

$= 1620.01\ \Omega$

$h_{FE1} = (r_{\pi 1})(g_{m1})$

$= 119.33$

$h_{FE2} = (r_{\pi 2})(g_{m2})$

$= 141.91$

2)

Keeping R_f to an open circuit the as in part 1, the bode plot is depicted in figure 3.2 and the values were obtained in table 3.3. The input ad output resistances were measured using a voltage and current test source.

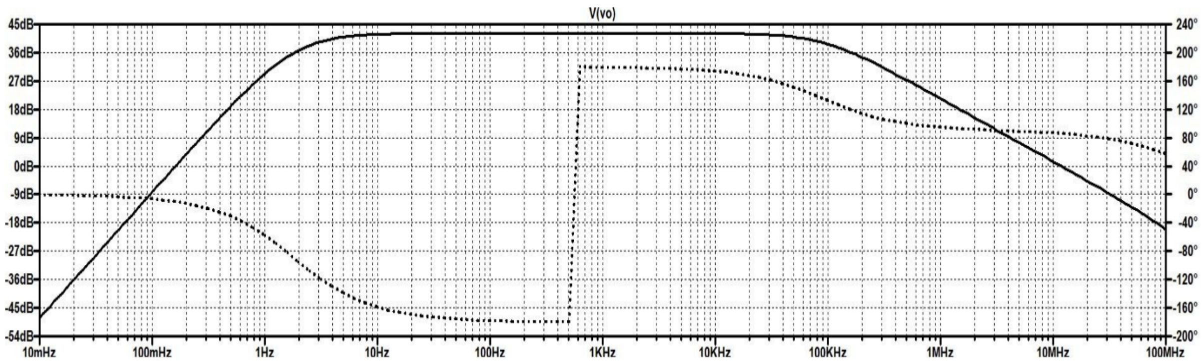

Figure 3.2: Bode plot for open loop

F_{L3dB}	F_{H3dB}	R_{in}	R_{out}	Open loop Gain (A_M)
2.94 Hz	90.31 kHz	2.57 kΩ	62.13 Ω	42.11 dB/ -125.55 V/V

Table 3.3: F_{L3dB} , F_{H3dB} , R_{in} , R_{out} and Open Loop Gain

The open-loop frequency response found above is used to calculate the following values: the predicted closed-loop frequency response and both the input and output resistance at 1kHz of our amplifier for R_f = 100kΩ. The calculation is shown below. Since the amplifier we are using is a shunt-shunt configuration, we use y-parameters for out calculations.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$\beta = y_{12} \qquad I_1 = y_{11}V_1 + y_{12}V_2$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-1}{R_f} = \beta = -10\,\mu\Omega$$

$\hookrightarrow V_1 = 0$

Open loop gain can be shown in terms of output voltage over input current as we have a shunt - shunt topology.

$$A' = \frac{V_o}{I_s} = R_S\left(\frac{V_o}{V_S}\right) = \left(-126.3\ V/V\right)\left(5\,k\Omega\right)$$

$$= -631.6\ \frac{kV}{A}$$

closed loop gain, A_f:

$$A_f = \frac{A'}{1 + A'\beta} = -86.33\ \frac{kV}{A}$$

Closed loop input and output resistances

$$R_{in,\,closed} = \frac{R_{in,\,open}}{1 + A'B} = 352\,\Omega$$

$$R_{out,\,closed} = \frac{R_{out,\,open}}{1 + A'B} = 8.43\,\Omega$$

Open and closed 3 dB frequencies:

$$f_{L3dB,closed} = \frac{f_{L3dB,open}}{1 + A'B} = 0.399\ Hz$$

$$f_{H3dB,closed} = \frac{f_{H3dB,open}}{1 + A'B} = 670.2\ kHz$$

The circuit in below in figure 3.3 is simulated and the closed loop characteristic values were measured and compared to the calculated values.

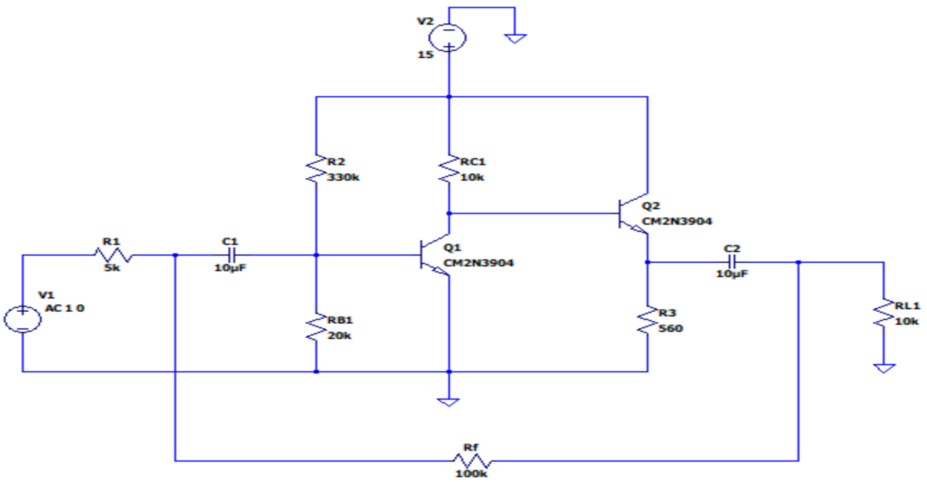

Figure 3.3: Closed loop circuit

	Calculated	Measured
f_{L3dB}	399 mHz	499.2 mHZ
f_{h3dB}	670.2 KHz	679.2 kHz
R_{in}	352 Ω	241.1 Ω
R_{out}	8.43 Ω	8.456 Ω

Table 3.4: Calculated versus measured closed loop values

3)

The magnitude bode plot for closed loop response by varying values of R_f is shown in figure 3.4 . The vared values of R_f are 1 kΩ, 10 kΩ, 100 kΩ, 1 MΩ and 10 MΩ. In figure 3.4, the top most graph is 10 MΩ and the subsequent one is 1 MΩ and so on.

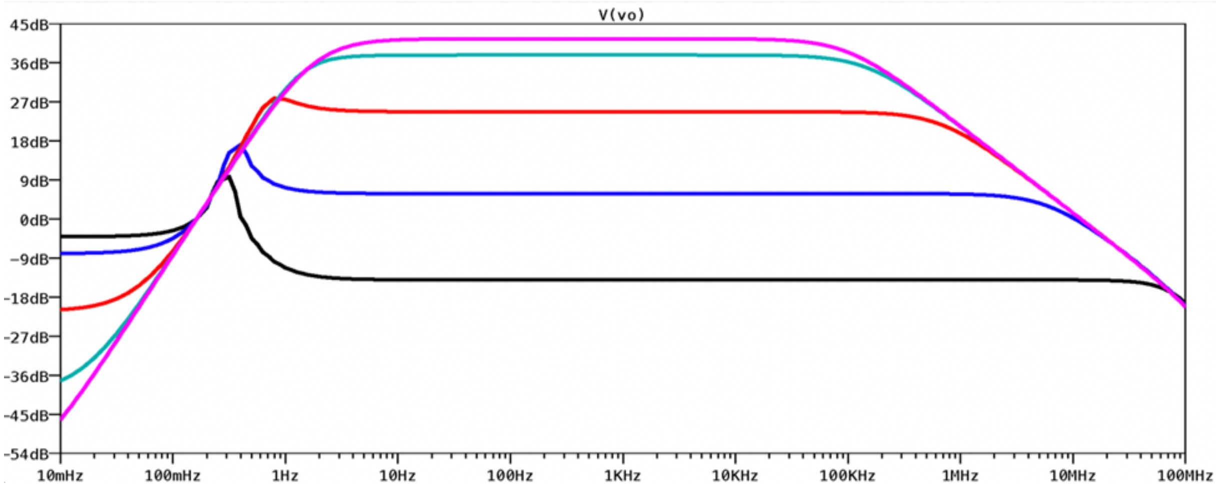

Figure 3.4: Closed loop magnitude plot with varying R_f

The equations to find the measured value of β is shown below:

$$A_f = \frac{A}{1 + A\beta}$$

Thus, $$\beta = \frac{A' - A_f}{A' A_f}$$

Since the open loop gain stay the same as the past part, the value that need to be calculated from the bode plot is A_f . Upon measuring the A_f value from the bod plot, it is noticed that the unit for this is V/V while we need a unit of V/A. Thus, we multiply it by 5 kΩ. The equation for finding the calculated value of β is shown below:

$$\beta = -\frac{1}{R_f}$$

R_f	A_f (kV/A)	Calculated β	Measured β
1 kΩ	-9.95	-1 m℧	-1.001 m℧
10 kΩ	-9.98	-100 μ℧	-100.270 μ℧
100 kΩ	-85.9	-10 μ℧	-10.021 μ℧
1 MΩ	-389.1	-1 μ℧	-1091 μ℧
10 mΩ	-598.9	-0.1 μ℧	-0.09781 μ℧

Table 3.5: calculated and measured β comparison with R_f varied

As seen from table 3.5 that the measure and calculated β are fairly close to one another. However, it becomes slightly less accurate as the value of R_f is seen to increase.

4)

The values below were found using the method described in past part.

R_f	R_{if}	R_{of}
10 kΩ	26.461 Ω	1.107 Ω
100 kΩ	241.159 Ω	8.462 Ω
1 MΩ	1.308 kΩ	37.30 Ω

Table 3.6: Input and output resistance at 1 kHz with varying values of R_f

To estimate the amount of feedback, 1 + Aβ, one can utilize the connection between input and output impedance and the amount of feedback in the shunt-shunt configuration.

$$R_{if} = \frac{R_i}{1+A\beta} \quad , \quad R_{of} = \frac{R_o}{1+A\beta}$$

$$\text{So, } 1+A\beta = \frac{R_i}{R_{if}} = \frac{R_o}{R_{of}}$$

For the predicted values we use the β values from from part 2 and the open loop gain A = -631.6 kV/A .

R_f	Predicted AOF	Measured AOF (R_{if})	Measured AOF (from R_{of})
10 kΩ	64.956	71.971	62.371
100 kΩ	7.309	10.01	8.312
1 MΩ	1.643	1.005	1.879

Table 3.7: Values of AOF

As noticed from table 3.7, the predicted and measured value of AOF are more closer to each other for larger values of R_f compared to smaller ones.

5)

The desensitivity factor can be found using the following equation.

$$\frac{dA_f}{dA} = \frac{1}{(1+A\beta)^2} \longrightarrow \frac{dA_f}{A_f} = \frac{1}{(1+A\beta)} \times \frac{dA}{A}$$

$$\therefore \text{desensitivity factor} = 1 + A\beta$$

From the equation of the desensitivity factor, we can understand that as R_f equates to infinity, β=0. Thus, the desensitivity factor equals 1. Table 3.8 and 3.9 shows the gain by varying RC when R_f equates to infinity and when R_f equates to 100 kiloohms.

R_C (kΩ)	A(V/V)
9.9	-126.765
10	-127.497
10.1	-128.086

Table 3.8: table of voltage gain with varied R_C and equating R_f equating to infinity

R_C (kΩ)	A(V/V)
9.9	-17.241
10	-17.250
10.1	-17.258

Table 3.8: table of voltage gain with varied R_C and equating R_f equating to 100 kΩ

From prior parts, we can calculate our theoretical desensitivity factor:

$$1 + (-631.6 \times 10^3 \times -10 \times 10^{-6})$$
$$= 7.316$$

Using the values obtaining in table 3.8 and 3.9, we can calculate the desensitivity factor and compare it with our theoretical value. The calculation for this shown below.

$$dA = \frac{(-127.497 + 126.765) + (-128.086 + 127.497)}{2}$$
$$= -0.6605$$

$$dA_f = \frac{(-17.298 + 17.199) + (-17.298 + 17.259)}{2}$$
$$= -0.0085$$

$$\text{desensitivity factor} = \sqrt{\frac{dA}{dA_f}} = 7.91$$

The desensitivity factor calculated using values in part 2 and 3 are 7.316 and the desensitivity factor calculated in this part is 7.91. From the two numbers, it is observed that the desentivity factor in both parts theoretical and measured are fairly close to each other.

Discussion:
This exercise provided a comprehensive exploration of the dynamics and characteristics of a feedback circuit featuring a two-stage cascaded transistor amplifier. Through practical implementation and testing, key insights were gained into the intricacies of DC bias adjustments for optimizing open-loop gain, the influence of feedback network parameters on amplifier performance, and the behavior of closed-loop frequency responses under various configurations. Analysis of transistor parameters, such as g_m, r_π, and h_{FE} offered a deeper understanding of amplifier behavior. The exercises further honed skills in measuring and calculating key amplifier parameters, interpreting frequency response plots, and assessing the effects of feedback on both input and output resistances, thereby enhancing overall comprehension of amplifier design and behavior in practical applications.

Additionally, it is observed that when employing smaller values for R_f, the amplifier gain is larger than at the mid-band. This may be due to the reason that for smaller values of R_f, a lesser negative feedback gain is obtained, increasing the overall gain of the system.

References:

[1] Miniproject 4 handout

[2] ELEC 301 course notes