Higher-Order Moment Inequality Restrictions for SVARs[1]

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[1] *The views expressed in this paper are those of the authors and are not necessarily reflective of views at the Federal Reserve Bank of Boston, the Federal Reserve Bank of Chicago, or the Federal Reserve System.*

Outline

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- Methods typically rely on second moments (signs/theoretical conditional correlations, correlation with narrative/proxies, max-share FE variance, exclusion restrictions).

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- Potentially higher moments of these structural shocks can be exploited to identify them.

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- Strike a balance between improving on the precision of identification and being robust to sample bias pervasive when estimating HOM.

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- Note 1: method can also be combined with constraints other than sign restrictions (zero, magnitude, narrative ...).
- Note 2: method can be applied to single out the shock that explains most of asymmetry or tailedness of the forecast error of an observed variable.

Literature Review

- Point identification through non-Gaussianity
  - Specific non-gaussian distribution for VAR errors or matching empirical innovations moments: Lanne, Meitz and Saikkonen (2017), Gourieroux, Monfort and Renne (2017, 2019) , Lanne, Liu and Luoto (2022).
  - Specific non-Gaussian distribution for structural shocks: Brunnermeier et al. (2021), Jarocinsky (2022).
  - We do not assume a specific distribution for structural shocks.

- Identified-set refinements through non-Gaussianity:
  - Montiel-Olea et al. (2022) call for robust approaches with non-Gaussian VAR.
  - Hoesch, Lee and Mesters (2022), exploit the weak non-Gaussianity of the VAR residuals to test if identified set can be narrowed.
  - Drautzburg and Wright (2023): discard rotations violating statistical independence of shocks. We do not impose independence.
  - We leverage on the non-Gaussianity of structural shocks.

- Sign restrictions and weak identification:
  - Kilian and Murphy (2012), Arias, Rubio-Ramirez and Waggoner (2018), Wolf (2020, 2022) show that imposing sign restrictions alone is often too weak to provide adequate identification of structural shocks.
  - Additional constraints: Antolin-Diaz and Rubio-Ramirez (2018) Arias, Caldara and Rubio-Ramirez (2018).
  - We use higher order moment restrictions.

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Static example

1/ ι empirical innovations (observed); ν structural shocks (unobserved). Linear mapping:

$$\iota = \begin{pmatrix} \iota_1 \\ \iota_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_o & -\sin\theta_o \\ \sin\theta_o & \cos\theta_o \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = A_o \nu$$

where θ_o is the 'true' unknown angle of rotation with $\theta_o \in (-\pi/2, \pi/2)$ and $\theta_o \neq 0$ (otherwise trivial).

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3/ 3^{rd} moments of structural shocks: $E(\nu_1^3) = 0$ and $E(\nu_2^3) = 1$, and and cross second and third moments are zero, i.e. $E(\nu_1\nu_2^2) = E(\nu_1^2\nu_2) = 0$,

$$E(\nu\nu' \otimes \nu') = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$\Rightarrow$ 3^{rd} moments of empirical innovations: $E(\iota_1^3) = -\sin^3\theta_o$, $E(\iota_2^3) = \cos^3\theta_o$, $E(\iota_1^2\iota_2) = \sin^2\theta_o \cos\theta_o$, $E(\iota_1\iota_2^2) = -\sin\theta_o \cos^2\theta_o$.

Eigenvalue decomposition

- First, the mapping between third moments of structural shocks and empirical innovations is given by

$$E(\iota\iota' \otimes \iota')E(\iota\iota' \otimes \iota')' = A_o E(\nu\nu' \otimes \nu')E(\nu\nu' \otimes \nu')' A_o' = A_o \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} A_o'.$$

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- Need precise sample estimates of $E(\iota_t\iota_t' \otimes \iota_t')$.

Inequality restriction on HOM

- Let $\breve{\nu} = A'\iota$ and let A be a generic rotation with angle θ, i.e. $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

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- Introduce the HOM inequality restriction of asymmetry, we have

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\begin{aligned}
&E(\breve{\nu}_2^3) > 0, \\
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- Easy to see that $\mathcal{I}_{hm} \subset \mathcal{I}_{ii}$ for any $\theta_o \neq 0$.

- No need to rely on sample estimates of $E(\iota_t\iota_t' \otimes \iota_t')$. Robust methods to compute $E(\breve{\nu}_2^3)$!

Estimation and Identification

- Let a $VAR(p)$ be:

$$y_t = \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_0 + u_t.$$

$u_t = \Sigma^{1/2} \iota_t = \Sigma^{1/2} \; A_o \; \nu_t$ and no distribution assumptions on v_t

- Let $\Sigma^{(j)}$ and $\Phi^{(j)}$ be the j^{th} draw from the BQML, as in Petrova (2023, JoE). Draw $\breve{\Omega}$ from a uniform distribution with the Rubio-Ramirez el al. (2010, RESTUD) algorithm
  I. compute the impulse response function and check if the sign (or any other economic) restrictions are verified,
  II. compute the implied structural shocks

$$\breve{\nu}_t^{(j)} = \breve{\Omega}' \left(\Sigma^{(j)}\right)^{-1/2} (y_t - \Phi_1^{(j)} y_{t-1} - \ldots - \Phi_p^{(j)} y_{t-p} - \Phi_0^{(j)}),$$

  II. compute $\mathcal{S}(\breve{\nu}_{n,t}^{(j)})$ and/or $\mathcal{K}(\breve{\nu}_{n,t}^{(j)})$ and check if the higher-order moment inequality restrictions are satisfied.

  If both [I] and [III] are satisfied, keep the draw $\Omega^{(j)} = \breve{\Omega}$. Else repeat [I], [II] and [III]. ▸ Gibbs Sampler

- Fourth and third sample moments sensitive. Use a robust non-parametric approach based on the distance across percentiles of the empirical distribution of the structural shocks of interest.

$$\mathcal{S}(x) = \frac{\bar{x} - F^{-1}(0.5)}{\text{std}(x)}, \quad \mathcal{K}(x) = \frac{F^{-1}(0.975) - F^{-1}(0.025)}{F^{-1}(0.75) - F^{-1}(0.25)} - 2.9,$$

where $F^{-1}(\alpha)$ is the α-percentile of the empirical distribution of x.

- These restrictions are modular, i.e. can be combined with sign (and zero), magnitude, narrative ... any restrictions that generate set-identification.

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- We suggest to use higher-order moment to improve identification.

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- In most of the analysis here we consider the SW posterior mode parameterization and the smoothed estimates of the shocks using postwar US data on output, consumption, investment, real wages, inflation, interest rate and hours worked as in their original work.

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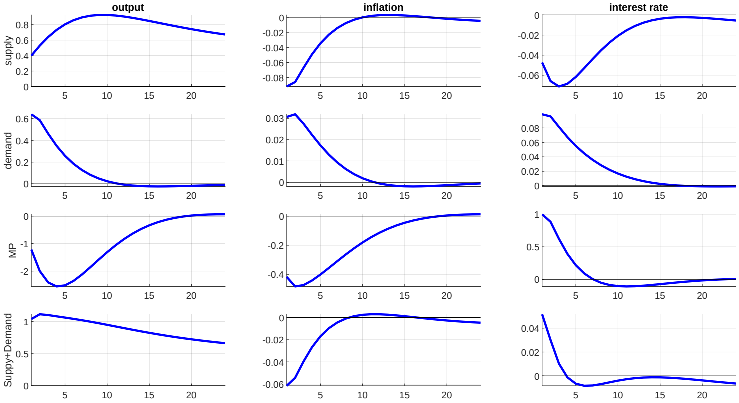
- Shocks: technology (supply), risk premium (demand) and MP
- Observables: output, inflation and interest rate

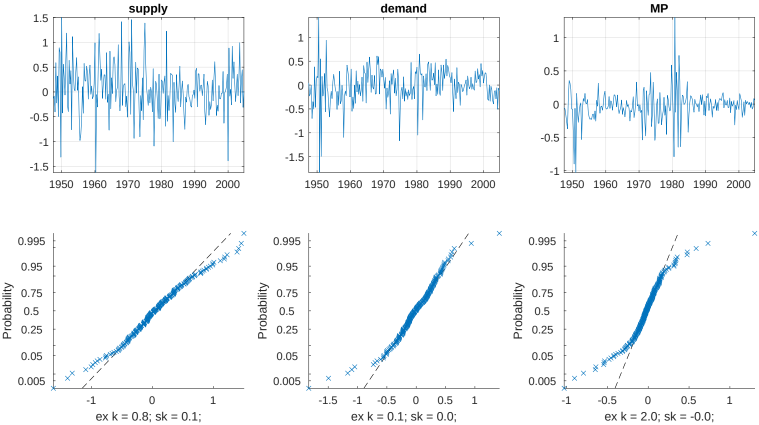
Estimated IRF

Figure: SW estimates of impulse response functions. From top to bottom technology, risk premium and monetary policy shocks and the sum of demand and supply shocks.



Estimated shocks

Figure: SW estimated shocks: from left to right technology, risk premium and monetary policy shocks. Top panels realizations, bottom panels probability distribution against the normal. ▸ other shocks



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- In particular, we assume that after a monetary policy shock,
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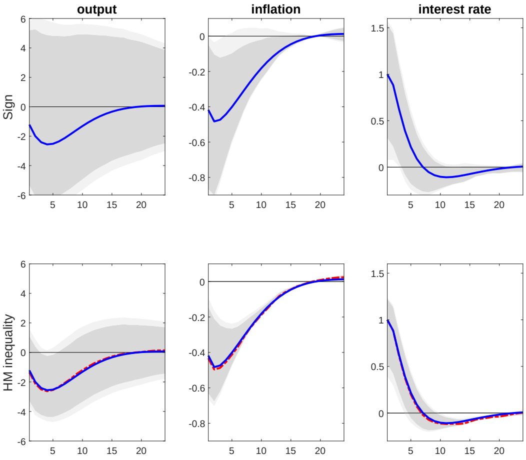
  - Monetary policy shocks are leptokrutic, i.e. monetary policy robust measure of excess kurtosis larger than 1.6

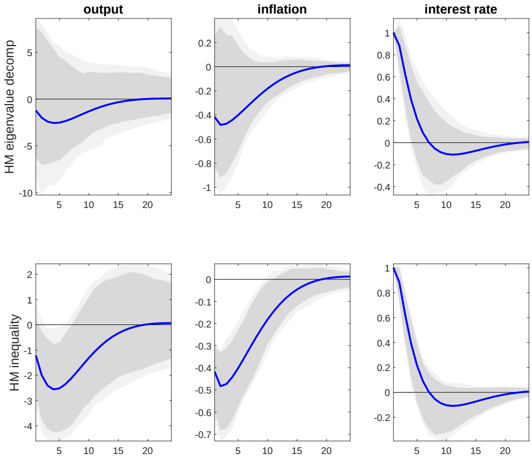
Large sample

Figure: IRF using sign (first row) and sign and higher moment inequality (second row) restrictions. The blue solid line is the true impulse response. The dark (light) gray areas report the 90% (99%) identified set using inequality restrictions on the fourth moment of the monetary policy shock.



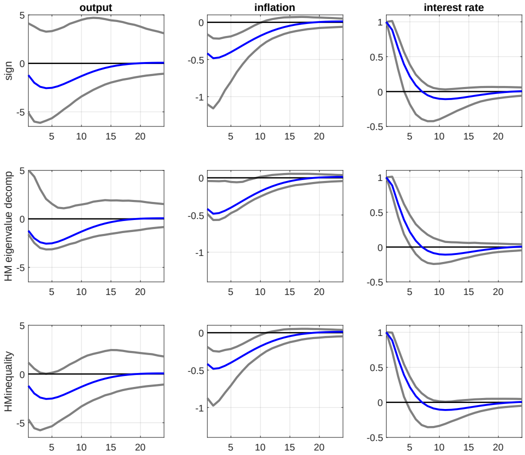
Short samples - Average Median

Figure: IRF using fourth moments eigenvalue decomposition and inequality restrictions (average median IRF across samples). The dark (light) gray areas report the 90% (99%) dispersion of the point estimates over repeated samples of 200 observation length. The blue line is the true impulse response.

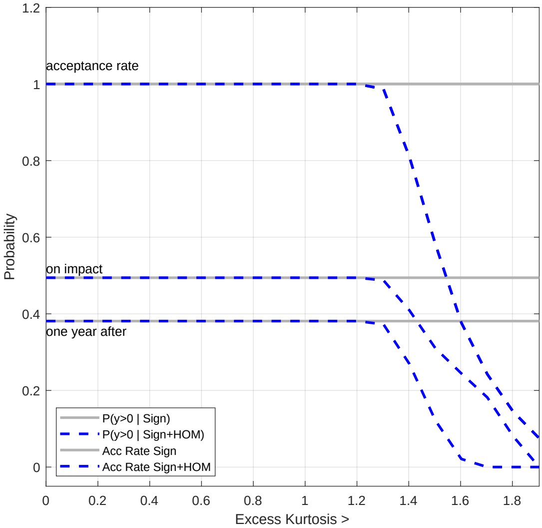


Short samples - Average Credible sets

Figure: The average upper and lower bounds of the 68% credible sets across Montecarlo simulations using signs, fourth moments eigenvalue decomposition and inequality restrictions.



$Prob(y > 0)$ as a function of the interval



Outline

Conventional MP in the U.S.

- The transmission mechanism of conventional monetary policy in the US using a minimal set of assumptions about the sign pattern of the impulse responses in the same spirit of Uhlig (2005).

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- higher-order mom: monetary policy shocks are drawn from a leptokurtic distributions, where most realizations are tiny but large deviations are more likely than with normal distributed shock.
- Is this a reasonable assumption? Look at estimates and proxies of monetary policy shocks.

Robust HM of MP shock proxies

- US MP surprises poorly correlated. Possibly span different info sets. All leptokurtic. corr
- Higher-order moment inequality interval → min and max, i.e. $\mathcal{I}_k \equiv [1.2, 12]$. $p(realization > 3\sigma) = 1\%$ (vs 0.15% for N)

	Ex-Kurtosis	Skewness	Sample Size
SW	2.0 [0.4, 3.2]	-0.0 [-0.1, 0.1]	179
SZ	3.8 [1.8, 6.3]	0.0 [-0.0, 0.1]	518
RR	3.2 [1.8, 5.1]	0.0 [-0.1, 0.1]	468
GK	11.3 [5.9, 18.2]	-0.3 [-0.3, -0.2]	269
MAR	3.3 [1.2, 5.9]	-0.1 [-0.2, 0.0]	228
JK	8.8 [5.4, 15.8]	-0.1 [-0.2, -0.0]	323
USf1	1.2 [0.3, 3.5]	0.1 [-0.0, 0.2]	204
USf2	3.0 [1.4, 7.1]	0.1 [-0.0, 0.2]	204
USf3	1.9 [0.5, 4.4]	0.0 [-0.1, 0.1]	204
AD	3.1[1.4, 7.0]	-0.0[-0.1, 0.1]	313
AF(target)	2.5 [0.6, 5.3]	-0.0 [-0.2, 0.1]	134
AF(delphic)	1.3 [0.2, 3.9]	-0.0 [-0.2, 0.1]	134
AF(FWG)	1.4 [0.2, 3.6]	0.0 [-0.1, 0.1]	134
EAf1	3.4 [1.4, 5.6]	-0.0 [-0.2, 0.1]	197
EAf2	1.5 [0.3, 3.9]	-0.1 [-0.2, 0.0]	197
EAf3	1.1 [0.2, 3.4]	0.0 [-0.1, 0.2]	197
CH	13 [5.9, 38]	0 [-0.1, 0.1]	348
GR(minutes)	2.5 [1.3, 4.9]	-0 [-0.2, 0.1]	211
GR(IR)	3.6 [2.8, 4.6]	-0.1 [-0.2, 0.0]	211
CBTV	3.4 [0.6, 7.5]	-0.1 [-0.2, 0.0]	212

Reduced form VAR residuals

- We consider the dataset studied in Uhlig (2005) with observations on real activity, prices and interest rates from 1965m1 to 2003m12
- 12 lags VAR residuals: reject normality for commodity price and interest rate (Kolmogorov-Smirnov test)

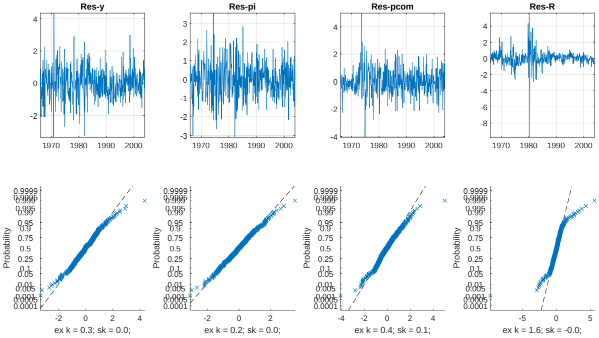
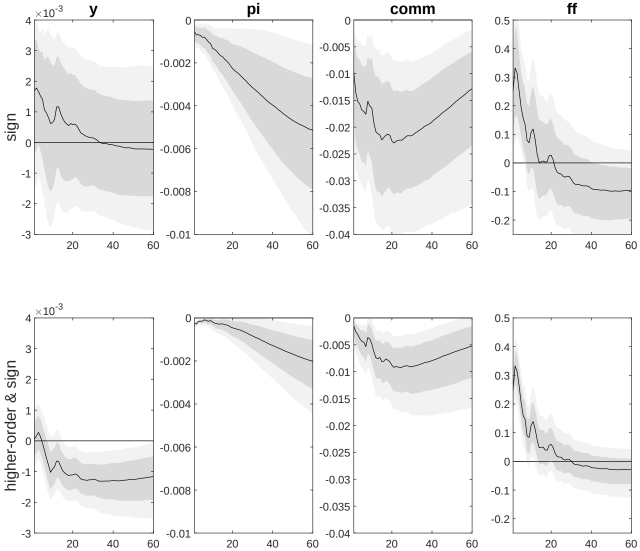


Figure: Orthogonalized LS residuals

Figure: Impulse responses to a monetary policy shock. Sign restrictions first row. Sign and kurtosis restrictions last row.



Spread Shocks in the E.A.

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- higher-order mom restrictions can be thought in this context as characterizing sovereign risk or spread shocks and used for identification.

Spread Shocks in the E.A.

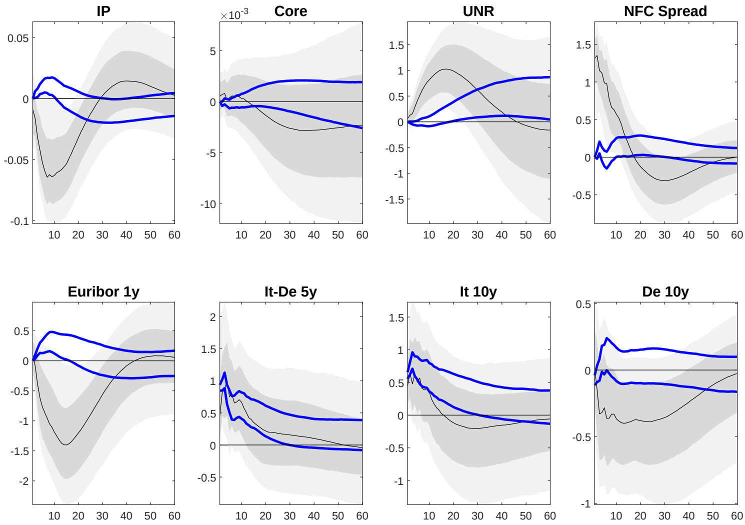
- Industrial production (IP), core HICP (Core), unemployment rate, a measure of borrowing costs (EBP), the one year Euribor, the spread between the 5 year Italian and German bond yield, and the 10 year Italian and German gov't bond yields from 1999m1 to 2019m12.

- Six lags VAR residuals. The K-S test rejects the null that the borrowing costs, the one year Euribor and the spread could have come from a standard normal distribution;

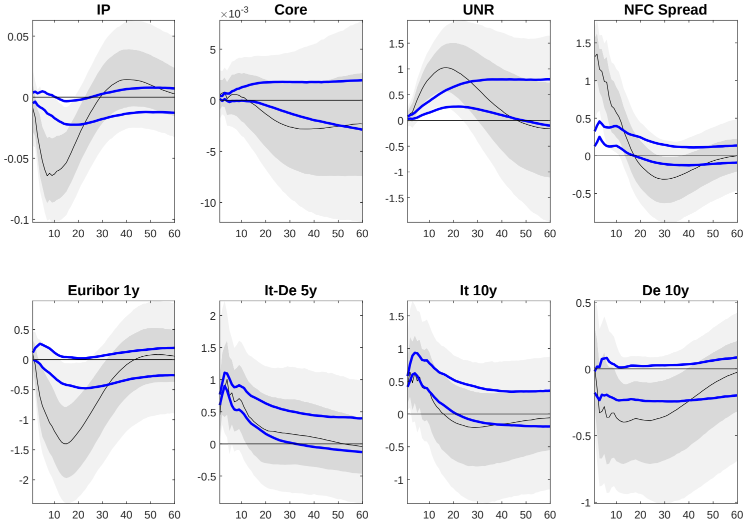
- Sign: spread shock $\uparrow$ the 5 spread, $\uparrow$ the 10 year Italian gov't bond yield and $\uparrow$ EBP on impact and for the following month; we assume further that the spread shock has skewness larger than one and excess kurtosis larger than one.

- Compare with Recursive, Max-Var (6 month, spread), Signs only.

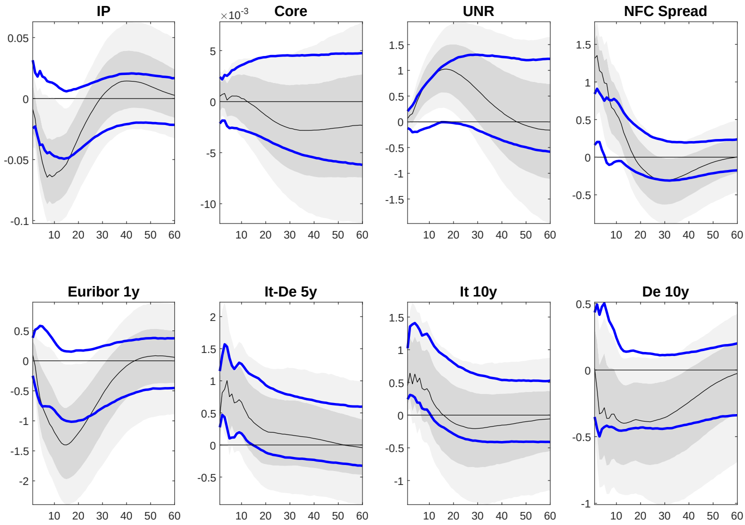
Recursive


IP
Core
$\times 10^{-3}$
UNR
NFC Spread
Euribor 1y
It-De 5y
It 10y
De 10y

Max-Var

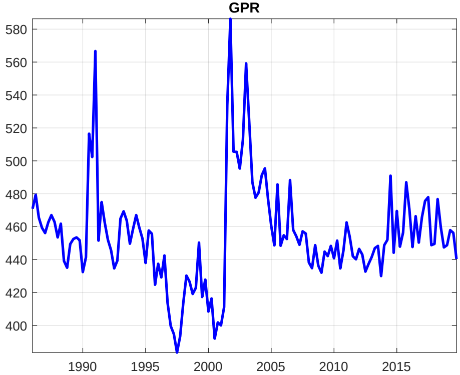


Signs


IP
0.05
0
-0.05
-0.1
10 20 30 40 50 60
Core
$\times 10^{-3}$
5
0
-5
-10
10 20 30 40 50 60
UNR
1.5
1
0.5
0
-0.5
-1
-1.5
10 20 30 40 50 60
NFC Spread
1.5
1
0.5
0
-0.5
10 20 30 40 50 60
Euribor 1y
0.5
0
-0.5
-1
-1.5
-2
10 20 30 40 50 60
It-De 5y
2
1.5
1
0.5
0
-0.5
10 20 30 40 50 60
It 10y
1.5
1
0.5
0
-0.5
-1
10 20 30 40 50 60
De 10y
0.5
0
-0.5
-1
10 20 30 40 50 60

Geopolitical Risk

- Caldara and Iacoviello (AER2022) Geopolitical Risk



Geopolitical Risk

- Caldara and Iacoviello (AER2022) Geopolitical Risk
- Recursive (exogenous ordered first) vs HOM restriction (fat tail and asymmetric) + sign (GPR $\uparrow$, S&P500 $\downarrow$ and Two-Year Yield $\uparrow$).

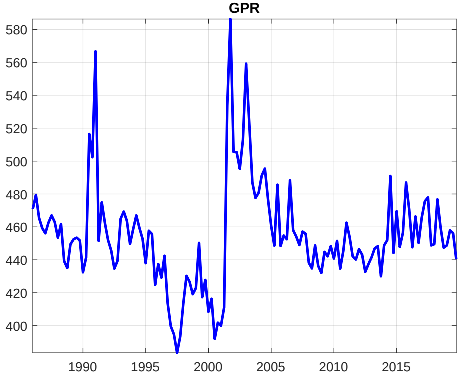


Figure: Impulse responses to a GPR shock. Recursive ordering. 2 standard deviation increase.

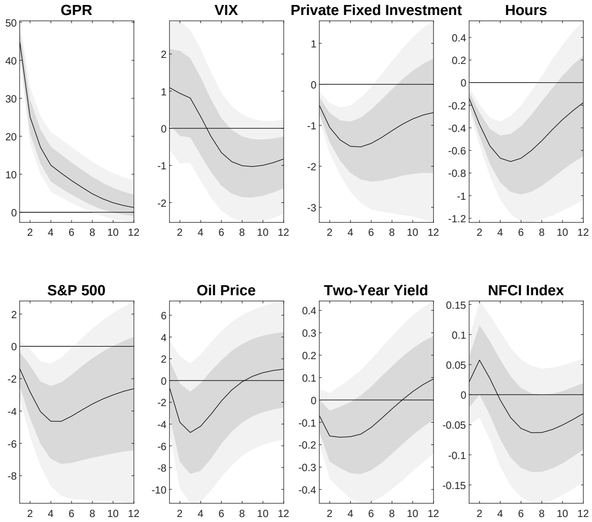
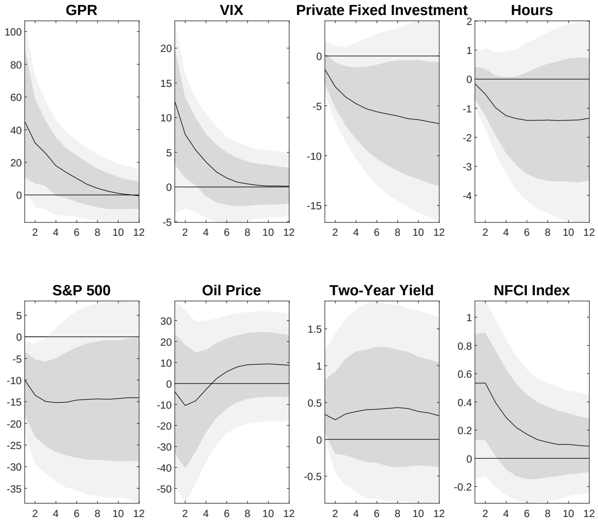


Figure: Impulse responses to a GPR shock. HOM restrictions. 2 standard deviation increase.



Outline

Conclusions

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Conclusions

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- Show how the excess kurtosis restriction can help isolating the impact of monetary policy shock on output from the supply and demand *masquerading* shock in an New Keynesian (NK) models.
- Using a Bayesian robust approach we apply our identification scheme to study the transmission of conventional monetary policy shocks in the U.S. before the financial crisis, of spread shocks in the Euro Area, and of Geopolitical risks to the macroeconomic aggregate.

Eigeinvalue decomposition - third mom

- Spectral decomposition of third mom

$$
\begin{aligned}
E(\iota_t\iota_t' \otimes \iota_t')E(\iota_t\iota_t' \otimes \iota_t')' &= A_o E(\nu_t\nu_t' \otimes \nu_t')(A_o \otimes A_o)'(A_o \otimes A_o)E(\nu_t\nu_t' \otimes \nu_t')' A_o' \\
&= A_o \left(\sum_{i=1}^{n} \zeta_i J_i \otimes \boldsymbol{e}_i\right)\left(\sum_{i=1}^{n} \zeta_i J_i \otimes \boldsymbol{e}_i\right)' A_o' \\
&= A_o \Lambda_\zeta A_o'
\end{aligned}
$$

where $\boldsymbol{e}_i$ is the $n \times 1$ vector with zeros everywhere except a one in the i^{th} position, J_i the $n \times n$ matrix of zeros everywhere except one in the i^{th} position of the main diagonal. Λ_ζ is a diagonal matrix collecting the squared third moments of the structural shocks.

- the eigenvalue $\rightarrow$ square of the third moments of the structural shock
- the eigenvector $\rightarrow$ coincides with the column of impact matrix, up to a sign switch and permutation of columns.
- Example $n = 2$,

$$
A_o \left(\begin{pmatrix} \zeta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_2 \end{pmatrix} \right) \left(\begin{pmatrix} \zeta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_2 \end{pmatrix} \right)' A_o'
$$
$$
= A_o \begin{pmatrix} \zeta_1^2 & 0 \\ 0 & \zeta_2^2 \end{pmatrix} A_o'
$$

return

Eigeinvalue decomposition - fourth mom

- Spectral decomposition of fourth mom

$$
\begin{aligned}
E(\iota_t\iota_t' \otimes \iota_t' \otimes \iota_t) - \mathcal{K}^z &= (A_o \otimes A_o)(E(\nu_t\nu_t' \otimes \nu_t' \otimes \nu_t) - \mathcal{K}^z)(A_o \otimes A_o)' \\
&= P\Lambda_\xi P'
\end{aligned}
$$

where Λ_ξ is a diagonal matrix

- the first n eigenvalues $\rightarrow$ fourth moments of the structural shock
- the first n elements of the first n eigenvectors divided by the absolute value of the first elements of the eigenvector, i.e. $P(1:n,j)/\sqrt{|P(1,j)|}$ for $j = 1, \ldots, n.\rightarrow$ impact matrix
- Example $n = 2$,

$$
\begin{aligned}
&(A_o \otimes A_o)\left(\begin{pmatrix} \xi_1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & \xi_2 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}\right)(A_o \otimes A_o) \\
&= (A_o \otimes A_o)\begin{pmatrix} \xi_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_2 \end{pmatrix}(A_o \otimes A_o)
\end{aligned}
$$

return

HM Eigeinvalue decomposition vs HM inequality restrictions

- Need to compute the full set of the third or fourth moments of the empirical innovations to retrieve the column of interest of the rotation matrix.

- Estimates of the fourth or third sample moments can be very sensitive to outliers or minor perturbation of the data and their estimates might be imprecise in short samples

- HOM inequality restrictions impose conditions on the higher moments of the structural shock itself using non-parametric robust methods based on the distance between different percentiles of the shock's empirical distribution.

- HOM inequality restrictions impose weaker conditions generating set-identification as opposed to point-identification and it can be coupled with other assumptions, such as signs, zeros, narrative, magnitude and/or statistical independence restrictions.

return

Gibbs Sampler ▸ return

Assuming a flat prior. Let $\widehat{S} = (Y - X\widehat{\Phi})'(Y - X\widehat{\Phi})$ and $\widehat{\Phi} = (X'X)^{-1}X'Y$, the steps of the Gibbs sampler are for $j = 1, \ldots, J$

- Draw $\Sigma^{(j)}$ from
$$N\left(\text{vech}(\widehat{S}), \widehat{\mathcal{C}}\right)$$
where $\widehat{\mathcal{C}} = \frac{1}{T} D_n^{+}\left(\widehat{S}^{1/2} \otimes \widehat{S}^{1/2}\right) D_n \left(\widehat{\mathcal{K}}^{\star} - \text{vech}(I_n)\text{vech}(I_n)'\right) D_n' \left(\widehat{S}^{1/2} \otimes \widehat{S}^{1/2}\right)' D_n^{+\prime}$ captures the fourth moments.
- Conditional on $\Sigma^{(j)}$, draw $\Phi^{(j)}$ from
$$N\left(\widehat{\Phi}, \Sigma^{(j)} \otimes (X'X)^{-1}\right)$$
  i. In case of an asymmetric distribution, the intercept, Φ_0, is drawn from
$$N(\widehat{\Phi}_0 + \widehat{\mathcal{S}}^{\star}\widehat{\mathcal{C}}^{-1}\text{vech}(\Sigma^{(j)} - \widehat{S}), \Sigma^{(j)} - 1/T\widehat{\mathcal{S}}_T^{\star}\widehat{\mathcal{C}}^{-1}\widehat{\mathcal{S}}_T^{\star\prime})$$
- Draw $\breve{\Omega}$ from a uniform distribution
  I. compute the impulse response function and check if the sign restrictions are verified
  II. compute the implied structural shocks
$$\breve{\nu}_t^{(j)} = \breve{\Omega}'\left(\Sigma^{(j)}\right)^{-1/2}(y_t - \Phi_1^{(j)}y_{t-1} - \ldots - \Phi_p^{(j)}y_{t-p} - \Phi_0^{(j)})$$
  and check if the higher moment inequality restrictions are satisfied

If both [I] and [II] are satisfied, keep the draw $\Omega^{(j)} = \breve{\Omega}$. Else repeat [I] and [II].

Fourth and third sample moments ▸ return

- the shrinkage estimator for the kurtosis is defined as

$$\widehat{\mathcal{K}}^{\star} = \frac{T}{T+\tau}\widehat{\mathcal{K}}_T + \frac{\tau}{T+\tau} D_n^{+}(I_n + K_{n,n} + \text{vec}(I_n)\text{vec}(I_n)')D_n^{+\prime} \quad (2)$$

where $\widehat{\mathcal{K}}_T$ represents the sample fourth moments of the empirical innovations, i.e. $\widehat{\mathcal{K}}_T = 1/T \sum \text{vech}(\iota_t \iota_t') \otimes \text{vech}(\iota_t \iota_t')$ with $\iota_t = \hat{\Sigma}^{-1/2} u_t$;

$K_{n,n}$ is a commutation matrix, which is a $(n^2 \times n^2)$ matrix consisting of $n \times n$ blocks where the $(j, i)-$element of the (i, j) block equals one, elsewhere there are all zeros;

D_n^{+} is the generalized inverse of the duplication matrix D_n.

- the shrinkage estimator skewness given by

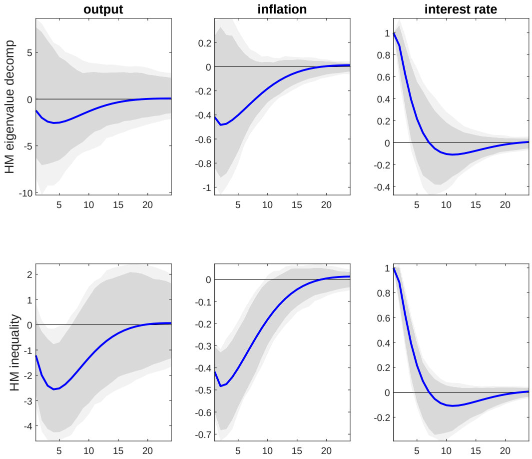
$$\widehat{\mathcal{S}}_T^{\star} = \frac{T}{T+\tau}\widehat{\mathcal{S}}_T$$

where $\widehat{\mathcal{S}}_T = \left(1/T \sum \text{vech}(u_t u_t') \otimes u_t\right)$

Short samples - Average Median

return

Figure: IRF using fourth moments eigenvalue decomposition and inequality restrictions (average median IRF across samples). The dark (light) gray areas report the 90% (99%) dispersion of the point estimates over repeated samples of 200 observation length. The blue line is the true impulse response.



Short samples - Average Credible sets

▸ return

Figure: The average upper and lower bounds of the 68% credible sets across Montecarlo simulations using signs, fourth moments eigenvalue decomposition and inequality restrictions.

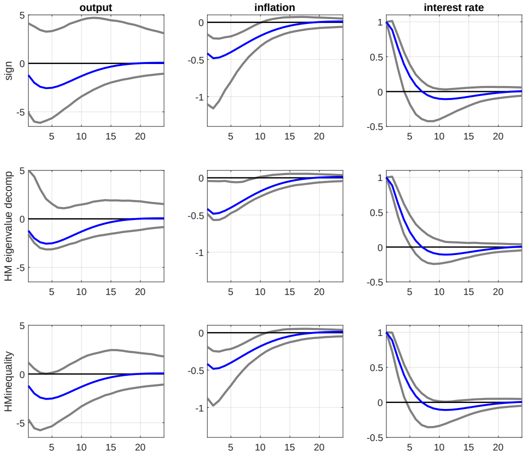
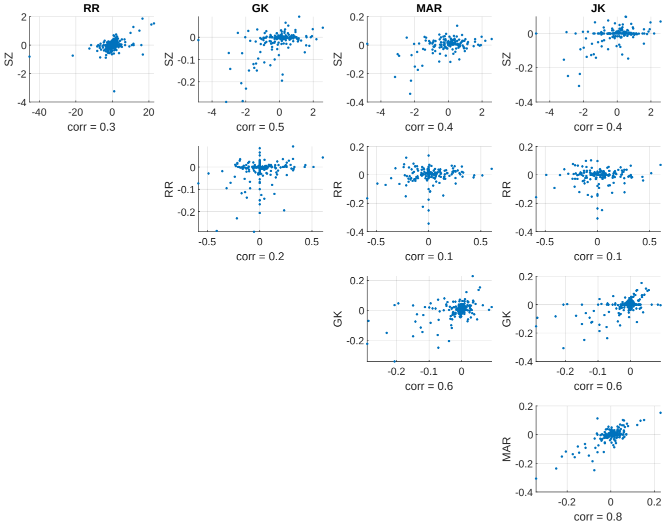


Figure: Correlations across measures of U.S. monetary policy shocks.



return

New Keynesian model ▸ return

- NK model

$$
\begin{aligned}
y_t =& y_{t+1|t} - (i_t - \pi_{t+1|t}) + \sigma_d \epsilon_t^d \\
\pi_t =& \beta \pi_{t+1|t} + \kappa y_t - \sigma_s \epsilon_t^s \\
i_t =& \phi_\pi \pi_t + \phi_y y_t + \sigma_m \epsilon_t^m
\end{aligned}
$$

New Keynesian model

return

- NK model

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\begin{aligned}
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i_t &= \phi_\pi \pi_t + \phi_y y_t + \sigma_m \epsilon_t^m
\end{aligned}
$$

- Solution of the model (in a linear model agents do not care about the shock's distribution)

$$
x_t = \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \frac{1}{1 + \kappa\phi_\pi + \phi_y} \begin{pmatrix} \sigma_d & \phi_\pi \sigma_s & -\sigma_m \\ \kappa\sigma_d & -(1+\phi_y)\sigma_s & -\kappa\sigma_m \\ (\phi_y + \kappa\phi_\pi)\sigma_d & -\phi_\pi\sigma_s & \sigma_m \end{pmatrix} \begin{pmatrix} \epsilon_t^d \\ \epsilon_t^s \\ \epsilon_t^m \end{pmatrix} = A_o \epsilon_t
$$

New Keynesian model return

- NK model

$$
\begin{aligned}
y_t &= y_{t+1|t} - (i_t - \pi_{t+1|t}) + \sigma_d \epsilon_t^d \\
\pi_t &= \beta \pi_{t+1|t} + \kappa y_t - \sigma_s \epsilon_t^s \\
i_t &= \phi_\pi \pi_t + \phi_y y_t + \sigma_m \epsilon_t^m
\end{aligned}
$$

- Solution of the model (in a linear model agents do not care about the shock's distribution)

$$
x_t = \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \frac{1}{1 + \kappa \phi_\pi + \phi_y} \begin{pmatrix} \sigma_d & \phi_\pi \sigma_s & -\sigma_m \\ \kappa \sigma_d & -(1+\phi_y)\sigma_s & -\kappa \sigma_m \\ (\phi_y + \kappa \phi_\pi)\sigma_d & -\phi_\pi \sigma_s & \sigma_m \end{pmatrix} \begin{pmatrix} \epsilon_t^d \\ \epsilon_t^s \\ \epsilon_t^m \end{pmatrix} = A_o \epsilon_t
$$

- Distribution assumption $\epsilon_t^d \sim N(0,1)$, $\epsilon_t^s \sim N(0,1)$, $\epsilon_t^m \sim Laplace(0,1)$. The Excess Kurtosis of the Laplace distribution is 3.

New Keynesian model

return

- NK model

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\begin{aligned}
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$$

- Distribution assumption $\epsilon_t^d \sim N(0,1)$, $\epsilon_t^s \sim N(0,1)$, $\epsilon_t^m \sim Laplace(0,1)$. The Excess Kurtosis of the Laplace distribution is 3.

- Simulate T =100,000 data points with parameters values: $\sigma_s = \sigma_d = \sigma_m = 1$, $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\kappa = 0.2$.

Masquerading - MP tightening

▸ return

The $S+D$ shock generate $\pi<0$ and $i>0$ (same as MP) and $y>0$

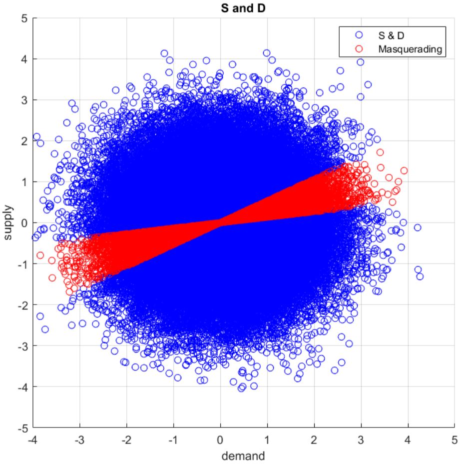
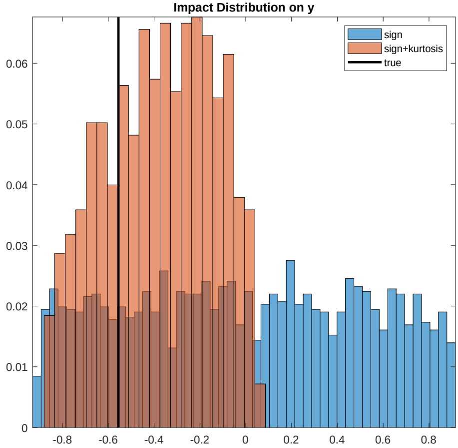


Figure: Realizations of demand and supply shocks: all (blue circles) and masqueraded MP (red circles).

Impact distribution on y return



$p(y < 0)$ with sign and sign+hom restrictions ▸ return

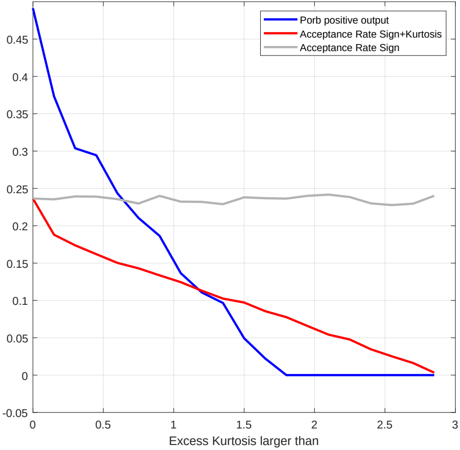


Figure: Probability of positive response of y at different restrictions on monetary policy excess kurtosis.

Estimated shocks - cont

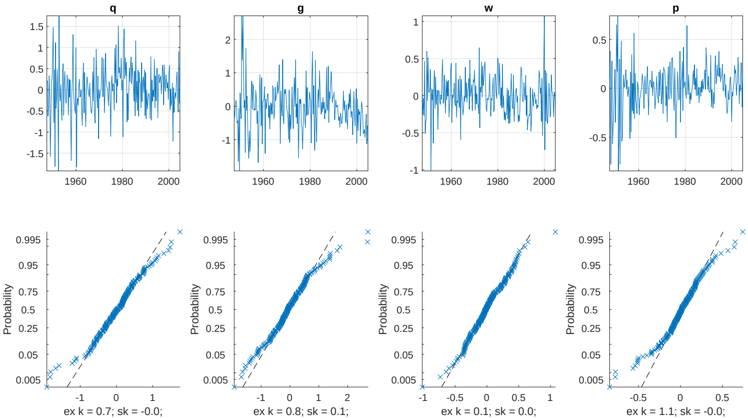


Figure: SW estimated shocks. return

Setup

- Econometrician observes a vector of *empirical innovations*, ι_t, where ι_t is a $n \times 1$ vector of innovations.

- Assume that empirical innovations are generated from the linear system,

$$\iota_t = A_o \nu_t$$

where ν_t is a $n \times 1$ vector of unobserved *structural shocks*. Let $\boldsymbol{\alpha}_i$ be the i^{th} column of A_o.
$A_o' A_o = A_o A_o' = I$.

- Structural shocks are i.i.d. over time and have the following properties return
    - $E(\nu_{i,t}^2) = 1$ and $E(\nu_{i,t}\nu_{j,t}) = 0$ for all i, j;
    - $E(\nu_{i,t}^3) = \zeta_i$ and $E(\nu_{i,t}\nu_{j,t}\nu_{k,t}) = 0$ for all $i \neq j, k$;
    - $E(\nu_{i,t}^4) = \xi_i$, $E(\nu_{i,t}^2\nu_{j,t}^2) = 1$ for all $i \neq j$, and $E(\nu_{i,t}\nu_{j,t}\nu_{k,t}\nu_{m,t}) = 0$ for all $i \neq j, k, m$.

- Gaussian case: $\zeta_i = 0$ and $\xi_i = 3$. We assume that the shock of interest has $\zeta \neq 0$ or $\xi \neq 3$.

Necessary conditions

- Gaussian: hom are not useful for identification since third mom are zero and fourth mom are invariant to orthonormal rotations, Ω, i.e.

$$(\Omega \otimes \Omega)' \, \mathcal{K}^z \, (\Omega \otimes \Omega) = \mathcal{K}^z,$$

  where $\mathcal{K}^z$ is the $n^2 \times n^2$ matrix of fourth mom of the standard MN.

- Non-Gaussian: the eigenvalue decomposition of the empirical innovation hom matrix recovers the impact matrix A_o.

  ▸ HOM egeinvalue decomposition

  Need many data points, else hom estimates imprecise $\rightarrow$ impact matrix estimate corrupted.

- Use a weaker result (Proposition):

  Let $\breve{\nu}_{n,t} = \boldsymbol{a}_n' \iota_t$ be a candidate structural shock with $\boldsymbol{a}_n$ a unit-length vector of weights.
  When $\boldsymbol{a}_n = \boldsymbol{\alpha}_n$ (where $\boldsymbol{\alpha}_n$ is the 'true' impact), then candidate shock ($\breve{\nu}_{n,t}$) has the same higher-order property of the 'true' structural shock.

- Necessary condition for identification:

  It is impossible to get the true rotation matrix without generating the correct higher moments $\rightarrow$ discard all rotations that do not satisfy the higher-order moment restriction.

Introducing dynamics

- Let a $VAR(p)$ be:

$$y_t = \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_0 + u_t.$$

- Reduced form errors, empirical innovations and structural shocks, ν_t properties

$$u_t = \Sigma^{1/2} \iota_t = \Sigma^{1/2}\ A_o\ \nu_t$$

- The Bayesian inference builds on the work by Petrova (2022, JoE); it exploits asymptotic normality of the Quasi Maximum Likelihood (QML) estimator of reduced form parameters.

- Asymptotically valid inference about Φ does not depend on the error term distribution: $\rightarrow u_t \checkmark$.
Asymptotically valid inference about Σ depends on fourth moments of the errors: $\rightarrow \iota_t \times$.
(assume no skewness for exposition simplicity)

- Asymptotic valid inference for Σ can be performed by drawing from the asymptotic normal distribution centered in the consistent estimator of Σ, i.e. the QML estimator $\hat{\Sigma}$, and with covariance matrix equal difference between the fourth mom and the 'squared' second moments.