April 18, 2023

$$\text{Area, } a = \int_{t_1}^{t_2} A e^{-\lambda t}\, dt = -\frac{A}{\lambda}\left[e^{-\lambda t_2} - e^{-\lambda t_1}\right]$$

$$= \frac{A}{\lambda}\left[e^{-\lambda t_1} - e^{-\lambda t_2}\right]$$

$$\boxed{\lambda = \frac{\ln(2)}{T_{eff}}}$$

$$\sigma_a^2 = \left(\frac{\partial a}{\partial A}\sigma_A\right)^2 + \left(\frac{\partial a}{\partial T}\sigma_T\right)^2 + 2\left(\frac{\partial a}{\partial A}\right)\left(\frac{\partial a}{\partial T}\right)\mathrm{cov}(A,T)$$

$$\frac{\partial a}{\partial A} = \frac{1}{\lambda}\left[e^{-\lambda t_1} - e^{-\lambda t_2}\right] = \frac{T}{\ln(2)}\left[e^{-\frac{t_1 \cdot \ln(2)}{T}} - e^{-\frac{t_2 \cdot \ln(2)}{T}}\right]$$

$$\frac{\partial a}{\partial T} = \frac{1}{\ln(2)}\left[e^{-\frac{t_1 \cdot \ln(2)}{T}} - e^{-\frac{t_2 \cdot \ln(2)}{T}}\right]$$

$$+ \frac{T}{\ln(2)}\left[\frac{t_1 \cdot \ln(2)}{T^2} e^{-\frac{t_1 \ln(2)}{T}} - \frac{t_2 \ln(2)}{T^2} e^{-\frac{t_2 \cdot \ln(2)}{T}}\right]$$

$$= \frac{1}{\ln(2)}\left[e^{-\frac{t_1 \cdot \ln(2)}{T}} - e^{-\frac{t_2 \ln(2)}{T}}\right]$$

$$+ \frac{1}{T}\left[t_1 e^{-\frac{t_1 \ln(2)}{T}} - t_2 e^{-\frac{t_2 \ln(2)}{T}}\right]$$

Simulation ($T_{eff} = 35.4$ h, $A = 2$)

t: from 0 to 48 hours

1. 5 points from 0 to 48 hours.
2. 10 unique combinations of points.
3. for each combination → 10,000 noise realizations.

Known { [0,1] → highest curvature along A direction
[2,3] → " " " T_{eff} "

Results:

[0,1] → most precise estimate for A
[2,3] → " " " " T_{eff}

Least bias from true A and true T_{eff} both achieved by [0,4]