Let covariance matrix Σ be diagonal.
So $\Sigma^{-1} = I$ is diagonal for "m" parameters.

Let $w_i = 1$ for $\forall i \in \{1, \dots, n\}$

$$\therefore \quad H_{a,b} = \frac{\partial^2 C}{\partial\theta_a \partial\theta_b}\bigg|_{\theta^*} = I_{ab} \approx \sum_{i=1}^{n} w_i \frac{\partial \rho_i}{\partial \theta_a} \frac{\partial \rho_i}{\partial \theta_b}\bigg|_{\theta^*}$$

Set "S_1" has only 1 data point

$$I_{S_1} = \begin{pmatrix} \left(\frac{\partial \rho_1}{\partial \theta_1}\right)^2 & & & 0 \\ & (\partial \rho_1/\partial \theta_2)^2 & & \\ & & \ddots & \\ 0 & & & (\partial \rho_1/\partial \theta_m)^2 \end{pmatrix} \rightarrow m \times m$$

$$I_{S_2}^{-1} = \begin{pmatrix} \sigma_{\theta_1}^2 & & & & 0 \\ & \sigma_{\theta_2}^2 & & & \\ & & \sigma_{\theta_3}^2 & & \\ & & & \ddots & \\ 0 & & & & \sigma_{\theta_m}^2 \end{pmatrix} \rightarrow m \times m$$

"σ_j" is NOT measurement error

$$\text{So, } I_{S_1} I_{S_2}^{-1} = \begin{pmatrix} \left(\frac{\partial \rho_1}{\partial \theta_1}\sigma_{\theta_1}\right)^2 & & & 0 \\ & \left(\frac{\partial \rho_1}{\partial \theta_2}\sigma_{\theta_2}\right)^2 & & \\ & & \ddots & \\ 0 & & & \left(\frac{\partial \rho_1}{\partial \theta_m}\sigma_{\theta_m}\right)^2 \end{pmatrix}$$

So, $\frac{1}{m}\operatorname{trace}\left(I_{S_1} I_{S_2}^{-1}\right)$

$$= \frac{1}{m}\left[\left(\frac{\partial \rho_1}{\partial \theta_1}\sigma_{\theta_1}\right)^2 + \dots + \left(\frac{\partial \rho_1}{\partial \theta_m}\sigma_{\theta_m}\right)^2\right]$$

With the w_i's put in (assuming H_{ab} is diagonal):

$$H_{ab}\Big|_{\theta^*} = I_{ab} \approx \sum_{i=1}^{n} w_i \left(\frac{\partial p_i}{\partial \theta_a}\right)^2 \rightarrow \text{for } a=b$$

Let $W^{1/2} = \begin{pmatrix} \sqrt{w_1} & & & \\ & \sqrt{w_2} & & \\ & & \ddots & \\ & & & \sqrt{w_n} \end{pmatrix}$

$$\therefore I_{aa} = \left(W^{1/2}\nabla_{\theta_a}\vec{p}\right)^T \left(W^{1/2}\nabla_{\theta_a}\vec{p}\right)$$

$$I_{s_1} = \begin{pmatrix} \left(\sqrt{w_1}\frac{\partial p_1}{\partial \theta_1}\right)^2 & & \\ & \ddots & \\ & & \left(\sqrt{w_1}\frac{d p_1}{d \theta_m}\right) \end{pmatrix}$$

$$I_{s_2}^{-1} = \begin{pmatrix} \left((\nabla_{\theta_1}\vec{p})^T W (\nabla_{\theta_1}\vec{p})\right)^{-1} & & \\ & \ddots & \\ & & \left((\nabla_{\theta_m}\vec{p})^T W (\nabla_{\theta_1}\vec{p})\right)^{-1} \end{pmatrix}$$

$$I_{s_1} I_{s_2}^{-1} = \begin{pmatrix} \left(\frac{\partial p_1}{\partial \theta_1}\right)^2 \frac{w_1}{\sum_{i=1}^{n} w_i \left(\frac{\partial p_i}{\partial \theta_1}\right)^2} & & 0 \\ & \ddots & \\ 0 & & \left(\frac{d p_1}{d \theta_m}\right)^2 \frac{w_1}{\sum_{i=1}^{n} w_i \left(\frac{\partial p_i}{d \theta_m}\right)^2} \end{pmatrix}$$

$$w_i > 0 \longrightarrow w_i = e^{-\xi_1}$$

$$\text{Normalize} \longrightarrow w_i = \frac{e^{-\xi_i}}{\sum_{j=1}^{n} e^{-\xi_j}}$$