$$p(\vec{y}\,|\,\lambda) = \prod_{i=1}^{N} \frac{1}{\sigma_i\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x_i - Ae^{-\lambda x_i}}{\sigma_i}\right)^2}$$

$$\ln\left[p(\vec{y}\,|\,\lambda)\right] = \sum_{i=1}^{N}\left[\ln\left(\frac{1}{\sigma_i\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma_i}\right)^2\right]$$

$$= \sum_{i=1}^{N}\left[-\ln\left(\sigma_i\sqrt{2\pi}\right) - \frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma_i}\right)^2\right]$$

$$= -\frac{N}{2}\ln(2\pi) - \sum_{i=1}^{N}\left[\ln(\sigma_i) + \frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma_i}\right)^2\right]$$

$$= -\frac{N}{2}\ln(2\pi) - \sum_{i=1}^{N}\left[\ln(\sigma_i)\right] - \frac{1}{2}\sum_{i=1}^{N}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma_i}\right)^2$$

Let $\sigma_i = \sigma$ for $\forall i \in \mathbb{Z}^+$,

$$\therefore\ -\ln\left[p(\vec{y}\,|\,\lambda)\right] = \frac{N}{2}\ln(2\pi) + N\ln(\sigma) + \frac{1}{2}\sum_{i=1}^{N}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2$$

$$I = E\left[\frac{\partial^2}{\partial\lambda^2}\left(-\ln\left[p(\vec{y}\,|\,\lambda)\right]\right)\right]$$

$$I = \mathbb{E}\left[\frac{\partial^2}{\partial\lambda^2}\left\{\frac{N}{2}\ln(2\pi) + N\ln(\sigma) + \frac{1}{2}\sum_{i=1}^{N}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2\right\}\right]_{\lambda^*}$$

$$I = E\left[\frac{\partial}{\partial\lambda}\left(\sum_{i=1}^{N}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)\left(\frac{Ax_i}{\sigma}\right)e^{-\lambda x_i}\right)^2\right]_{\lambda=\lambda^*}$$

$l_i = \log p(y_i|\lambda)$ for 1 sample

~~Set~~ $N=1$

$$(2)\ \frac{\partial^2 l_i}{\partial \lambda^2} = \frac{\partial}{\partial \lambda}\left[\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)\left(\frac{Ax_i}{\sigma}\right)e^{-\lambda x_i}\right]$$

$$= \left(\frac{A}{\sigma^2}\right)\frac{\partial}{\partial \lambda}\left[(y_i - Ae^{-\lambda x_i})\, x_i \cdot e^{-\lambda x_i}\right]$$

$$= \left(\frac{A}{\sigma^2}\right)\left[(y_i - Ae^{-\lambda x_i})\frac{\partial}{\partial \lambda}(x_i e^{-\lambda x_i}) + \left(\frac{\partial}{\partial \lambda}(y_i - Ae^{-\lambda x_i})\right)(x_i e^{-\lambda x_i})\right]$$

$$= \left(\frac{A}{\sigma^2}\right)\left[(y_i - Ae^{-\lambda x_i})(-x_i)(x_i e^{-\lambda x_i}) + \left(Ax_i e^{-\lambda x_i}\right)x_i e^{-\lambda x_i}\right]$$

$$= \left(\frac{A}{\sigma^2}\right)\left[-y_i x_i^2 e^{-\lambda x_i} + \ldots + Ax_i^2 e^{-2\lambda x_i}\right]$$

$$= \left(\frac{A}{\sigma^2}\right)\left[-y_i x_i^2 e^{-\lambda x_i} + 2Ax_i^2 e^{-2\lambda x_i}\right]$$

$$I_i = \mathbb{E}\left[\left(\frac{A}{\sigma^2}\right)x_i^2\left[-y_i e^{-\lambda x_i} + 2Ae^{-2\lambda x_i}\right]\right]$$

$$\Rightarrow I_i = \int_{-\infty}^{\infty}\left(+\frac{A}{\sigma^2}\right)x_i^2\left[-y_i e^{-\lambda x_i} + 2Ae^{-2\lambda x}\right] \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2} dy_i$$

$$\Rightarrow I_1 = \left(+\frac{A}{\sigma^2}\right)\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} dy_i \; x_i^2\left[2Ae^{-2\lambda x_i} - y_i e^{-\lambda x_i}\right] \cdot \exp\left[-\frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2\right]$$

$x_i = x$

$$\Rightarrow I_{1_{/\lambda}} = \left(+\frac{A}{\sigma^2}\right)\frac{1}{\sigma\sqrt{2\pi}}\left\{\int_{-\infty}^{\infty} dy_i \; 2A x_i^2 e^{-2\lambda x_i} e^{-\frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2} - \int_{-\infty}^{\infty} dy_i \,(x_i^2 y_i) e^{-\lambda x_i} e^{-\frac{1}{2}\left(\frac{y_i - Ae^{-\lambda x_i}}{\sigma}\right)^2}\right\}$$

$= I_1(\lambda, x_i)$ for a single y_i

$$I(\lambda) = \sum_{i=1}^{N} I_i(\lambda, x_i)$$

Incorporate weights by: $\sigma \rightarrow \sigma/w_i$ for $i = 1 \ldots N$