Exercise set 1.2.3

1. (a) $f(x) = x + x^3$

$$f(-x) = (-x) + (-x)^3$$
$$= -x - (1)^3 x^3$$
$$= -x - x^3$$
$$= -(x + x^3)$$
$$\therefore f(-x) = -f(x)$$

So $f(x)$ is an odd function.

(b) $f(x) = (x^2+25)^2$

$$f(-x) = \{(-x)^2+25\}^2$$
$$\therefore f(-x) = (x^2+25)^2$$
$$\therefore f(-x) = f(x)$$

So, $f(x)$ is an even function

(c) $f(x) = x^6 + e^{4x}$

$$f(-x) = (-x)^6 + e^{4(-x)}$$
$$= (-1)^6 x^6 + e^{-4x}$$
$$\therefore f(-x) = x^6 + e^{-4x}$$

So, $f(x)$ is nither odd nor even function.

(d) $f(x) = \sin^3 x \cos^6 x$

$f(-x) = \sin^3(-x) \cos^6(-x)$

$= -\sin^3 x \cos^6 x$

$\therefore f(-x) = -f(x)$

So, $f(x)$ is an odd function.

(e) $f(x) = \sin^4 x \cos^5 x$

$f(-x) = \sin^4(-x) \cos^5(-x)$

$= \sin^4 x \cos^5 x$

$\therefore f(-x) = f(x)$

So, $f(x)$ is an even function.

(f) $f(x) = \tan x + \cot x$

$f(-x) = \tan(-x) + \cot(-x)$

$= -\tan x - \cot x$

$= -(\tan x + \cot x)$

$\therefore f(-x) = -f(x)$

So, $f(x)$ is an odd function.

(2)(a) $\int_{-1}^{1}(x^3+5x^4)\,dx$

Here $f(x) = x^3+5x^4$

$f(-x) = (-x)^3+5(-x)^4$

$= (-1)^3x^3+5(-1)^4x^4$

$\therefore f(-x) = -x^3+5x^4$

so $f(x)$ is nither odd nor even function

$\therefore \int_{-1}^{1}(x^3+5x^4)\,dx$

$= \left[\frac{x^4}{4}+\frac{5x^5}{5}\right]_{-1}^{1}$

$= \left(\frac{1^4}{4}+\frac{5.1^5}{5}\right)-\left(\frac{(-1)^4}{4}+\frac{5.(-1)^5}{5}\right)$

$= \frac{1}{4}+1-\left(\frac{1}{4}-1\right)$

$= \frac{1}{4}+1-\frac{1}{4}+1$

$= 2$

(b) $\int_{-2}^{2}x(1+x+x^2)\,dx$

Here $f(x) = x(1+x+x^2)$

$= x+x^2+x^3$

$f(-x) = (-x)+(-x)^2+(-x)^3$

$= -x+(-1)^2x^2+(-1)^3x^3$

$\therefore f(-x) = -x+x^2-x^3$

So $f(x)$ is nither odd nor even function.

$\therefore \int_{-2}^{2}x(1+x+x^2)\,dx$

$= \int_{-2}^{2}(x+x^2+x^3)\,dx$

$= \left[\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}\right]_{-2}^{2}$

$= \left[\frac{2^2}{2}+\frac{2^3}{3}+\frac{2^4}{4}\right]-\left[\frac{(-2)^2}{2}+\frac{(-2)^3}{3}+\frac{(-2)^4}{4}\right]$

$= \left(2+\frac{8}{3}+4\right)-\left(2-\frac{8}{3}+4\right)$

$= \frac{16}{3}$

(c) $\int_{-4}^{4} (2+3x^2)\,dx$

Here $f(x) = 2+3x^2$

$f(-x) = 2+3(-x)^2$

$= 2+3(-1)^2x^2$

$= 2+3x^2$

$\therefore f(-x) = f(x)$ [Even function]

$\therefore \int_{-4}^{4}(2+3x^2)dx = 2\int_{0}^{4}(2+3x^2)dx$,

since $(2+3x^2)$ is an even function.

$\therefore \int_{-4}^{4}(2+3x^2)\,dx$

$= 2\int_{0}^{4}(2+3x^2)\,dx$

$= 2\left[2x+\frac{3x^3}{3}\right]_0^4$

$= 2\left[(2.4+4^3)-0\right]$

$= 2(8+64)$

$= 144$

(d) $\int_{-5}^{5} x^5 e^{x^4}\,dx$

Here $f(x) = x^5 e^{x^4}$

$f(-x) = (-x)^5 e^{(-x)^4}$

$= (-1)^5 x^5 e^{(-1)^4 x^4}$

$= -x^5 e^{x^4}$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-5}^{5} x^5 e^{x^4}\,dx = 0$, since $x^5 e^{x^4}$ is an odd function.

(e) $\int_{-\pi}^{\pi} x^8 \sin x \, dx$

Here, $f(x) = x^8 \sin x$

$f(-x) = (-x)^8 \sin(-x)$

$= (-1)^8 x^8 (-\sin x)$

$= -x^8 \sin x$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-\pi}^{\pi} x^8 \sin x \, dx = 0$; since $x^8 \sin x$ is an odd function.

(f) $\int_{-\pi}^{\pi} x \cos x \, dx$

Here, $f(x) = x \cos x$

$f(-x) = (-x) \cos(-x)$

$= -x \cos x$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-\pi}^{\pi} x \cos x \, dx = 0$, Since $x \cos x$ is an odd function.

(g) $\int_{-\pi}^{\pi} \sin^3 x \cos^5 x \, dx$

Here, $f(x) = \sin^3 x \cos^5 x \, \cancel{dx}$

$f(-x) = \sin^3(-x) \cos^5(-x)$

$= -\sin^3 x \cos^5 x$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-\pi}^{\pi} \sin^3 x \cos^5 x \, dx = 0$, since $\sin^3 x \cos^5 x$ is an odd function.

(h) $\int_{-\pi/2}^{\pi/2} x^4 \sin^3 x \cos^3 x \, dx$

Here $f(x) = x^4 \sin^3 x \cos^3 x$

$$f(-x) = (-x)^4 \sin^3(-x) \cos^3(-x)$$

$$= x^4 (-\sin^3 x) \cos^3 x$$

$$= -x^4 \sin^3 x \cos^3 x$$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-\pi/2}^{\pi/2} x^4 \sin^3 x \cos^3 x \, dx$ ~~is~~ $= 0$, since $x^4 \sin^3 x \cos^3 x$ is an odd function.

(i) $\int_{-\pi}^{\pi} \frac{x^3}{\sqrt{1+x^2}} dx$

Here, $f(x) = \frac{x^3}{\sqrt{1+x^2}}$

$$f(-x) = \frac{(-x)^3}{\sqrt{1+(-x)^2}}$$

$$= -\frac{x}{\sqrt{1+x^2}}$$

$\therefore f(-x) = -f(x)$

$\therefore \int_{-\pi}^{\pi} \frac{x^3}{\sqrt{1+x^2}} dx = 0$, Since $\frac{x^3}{\sqrt{1+x^2}}$ is an odd function.