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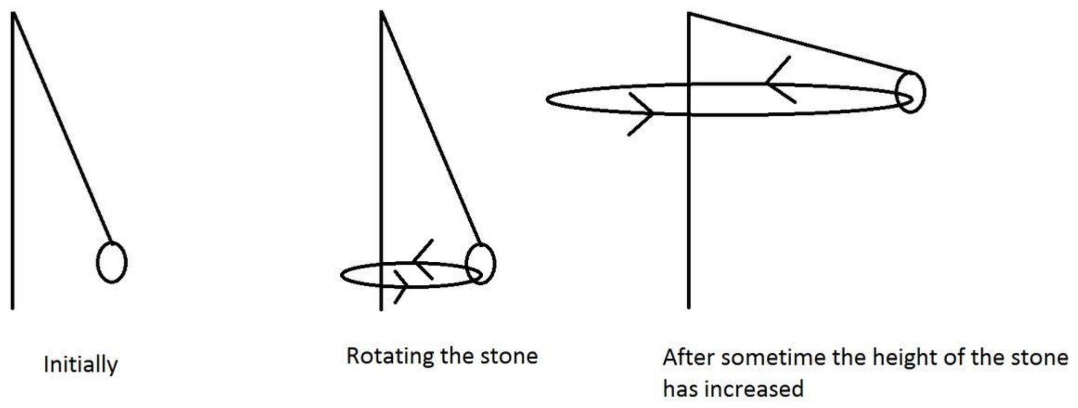
Chapter 15: Oscillations

Simple Harmonic Motion and Uniform Circular Motion:

If we tie a stone to the end of a string and move it with a constant angular speed in a horizontal plane about fixed point, the stone will perform a uniform circular motion in the plane. If we observe the stone sideways, the stone will appear to perform a to and fro motion along the horizontal line with the other end of the string as the midpoint.



Similarly, the projection of the motion or the shadow of the stone would appear to perform a to and fro motion perpendicular to the plane of the circle. Similar case was observed by Galileo, who discovered the four principle moons of the planet Jupiter moved back and forth relative to the planet, executing a simple harmonic motion

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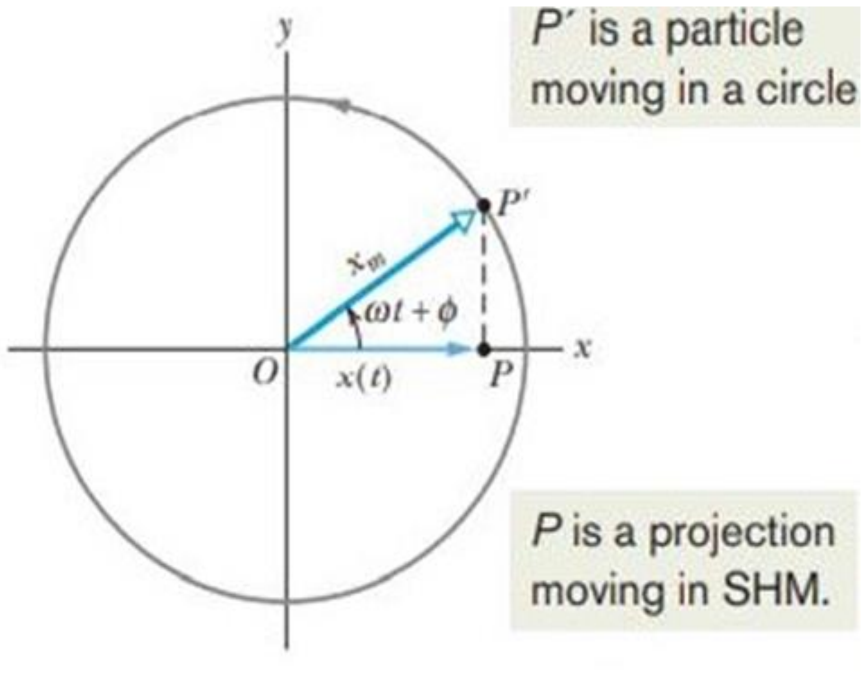
- Figure "a" gives an example. It shows a reference particle **P′** moving in uniform circular motion with (constant) angular speed ω in a reference circle.
- The radius x_m of the circle is the magnitude of the particle's position vector.
- At any time t, the angular position of the particle is

$(\omega t + \varphi)$, where φ is its angular position at t = 0.



(a)

Position

- The projection of particle **P′** onto the x axis is a point **P**, which we take to be a second particle.
- The projection of the position vector of particle **P′**. onto the x axis gives the location x(t) of ***P***. Thus, we find

$$\cos(\omega t + \varphi) = \frac{x(t)}{x_m}$$

$$\rightarrow \quad x(t) = x_m \cos(\omega t + \varphi),$$

If reference particle **P′** , moves in uniform circular motion, its projection particle **P** moves in simple harmonic motion along a diameter of the circle.

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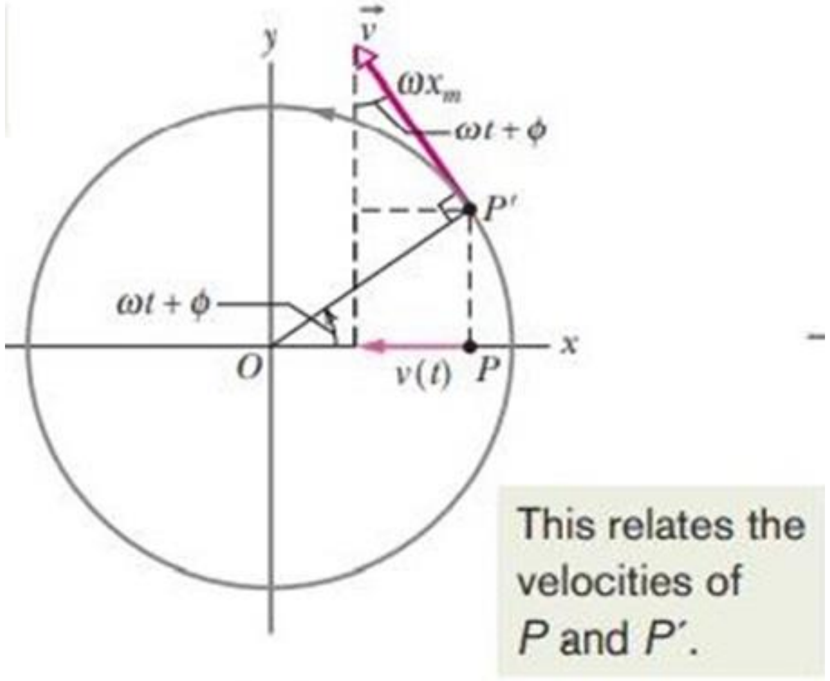
Velocity

- Figure "b" shows the velocity $\vec{v}$ of the reference particle.
- From the relation , $V = \omega r$ the magnitude of the velocity vector is ωx_m ; its projection on the **x** axis is

$$\sin(\omega t + \varphi) = -\frac{v(t)}{\omega x_m}$$

$$v(t) = -\omega x_m \sin(\omega t + \varphi),$$

- The minus sign appears because the velocity component of **P** in Fig. "b" is directed to the left, in the negative direction of **x**.



(b)

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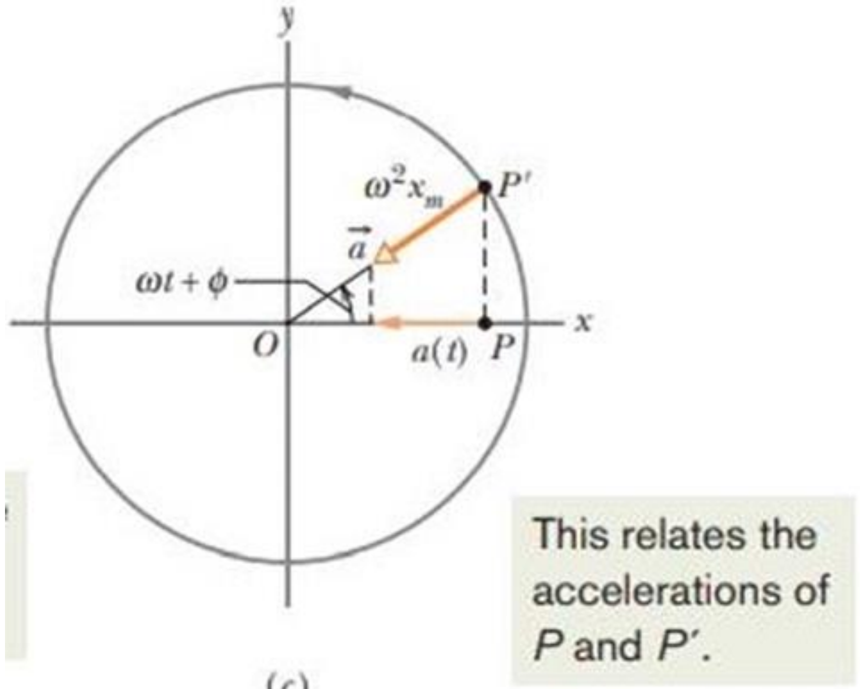
Acceleration

- Figure "c" shows the radial acceleration $\vec{a}$ of the reference particle.

- From the relation $\boldsymbol{a_r} = \boldsymbol{\omega}^2\mathbf{r}$ the magnitude of the radial acceleration vector is $\boldsymbol{\omega}^2\ \mathbf{x_m}$; projection on the x axis is

$$a(t) = -\,\omega^2\, x_m \cos(\omega t + \varphi),$$



(c)

Thus, whether we look at the displacement, the velocity, or the acceleration, the projection of uniform circular motion is indeed simple harmonic motion.

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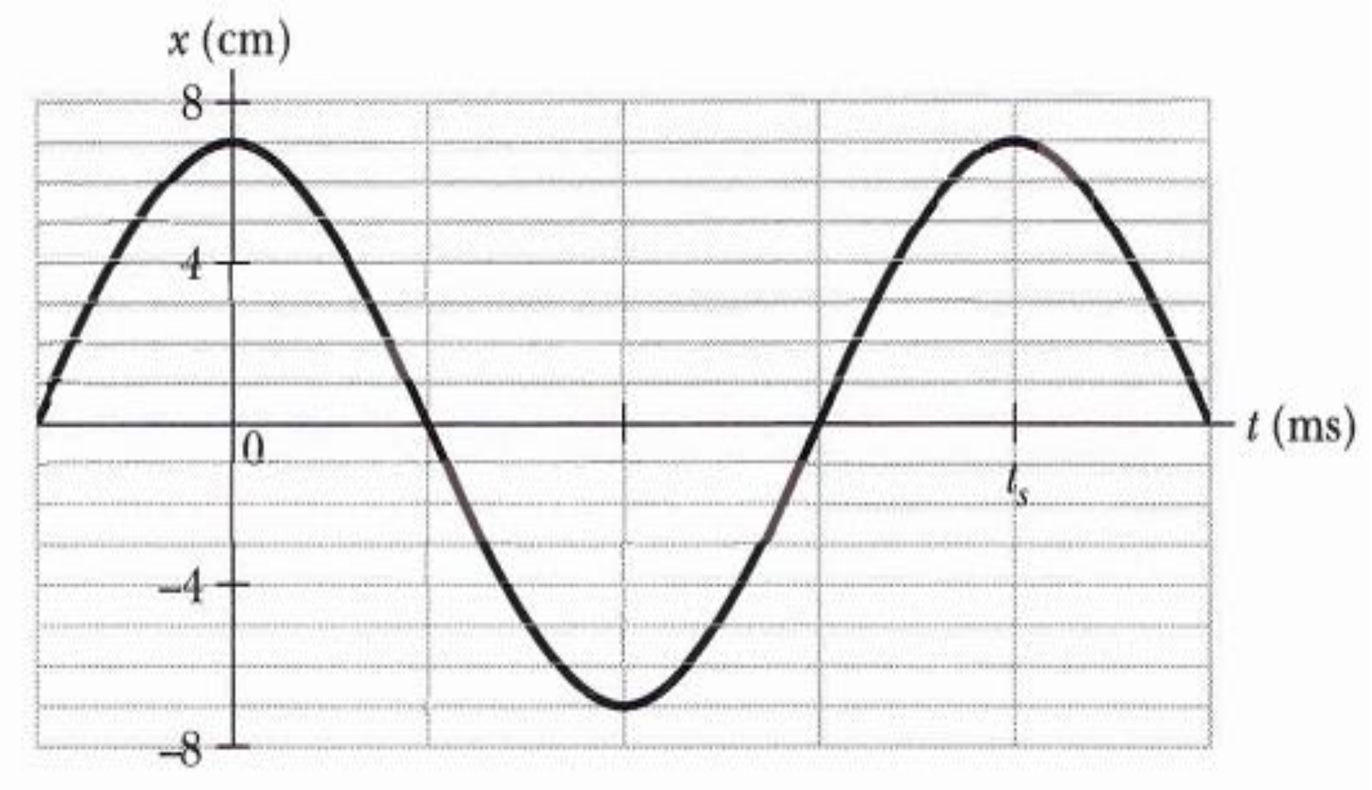
Problem 77: ***Figure*** *15-53 gives the position of a 20 g block oscillating in SHM on the end of a spring. The horizontal axis scale is set by t_s = 40.0 ms. What are (a) the maximum kinetic energy of the block and (b) the number of times per second that maximum is reached? (Hint: Measuring a slope will probably not be very accurate. Find another approach)*



Given , m = 20 g = 0.02 Kg ;

$\mathbf{t_s} = \mathbf{T}$ = 40 ms = 0.04 s

and

x_m = 7 cm = 0.07 m

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Solution : Given , m = 20 g = 0.02 Kg ; t_s = T = 40 ms = 0.04 s and $\boldsymbol{x_m}$ *= 7 cm = 0.07 m*

(a) The maximum Kinetic energy K_{max} is obtained at the maximum velocity v_m by

$$\boldsymbol{K_{max} = \frac{1}{2} m {v_m}^2}$$

The maximum velocity is given by $\boldsymbol{v_m = \omega x_m}$

$$K_{max} = \frac{1}{2} m {v_m}^2$$

$$= \frac{1}{2} m \omega^2 {x_m}^2$$

$$= \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 {x_m}^2$$

$$= \frac{1}{2} \times 0.02\ kg \times \left(\frac{2\pi}{0.04 s}\right)^2 (0.07\ m)^2$$

$$= 1.2\ \text{J}$$

(Ans)

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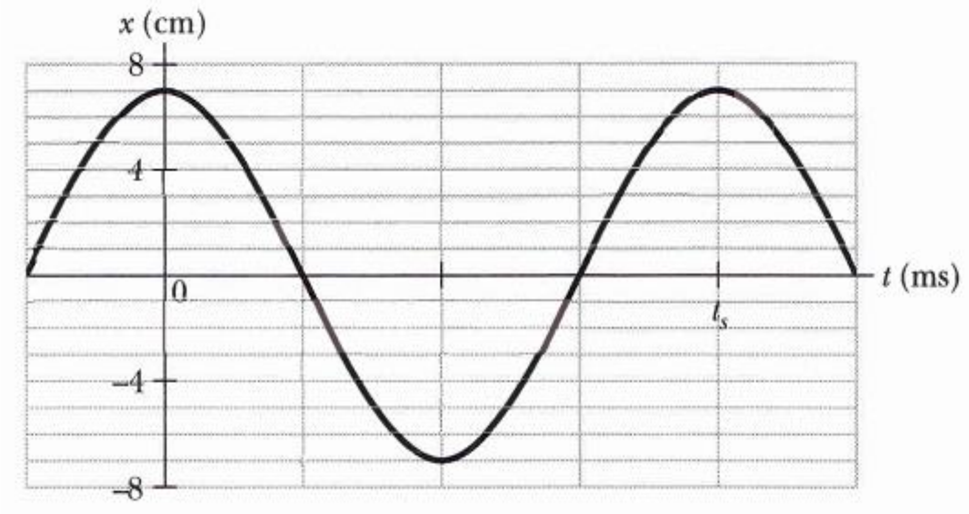

(b) As shown by the graph ,

the number per times the maximum is reached within $\mathbf{t_s}$ = 40 ms is 2.

So for 1 second the maximum reached will be

$$f = \left(\frac{2}{0.04}\right) \times 1 = 50 \text{ oscillation per second} = 50 \text{ hertz.}$$

(Ans)

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Problem 78 :*Figure 15-53 gives the position x(t) of a block oscillating in SHM on the end of a spring (t_s = 40.0 ms).What are (a) the speed and (b) the magnitude of the radial acceleration of a particle in the corresponding uniform circular motion?*

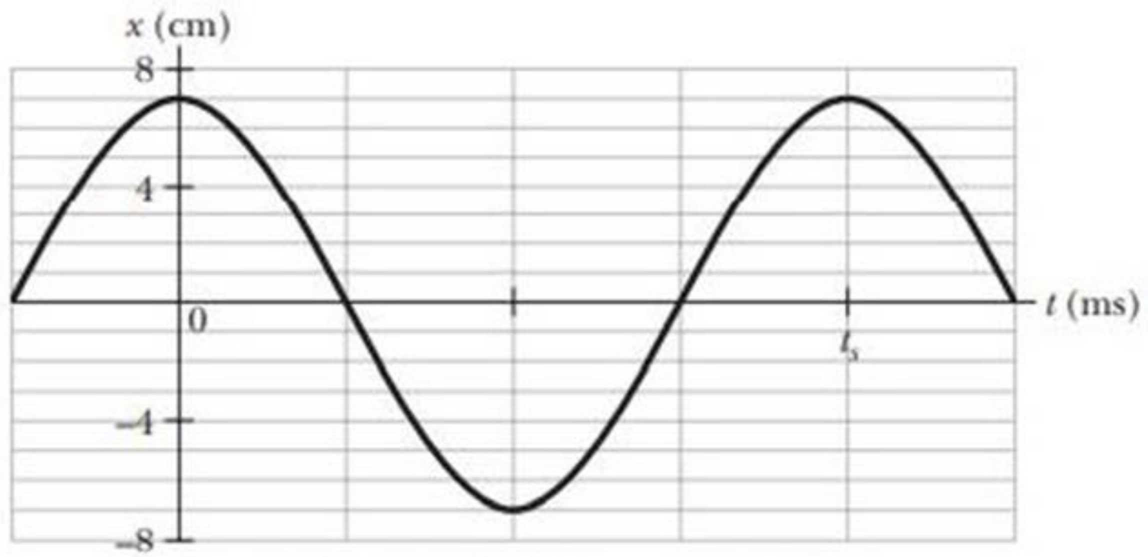

Figure 15-53 Problems 77 and 78.

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Solution:

The time period T = t_s = 40 ms = 0.04 sec

(a) So the maximum velocity is given by $\boldsymbol{v_m = \omega x_m = \frac{2\pi}{T} x_m}$

$$= \left(\frac{2\pi}{0.04\ s}\right) \times (0.07\ \text{m})$$

$$= 11\ \text{m/s}$$

(b) The radial acceleration of the particle, $\boldsymbol{a_m = \omega^2 x_m = \left(\frac{2\pi}{T}\right)^2 x_m}$

$$= \left(\frac{2\pi}{0.04\ s}\right)^2 \times (0.07\ \text{m})$$

$$= 1727\ \ \text{m/s}^2$$

(Ans)