Example set 1.1.2

1. $\int (1+x^{3/2})^3 x^{1/2}\,dx$

Set, $u = 1 + x^{3/2}$

$du = \frac{3}{2} x^{\frac{3}{2}-1} dx$

$= \frac{3}{2} x^{\frac{3-2}{2}} dx$

$du = \frac{3}{2} x^{1/2} dx$

$\therefore dx = \frac{2}{3} \frac{du}{x^{1/2}}$

Then $\int (1+x^{3/2})^3 x^{1/2}\,dx$

$= \int u^3 \; x^{1/2} \; \frac{2}{3} \frac{du}{x^{1/2}}$

$= \frac{2}{3} \int u^3\,du$

$= \frac{2}{3} \; \frac{u^{3+1}}{3+1} + c$

$= \frac{2}{3} \; \frac{u^4}{4} + c$

$= \frac{2}{12} \; u^4 + c$

$= \frac{1}{6} (1+x^{3/2})^4 + c$

2. $\int \frac{x}{4+x^4}\,dx$

$= \int \frac{x}{2^2+u^2} \cdot \frac{du}{2x}$

$= \frac{1}{2} \int \frac{du}{2^2+u^2}$

$= \frac{1}{2} \cdot \frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right) + c$

$= \frac{1}{4} \tan^{-1}\left(\frac{x^2}{2}\right) + c$

Set $u = x^2$

$du = 2x\,dx$

$\therefore dx = \frac{du}{2x}$

3. $\int \sqrt{2x+1}\,dx$

$= \int \sqrt{u}\,\frac{du}{2}$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\,\frac{u^{1/2+1}}{\frac{1}{2}+1} + c = \frac{1}{2}\,\frac{u^{\frac{1+2}{2}}}{\frac{1+2}{2}} + c$

$= \frac{1}{2}\,\frac{u^{3/2}}{\frac{3}{2}} + c = \frac{1}{2}\,\frac{2}{3}\,u^{3/2} + c$

$= \frac{1}{3}(2x+1)^{3/2} + c$

set, $u = 2x+1$

$du = 2\,dx$

$\therefore dx = \frac{du}{2}$

4. $\int \frac{x}{\sqrt{1-4x^2}}\,dx$

$= \int \frac{x}{\sqrt{u}}\left(-\frac{du}{8x}\right)$

$= -\frac{1}{8}\int \frac{du}{u^{1/2}} = -\frac{1}{8}\int u^{-1/2}\,du$

$= -\frac{1}{8}\,\frac{u^{-1/2+1}}{-\frac{1}{2}+1} + c = -\frac{1}{8}\,\frac{u^{1/2}}{\frac{1}{2}} + c$

$= -\frac{1}{8}\cdot 2\,u^{1/2} + c = -\frac{1}{4}(1-4x^2)^{1/2} + c$

set, $u = 1-4x^2$

$du = -8x\,dx$

$\therefore dx = -\frac{du}{8x}$

5. $\int \frac{2x+3}{x^2+3x+5}\,dx$

$= \int \frac{2x+3}{u}\cdot\frac{du}{2x+3}$

$= \int \frac{du}{u} = \ln u + c$

$= \ln(x^2+3x+5) + c$

set, $u = x^2+3x+5$

$du = (2x+3)\,dx$

$\therefore dx = \frac{du}{2x+3}$

6. $\int \frac{\cos x - \sin x}{\sin x + \cos x}\,dx$

set, $u = \sin x + \cos x$

$du = (\cos x - \sin x)\,dx$

$\therefore dx = \frac{du}{\cos x - \sin x}$

$= \int \frac{\cos x - \sin x}{u} \cdot \frac{du}{\cos x - \sin x}$

$= \int \frac{du}{u} = \ln u + c = \ln(\sin x + \cos x) + c$

7. $\int \frac{\sin 3x}{1+\cos 3x}\,dx$

set, $u = 1 + \cos 3x$

$du = -3\sin 3x\,dx$

$\cancel{\therefore dx = -3\sin 3x}$

$dx = -\frac{du}{3\sin 3x}$

$= \int \frac{\sin 3x}{u} \cdot \left(-\frac{du}{3\sin 3x}\right)$

$= -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln u + c = -\frac{1}{3}\ln(1+\cos 3x) + c$

8. $\int \frac{\sec^2 2x}{5+\tan 2x}\,dx$

set, $u = 5 + \tan 2x$

$du = 2\sec^2 2x\,dx$

$\therefore dx = \frac{du}{2\sec^2 2x}$

$= \int \frac{\sec^2 2x}{u} \cdot \frac{du}{2\sec^2 2x}$

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln(5+\tan 2x) + c$

9. $\int \tan x \, dx$

$= \int \frac{\sin x}{\cos x} \, dx$

$= \int \frac{\sin x}{u} \cdot \left(-\frac{du}{\sin x}\right) = -\int \frac{du}{u}$

$= -\ln u + c = -\ln(\cos x) + c$

set, $u = \cos x$

$du = -\sin x \, dx$

$\therefore dx = -\frac{du}{\sin x}$