Lesson plan 18

16-2: Wave speed on a stretched string, v = $\sqrt{\frac{\tau}{\mu}}$

- Speed of a wave is set by the properties of the medium (stretched string).
- If a wave is to travel through a medium, it must cause the particles of that stretched string (medium) to oscillate as it passes.
- It requires both mass (for kinetic energy, K= $\frac{1}{2}mv^2$) and elasticity (for potential energy, U = $\frac{1}{2}kx^2$) properties.
- Thus the mass and elasticity properties of the medium determine how fast the wave can travel in the medium.

Derivation from Newton's second law of motion, $v = \sqrt{\frac{\tau}{\mu}}$

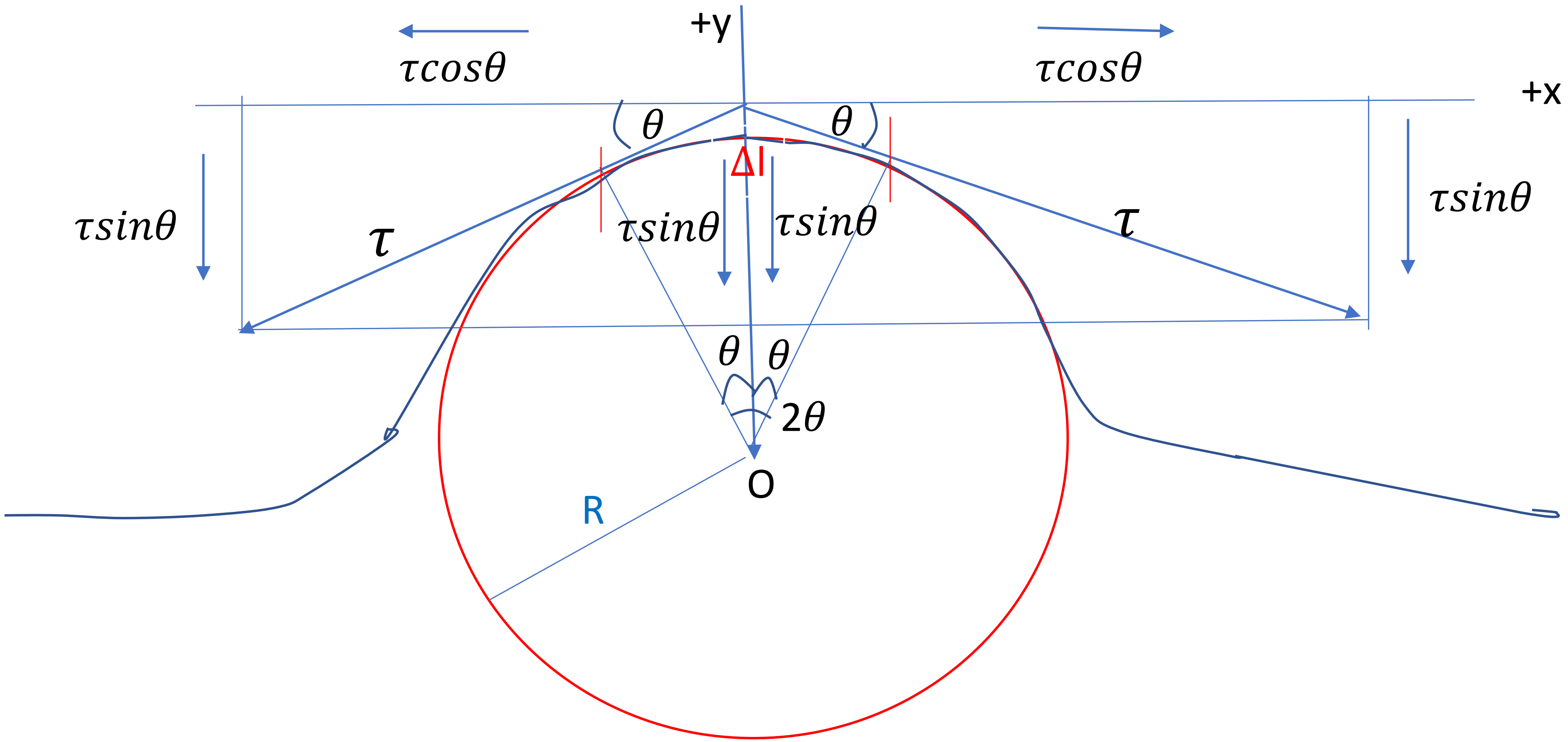
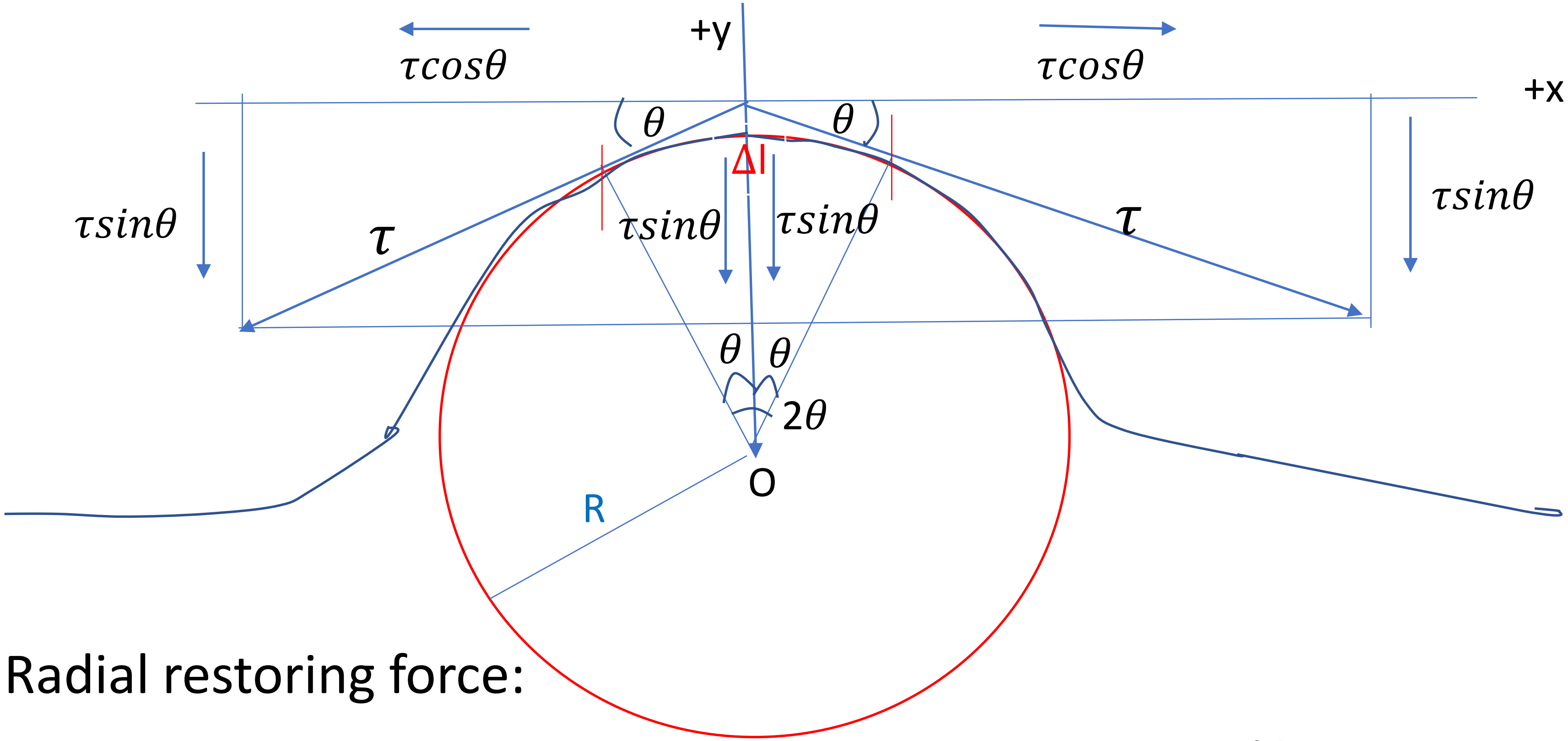


Fig.: A symmetrial pulse is stationary viewed from a reference frame.
String appears to move from right ot left with speed v.
String element of length Δl located at the top of the pulse



1. Radial restoring force:

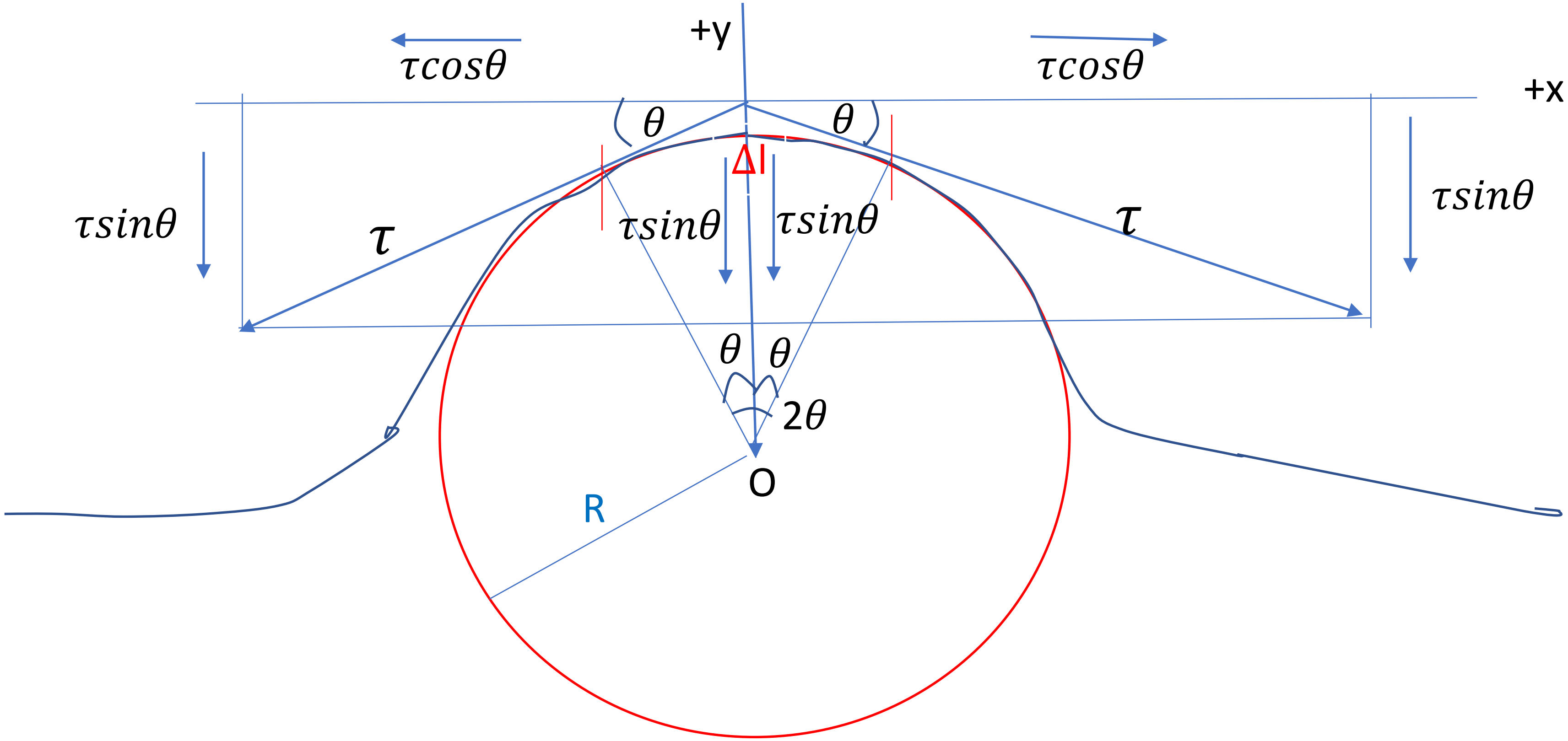
$$F = \tau \sin\theta + \tau \sin\theta = 2\tau \sin\theta = 2\tau\,\theta = \tau(2\,\theta) = \tau\ \left(\frac{\Delta l}{R}\right)$$

[If θ is very small, Sin$\theta \cong tan\theta \cong \theta\ and\ 2\theta = \frac{\Delta l}{R}$]

2. Mass of the element: *Linear density of the string* = $\frac{mass}{length}$

$$\mu = \frac{\Delta m}{\Delta l}$$

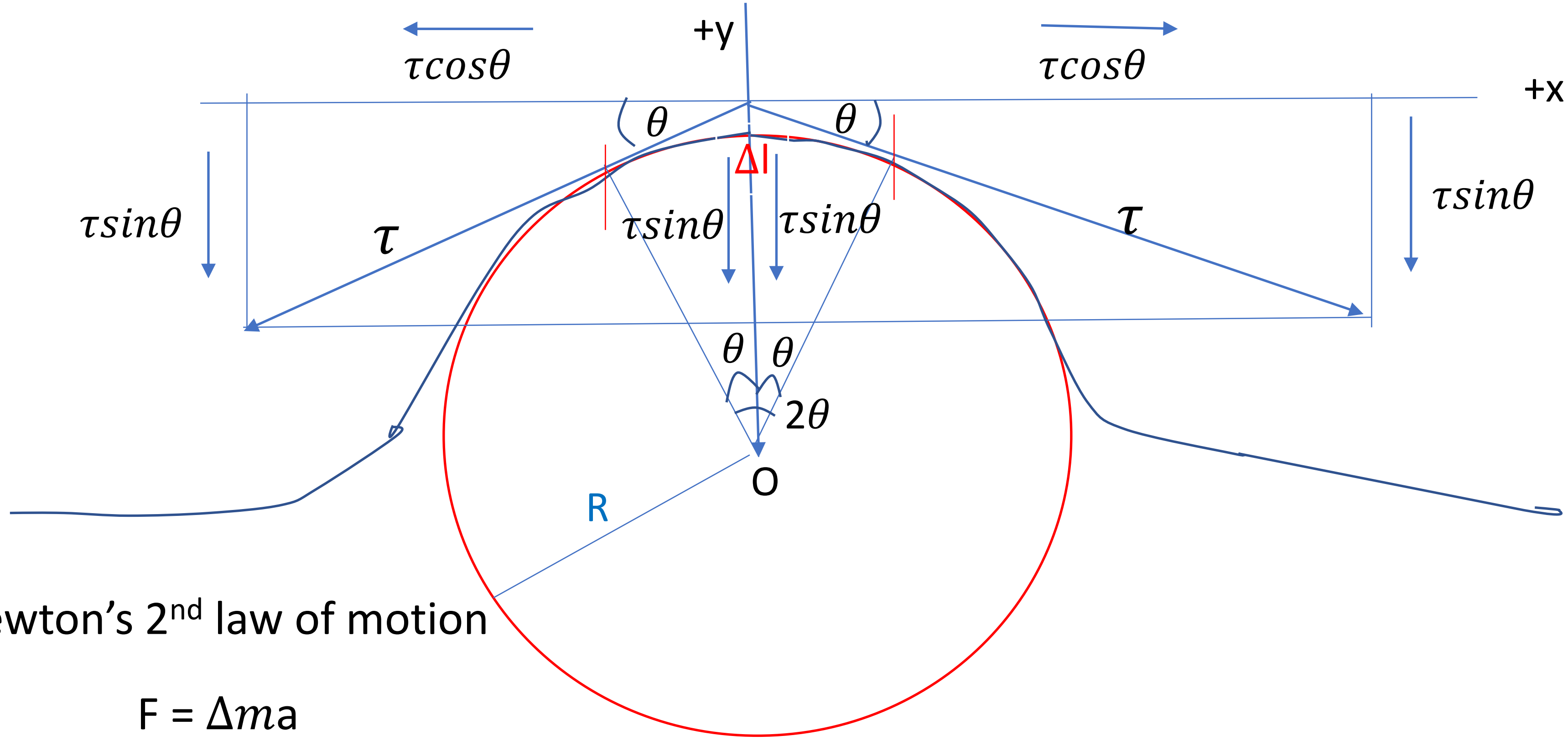
$$\Delta m = \mu\ \Delta l$$



3. Centripetal acceleration:

String element moves in an arc of a circle. It has a centripetal acceleration toward the center of the circle.

Centripetal acceleration is given by $a = \frac{v^2}{R}$



Newton's 2nd law of motion

F = $\Delta m a$

$$\tau\left(\frac{\Delta l}{R}\right) = \mu \Delta l\left(\frac{v^2}{R}\right)$$

$$\tau = \mu v^2$$

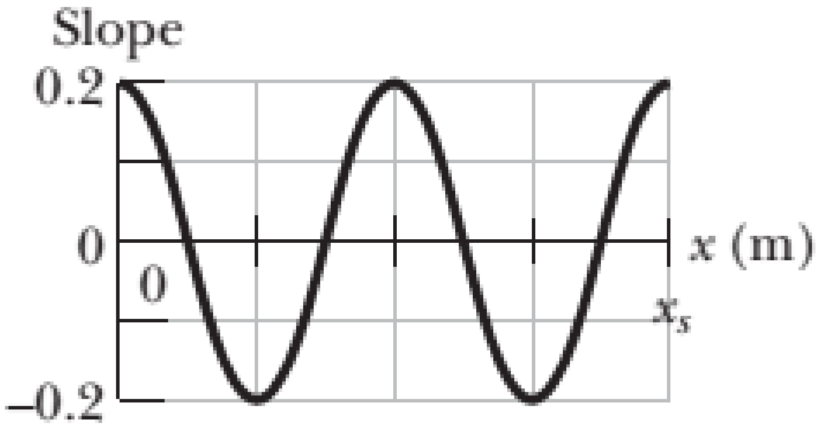
$$v^2 = \frac{\tau}{\mu}$$

$$v = \sqrt{\frac{\tau}{\mu}}$$

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

6. A sinusoidal wave travels along a string under tension. Figure gives the slopes along the string at time t = 0. The scale of the x axis is set by x_s = 0.80 m. What is the amplitude of the wave?

$$y(x, t) = y_m \sin(kx - \omega t)$$



$$\frac{dy}{dx} = \frac{d}{dx}\{y_m \sin(kx - \omega t)\}$$

$$\frac{dy}{dx} = y_m \frac{d}{dx}\{\sin(kx - \omega t)\}$$

$$\frac{dy}{dx} = k y_m \cos(kx - \omega t)$$

At t = 0 and x =0, $\frac{dy}{dx} = k y_m \cos\{k(0) - \omega(0)\}$

$$\frac{dy}{dx} = k y_m \cos 0$$

$$\frac{dy}{dx} = k y_m$$

$$\frac{dy}{dx} = k y_m$$

$$0.2 = k y_m$$

$$0.2 = \frac{2\pi}{\lambda} y_m$$

$$y_m = \frac{0.2\lambda}{2\pi}$$

From the Fig., $\lambda = \frac{x_s}{2}$

$$\lambda = \frac{0.8}{2}$$

$$\lambda = 0.4 \text{ m}$$

$$y_m = \frac{0.2(0.4)}{2\pi}$$

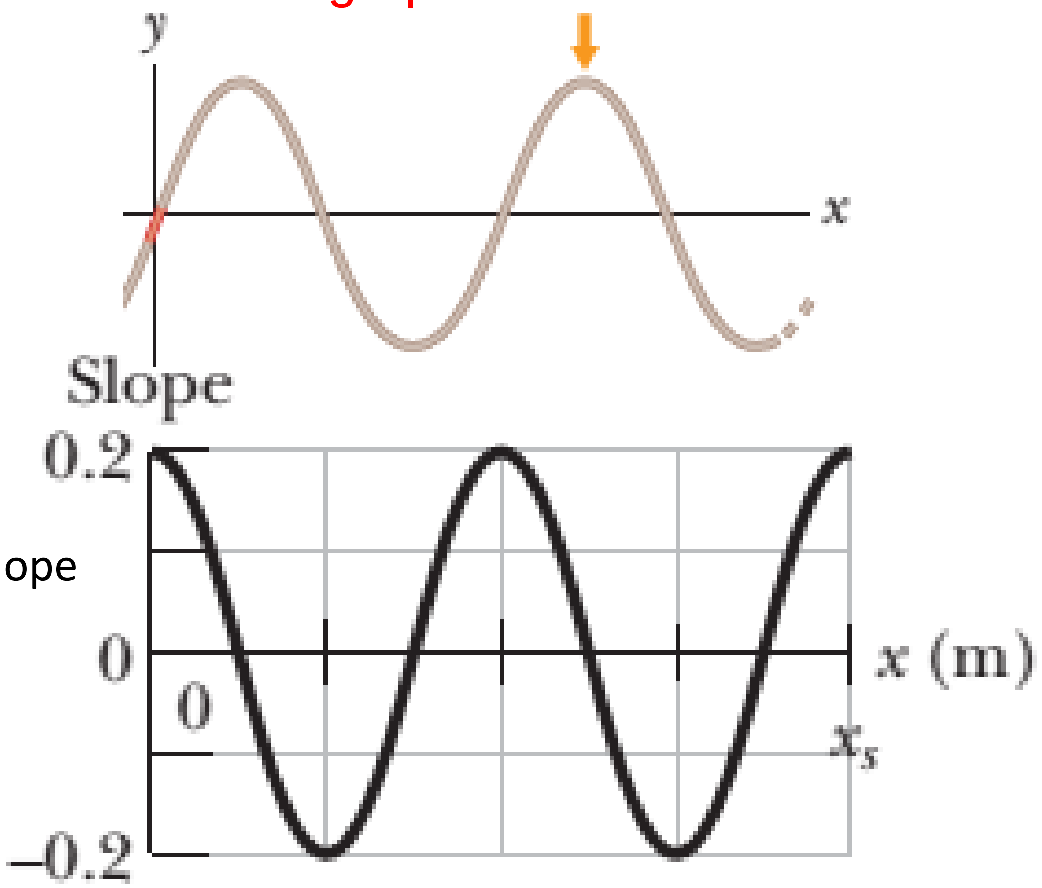
y_m = 0.01273 m = 1.17 cm

$y(x, 0) = y_m \sin kx$

$y(0,0) = 0$

Both graph t = 0

$\frac{dy}{dx}$ = slope



14. The equation of a transverse wave on a string is $y = (2.0\ m)\ \sin[(20\ m^{-1})x - (600\ s^{-1})t]$. The tension in the string is 15 N. (a) What is the wave speed? (b) Find the linear density of this string in grams per meter.

$y = (2.0\ m)\ \sin[(20\ m^{-1})x - (600\ s^{-1})t]$

$y = y_m \sin(kx - \omega t)$

Given, $y_m = 2.0$ m

$k = 20$ rad/m

$\omega = 600$ rad/s

$\tau = 15$ N

(a) $v = \frac{\omega}{k} = \frac{600}{20} = 30$ m/s

(b) $v = \sqrt{\frac{\tau}{\mu}}$

$v^2 = \frac{\tau}{\mu}$

$\mu = \frac{\tau}{v^2} = \frac{15}{(30)^2} = 1.67 \times 10^{-2}$ kg/m = 16.7 gm/m