1. $\frac{dy}{dt} = (t-1)$, $y(0) = 2$

$\Rightarrow dy = (t-1)dt$

$\Rightarrow \int dy = \int (t-1)\, dt$

$\Rightarrow y = \frac{t^2}{2} - t + C$

Using initial condition,

$2 = C$

Therefore, $y = \frac{t^2}{2} - t + 2$

2. $\frac{dy}{dt} = \frac{y}{1+y^2}$

$\Rightarrow \left(\frac{1+y^2}{y}\right) dy = dt$

$\Rightarrow \int\left(\frac{1}{y} + y\right) dy = \int dt$

$\Rightarrow \ln y + \frac{y^2}{2} = t + C$

$\Rightarrow 2 \ln y + y^2 = 2t + 2C$

$\Rightarrow y^2 + \ln y^2 = 2t + C$ [$\because$ 2C is a constant]

3. $\frac{dy}{dt} = \frac{t}{y^2}$, $y(1) = 1$

$\Rightarrow \int y^2\,dy = \int t\,dt$

$\Rightarrow \frac{y^3}{3} = \frac{t^2}{2} + c$

$\Rightarrow 2y^3 = 3t^2 + 6c$

Using initial condition:

$2 = 3 + 6c \Rightarrow c = -\frac{1}{6}$

Therefore, $2y^3 = 3t^2 - 1$

$\Rightarrow y^3 = \frac{3}{2}t^2 - \frac{1}{2}$

$\Rightarrow \sqrt[3]{y^3} = \sqrt[3]{\frac{3}{2}t^2 - \frac{1}{2}}$

$\therefore\ y = \sqrt[3]{\frac{3}{2}t^2 - \frac{1}{2}}$

4. $xy^3dx + e^{x^2}dy = 0$

$\Rightarrow xy^3dx = -e^{x^2}dy$

$\Rightarrow \int \frac{dy}{y^3} = -\int \frac{x}{e^{x^2}}\,dx$

Let, $x^2 = u$

$\Rightarrow 2x\,dx = du$

$\Rightarrow dx = \frac{du}{2x}$

$\Rightarrow -\frac{y^{-2}}{2} = -\int \frac{x}{e^u} \cdot \frac{du}{2x}$

$\Rightarrow \frac{1}{y^2} = \int e^{-u}\,du$

$\Rightarrow \frac{1}{y^2} = -e^{-u} + c$

$\Rightarrow y^2 = \frac{1}{-e^{-u} + c} = \frac{1}{-e^{-x^2} + c}$

1. $\frac{dy}{dt} + \frac{2}{t}y = t-1$

Here $P(t) = \frac{2}{t}$

Now $\mu(t) = e^{\int \frac{2}{t} dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$\therefore y = \frac{1}{t^2}\int (t-1).t^2\, dt + C$

$= \frac{1}{t^2}\int (t^3 - t^2)\, dt + c$

$= \frac{1}{t^2}\left(\frac{t^4}{4} - \frac{t^3}{3}\right) + C$

$\therefore y = \frac{t^2}{4} - \frac{t}{3} + C$

2. $\frac{dy}{dt} - \frac{2y}{t} = 2t^2$, $y(-2) = 4$

Here $P(t) = -\frac{2}{t}$

Now $\mu(t) = e^{\int -\frac{2}{t} dt} = e^{-2\ln t} = e^{-\ln t^2} = \frac{1}{e^{\ln t^2}} = \frac{1}{t^2}$

$\therefore y = \frac{1}{\frac{1}{t^2}}\int 2t^2 . \frac{1}{t^2}\, dt + c$

$\therefore y = t^2\int 2 dt + c = t^2 . 2t + c = 2t^3 + c$

3. $\frac{dy}{dt} = -2ty + 4e^{-t^2}$, $y(0) = 3$

Here, $P(t) = 2t$

Now $\mu(t) = e^{\int 2t\, dt} = e^{2 \cdot \frac{t^2}{2}} = e^{t^2}$

$\therefore y = \frac{1}{e^{t^2}}\int 4e^{-t^2} . e^{t^2} dt + c$

$\therefore y = \frac{1}{e^{t^2}}\int 4 . e^0\, dt + c = \frac{4t}{e^{t^2}} + c$ —— (1)

Using initial values in eqn (1) we get,

$\cancel{7}\ 3 = c$

Then eqn (1) becomes: $y = \frac{4t}{e^{t^2}} + 3$

4. $\frac{dy}{dt} = -\frac{y}{1+t} + t^2$

$$\Rightarrow \frac{dy}{dt} + \frac{y}{1+t} = t^2$$

Here, $P(t) = \frac{1}{1+t}$

Now, $\mu(t) = e^{\int \frac{1}{1+t} dt} = e^{\ln(1+t)} = 1+t$

$$\therefore y = \frac{1}{1+t} \int \left[t^2 \cdot (1+t)\right] dt + c$$

$$= \frac{1}{1+t} \int (t^2 + t^3)\, dt + c$$

$$= \frac{1}{1+t} \left(\frac{t^3}{3} + \frac{t^4}{4}\right) + c$$

$$= \frac{1}{1+t} \; \frac{4t^3 + 3t^4}{12} + c$$

$$\therefore y = \frac{4t^3 + 3t^4}{12(1+t)} + c$$