Exercise set 1.1.4

(a) $\int 2x(x^2+1)^{21}\,dx$

$= \int u \cdot 2x \cdot \frac{du}{2x}$

$= \int u\,du$

$= \frac{u^{1+1}}{1+1} + c = \frac{u^2}{2} + c = \frac{1}{2}(x^2+1) + c$

Set, $u = x^2+1$

Then $du = 2x\,dx$

$dx = \frac{du}{2x}$

(b) $\int \frac{\sec^2(\ln x)}{x}\,dx$

$= \int \frac{\sec^2(u)}{x} \cdot x\,du$

$= \int \sec^2(u)\,du$

$= \tan u + c = \tan(\ln x) + c$

Set, $u = \ln x$

Then $du = \frac{dx}{x}$

$\therefore dx = x \cdot du$

(c) $\int \frac{e^{3x}}{e^{3x}+5}\,dx$

$= \int \frac{e^{3x}}{u} \cdot \frac{3\,du}{e^{3x}}$

$= 3\int \frac{du}{u} = 3\ln u + c = 3\ln(e^{3x}+5) + c$

Set, $u = e^{3x}+5$

$du = \left(\frac{e^{3x}}{3}\right)dx$

$\therefore dx = \frac{3\,du}{e^{3x}}$

(d) $\int \cos^3 x \sin x \, dx$

set, $u = \cos x$

$du = \sin x \, dx$

$\therefore dx = \frac{du}{\sin x}$

$= \int u^3 \sin x \frac{du}{\sin x}$

$= \int u^3 du = \frac{u^{3+1}}{3+1} + c = \frac{u^4}{4} + c$

$= \frac{1}{4} \cos^4 x + c$

(e) $\int \frac{x^3}{(x^4+1)^5} dx$

let, $u = x^4 + 1$

$du = 4x^3 dx$

$\therefore dx = \frac{du}{4x^3}$

$= \int \frac{x^3}{u^5} \cdot \frac{du}{4x^3}$

$= \frac{1}{4} \int \frac{du}{u^5} = \frac{1}{4} \int u^{-5} du \cancel{= \frac{1}{4}\int}$

$= \frac{1}{4} \frac{u^{-5+1}}{-5+1} + c$

$= -\frac{1}{4} \frac{u^{-4}}{4} + c$

$= -\frac{1}{16} u^{-4} + c$

$= -\frac{1}{16} (x^4+1)^{-4} + c$

(f) $\int \frac{(1+\ln x)^3}{x} dx$

set,

$u = 1 + \ln x$

$du = \frac{dx}{x}$

$\therefore dx = x \, du$

$= \int \frac{u^3}{x} \cdot x \, du$

$= \int u^3 du = \frac{u^{3+1}}{3+1} + c$

$= \frac{1}{4} u^4 + c = \frac{1}{4} (1 + \ln x)^4 + c$

(g) $\int \frac{\cos x}{(1+\sin x)^5}\,dx$

set,
$u = 1+\sin x$
$du = \cos x\,dx$
$\therefore dx = \frac{du}{\cos x}$

$= \int \frac{\cos x}{u^5}\,\frac{du}{\cos x}$

$= \int \frac{du}{u^5} = \int u^{-5}\,du$

$= \frac{u^{-5+1}}{-5+1} + c = -\frac{1}{4}u^{-4} + c$

$= -\frac{1}{4}(1+\sin x)^{-4} + c$

(h) $\int \sin 3x \sqrt{2+\cos 3x}\,dx$

set,
$u = 2+\cos 3x$
$du = -3\sin 3x\,dx$
$\therefore dx = -\frac{du}{3\sin 3x}$

$= \int \sin 3x \sqrt{u}\left(-\frac{du}{3\sin 3x}\right)$

$= -\frac{1}{3}\int u^{1/2}\,du$

$= -\frac{1}{3}\frac{u^{1/2+1}}{\frac{1}{2}+1} + c = -\frac{1}{3}\frac{u^{\frac{1+2}{2}}}{\frac{1+2}{2}} + c$

$= -\frac{1}{3}\frac{u^{3/2}}{\frac{3}{2}} + c = -\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + c$

$= -\frac{2}{9}(2+\cos 3x)^{3/2} + c$

(i) $\int \frac{\cos(2/x)}{x^2}\,dx$

Set, $u = \frac{2}{x} = 2x^{-1}$
$du = 2(-x^{-2})\,dx$
$\therefore dx = -\frac{du}{2x^{-2}}$
$\therefore dx = -\frac{x^2 du}{2}$

$= \int \frac{\cos u}{x^2}\cdot\left(-\frac{x^2 du}{2}\right)$

$= -\frac{1}{2}\int \cos u\,du$

$= -\frac{1}{2}\sin u + c = -\frac{1}{2}\sin(2/x) + c$

(j) $\int \left(\frac{e^{\sqrt{x}}}{\sqrt{x}} \right) dx$

$\cancel{= \int \frac{e^u}{\sqrt{x}} \cdot \frac{2\sqrt{x}}{du}}$

$= \int \frac{e^u}{\sqrt{x}} \cdot 2\sqrt{x}\, du$

$= 2\int e^u\, du$

$= 2e^u + c = 2e^{\sqrt{x}} + c$

set, $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{1/2 - 1} dx$

$= \frac{1}{2} x^{-1/2} dx$

$du = \frac{1}{2} \frac{dx}{\sqrt{x}}$

$\cancel{\therefore dx = \frac{2\sqrt{x}}{du}}$

$\therefore dx = 2\sqrt{x}\, du$

(k) $\int \frac{1}{x(1+\ln x)^3} dx$

$= \int \frac{1}{x \cdot u^3} \cdot x\, du$

$= \int \frac{du}{u^3} = \int u^{-3} du$

$= \frac{u^{-3+1}}{-3+1} + c = -\frac{1}{2} u^{-2} + c$

$= -\frac{1}{2} \frac{1}{(1+\ln x)^2} + c$

set, $u = 1 + \ln x$

$du = \frac{1}{x} dx$

$\therefore dx = x\, du$

(l) $\int \frac{e^{-3x}}{\sqrt{3+e^{-3x}}} dx$

$= \int \frac{e^{-3x}}{\sqrt{u}} \left(-\frac{du}{3e^{-3x}} \right)$

$= -\frac{1}{3} \int \frac{du}{\sqrt{u}} = -\frac{1}{3} \int u^{-1/2} du$

$= -\frac{1}{3} \frac{u^{-1/2+1}}{-\frac{1}{2}+1} + c = -\frac{1}{3} \frac{u^{1/2}}{\frac{1}{2}} + c$

$= -\frac{1}{3} \cdot 2 u^{1/2} + c$

$= -\frac{2}{3} (3+e^{-3x})^{1/2} + c$

set, $u = 3 + e^{-3x}$

$du = -3e^{-3x} dx$

$\therefore dx = -\frac{du}{3e^{-3x}}$

(m) $\int \frac{e^{m(\arctan x)}}{1+x^2}\,dx$

set, $u = \arctan x$

$= \tan^{-1} x$

$du = \frac{dx}{1+x^2}$

$dx = (1+x^2)\,du$

$= \int \frac{e^{mu}}{1+x^2} \cdot (1+x^2)\,du$

$= \int e^{mu}\,du$

$= \frac{1}{m} e^{mu} + c$

$= \frac{1}{m} e^{m(\arctan x)} + c$

(n) $\int \frac{e^x}{e^x+1}\,dx$

set $u = e^x + 1$

$du = e^x\,dx$

$\therefore dx = \frac{du}{e^x}$

$= \int \frac{e^x}{u} \cdot \frac{du}{e^x}$

$= \int \frac{du}{u} = \ln u + c = \ln(e^x+1) + c$

(o) $\int 4 \tan^3 x \sec^2 x\,dx$

set $u = \tan x$

$du = \sec^2 x\,dx$

$\therefore dx = \frac{du}{\sec^2 x}$

$= \int 4 u^3 \sec^2 x \cdot \frac{du}{\sec^2 x}$

$= 4 \int u^3\,du$

$= 4 \frac{u^{3+1}}{3+1} + c = u^4 + c = \tan^4 x + c$