1 (a) Given, $y = 3 - 2x$

$$\therefore x = \frac{3-y}{2}$$

So the volume is,

$$V_y = \int_0^2 \pi \left(\frac{3-y}{2}\right)^2 dy$$

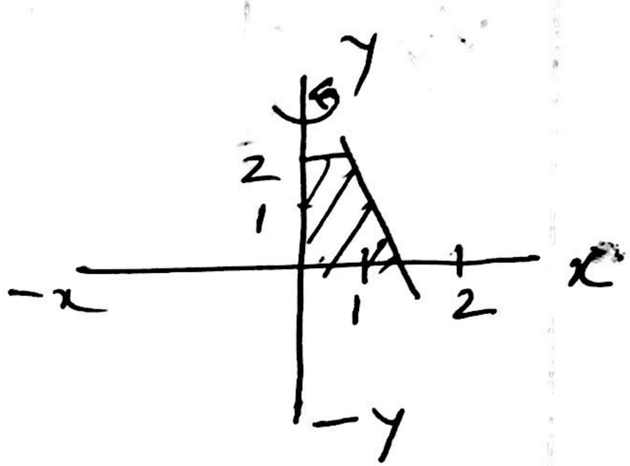
$$= \frac{\pi}{4}\int_0^2 (9 - 6y + y^2)\, dy$$

$$= \frac{\pi}{4}\left[9y - \frac{6y^2}{2} + \frac{y^3}{3}\right]_0^2$$

$$= \frac{\pi}{4}\left[9.2 - 3.2^2 + \frac{2^3}{3}\right]$$

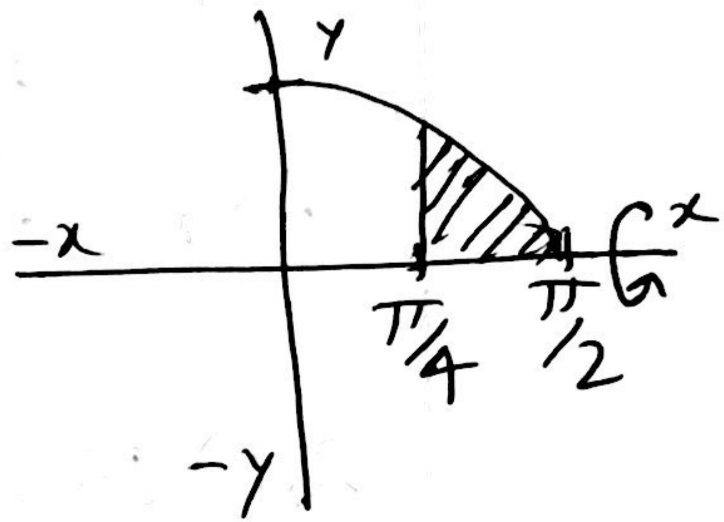
$$= \frac{\pi}{4}\left(18 - 12 + \frac{8}{3}\right)$$

$$= \frac{13\pi}{6}$$



1(b) Given, $y = \sqrt{\cos x}$

So the volume is,



$$V_x = \int_{\pi/4}^{\pi/2} \pi \left(\sqrt{\cos x}\right)^2 dx$$

$$= \int_{\pi/4}^{\pi/2} \pi \cos x \, dx$$

$$= \pi \left[\sin x\right]_{\pi/4}^{\pi/2} = \pi\left(1 - \frac{1}{\sqrt{2}}\right)$$

1 (c) Given, $x = \sqrt{1+y}$

So the volume is

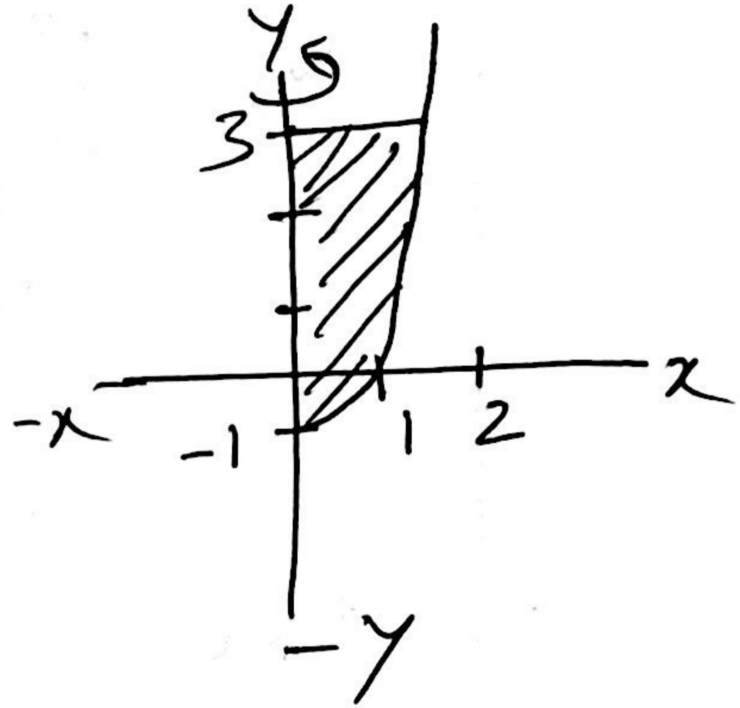
$$V_y = \int_{-1}^{3} \pi \left(\sqrt{1+y}\right)^2 dy$$

$$= \pi \int_{-1}^{3} (1+y)\, dy$$

$$= \pi \left[y + \frac{y^2}{2} \right]_{-1}^{3}$$

$$= \pi \left[\left(3 + \frac{3^2}{2}\right) - \left((-1) + \frac{(-1)^2}{2}\right) \right]$$

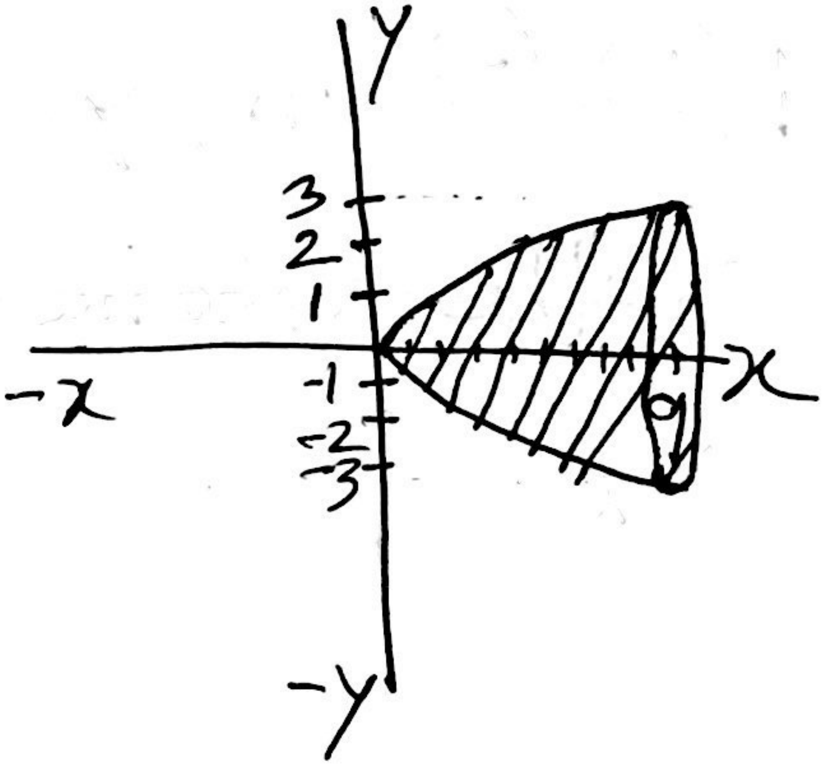
$$= 8\pi$$



2(a) Given, $y = \sqrt{x}$ ——— (1)

and $x = 0$, $x = 9$

x	0	9
y	0	3



So the volume is,

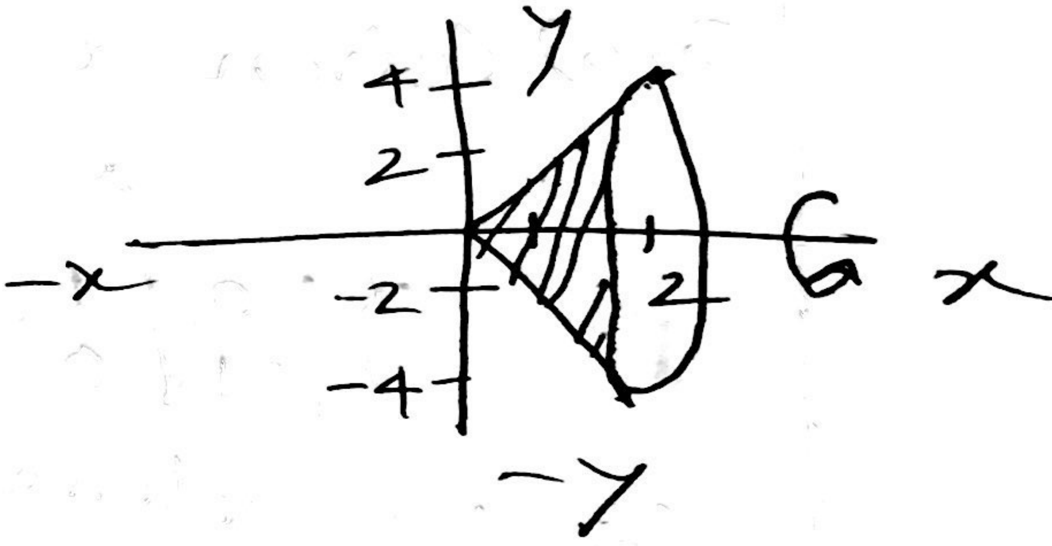
$$V_x = \int_0^9 \pi(\sqrt{x})^2 dx = \pi\int_0^9 x\, dx = \pi\left[\frac{x^2}{2}\right]_0^9$$

$$\therefore V_x = \frac{81\pi}{2}$$

2(b) Given, $y = x^2$

and, $x=0$, $x=2$

x	0	2
y	0	4



So the volume is,

$$V_x = \int_0^2 \pi (x^2)^2 dx$$

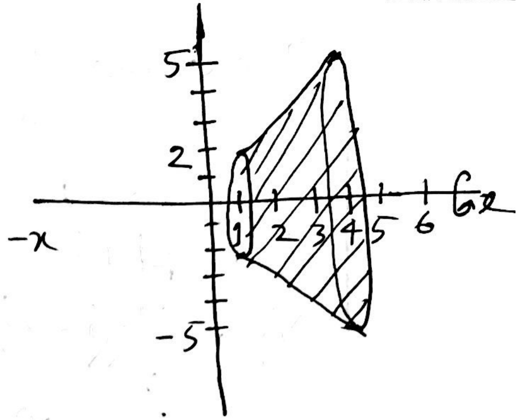
$$= \pi \int_0^2 x^4 dx = \pi \left[\frac{x^5}{5} \right]$$

$$= \frac{\pi}{5} (2^5 - 0)$$

$$\therefore V_x = \frac{32\pi}{5}$$

2(c) Given, $y = x^2 - 4x + 5$

and $x = 1$, $x = 4$



~~So the volume is,~~

~~x~~	~~1~~	~~4~~
~~y~~	~~2~~	~~5~~

So the volume is,

$$V_x = \int_1^4 \pi (x^2 - 4x + 5)^2 dx$$

$$= \pi \int_1^4 \left((x^2)^2 - 2.x^2(4x-5) + (4x-5)^2\right) dx$$

$$= \pi \int_1^4 (x^4 - 8x^3 + 10x^2 + 16x^2 - 40x + 25) dx$$

$$= \pi \int_1^4 (x^4 - 8x^3 + 26x^2 - 40x + 25) dx$$

$$= \pi \left[\int_1^4 x^4 dx - \int_1^4 8x^3 dx + \int_1^4 26x^2 dx - \int_1^4 40x dx + \int_1^4 25 dx \right.$$

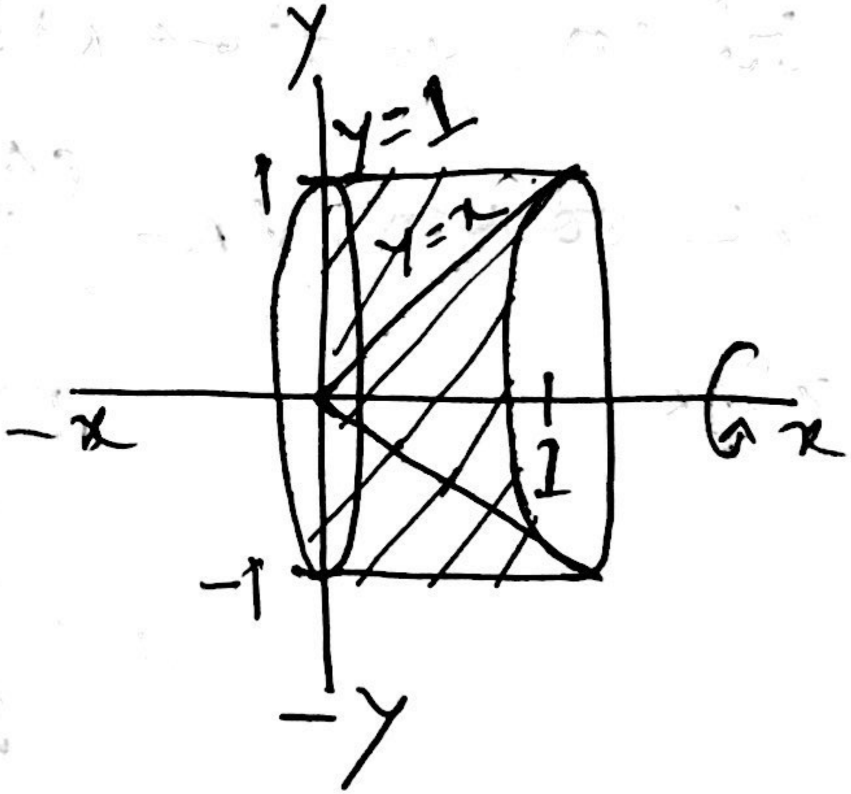
$$= \pi \left[\left[\frac{x^5}{5}\right]_1^4 - \left[\frac{8x^4}{4}\right]_1^4 + \left[\frac{26x^3}{3}\right]_1^4 - \left[\frac{40x^2}{2}\right]_1^4 + \left[25x\right]_1^4 \right]$$

$$= \frac{78\pi}{5}$$

(d) Given, $y = x$ —— (1)

$y = 1$ —— (2)

and $x = 0$



So the volume

$$V_x = \int_0^1 \pi(1^2 - x^2)\,dx$$

$$= \pi\left[x - \frac{x^3}{3}\right]_0^1$$

$$= \pi\left[1 - \frac{1}{3}\right]$$

$$\therefore V_x = \frac{2\pi}{3}$$

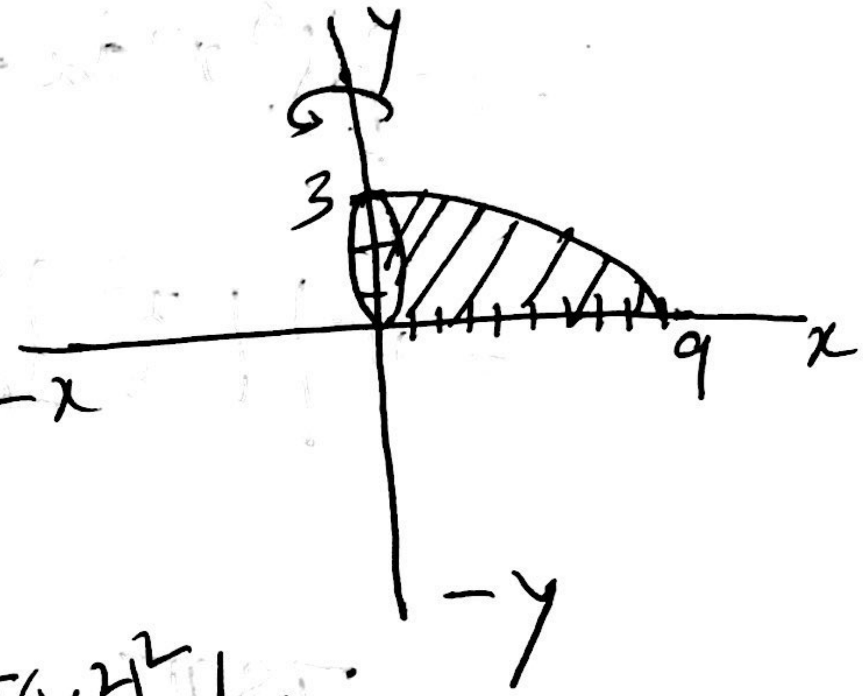
3(a) Given, $y=\sqrt{x}$

$x = y^2$ —— (1)

$x = 0$ —— (2)

and $y = 3$

x	0	9
y	0	3



So the volume is, $V_y = \int_0^3 \pi (y^2)^2 \, dy$

$= \pi \int_0^3 y^4 \, dy = \pi \left[\frac{y^5}{5} \right]_0^3$

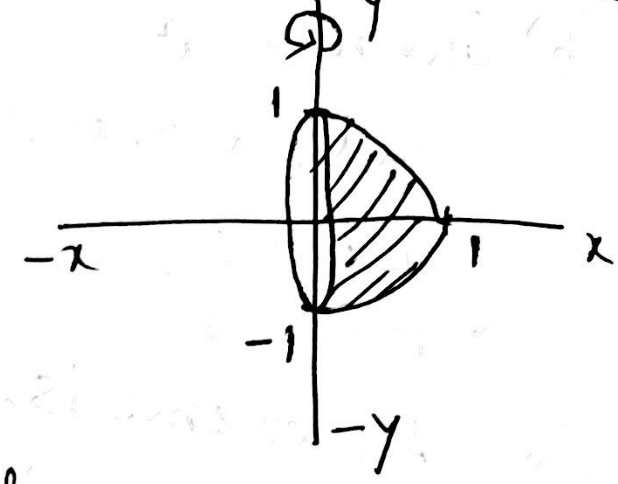
$= \frac{\pi}{5} \cdot 243$

$\therefore V_y = \frac{243\pi}{5}$

3(b) Given, $x = 1 - y^2$ —— ①

$x = 0$ —— ②

x	0	1
y	± 1	0



So the volume is,

$$V_y = \int_{-1}^{1} \pi\left[(1-y^2)^2 - 0^2\right] dy$$

$$= \pi \int_{-1}^{1} (1 - 2y^2 + y^4)\, dy$$

$$= 2\pi \int_{0}^{1} (1 - 2y^2 + y^4)\, dy$$

$$= 2\pi \left[y - 2\frac{y^3}{3} + \frac{y^5}{5} \right]_0^1$$

$$= 2\pi \left(1 - \frac{2}{3} + \frac{1}{5}\right)$$

$$= 2\pi \left(\frac{15 - 10 + 3}{15} \right)$$

$$V_y = \frac{16\pi}{15}$$