1. $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 0$

Let $D \equiv \frac{d}{dt}$

$\therefore (D^2 + 7D + 10)y = 0$ —— (1)

Therefore the auxiliary eqn is:

$m^2 + 7m + 10 = 0$

$\Rightarrow m^2 + 5m + 2m + 10 = 0$

$\Rightarrow m(m+5) + 2(m+5) = 0$

$\Rightarrow (m+5)(m+2) = 0$

$\therefore m = -2, -5$

Thus the general solution is:

$y = Ae^{-2t} + Be^{-5t}$

2. $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$

Let, $D \equiv \frac{d}{dt}$

$\therefore (D^2 + 5D + 6)y = 0$

Therefore the auxiliary eqn is:

$m^2 + 5m + 6 = 0$

$\Rightarrow m^2 + 3m + 2m + 6 = 0$

$\Rightarrow m(m+3) + 2(m+3) = 0$

$\Rightarrow (m+3)(m+2) = 0$

$\therefore m = -3, -2$

Thus the general solution is:

$y = Ae^{-3t} + Be^{-2t}$

3. $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0$

let, $D \equiv \frac{d}{dt}$

$\therefore (D^2 + 4D + 1)y = 0$

Therefore the auxiliary eqn is:

$m^2 + 4m + 1 = 0$

$\Rightarrow \cancel{m^2 + 2m + 2m + 1 = 0}$

$\Rightarrow m = \frac{-4 \pm \sqrt{16 - 4}}{2}$

$\therefore m = -2 \pm \sqrt{3}$

Thus the general solution is,

$y = Ae^{(-2+\sqrt{3})t} + Be^{(-2-\sqrt{3})t}$

4. $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$

let, $D \equiv \frac{d}{dt}$

$\therefore (D^2 + 3D + 2)y = 0$

Therefore the auxiliary eqn is:

$m^2 + 3m + 2 = 0$

$\Rightarrow m^2 + 2m + m + 2 = 0$

$\Rightarrow m(m+2) + 1(m+2) = 0$

$\Rightarrow (m+2)(m+1) = 0$

$\therefore m = -1, -2$

Thus the general solution is:

$y = Ae^{-t} + Be^{-2t}$ ——— (1)

Scanned with CamScanner

Using the initial condition,

$y(0) = 2$, we get,

$$2 = Ae^{0} + Be^{0}$$

$$\therefore A + B = 2 \quad \text{—— (2)}$$

From eqn (1)

$$y = Ae^{-t} + Be^{-2t}$$

$$\therefore y' = -Ae^{-t} - 2Be^{-2t} \quad \text{—— (3)}$$

Using $y'(0) = 3$ in eqn (3), we get,

$$-3 = -Ae^{0} - 2Be^{0}$$

$$\Rightarrow A + 2B = +3 \quad \text{—— (4)}$$

$$\therefore A = +3 - 2B$$

Using the value of A into (2)

$$+3 - 2B + B = 2$$

$$\Rightarrow -B = \cancel{8} -1$$

$$\therefore B = \cancel{5}\ 1$$

Using the value of B into (2)

$$A + (1\cancel{5}) = 2 \quad \therefore A = \cancel{7}\ 1$$

Hence the solution becomes:

$$\cancel{y = 7e^{-t} + 5e^{-2t}}$$

$$y = e^{-t} + e^{-2t}$$

1. $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = t^2$

Let, $D \equiv \frac{d}{dt}$ $\therefore (D^2+3D+2)y = t^2$ —— (1)

Consider, $(D^2+3D+2)y = 0$ —— (2)

Let, $y = e^{mt}$ is the trial solⁿ of (2)

Therefor the auxiliary eqⁿ becomes:

$m^2+3m+2=0 \Rightarrow m^2+2m+m+2=0$

$\Rightarrow m(m+2)+1(m+2)=0 \Rightarrow (m+1)(m+2)=0.$

$\therefore \quad m = -1, -2$

Hence the complimentary function becomes:

$Y_c = Ae^{-t} + Be^{-2t}$

Again let, $y_p = a_0 + a_1 t + a_2 t^2$ —— (3)

$\Rightarrow Dy_p = a_1 + 2a_2 t$

& $D^2 y_p = 2a_2$

putting the values of y_p & its derivatives into eqn (1) we get:

$2a_2 + 3(a_1 + 2a_2 t) + 2(a_0 + a_1 t + a_2 t^2) = t^2$

$\Rightarrow 2a_2 + 3a_1 + 6a_2 t + 2a_0 + 2a_1 t + 2a_2 t^2 = t^2$

$\Rightarrow (2a_0 + 3a_1 + 2a_2) + (2a_1 + 6a_2)t + 2a_2 t^2 = t^2$

Equating the like terms we have:

$2a_2 = 1$, $\quad 2a_1 + 6a_2 = 0$ & $2a_0 + 3a_1 + 2a_2 = 0$

$\therefore a_2 = \frac{1}{2}$ $\quad \Rightarrow 2a_1 + 6 \cdot \frac{1}{2} = 0$ $\quad \Rightarrow 2a_0 + 3\left(-\frac{3}{2}\right) + 2 \cdot \frac{1}{2} = 0$

$\therefore a_1 = -\frac{3}{2}$ $\quad \Rightarrow 2a_0 - \frac{9}{2} + 1 = 0$

$\therefore a_0 = \frac{7}{4}$

putting the values of a_0, a_1 & a_2 into eqn (3);

$y_p = \frac{7}{4} - \frac{3}{2}t + \frac{1}{2}t^2$

Hence the general soln becomes:

$y = y_c + y_p$

$= Ae^{-t} + Be^{-2t} + \frac{7}{4} - \frac{3}{2}t + \frac{1}{2}t^2$

2. $\frac{d^2y}{dt^2} + 4y = t - \frac{t^2}{20}$

let, $D \equiv \frac{d}{dt}$ $\therefore (D^2+4)y = t - \frac{t^2}{20}$ —— (1)

consider, $(D^2+4)\,y = 0$ —— (2)

Let, $y = e^{mt}$ is the trial sol^n of eq^n (2)

Therefore the auxiliary eq^n becomes:

$m^2+4=0 \Rightarrow m^2 = -4 \quad \therefore m = \pm 2i$

Hence the complimentary function becomes:

$y_C = A\cos 2t + B\sin 2t$

Again let, $y_p = a_0 + a_1 t + a_2 t^2$ ——— (3)

$\Rightarrow Dy_p = a_1 + 2a_2 t$

& $D^2 y_p = 2a_2$

putting the values of y_p & its derivatives into eqn (1) we get:

$2a_2 + 4(a_0 + a_1 t + a_2 t^2) = t - \frac{t^2}{20}$

$\Rightarrow 2a_2 + 4a_0 + 4a_1 t + 4a_2 t^2 = t - \frac{t^2}{20}$

$\Rightarrow (4a_0 + 2a_2) + 4a_1 t + 4a_2 t^2 = t - \frac{t^2}{20}$

Equating the like terms we have:

$4a_2 = -\frac{1}{20}$, $\quad 4a_1 = 1$ $\quad$ & $4a_0 + 2a_2 = 0$

$\therefore a_2 = -\frac{1}{80}$ $\quad \therefore a_1 = \frac{1}{4}$ $\quad \Rightarrow 4a_0 + 2(-\frac{1}{80}) = 0$

$\Rightarrow 4a_0 = \frac{1}{40}$

$\therefore a_0 = \frac{1}{160}$

putting the values of a_0, a_1 & a_2 in eqn (3) we get:

$y_p = \frac{1}{160} + \frac{1}{4}t - \frac{1}{80}t^2$

Hence the general sol^n becomes:

$y = y_c + y_p$

$= A\cos 2t + B\sin 2t + \frac{1}{160} + \frac{1}{4}t - \frac{1}{80}t^2$

3. $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 2e^{-3t}$

let, $D \equiv \frac{d}{dt}$ $\therefore (D^2+6D+8)y = 2e^{-3t}$ —— ①

Consider, $(D^2+6D+8)y = 0$ —— ②

let, $y = e^{mt}$ is the trial function of eqn ②

Therefore the auxiliary function becomes:

$m^2+6m+8=0 \Rightarrow m^2+4m+2m+8=0$

$\Rightarrow m(m+4)+2(m+2)=0 \Rightarrow (m+2)(m+4)=0$

$\therefore m = -2, -4$

Therefore the complimentary function becomes:

$y_c = Ae^{-2t} + Be^{-4t}$

Again let, $y_p = a_0 e^{-3t}$ —— ③

$\Rightarrow Dy_p = -3a_0 e^{-3t}$

& $D^2 y_p = 9a_0 e^{-3t}$

Substituting the values of y_p & its derivatives into eqn ① we get:

$9a_0e^{-3t} + 6.(-3a_0e^{-3t}) + a_0e^{-3t} = 2e^{-3t}$

$\Rightarrow 9a_0e^{-3t} - 18a_0e^{-3t} + a_0e^{-3t} = 2e^{-3t}$

$\Rightarrow -8a_0e^{-3t} = 2e^{-3t}$

$\therefore a_0 = -\frac{1}{4}$

Putting the value of a_0 into eqn ③ the particular integral becomes $y_p = -\frac{1}{4}e^{-3t}$

Hence the general soln becomes:

$y = y_c + y_p$

$= Ae^{-2t} + Be^{-4t} - \frac{1}{4}e^{-3t}$

Scanned with CamScanner

4. $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = e^{-t}$, $y(0) = y'(0) = 0$

let, $\frac{d}{dt} \equiv D$ $\therefore (D^2+6D+8)y = e^{-t}$ ——— (1)

Consider, $(D^2+6D+8)y = 0$ ——— (2)

let, $y = e^{mt}$ is the trial sol^n of eq^n (2)

Therefore the auxiliary eq^n becomes:

$m^2+6m+8=0 \Rightarrow \cancel{m^2}\ m = -2, -4$

$\therefore y_c = Ae^{-2t} + Be^{-4t}$

Again let, $y_p = a_0 e^{-t}$ ——— (3)

$\Rightarrow Dy_p = -a_0 e^{-t}$

& $D^2 y_p = a_0 e^{-t}$

putting the values of y_p & its derivatives into eq^n (1):

$a_0 e^{-t} + 6.(-a_0 e^{-t}) + 8.a_0 e^{-t} = e^{-t}$

$\Rightarrow a_0 e^{-t} - 6a_0 e^{-t} + 8a_0 e^{-t} = e^{-t}$

$\Rightarrow 3a_0 e^{-t} = e^{-t}$ $\therefore a_0 = \frac{1}{3}$

Putting the value of a_0 into eq^n (3) we get the particular integral:

$y_p = \frac{1}{3}e^{-t}$

Hence the general soln becomes:

$y = y_c + y_p$

$\therefore y = Ae^{-2t} + Be^{-4t} + \frac{1}{3}e^{-t}$ ——— (4)

& $y' = -2Ae^{-2t} - 4Be^{-4t} - \frac{1}{3}e^{-t}$ ——— (5)

putting the initial value $y(0)=0$ into eqn ④

$0 = A + B + \frac{1}{3} \Rightarrow A+B = -\frac{1}{3}$ ——— ⑥

$\Rightarrow A = -\frac{1}{3} - B$ ——— ⑦

putting the initial value $y'(0)=0$ into eqn ⑤

$0 = -2A - 4B - \frac{1}{3}$

$\Rightarrow 2A + 4B = -\frac{1}{3}$

$\Rightarrow 2\left(-\frac{1}{3} - B\right) + 4B = -\frac{1}{3}$ [From eqn ⑦]

$\Rightarrow -\frac{2}{3} - 2B + 4B = -\frac{1}{3}$

$\Rightarrow 2B = \frac{1}{3} \quad \therefore B = \frac{1}{6}$

Putting the value of B into eqn ⑦

$A = -\frac{1}{3} - \frac{1}{6} \quad \therefore A = -\frac{1}{2}$

Putting the values of A & B into eqn ④:

$y = -\frac{1}{2}e^{-2t} + \frac{1}{6}e^{-4t} + \frac{1}{3}e^{-t}$

5. $\frac{d^2y}{dt^2} + 9y = e^{-t}$

Let, $D \equiv \frac{d}{dt} \therefore (D^2+9)y = e^{-t}$ ——— ①

consider, $(D^2+9)y = 0$ ——— ②

let $y = e^{mt}$ is the trial soln of eqn ②:

Hence the auxiliary eqn becomes:

$m^2+9=0 \Rightarrow m^2 = -9 \therefore m = \pm 3i$

Therefore the complimentary function becomes;

$y_c = A\cos 3t + B\sin 3t$

Scanned with CamScanner

Again let, $y_p = a_0 e^{-t}$ —— (3)

$\Rightarrow Dy_p = -a_0 e^{-t}$

& $D^2 y_p = a_0 e^{-t}$

putting the values of y_p & its derivatives into eqn (1) we get:

$a_0 e^{-t} + 9a_0 e^{-t} = e^{-t}$

$\Rightarrow 10a_0 e^{-t} = e^{-t} \quad \therefore a_0 = \frac{1}{10}$

putting the value of a_0 into eqn (3): $y_p = \frac{1}{10} e^{-t}$

Therefore the general sol^n becomes

$y = y_c + y_p$

$\therefore y = A\cos 3t + B\sin 3t + \frac{1}{10} e^{-t}$

6. $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = \cos t \Rightarrow (D^2 + 6D + 8)y = \cos t$ —— (1)

similar to no (3) & (4), $y_c = c_1 e^{-2t} + c_2 e^{-4t}$

Again let, $y_p = A\cos t + B\sin t$ —— (2)

$\Rightarrow Dy_p = -A\sin t + B\cos t$

& $D^2 y_p = -A\cos t - B\sin t$

putting the values of y_p & the derivatives of y_p into eqn (1) we get:

$-A\cos t - B\sin t + 6(-A\sin t + B\cos t) + 8(A\cos t + B\sin t) = \cos t$

$\Rightarrow -A\cos t - B\sin t - 6A\sin t + 6B\cos t + 8A\cos t + 8B\sin t = \cos t$

$\Rightarrow (7A + 6B)\cos t + (-6A + 7B)\sin t = \cos t$

Equating like terms we get,

$-6A + 7B = 0$ & $7A + 6B = 1$

$\Rightarrow A = \frac{7B}{6}$ —— (3) $\quad \Rightarrow 7 \cdot \frac{7B}{6} + 6B = 1 \Rightarrow \frac{11B}{6} = 1 \therefore B = \frac{6}{11}$

putting the value of B into eq^n 3 :

$A = \frac{7}{6} \cdot \frac{6}{11} \Rightarrow A = \frac{7}{11}$

putting the values of A & B into eqn 2 :

$y_p = \frac{7}{11}\cos t + \frac{6}{11}\sin t$

Therefore the general sol^n becomes :

$y = y_c + y_p$

$\therefore y = a_1 e^{-2t} + a_2 e^{-4t} + \frac{7}{11}\cos t + \frac{6}{11}\sin t$