Assignment 1

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Section: M

1. a) $\int_{2}^{4}\int_{0}^{3} (x+y)\,dx\,dy$

$= \int_{2}^{4} \left[\frac{x^2}{2} + xy\right]_{0}^{3} dy$

$= \int_{2}^{4} \left(\frac{9}{2} + 3y\right) - 0 \; dy$

$= \left[\frac{9}{2}y + \frac{3y^2}{2}\right]_{2}^{4}$

$= (18+24) - (9+6)$

$= 27$ (Ans:)

b) $\int_0^1 \int_x^y xy \, dy \, dx$

$= \int_0^1 \left[\frac{xy^2}{2}\right]_x^y dx$

$= \int_0^1 \left(\frac{xy^2}{2} - \frac{x^3}{2}\right) dx$

$= \left[y^2 \frac{x^2}{4} - \frac{x^4}{8}\right]_0^1$

$= \frac{y^2}{4} - \frac{1}{8}$ (Ans:)

c) $\int_0^1 \int_{y^2}^y (x^2y + xy^2) \, dx \, dy$

$= \int_0^1 \left[\frac{yx^3}{3} + \frac{y^2x^2}{2}\right]_{y^2}^y dy$

$$= \int_0^1 \left(\frac{y^4}{3} + \frac{y^4}{2}\right) - \left(\frac{y^7}{3} + \frac{y^6}{2}\right) dy$$

$$= \int_0^1 \left(\frac{5y^4}{6} - \frac{y^2}{3} - \frac{y^6}{2}\right) dy$$

$$= \left[\frac{5}{6} \cdot \frac{y^5}{5} - \frac{1}{3} \cdot \frac{y^3}{3} - \frac{1}{2} \cdot \frac{y^7}{7}\right]_0^1$$

$$= \left(\frac{1}{6} - \frac{1}{24} - \frac{1}{14}\right) - 0$$

$$= \frac{3}{56} \text{ (Ans:)}$$

d) $$\int_1^2 \int_1^y \left(\frac{1}{x} + \frac{1}{y}\right) dx\, dy$$

$$= \int_1^2 \left[\ln x + \frac{x}{y}\right]_1^y dy$$

$$= \int_1^2 \left(\ln y + 1 - \frac{1}{y}\right) dy$$

$$= \left[y \ln y - y + y - \ln y\right]_1^2$$

$$= (2\ln 2 - \ln 2) - (\ln 1 - \ln 1)$$

$$= \ln 2 \quad \text{(Ans:)}$$

e) $$\int_0^1 \int_0^{\sqrt{x}} y \cdot e^{x^2} \, dy \, dx$$

x	0	1
y	0	1

$$= \int_0^1 \left[e^{x^2} \cdot \frac{y^2}{2}\right]_0^{\sqrt{x}} dx$$

$$= \int_0^1 \frac{1}{2} x \cdot e^{x^2} \, dx$$

$$= \left[\frac{1}{4} e^{x^2}\right]_0^1$$

Since,

$$\int x e^{x^2} dx \quad \begin{array}{|l} u = x^2 \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx \end{array}$$

$$= \int \frac{1}{2} e^u dx$$

$$= \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

$$= \frac{1}{4} e^1 - \frac{1}{4} e^0$$

$$= \frac{1}{4}(e-1)$$

(Ans:)

f) $\int_0^{\sqrt{\pi/2}} \int_0^{x^2} x \cos y \, dy \, dx$

Since,

$$\int x \sin x^2 dx \quad \begin{array}{|l} x^2 = u \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx \end{array}$$

$$= \int \frac{1}{2} \sin u \, du$$

$$= -\frac{1}{2} \cos x^2 + C$$

$$= \int_0^{\sqrt{\pi/2}} \left[x \sin y\right]_0^{x^2} dx$$

$$= \int_0^{\sqrt{\pi/2}} x \sin x^2 \, dx$$

x	0	$\sqrt{\pi/2}$
u	0	$\pi/2$

$$= \left[-\frac{1}{2} \cos x^2\right]_0^{\sqrt{\pi/2}}$$

$= -\frac{1}{2}\left(\cos\frac{\cancel{\pi}}{2} - \cos 0\right)$

$= -\frac{1}{2}$ (Ans:)

8) $\int_0^1 \int_0^{x^2} (x^2+y)\,dy\,dx$

$= \int_0^1 \left[x^2 y + \frac{y^2}{2}\right]_0^{x^2} dx$

$= \int_0^1 \left(x^4 + \frac{x^4}{2}\right) dx$

$= \int_0^1 \frac{3x^4}{2}\, dx$

$= \frac{3}{2}\left[\frac{x^5}{5}\right]_0^1 = \frac{3}{10}$ (Ans:)

h) $\displaystyle\int_0^{\pi/2}\int_0^2 r\sqrt{4-r^2}\,dr\,d\theta$

$$=\int_0^{\pi/2}\left[-\frac{1}{3}(4-r^2)^{3/2}\right]_0^2 d\theta$$

$$=\int_0^{\pi/2}-\frac{1}{3}\left[(4-2^2)^{3/2}-4^{3/2}\right]d\theta$$

Since,

$$\int r\sqrt{4-r^2}\,dr$$

$$=\int -\frac{1}{2}\sqrt{u}\,du$$

$$=-\frac{1}{2}\frac{u^{3/2}}{3/2}$$

$$=-\frac{1}{3}(4-r^2)^{3/2}$$

$4-r^2=u$

$du=-2r\,dr$

$\Rightarrow -\frac{du}{2}=r\,dr$

$$=\int_0^{\pi/2}\frac{8}{3}\,d\theta$$

$$=\left[\frac{8\theta}{3}\right]_0^{\pi/2}$$

$$=\frac{4\pi}{3}$$ (Ans.)

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i) $\int_{0}^{1}\int_{-x}^{x}(x^2-y^2)\,dy\,dx$

$$=\int_{0}^{1}\left[x^2y-\frac{y^3}{3}\right]_{-x}^{x}dx$$

$$=\int_{0}^{1}\left(x^3-\frac{x^3}{3}\right)-\left(-x^3+\frac{x^3}{3}\right)dx$$

$$=\int_{0}^{1}2x^3\,dx$$

$$=\left[\frac{2x^4}{4}\right]_{0}^{1}$$

$$=\frac{1}{2}\text{ (Ans:)}$$

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j) $\int_0^{\pi/2} \int_0^{\sin\theta} r\cos\theta \, dr \, d\theta$

$= \int_0^{\pi/2} \left[\frac{r^2}{2} \cdot \cos\theta\right]_0^{\sin\theta} d\theta$

$= \int_0^{\pi/2} \cos\theta \cdot \sin^2\theta/2 \; d\theta$

$= \frac{1}{2}\int_0^{\pi/2} \sin^2\theta \cdot \cos\theta \cdot d\theta$

$= \frac{1}{2}\left[\frac{\sin^3\theta}{3}\right]_0^{\pi/2}$

$= \frac{1}{6}$ (Ans:)

Since,

$\int \sin^2\theta \cdot \cos\theta \, d\theta$

$= u^2 du$

$= u^3/3 + C$

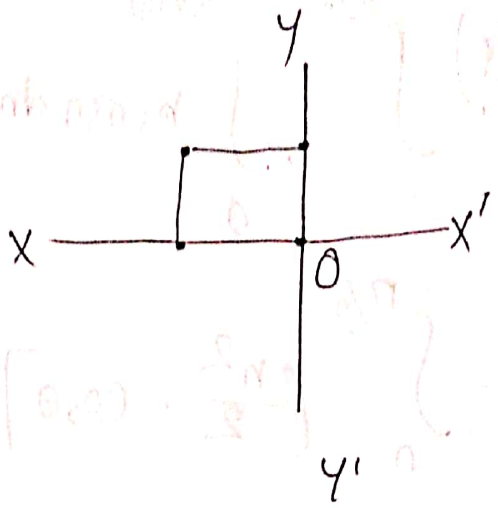
$= \frac{\sin^3\theta}{3}$

$\sin\theta = u$

$\frac{du}{d\theta} = \cos\theta$

$\Rightarrow du = \cos\theta \, d\theta$

2.a) $\iint\limits_R (xy - y^2)$



$$= \int_{-1}^{0} \int_{0}^{1} (xy - y^2)\, dy\, dx$$

here, $-1 \leq x \leq 0$
$0 \leq y \leq 1$

$$= \int_{-1}^{0} \left[\frac{xy^2}{2} - \frac{y^3}{3} \right]_0^1 dx$$

$$= \int_{-1}^{0} \left(\frac{x}{2} - \frac{1}{3} \right) dx$$

$$= \frac{1}{6} \int_{-1}^{0} (3x - 2)\, dx$$

$$= \frac{1}{6} \left[\frac{3x^2}{2} - 2x \right]_{-1}^{0}$$

$$= \frac{1}{6} \left(-\frac{3}{2} + 2 \right) = \frac{1}{12} \text{ (Ans.)}$$

b) $\iint_R (2x+y)\,dA$ $R=\{(x,y) \mid 3\le x\le 5,\ 1\le y\le 2\}$

$$= \int_3^5 \int_1^2 (2x+y)\,dy\,dx$$

$$= \int_3^5 \left[2xy + \frac{y^2}{2}\right]_1^2 dx$$

$$= \int_3^5 \left(4x+2-2x-\frac{1}{2}\right) dx$$

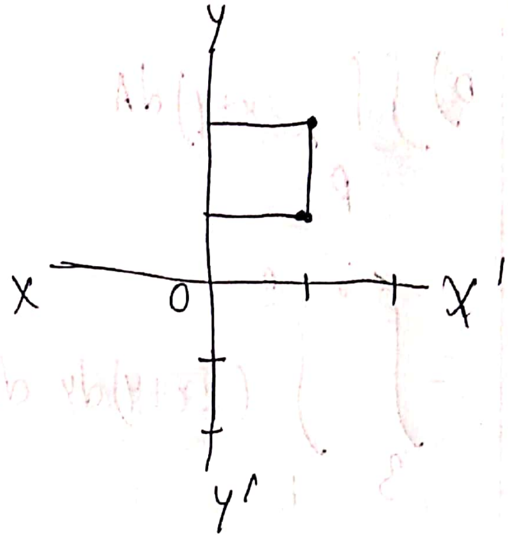
$$= \cancel{5} \int_3^5 \left(2x+\frac{3}{2}\right) dx$$

$$= \left[\frac{2x^2}{2} + \frac{3}{2}x\right]_3^5$$

$$= 25 + \frac{15}{2} - 9 - \frac{9}{2}$$

$$= 19 \text{ (Ans:)}$$

c) $\iint_R (x^2+y^2)\,dA$



$= \int_0^1 \int_1^2 (x^2+y^2)\,dy\,dx$

here, $0 \le x \le 1$
$1 \le y \le 2$

$= \int_0^1 \left[x^2y + \frac{y^3}{3}\right]_1^2 dx$

$= \int_0^1 \left(2x^2 + 8/3 - x^2 - 1/3\right) dx$

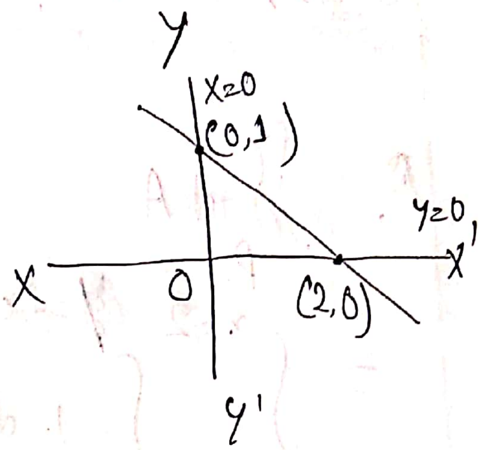
$= \int_0^1 \left(x^2 + 7/3\right) dx$

$= \left[\frac{x^3}{3} + 7/3\, x\right]_0^1$

$= \frac{1}{3} + 7/3 = 8/3$ (Ans:)

$x+2y=2, \ x=0, \ y=0$

d) $\iint_R x \, dA$



$$= \int_0^2 \int_0^{1-X/2} x \, dy \, dx$$

$$= \int_0^2 [xy]_0^{1-X/2} \, dx$$

$$= \int_0^2 \left(x - \frac{x^2}{2}\right) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2$$

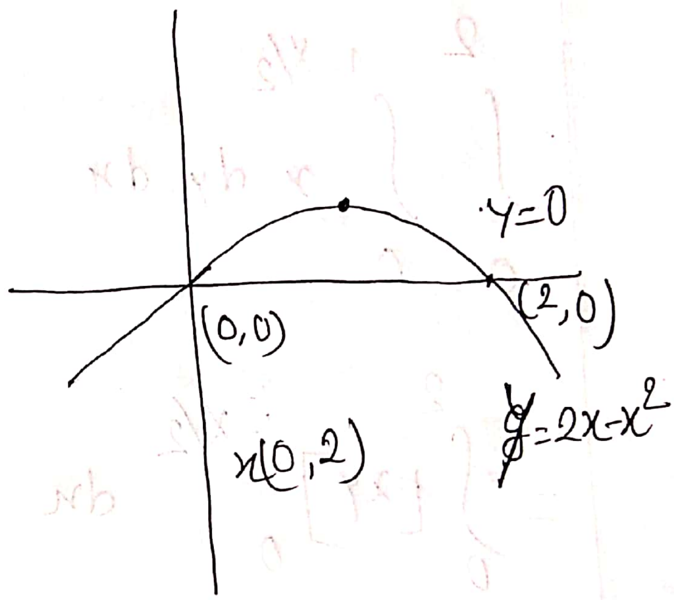
$$= \frac{4}{2} - \frac{8}{6}$$

$$= \frac{10}{3} \text{ (Ans:)}$$

3. a) $y = 2x - x^2, \quad y = 0$

$$A = \iint_R dA$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=2x-x^2} 1 \cdot dy\, dx$$



$$= \int_0^2 [y]_0^{2x-x^2}\, dx$$

$$= \int_0^2 2x - x^2\, dx$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= 4 - 8/3$$

$$= 4/3 \text{ (Ans.)}$$

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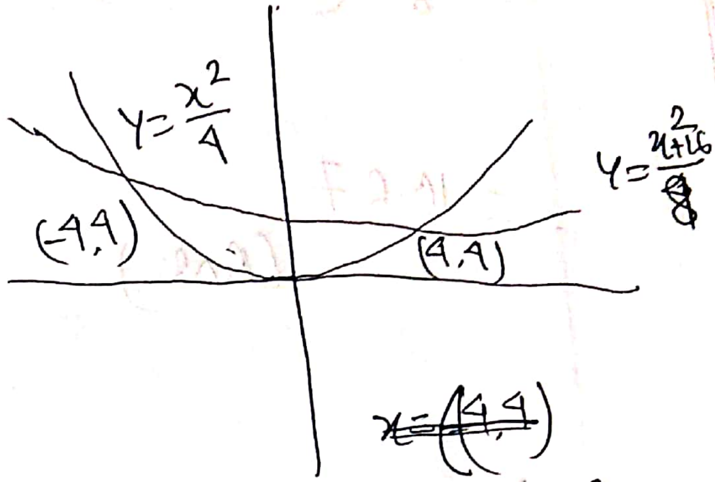
b) $x^2 = 4y,\ 8y = x^2+16$



$$A = \iint_R dA$$

$$= \int_{-4}^{4} \int_{y=\frac{x^2}{4}}^{y=\frac{x^2+16}{8}} dy\, dx$$

$$= \int_{-4}^{4} \left(\frac{x^2+16}{8} - \frac{x^2}{4} \right) dx$$

$$= \int_{-4}^{4} \frac{16-x^2}{8}\, dx$$

$$= \frac{1}{8} \left[16x - \frac{x^3}{3} \right]_{-4}^{4}$$

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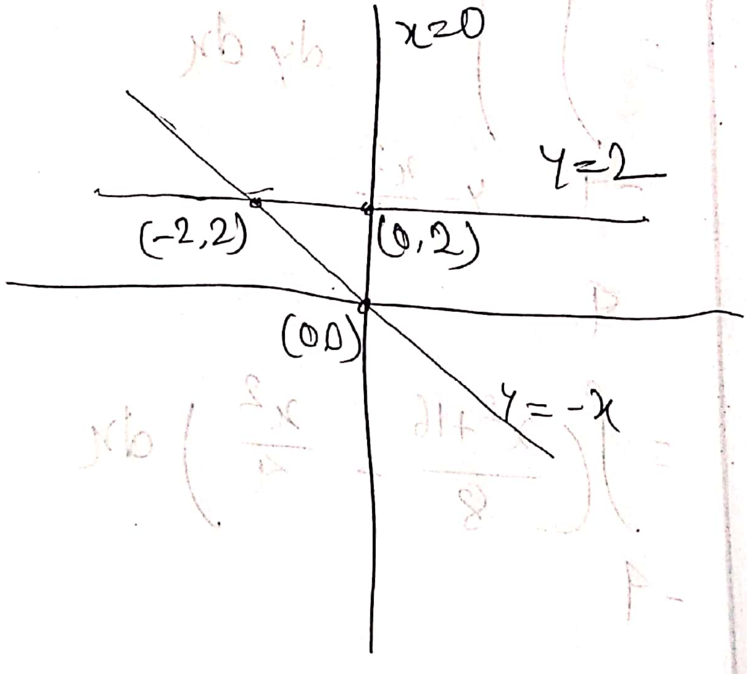
$= \frac{1}{8}\left(64 - \frac{64}{3} + 64 - \frac{64}{3}\right)$

$= 10.67$

(Ans:)

c) $y = -x,\ x = 0,\ y = 2$



$A = \iint_R dA$

$= \int_{-2}^{0} \int_{y=-x}^{y=2} dy\, dx$

here, $-2 \le x \le 0$

$= \int_{2}^{0} [y]_{y=-x}^{y=2}\, dx$

$= \int_{2}^{0} (2+x)\, dx$

$$= \left[2x + \frac{x^2}{2}\right]_{-2}^{0}$$

$$= 0 - \left(-4 + \frac{4}{2}\right)$$

$$= 2 \quad \text{(Ans:)}$$

4. a) $\int_1^2 \int_0^1 \int_{-1}^{1} (x^2+y^2+z^2)\, dx\, dy\, dz$

$$= \int_1^2 \int_0^1 \left[\frac{x^3}{3} + y^2x + z^2x\right]_{-1}^{1} dy\, dz$$

$$= \int_1^2 \int_0^1 \left(\frac{1}{3} + y^2 + z^2\right) - \left(-\frac{1}{3} - y^2 - z^2\right) dy\, dz$$

$$= \int_1^2 \int_0^1 \left(\frac{2}{3} + 2y^2 + 2z^2 \right) dy\, dz$$

$$= \int_1^2 \left[\frac{2y}{3} + \frac{2y^3}{3} + 2yz^2 \right]_0^1 dz$$

$$= \int_1^2 \left(\frac{2}{3} + \frac{2}{3} + 2z^2 \right) dz$$

$$= \int_1^2 \left(\frac{4}{3} + 2z^2 \right) dz$$

$$= \left[\frac{4}{3} z + 2 \frac{z^3}{3} \right]_1^2$$

$$= \frac{8}{3} + \frac{16}{3} - \frac{4}{3} - \frac{2}{3}$$

$= 6$ (Ans:)

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b) $\int_0^1 \int_0^{y^2} \int_0^{x+y} x \, dz \, dx \, dy$

$$= \int_0^1 \int_0^{y^2} \left[zx \right]_0^{x+y} dx \, dy$$

$$= \int_0^1 \int_0^{y^2} \left(x^2 + xy \right) dx \, dy$$

$$= \int_0^1 \left[\frac{x^3}{3} + \frac{x^2 y}{2} \right]_0^{y^2} dy$$

$$= \int_0^1 \left(\frac{y^6}{3} + \frac{y^5}{2} \right) dy$$

$$= \left[\frac{y^7}{21} + \frac{y^6}{12}\right]_0^1$$

$$= \frac{1}{21} + \frac{1}{12}$$

$$= \frac{11}{84} \quad \text{(Ans:)}$$

c) $$\int_0^1 \int_0^x \int_0^{x-y} x \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^x [xz]_0^{x-y} \, dy \, dx$$

$$= \int_0^1 \int_0^x (x^2 - xy) \, dy \, dx$$

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$$= \int_0^1 \left[x^2y - \frac{xy^2}{2}\right]_0^x dx$$

$$= \int_0^1 \left(x^3 - \frac{x^3}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{8} \text{ (Ans.)}$$

d) $$\int_0^2 \int_{-1}^{y^2} \int_{-1}^{z} yz \, dx \, dz \, dy$$

$$= \int_0^2 \int_{-1}^{y^2} \left[xyz\right]_{-1}^{z} dz \, dy$$

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$$= \int_0^2 \int_{-1}^{y^2} (yz^2 + yz)\, dz\, dy$$

$$= \int_0^2 \left[\frac{yz^3}{3} + \frac{yz^2}{2} \right]_{-1}^{y^2} dy$$

$$= \int_0^2 \left(\frac{y^7}{3} + \frac{y^5}{2}\right) - \left(-y/3 + y/2\right) dy$$

$$= \int_0^2 \left(\frac{y^7}{3} + \frac{y^5}{2} - \frac{y}{6}\right) dy$$

$$= \left[\frac{y^8}{24} + \frac{y^6}{12} - \frac{y^2}{12} \right]_0^2$$

$$= \frac{1}{24} [256 + 128 - 8] = \frac{47}{3} \text{ (Ans:)}$$

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e) $$\int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} zr\,dz\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2}\left[\frac{z^2 r}{2}\right]_0^{4-r^2} dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2}\frac{r}{2}\left(16-8r^2+r^4\right) dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{2}\left(8r-4r^3+\frac{r^5}{2}\right) dr\,d\theta$$

$$=\int_0^{2\pi}\left[\frac{8r^2}{2}-4\frac{r^4}{4}+\frac{r^6}{12}\right]_0^{2} d\theta$$

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$$= \int_0^{2\pi} \left(16 - 16 + \frac{64}{12}\right) d\theta$$

$$= \int_0^{2\pi} \frac{16}{3} \, d\theta$$

$$= \left[\frac{16\theta}{3}\right]_0^{2\pi}$$

$$= \frac{32}{3}\pi \text{ (Ans:)}$$

f) $$\int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^3 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \left[\frac{r^4}{4} \sin\theta\right]_0^a d\theta \, d\phi$$

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$$= \int_0^{2\pi}\int_0^{\pi} \frac{a^4}{4} \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \left[-\frac{a^4}{4} \cos\theta \right]_0^{\pi} d\phi$$

$$= \int_0^{2\pi} \left(\frac{a^4}{4} + \frac{a^4}{4} \right) d\phi$$

$$= \int_0^{2\pi} \frac{a^4}{2} \, d\phi$$

$$= \left[\frac{a^4}{2} \right]_0^{2\pi}$$

$$= a^4 \pi$$

(Ans:)

5. $D = \{(x,y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}$.

$\rho(x,y) = y^2$

$$m = \int_0^2 \int_0^1 y^2 \, dx \, dy$$

$$= \int_0^2 [x y^2]_0^1 \, dy$$

$$= \int_0^2 y^2 \, dy$$

$$= \left[\frac{y^3}{3}\right]_0^2$$

$$= \frac{8}{3}$$

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$$\bar{x} = \frac{1}{m} \iint_D x \cdot \rho(x,y)\, dA$$

$$= \frac{1}{8/3} \int_0^2 \int_0^1 x y^2 \, dx \, dy$$

$$= \frac{3}{8} \int_0^2 \left[\frac{x^2 y^2}{2} \right]_0^1 dy$$

$$= \frac{3}{8} \int_0^2 \frac{y^4}{2} \, dy$$

$$= \frac{3}{16} \left[\frac{y^3}{3} \right]_0^2$$

$$= \frac{3}{16} \left(\frac{8}{3} - 0 \right)$$

$$= \frac{1}{2}$$

$$\bar{y} = \frac{1}{m} \iint_D y \cdot \rho(x,y)\, dA$$

$$= \frac{1}{8/3} \int_0^2 \int_0^1 y \cdot y^2 \, dx\, dy$$

$$= \frac{3}{8} \int_0^2 \int_0^1 y^3 \, dx\, dy$$

$$= \frac{3}{8} \int_0^2 \left[x y^3 \right]_0^1 dy$$

$$= \frac{3}{8} \int_0^2 y^3 \, dy$$

$$= \frac{3}{8} \left[\frac{y^4}{4} \right]_0^2$$

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$$= \frac{3}{8} \times \left(\frac{2^4}{4} - 0\right)$$

$$= \frac{3}{2}$$

$\therefore m = \frac{8}{3}, \quad \bar{x} = \frac{1}{2}, \quad \bar{y} = \frac{3}{2}$ (Ans:)

6. $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1\}, \quad \rho(x, y) = 2x$

$$m = \int_0^1 \int_0^1 2x \, dx \, dy$$

$$= \int_0^1 \left[\frac{2x^2}{2}\right]_0^1 dy$$

$$= \int_0^1 1 \, dy$$

$$= [y]_0^1$$

$$= 1 - 0$$

$$= 1$$

$$\bar{x} = \frac{1}{m} \iint_D x \cdot \rho(x, y) \, dA$$

$$= \frac{1}{1} \int_0^1 \int_0^1 x \cdot 2x \, dx \, dy$$

$$= \int_0^1 \int_0^1 2x^2 \, dx . dy$$

$$= \int_0^1 \left[\frac{2x^3}{3} \right]_0^1 dy$$

$$= \int_0^1 \frac{2}{3} \, dy$$

$$= \frac{2}{3} \int_0^1 dy$$

$$= \frac{2}{3} \left[y \right]_0^1$$

$$= \frac{2}{3} (1-0) = \frac{2}{3}$$

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$$\bar{y} = \frac{1}{m} \iint_D y \cdot \rho(x,y) \, dA$$

$$= \frac{1}{1} \int_0^1 \int_0^1 y \cdot 2x \cdot dx \, dy$$

$$= 1 \int_0^1 \int_0^1 2xy \, dx \, dy$$

$$= \int_0^1 \left[\frac{2x^2 y}{2} \right]_0^1 dy$$

$$= \int_0^1 y \cdot dy$$

$$= \left[\frac{y^2}{2} \right]_0^1$$

$= \frac{1}{2} - 0$

$= \frac{1}{2}$

$m = 1, \quad \bar{x} = \frac{2}{3}, \quad \bar{y} = \frac{1}{2}$ (Ans.)

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