Welcome!

Subject : PHYSICS – 2 [1203]

Heat & Thermodynamics, Oscillations, Waves & Optics

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Chapter – 16
(Waves-I)
Lesson- 17

16-1 TRANSVERSE WAVE

Types of Waves

Waves are of three main types:

1. **<u>Mechanical waves :</u>**

- most familiar waves because we encounter them almost constantly;
- common examples include water waves, sound waves, and seismic waves.
- All these waves have two central features:

  (i) They are governed by Newton's laws, and

  (ii) they can exist only within a material medium, such as water, air, and rock.

2. Electromagnetic waves

- less familiar waves, but we use them constantly;
- common examples include visible and ultraviolet light, radio and television waves, These waves require no material medium to exist.
- Light waves from stars, for example , travel through the vacuum of space to reach us.
- All electromagnetic waves travel through a vacuum at the same speed c = 2.998×10^8 m/s.

3. Matter waves

- commonly used in modern technology, but unfamiliar.
- These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules.
- Because we commonly think of these particles as constituting matter , such waves are called matter waves.

There are two types of *MECHANICAL WAVES* such as

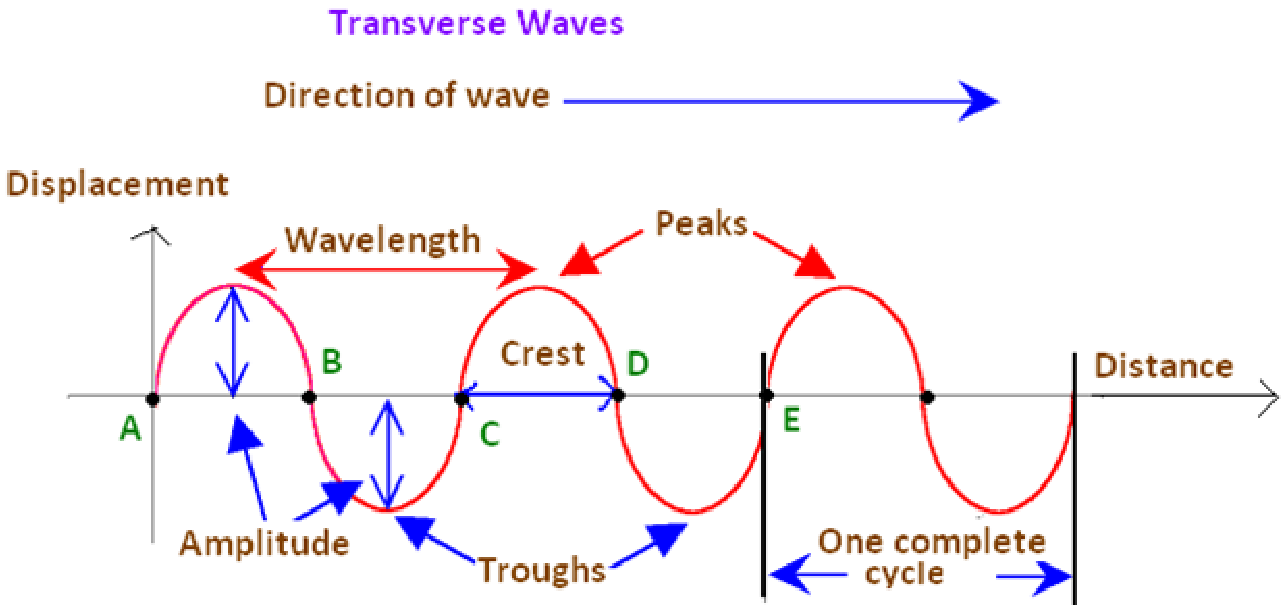
- ***Transverse Wave***
- ***Longitudinal Wave***

1. Transverse Wave

- When the particles of the medium vibrate about their mean positions perpendicular to the direction of propagation, then the wave is called transverse wave.

- The Figure shows a transverse wave. The particles of the medium in transverse wave, move up and down and the wave travels in a horizontal direction.

- Example – When a stone is thrown in water of a pond.



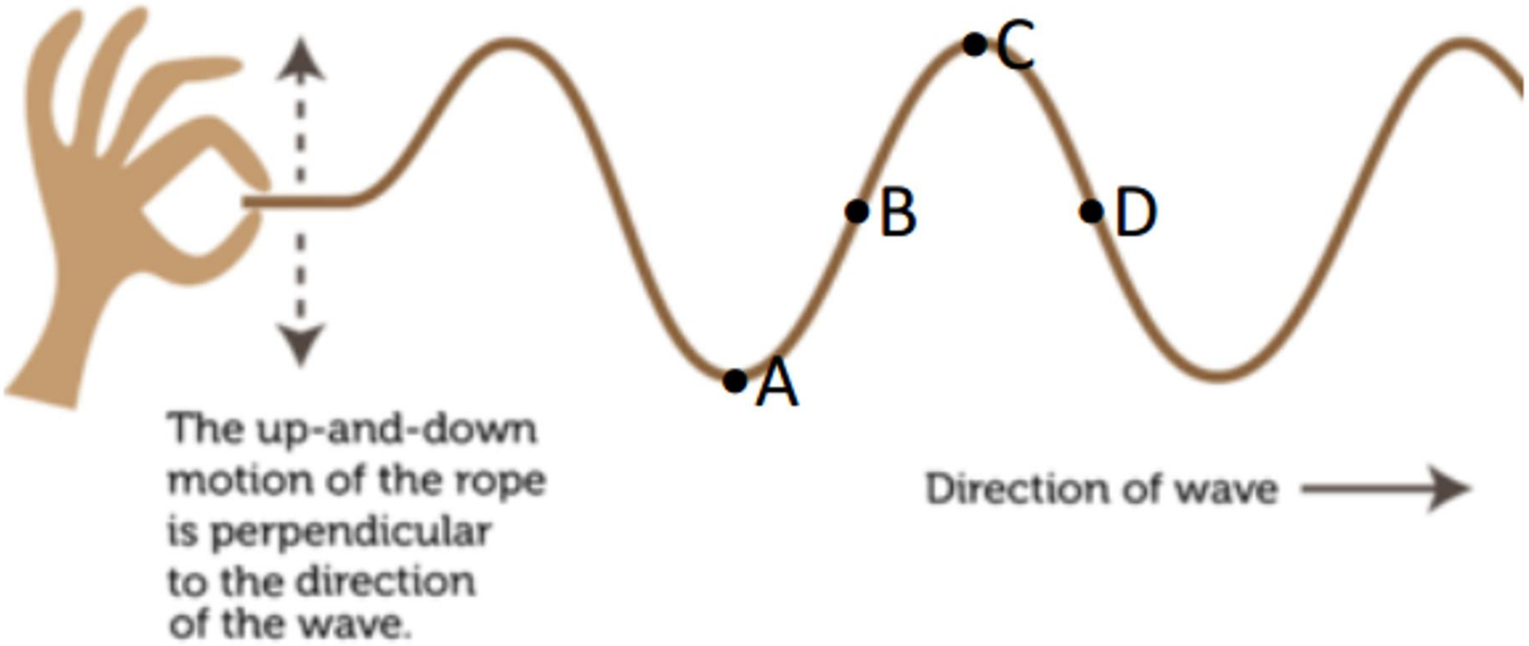
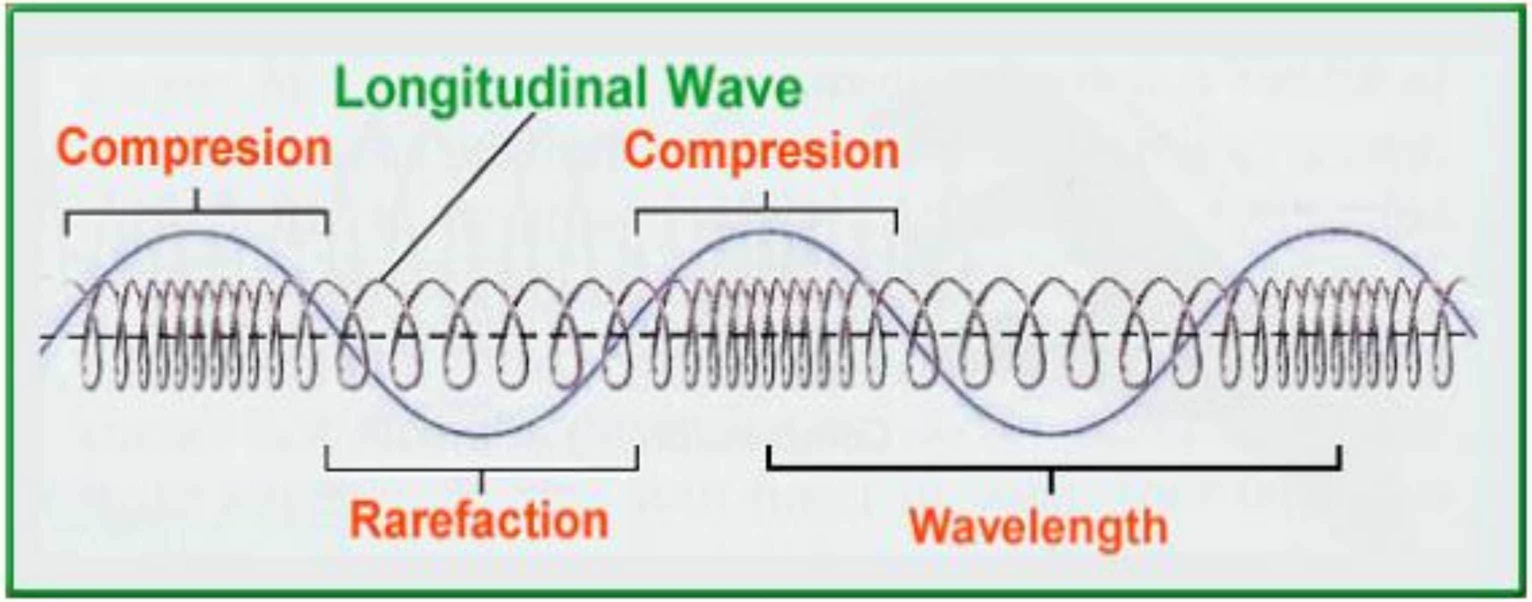


Figure 1: Generating transverse waves on a string

2. Longitudinal Waves

- When particles of the medium vibrate about their mean position parallel to the direction of propagation of the disturbance, the wave is called longitudinal wave.
- The figure shows a longitudinal wave. During flow of the wave compression and refraction of the medium take place
- Example – Waves in a spring, Sound wave etc



Traveling Wave

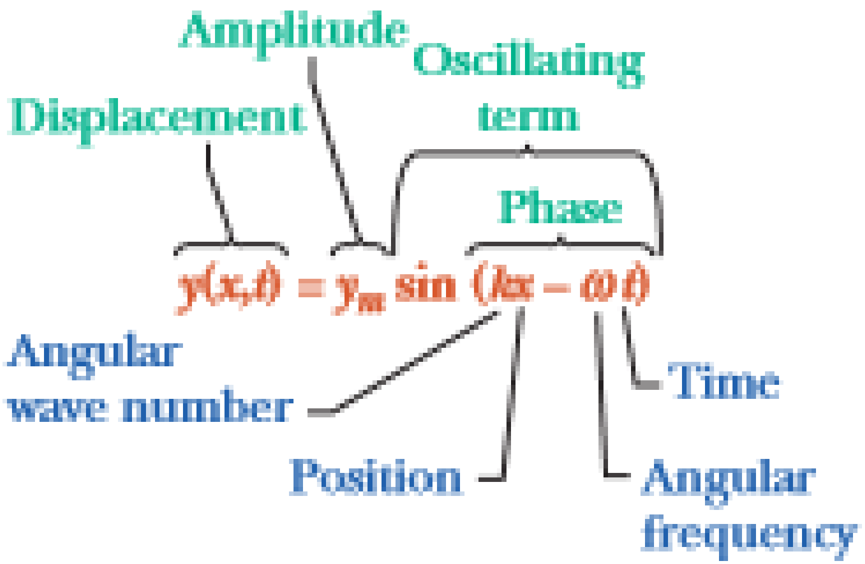
Both a transverse wave and a longitudinal wave are said to be traveling waves because the both travel from one point to another.

In this chapter we will focus on ***Transverse Waves.***

The displacement of a transverse sinusoidal wave :

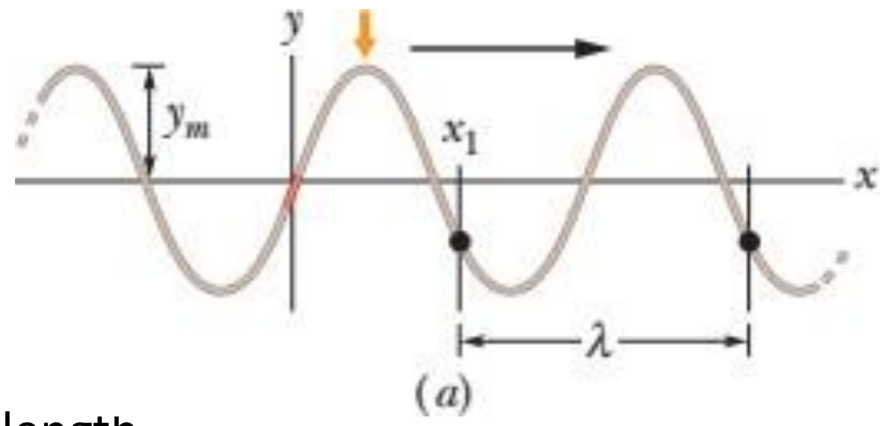
At time *t*, the displacement *y* of the element located at position *x* is given by

$$y(x, t) = y_m \sin(kx - \omega t) \quad \ldots\ldots\ldots\ldots \quad (1)$$



- **Question -** ***From the wave function of a traveling wave, $y(x,t) = y_m \sin(kx - \omega t)$, prove that (i) $k = 2\pi/\lambda$, (ii) $\omega = 2\pi/T$ (iii) $v = +\omega/k$ and (iv) $v = -\omega/k$.***

Solution :



Wavelength and Angular Wave Number :

At time t = 0, Equation (1) becomes

$$y(x, 0) = y_m \sin kx \qquad \text{……….} \ (2)$$

By definition, the displacement y is the same at both ends of this wavelength

— that is,at $x = x_1$ and $x = x_1 + \lambda$ Thus, by Eq.2

$$y_m \sin k\, x_1 = y_m \sin k\,(x_1 + \lambda)$$

$$= y_m \sin(k\, x_1 + k\lambda) \qquad \text{………}(3)$$

A sine function begins to repeat itself when its angle (or argument) is increased by **2 π rad** ,

i. e **$\sin k\, x_1 = \sin(k\, x_1 + 2\pi)$…….. *(a)***

Comparing Eq (3) and Eq (a) we must have **$k\lambda = 2\pi$ *rad*** , or

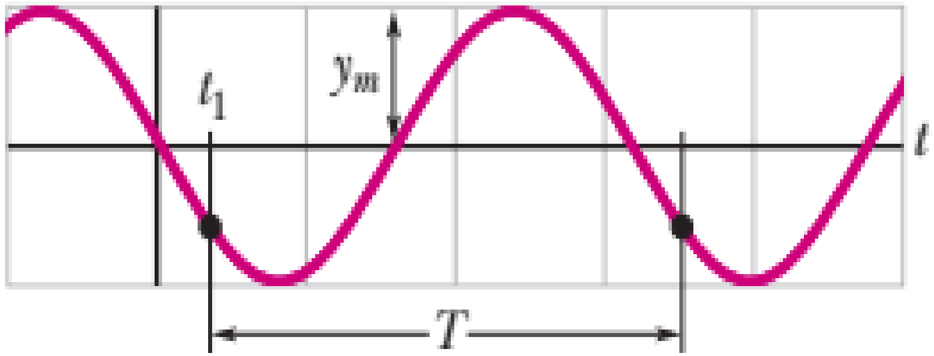
$$k = \frac{2\pi}{\lambda} \qquad \textit{(Angular Wave Number)} \quad \text{……} \ (4)$$

k is called the ***Angular Wave Number*** *of the wave.* ***S.I Unit*** *– Radian/meter.*

$$[\, y(x, t) = y_m \sin(kx - \omega t) \;\ldots\ldots\ldots\ldots\; (1)\,]$$

Angular Frequency

Figure shows a graph of the displacement y of Eq.1 vs time t at a certain position along the string, taken to be x = 0.



Then Eq (1) becomes

$$y(0, t) = y_m \sin(-\omega t)$$

$$= -\; y_m \sin \omega t \;\; ; (x = 0) \ldots. \quad (5)$$

Where , $\sin(-\alpha) = -\sin \alpha$

Period : We define the period of oscillation T of a wave to be the time any string element takes to move through one full oscillation. Applying Eq.5 to both ends of this time interval and equating the results yield

$$-\; y_m \sin \omega t_1 = -\; y_m \sin \omega (t_1 + T)$$

$$= -\; y_m \sin(\omega t_1 + \omega T) \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

This can only be true if $\omega T = 2\pi$ *or if* $\quad [\, \sin \omega t_1 = \sin(\omega t_1 + 2\pi)\,]$ — printed as *[since sin ωt₁ = sin (ωt₁ + 2π)]*

$$\omega = \frac{2\pi}{T} \; ; \; (\textit{Angular Frequency}) \qquad \ldots\ldots.(7)$$

Where, ***ω = Angular Frequency*** *of the wave. S.I Unit –* ***Radian/ second.***

Wave speed

$$kx - \omega t = a \text{ constant.} \quad \ldots\ldots\ldots\ldots. \quad (8)$$

To find the *wave speed v* , we take the derivative of Eq. 8 ,getting

$$\frac{d}{dt}\,[\,kx - \omega t\,] = \frac{d}{dt}\,[\text{constant}\,]$$

$$k\,\frac{dx}{dt} - \omega = 0$$

$$\text{or,} \quad \frac{dx}{dt} = \frac{\omega}{k} = v \quad \ldots\ldots\ldots\ldots \quad (9)$$

Using Eq (4) $[\,k = \frac{2\pi}{\lambda}\,]$ *and Eq (7)* $[\omega = \frac{2\pi}{T}]$, *we can rewrite the wave speed as,*

$$v = \frac{\omega}{k} = \frac{\frac{2\,\pi}{T}}{\frac{2\,\pi}{\lambda}} = \frac{\lambda}{T} = \lambda f \quad (\text{ wave speed}) \quad \ldots\ldots\ldots\ldots \quad (10)$$

- Equation (1) [$y(x, t) = y_m \sin(kx - \omega t)$] describes a wave moving in the positive direction of x.
- We can find the equation of a wave traveling in the opposite direction by replacing ***t*** in Eq.(1) with ***– t*** .This corresponds to the condition

$$kx + \omega t = a\ constant \quad .... (11)$$

which (compare 9) requires that x decrease with time.

- Thus,a wave traveling in the negative direction of x is described by the equation

$$y(x, t) = y_m \sin(kx + \omega t) \quad .... (12)$$

To find the *wave speed v* , we take the derivative of Eq.12 ,getting

$$k\frac{dx}{dt} + \omega = 0 \ ; Or, \ \frac{dx}{dt} = -\frac{\omega}{k} = v$$

$$so. \quad \frac{dx}{dt} = -\frac{\omega}{k} \quad .... (13)$$

The minus sign (compare Eq.10) verifies that the wave is indeed moving in the negative direction of x .

Problem- 1 :

If a wave y(x, t) = (6.0 mm) sin(kx + (600 rad/s)t + φ) travels along a string, how much time does any given point on the string take to move between displacements y = + 2.0 mm and y = - 2.0 mm?

Solution : $y = y_m \sin (kx + \omega t + \varphi)$

1st we have to write both equations

$$2 = 6 \sin(kx + 600\, t_1 + \varphi)$$

$$-2 = 6 \sin(kx + 600\, t_2 + \varphi)$$

Taking arc sin of Both Equations

$$kx + 600\, t_1 + \varphi = \frac{\pi}{180} sin^{-1}(1/3)$$

$$kx + 600\, t_2 + \varphi = \frac{\pi}{180} sin^{-1}(-1/3) = -\frac{\pi}{180} sin^{-1}(1/3)$$

By Subtracting these 2 equations

$$600\ (t_1 - t_2) = \frac{2\pi}{180} sin^{-1}(1/3)$$

Therefore the time taken to move between these 2 displacements is

$$t_1 - t_2 = 0.0011\ s$$

Thank you

For Your ATTENTION!!

(Questions and Answers)