Exercise set 1.1.1

(a) $\int dx = x + c$

(b) $\int x^5 \, dx = \frac{x^{5+1}}{5+1} + c = \frac{x^6}{6} + c$

(c) $\int x^{3/2} \, dx = \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$

$$= \frac{x^{\frac{3+2}{2}}}{\frac{3+2}{2}} + c$$

$$= \frac{x^{5/2}}{5/2} + c$$

$$= \frac{2}{5} x^{5/2} + c$$

(d) $\int \sin(-3x)\,dx = -\frac{1}{-3}\cos(-3x) + c$

$= \frac{1}{3}\cos 3x + c \quad [\because \cos(-\theta) = \cos\theta]$

(e) $\int \cos(2x)\,dx = \frac{1}{2}\sin 2x + c$

(f) $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + c$

(g) $\int e^{2x/3}\,dx = \frac{1}{\frac{2}{3}} e^{2x/3} + c$

$= \frac{3}{2} e^{2x/3} + c$

(h) $\int \exp(-3x)\,dx = -3\exp(-3x) + c$

$$= 3\exp(-3x) + c$$

(i) $\int x^{-1}\,dx = \ln x + c$

(j) $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-3+1}}{-3+1} + c = \frac{x^{-2}}{-2} + c$

$$= -\frac{1}{x^2} + c$$

(k) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$

(l) $$\int \sqrt[3]{y^2}\,dy = \int y^{2\cdot\frac{1}{3}}\,dy = \int y^{2/3}\,dy$$

$$= \frac{y^{2/3+1}}{\frac{2}{3}+1} + c$$

$$= \frac{y^{\frac{2+3}{3}}}{\frac{2+3}{3}} + c$$

$$= \frac{y^{5/3}}{5/3} + c$$

$$= \frac{3}{5} y^{5/3} + c$$

(m) $$\int \frac{1}{r}\,dr = \int r^{-1}\,dr = \ln r + c$$

(n) $$\int \sinh(2x)\,dx = \frac{1}{2}\cosh(2x) + c$$

(o) $\int \cosh(-3x)\,dx = \frac{1}{-3}\sinh(-3x) + c$

$= \frac{1}{3}\sinh(3x) + c \quad [\because \sinh(-\theta) = -\sinh\theta$

(p) $\int (2x+3)^{3/2}\,dx = \frac{(2x+3)^{3/2+1}}{(\frac{3}{2}+1)\times 2} + c$

$= \frac{(2x+3)^{\frac{3+2}{2}}}{(\frac{3+2}{2})\times 2} + c$

$= \frac{(2x+3)^{5/2}}{\frac{5}{\cancel{2}}\times \cancel{2}} + c$

$= \frac{1}{5}(2x+3)^{5/2} + c$

(q) $\int (1-2x)^5 dx = \dfrac{(1-2x)^{5+1}}{(5+1)(-2)} + c$

$= \dfrac{(1-2x)^6}{6(-2)} + c$

$= -\dfrac{1}{12}(1-2x)^6 + c$

(r) $\int \left(\dfrac{x^3+3x^2+3}{x}\right) dx$

$= \int x^2 dx + \int 3x\, dx + 3\int \frac{1}{x} dx$

$= \dfrac{x^{2+1}}{2+1} + 3\dfrac{x^{1+1}}{1+1} + 3\ln(x) + c$

$= \frac{1}{3}x^3 + \frac{3}{2}x^2 + 3\ln(x) + c$

(s) $\int (x+2)^{-3/2}dx = \dfrac{(x+2)^{-3/2+1}}{-\frac{3}{2}+1}+c$

$$= \dfrac{(x+2)^{\frac{-3+2}{2}}}{\frac{-3+2}{2}} + c$$

$$= \dfrac{(x+2)^{-1/2}}{-\frac{1}{2}} + c$$

$$= -2\,(x+2)^{-1/2} + c$$

(t) $\int \dfrac{1}{3x-1}\,dx = \dfrac{\ln(3x-1)}{3} + c$

(u) $\int \cos(3x-2)\,dx = \frac{1}{3}\sin(3x-2)+c$

(v) $\int \sin(1-2x)\,dx = -\dfrac{1}{-2}\cos(1-2x)+c$

$$= \frac{1}{2}\cos(1-2x)+c$$

(w) $\int \exp(-3x+1)\,dx = -\frac{1}{3}\exp(-3x+1)+c$

(x) $\int \frac{3}{9+(x-2)^2}\,dx$

$= \int \frac{3}{3^2+(x-2)^2}\,dx$

$= \frac{3}{3}\tan^{-1}\left(\frac{x-2}{3}\right)+c$

$= \tan^{-1}\left(\frac{x-2}{3}\right)+c$

(y) $\int \frac{1}{\sqrt{4-(x+1)^2}}\,dx = \int \frac{1}{\sqrt{(2)^2-(x+1)^2}}\,dx$

$= \sin^{-1}\frac{x+1}{2}+c$

(2) $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

$$= \int \frac{1}{\sqrt{2^2-(3x)^2}}\,dx$$

$$= \frac{\sin^{-1}\frac{3x}{2}}{3} + C$$