1. $\frac{dx}{dt} = 2x+2y$ —— ①

$\frac{dy}{dt} = x+3y$ —— ②

Using differential operator $D \equiv \frac{d}{dt}$, we can write

$Dx = 2x+2y$

$Dy = x+3y$

This system can be written as:

$(D-2)x - 2y = 0$ —— ③

$x - (D-3)y = 0$ —— ④

Multiplying eqn ④ by $(D-2)$

$(D-2)x - (D-2)(D-3)y = 0$

$\Rightarrow (D-2)x - (D^2-5D+6)y = 0$ —— ⑤

Subtracting ⑤ from ③, we have:

$-2y + (D^2-5D+6)y = 0$

$\Rightarrow (D^2-5D+4)y = 0$ —— ⑥

$\Rightarrow$ Now the auxiliary eqn of ⑥ is

$m^2 - 5m + 4 = 0$

$\Rightarrow m^2 - 4m - m + 4 = 0$

$\Rightarrow m(m-4) - 1(m-4) = 0$

$\Rightarrow (m-4)(m-1) = 0$

$\therefore m = 1, 4$

Thus $y(t) = Ae^{t} + Be^{4t}$

substituting y in eq^n ② we get

$$\frac{d}{dt}(Ae^{t}+Be^{4t}) = x+3(Ae^{t}+Be^{4t})$$

$$\Rightarrow Ae^{t}+4Be^{4t} = x+3Ae^{t}+3Be^{4t}$$

$\Rightarrow \cancel{(Ae}$

$$\Rightarrow x = Ae^{t}+4Be^{4t}-3Ae^{t}-3Be^{4t}$$

$$\therefore x(t) = Be^{4t}-2Ae^{t}$$

Therefore the required sol^n are:

$$x(t) = Be^{4x}-2Ae^{t}$$

$$y(t) = Ae^{t}+Be^{4t}$$

2. $\frac{dx}{dt} = 2x+3y$ ——— ①

$\frac{dy}{dt} = -4y$ ——— ②

Using differential operator $D \equiv \frac{d}{dt}$, we can write

$$Dx = 2x+3y$$

$$Dy = -4y$$

This system can be written as:

$(D-2)x-3y=0$ ——— ③

$(D+4)y = 0$ ——— ④

Multiplying eqn 4 by 3 & eqn 3 by $(D+4)$ we get:

$$(D+4)(D-2)x-(D+4)3y=0 \quad \text{—— } 5$$

$$(D+4)3y=0 \quad \text{—— } 6$$

Adding eqn 5 & 6 we get:

$$(D+4)(D-2)x=0$$

$$\Rightarrow (D^2+4D-2D-6)x=0$$

$$\Rightarrow (D^2+2D-6)x=0$$

So the auxiliary eqn becomes

$$m^2+2m-6=0$$

$$\therefore m=-4,2$$

$$\therefore x(t)=c_1e^{-4t}+c_2e^{2t}$$

substuting this value in eqn 1 we get:

$$\frac{d}{dt}[c_1e^{-4t}+c_2e^{2t}]=2(c_1e^{-4t}+c_2e^{2t})+3y$$

$$\Rightarrow -4c_1e^{-4t}+2c_2e^{2t}-2c_1e^{-4t}-2c_2e^{2t}=3y$$

$$\Rightarrow 3y=-6c_1e^{-4t}$$

$$\therefore y(t)=-2c_1e^{-4t}$$

so the required solⁿ are

$$x(t)=c_1e^{-4t}+c_2e^{2t}$$

$$y(t)=-2c_1e^{-4t}$$

3. $\frac{dx}{dt} = -x+y$ ——— (1)

$\frac{dy}{dt} = -3x-5y$ ——— (2)

Using differential operator $D \equiv \frac{d}{dt}$, we can write

$Dx = -x+y$ ——— (3)

$Dy = -3x-5y$ ——— (4)

This system can be written as:

$$(D+1)x - y = 0 \quad \text{——— (5)}$$

$$3x + (D+5)y = 0 \quad \text{——— (6)}$$

Multiplying eqn (5) by 3 & eqn (6) by $(D+1)$ we get:

$$(D+1)3x - 3y = 0 \quad \text{——— (7)}$$

$$(D+1)3x + (D+1)(D+5)y = 0 \quad \text{——— (8)}$$

Subtracting eqn ⑦ from eqn ⑧ we get:

$$(D+1)(D+5)y+3y=0$$

$$\Rightarrow (D^2+6D+5+3)y=0$$

$$\Rightarrow (D^2+6D+8)y=0$$

So the auxiliary eqn becomes:

$$m^2+6m+8=0$$

$$\Rightarrow m^2+4m+2m+8=0$$

$$\Rightarrow m(m+4)+2(m+4)=0$$

$$\Rightarrow (m+2)(m+4)=0$$

$$\therefore \quad m=-2,-4$$

So $y(t)=Ae^{-2t}+Be^{-4t}$

putting the value of $y(t)$ in eqn ② we get:

$$\frac{d}{dt}\left(Ae^{-2t}+Be^{-4t}\right)=-3x-5\left(Ae^{-2t}+Be^{-4t}\right)$$

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$$\Rightarrow -\cancel{2}Ae^{-2t} - 4Be^{-4t} = -3x - 5Ae^{-2t} - \cancel{5}Be^{-4t}$$

$$\Rightarrow 3x = 2Ae^{-2t} + 4Be^{-4t} - 5Ae^{-2t} - \cancel{5}Be^{-4t}$$

$$\Rightarrow 3x = -\cancel{1}Be^{-4t} - 3Ae^{-2t}$$

$$\therefore \; x(t) = -\frac{\cancel{1}}{\cancel{3}3}Be^{-4t} - Ae^{-2t}$$

So the required sol^{n}s are

$$\cancel{x(t) = -\frac{1}{3}Be^{-4t} - Ae^{-2t}}$$

$$y(t) = Ae^{-2t} + Be^{-4t}$$

$$x(t) = -\frac{1}{3}Be^{-4t} - Ae^{-2t}$$