Exercise set 1.1.3

(a) $\int \frac{1}{x+2}\,dx = \ln(x+2) + C$

(b) $\int \frac{3x^2+2}{x^3+2x}\,dx$

Set, $u = x^3+2x$

Then $du = (3x^2+2)\,dx$

$\therefore dx = \frac{du}{3x^2+2}$

Thus, $\int \frac{3x^2+2}{x^3+2x}\,dx$

$= \int \frac{(3x^2+2)}{u} \cdot \frac{du}{(3x^2+2)}$

$= \int \frac{du}{u}$

$= \ln u + c$

$= \ln(x^3+2x) + c$

(c) $\int \frac{2x-1}{x^2-x+3}\,dx$

$= \int \frac{(2x-1)}{u} \cdot \frac{du}{(2x-1)}$

$= \int \frac{du}{u} = \ln u + C = \ln(x^2-x+3) + c$

Set, $u = x^2-x+3$

$du = (2x-1)\,dx$

$\therefore dx = \frac{du}{2x-1}$

(d) $\int \frac{2x+\sin x}{x^2-\cos x}\,dx$

$= \int \frac{(2x+\sin x)}{u} \cdot \frac{du}{(2x+\sin x)}$

$= \int \frac{du}{u} = \ln u + c$

$= \ln(x^2-\cos x)+c$

Set, $u = x^2 - \cos x$

$du = (2x+\sin x)\,dx$

$\therefore dx = \frac{du}{2x+\sin x}$

(e) $\int \frac{1+e^{-t}}{t-e^{-t}}\,dt$

$= \int \frac{(1+e^{-t})}{u} \cdot \frac{du}{(1+e^{-t})}$

$= \int \frac{du}{u} = \ln u + c = \ln(t-e^{-t})+c$

Set, $u = t - e^{-t}$

$du = (1+e^{-t})\,dt$

$\therefore dt = \frac{du}{(1+e^{-t})}$

(f) $\int \frac{1}{2x+3}\,dx$

$= \int \frac{1}{2x+3} \cdot \frac{du}{2}$

$= \int \frac{1}{u} \cdot \frac{du}{2}$

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln(2x+3)+c$

Set, $u = 2x+3$

$du = 2\,dx$

$dx = \frac{du}{2}$

(g) $\int \frac{x^2+2x}{x^3+3x^2+1}\,dx$

$= \int \frac{(x^2+2x)}{u} \cdot \frac{du}{3(x^2+2x)}$

$= \frac{1}{3} \int \frac{du}{u}$

$= \frac{1}{3} \ln u + c$

$= \frac{1}{3} \ln(x^3+3x^2+1) + c$

Set, $u = x^3+3x^2+1$

$du = (3x^2+3.2.x)dx$

$du = 3(x^2+2x)dx$

$\therefore dx = \frac{du}{3(x^2+2x)}$

(h) $\int \frac{\cos 3x}{3+\sin 3x}\,dx$

Set, $u = 3+\sin 3x$

Then, $du = 3\cos 3x\, dx$

$\therefore dx = \frac{du}{3\cos 3x}$

Thus. $\int \frac{\cos 3x}{3+\sin 3x}\,dx = \int \frac{\cos 3x}{u} \cdot \frac{du}{3\cos 3x}$

$= \int \frac{du}{3u} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln u + c$

$= \frac{1}{3} \ln(3+\sin 3x) + c$

(i) $\displaystyle\int \frac{\sec^2 3x}{2+\tan 3x}\,dx$

Set $\quad u = 2+\tan 3x$

Then, $\quad du = 3\sec^2 3x\,dx$

$\quad\therefore\; dx = \dfrac{du}{3\sec^2 3x}$

Thus $\displaystyle\int \frac{\sec^2 3x}{2+\tan 3x}\,dx = \int \frac{\sec^2 3x}{u}\cdot\frac{du}{3\sec^2 3x}$

$\displaystyle = \int \frac{du}{3u} = \frac{1}{3}\int\frac{du}{u} \cancel{= \frac{1}{3}\ln u}$

$= \frac{1}{3}\ln u + c$

$= \frac{1}{3}\ln(2+\tan 3x) + c$

(j) $\displaystyle\int \frac{e^{3x}}{3-2e^{3x}}\,dx$

Set, $\quad u = 3-2e^{3x}$

Then, $\quad du = -2.3.e^{3x}\,dx$

$\quad\therefore\; dx = -\dfrac{du}{6e^{3x}}$

Thus, $\displaystyle\int \frac{e^{3x}}{3-2e^{3x}}\,dx = \int \frac{e^{3x}}{u}\cdot\left(-\frac{du}{6e^{3x}}\right)$

$\displaystyle = -\frac{1}{6}\int\frac{du}{u}) = -\frac{1}{6}\ln u + c$

$= -\frac{1}{6}\ln(3-2e^{3x}) + c$

(k) $\displaystyle\int \cot 3z\,dz = -\frac{1}{3}\operatorname{cosec}^2 3z + c$

(2) $\int \frac{1}{y(1+\ln y)} dy$

$= \int \frac{1}{y\,u} \cdot y\,du$

$= \int \frac{du}{u}$

$= \ln u + c$

$= \ln(1+\ln y) + c$

set, $u = 1 + \ln y$

$du = \frac{dy}{y}$

$\therefore dy = y\,du$