Example 1.1.2

5. $\int (2x+3)^3\, dx$

$= \dfrac{(2x+3)^{3+1}}{(3+1) \times 2} + c$

$= \dfrac{(2x+3)^4}{4 \cdot 2} + c$

$= \dfrac{(2x+3)^4}{8} + c$

Example set 1.1.1

1. $\int \left(3x^4 - \dfrac{1}{\sqrt{x}} + 1\right) dx$

$= \int 3x^4\, dx - \int \dfrac{1}{\sqrt{x}}\, dx + \int dx$

$= 3\dfrac{x^{4+1}}{4+1} - \int x^{-1/2}\, dx + x + c$

$= \dfrac{3x^5}{5} - \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + x + c$

$= \dfrac{3x^5}{5} - \dfrac{x^{-\frac{1+2}{2}}}{\frac{-1+2}{2}} + x + c$

$= \dfrac{3x^5}{5} - \dfrac{x^{1/2}}{\cdot \frac{1}{2}} + x + c$

$= \dfrac{3x^5}{5} - 2\sqrt{x} + x + c$

2. $\int (3\cos 4x - 5e^{3x})dx$

$= 3\int \cos 4x - 5\int e^{3x}dx$

$= \frac{3}{4}\sin 4x - \frac{5}{3}e^{3x}\cancel{dx} + c$

3. $\int (10x^4 - 2\sec^2 x\, dx$

$= 10\int x^4 dx - 2\int \sec^2 x\, dx$

$= 10\frac{x^{4+1}}{4+1} - 2\tan x + c$

$= \frac{10x^5}{5} - 2\tan x + c$

$= 2x^5 - 2\tan x + c$

4. $\int \frac{\cos\theta}{\sin^2\theta}d\theta = \int \frac{1}{\sin\theta}\cancel{d\theta}\cdot\frac{\cos\theta}{\sin\theta}d\theta$

$= \int \csc\theta\cdot\cot\theta\, d\theta$

$= -\csc\theta + c$

5. $\int 2x^3 - 6x + \frac{3}{x^2+1} \, dx$

$= 2\int x^3 dx - 6\int x \, dx + 3\int \frac{1}{x^2+1} dx$

$= 2\frac{x^{3+1}}{3+1} - 6\frac{x^{1+1}}{1+1} + 3 \cdot \tan\left(\frac{x}{1}\right) + c$

$= \frac{2x^4}{4} - 6\frac{x^2}{2} + 3\tan^{-1}x + c$

$= \frac{x^4}{2} - 3x^2 + 3\tan^{-1}x + c$

6. $\int \left(\frac{2+t^2 + t^2\sqrt{t} + 2}{t^2}\right) dt$

$= 2\int \frac{t^2}{t^2} dt + \int \frac{t^2 t^{1/2}}{t^2} dt + 2\int \frac{dt}{t^2}$

$= 2\int dt + \int t^{1/2} dt + 2\int t^{-2} dt$

$= 2t + \frac{t^{1/2+1}}{t^{1/2+1}} + 2 \cdot \frac{t^{-2+1}}{-2+1} + c$

$= 2t + \frac{t^{\frac{1+2}{2}}}{\frac{1+2}{2}} + 2 \cdot \frac{t^{-1}}{-1} + c$

$= 2t + \frac{t^{3/2}}{\frac{3}{2}} - 2\frac{1}{t} + c$

$= 2t + \frac{2}{3}t^{3/2} - \frac{2}{t} + c$