Exercise 2.1

1(a) $y = x$ —— ① & $y = 0$; —— ②

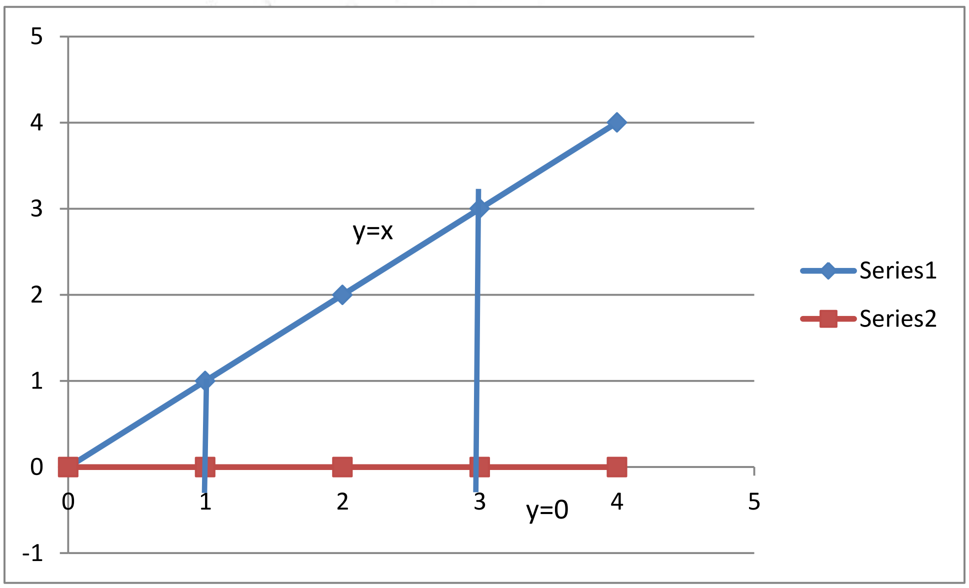
So the area is,

$$A = \int_1^3 (x - 0)\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^3$$

$$= \frac{3^2}{2} - \frac{1}{2}$$

$$= 4$$



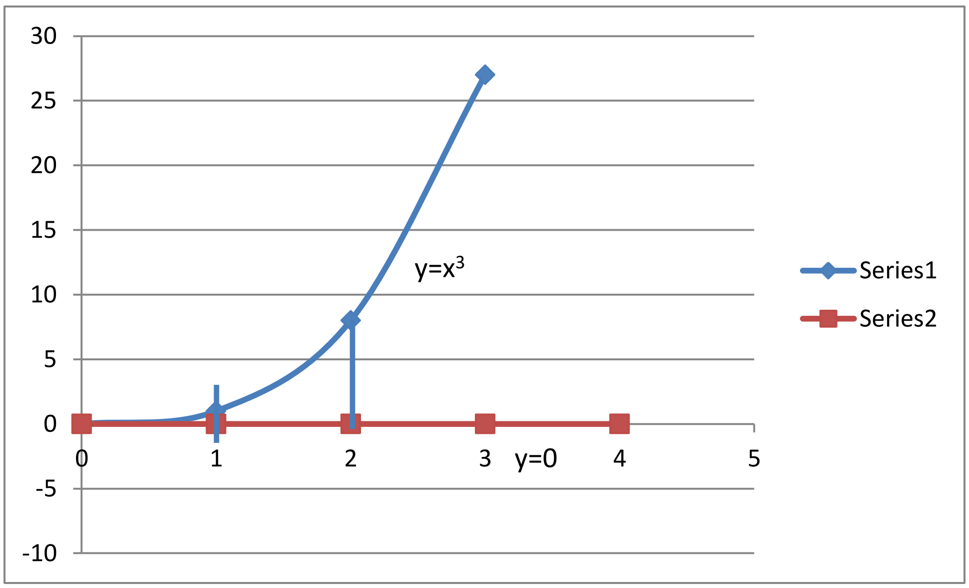
(b) $y = x^3$ —— ① $y = 0$ —— ②

So the area is,

$$A = \int_1^2 (x^3 - 0)\,dx$$

$$= \left[\frac{x^4}{4}\right]_1^2$$

$$= \frac{2^4}{4} - \frac{1^4}{4} = \frac{15}{4}$$



(c) $y = x^2+x+4 \quad , 1 \leq x \leq 3$

So the area is,

$$A = \int_1^3 (x^2+x+4)\,dx$$

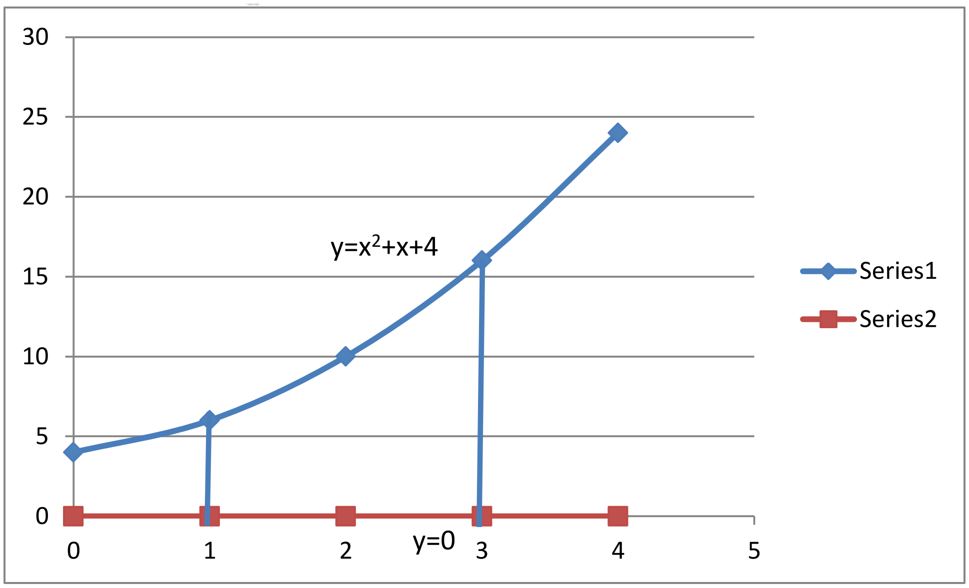
$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + 4x\right]_1^3$$

$$= \left(\frac{3^3}{3} + \frac{3^2}{2} + 4.3\right) - \left(\frac{1^3}{3} + \frac{1^2}{2} + 4.1\right)$$

$$= \left(9 + \frac{9}{2} + 12\right) - \left(\frac{1}{3} + \frac{1}{2} + 4\right)$$

$$= \frac{54+27+72-2-3-24}{6}$$

$$= \frac{124}{6} = 20'66$$



(d) $y = \sin x$ — ① $0 \le x \le \frac{3\pi}{2}$

$y = 0$ — ②

So the area is,

$$A = \int_0^{\pi} (\sin x - 0)\,dx + \int_{\pi}^{3\pi/2} (0 - \sin x)\,dx$$

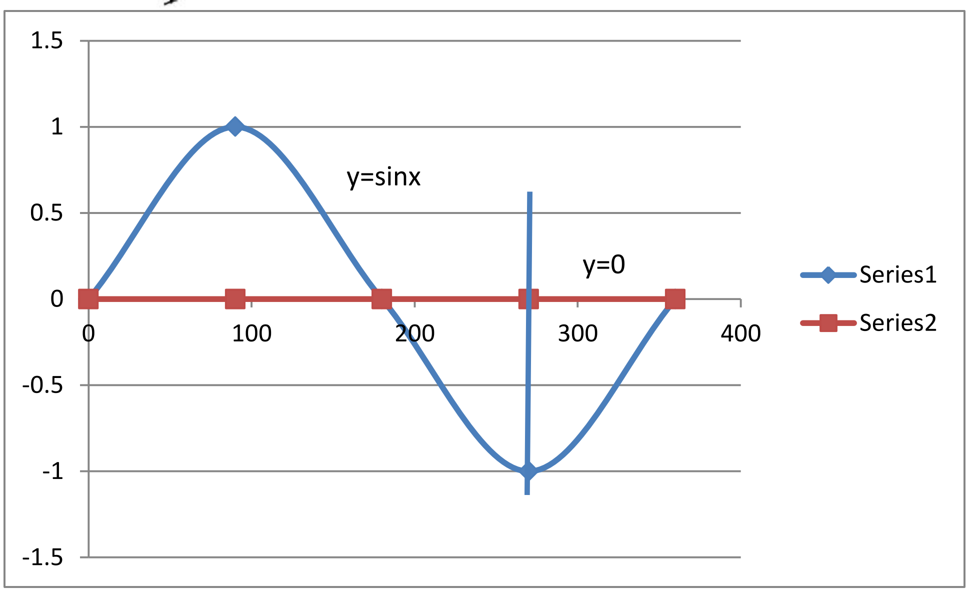
$$= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{3\pi/2} \sin x\,dx$$

$$= \left[-\cos x\right]_0^{\pi} + \left[-\cos x\right]_{\cancel{\pi}}^{3\pi/2}$$

$$= -\cos\pi + \cos 0 - \cos\frac{3\pi}{2} + \cancel{\cos 0}\ \cos\pi$$

$$= 1 - 0$$

$$\therefore A = 1$$



(f) $y = x^2 - 4$ —① & $y = 0$ —②

From eqn ① & ②

$x^2 - 4 = 0$

$x = -2, 2$

So the area is,

$$A = \int_{-2}^{2} [0 - (x^2 - 4)]\,dx$$

$$= \int_{-2}^{2} (-x^2 + 4)\,dx$$

$$= 2\int_{0}^{2} (-x^2 + 4)\,dx$$

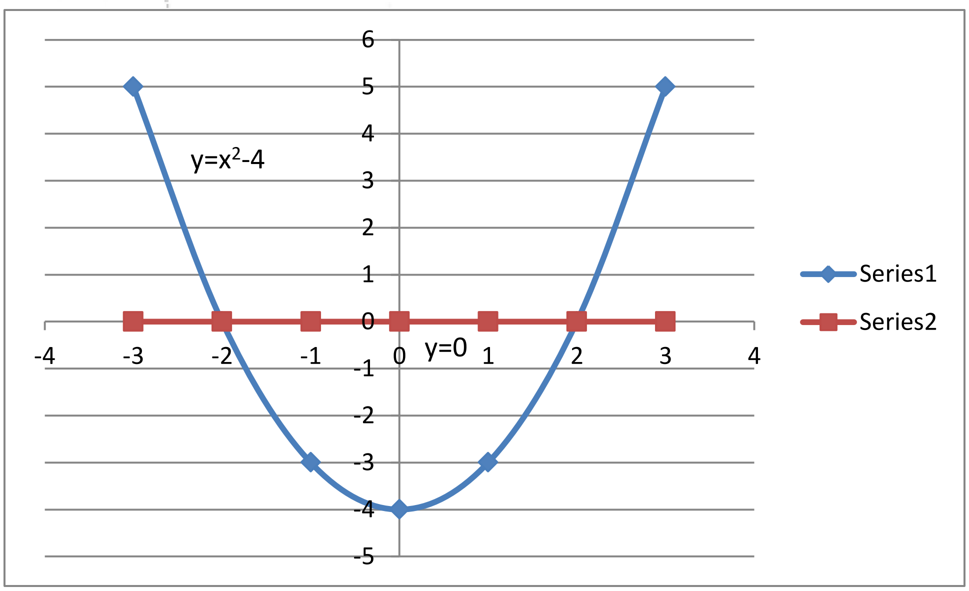
$$= 2\left[-\frac{x^3}{3} + 4x\right]_0^2$$

$$= 2\left(-\frac{2^3}{3} + 4.2\right)$$

$$= 2\left(\frac{-8 + 24}{3}\right)$$

$$= 2 \cdot \frac{16}{3}$$

$$= \frac{32}{3}$$



(9) $x = 1 - y^2$ —① $x = 0$ —②

From eqn ① & ②

$1 - y^2 = 0 \quad \therefore y = -1, 1$

So the area is

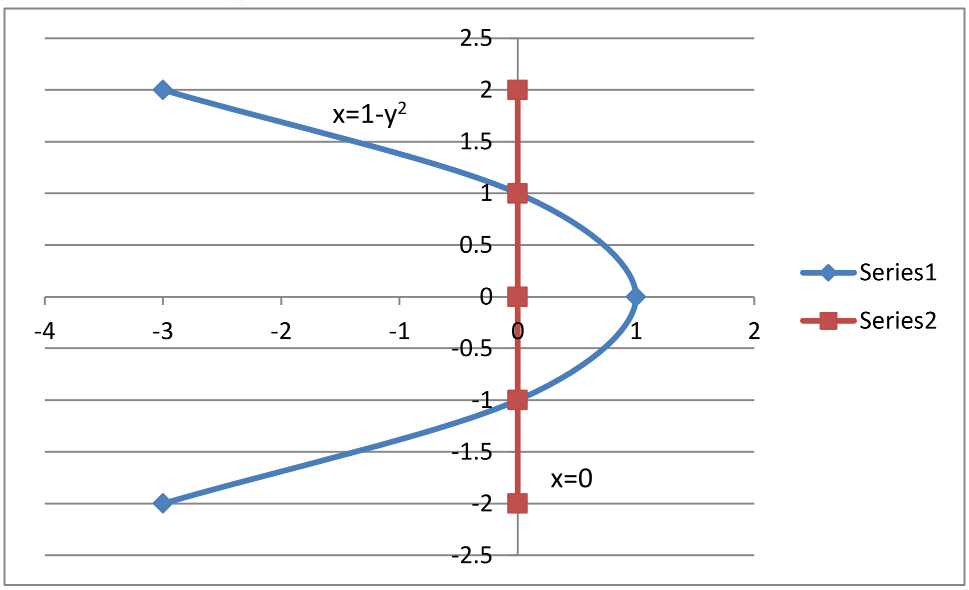
$$A = \int_{-1}^{1} (1 - y^2 - 0)\,dy$$

$$= 2\int_0^1 (1 - y^2)\,dy$$

$$= 2\left[y - \frac{y^3}{3}\right]_0^1$$

$$= 2\left(1 - \frac{1^3}{3}\right)$$

$$\therefore A = \frac{4}{3}$$



2(a) $y = x^2$ —— ① $y = x$ —— ②

From eqn ① & ②

$x^2 = x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0 \quad \therefore x = 0, 1$

So the area is

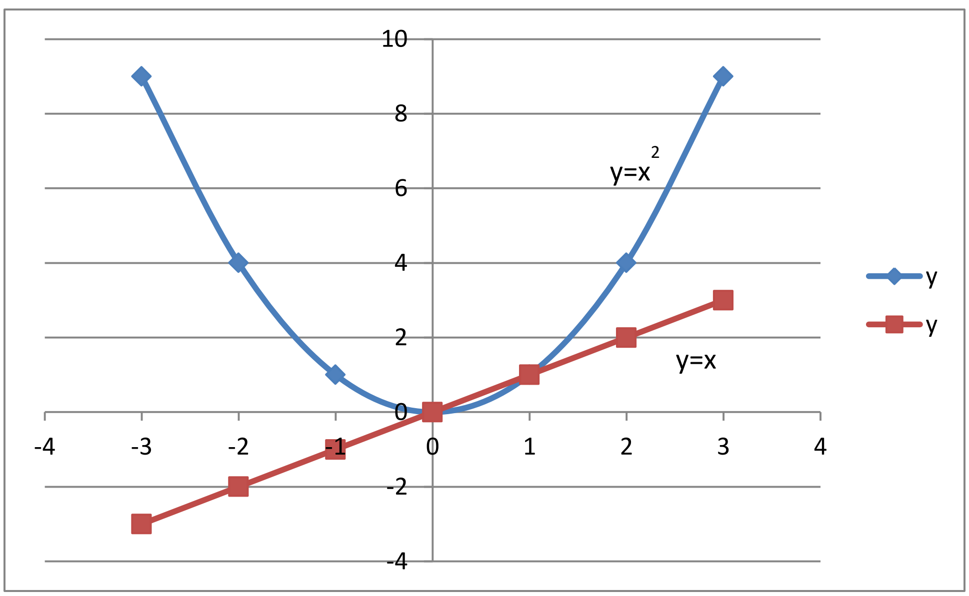
$A = \int_0^1 (\text{upper function}) - (\text{lower function})\,dx$

$= \int_0^1 (x - x^2)\,dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$\therefore A = \frac{1}{6}$



(b) $y = x(x-3)$ —① & $y = 0$ —②

From, eqn ① & ②

$x(x-3) = 0 \quad \therefore x = 0, 3$

so the area is,

$$A = \int_0^3 (0 - x(x-3))\,dx + \int_3^5 (x(x-3) - 0)\,dx$$

$$= \int_0^3 (-x^2 + 3x)\,dx + \int_3^5 (x^2 - 3x)\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3 + \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_3^5$$

$$= -\frac{3^3}{3} + \frac{3.3^2}{2} + \frac{5^3}{3} - \frac{3.5^2}{2} - \frac{3^3}{3} + \frac{3.3^2}{2}$$

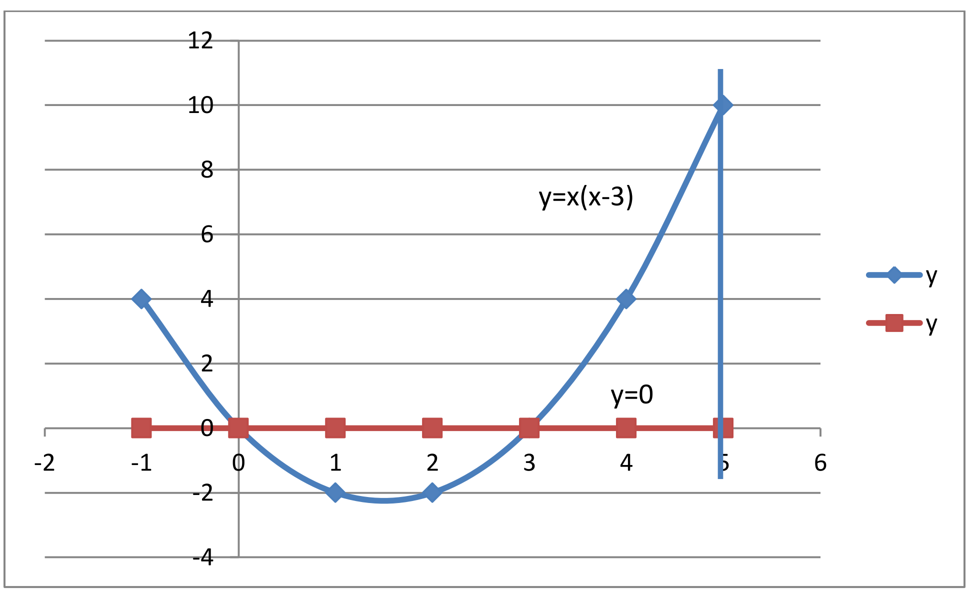
$$= -9 + \frac{27}{2} + \frac{125}{3} - \frac{75}{2} - 9 + \frac{27}{2}$$

$$= \cancel{\frac{-56 + 27 + 125 -}{6}}$$

$$= \frac{-54 + 81 + 250 - 225 - 54 + 81}{6}$$

$$= \frac{412 - 333}{6}$$

$$\therefore A = \frac{79}{6}$$



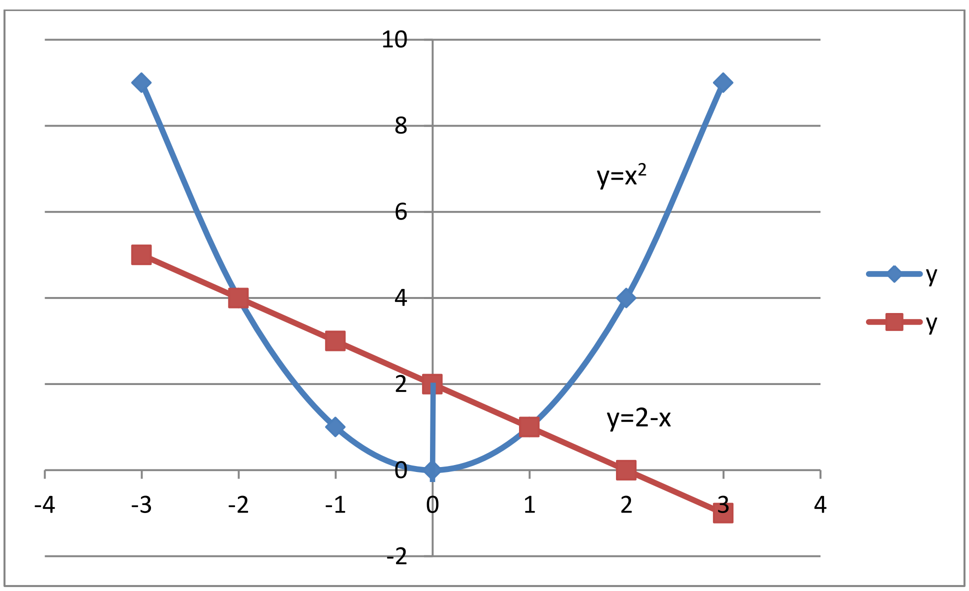
(c) $y = x^2$ —① & $y = 2 - x$ —②

From eqⁿ ① & ②

$$x^2 = 2 - x$$
$$\Rightarrow x^2 + x - 2 = 0$$
$$\Rightarrow x^2 + 2x - x - 2 = 0$$
$$\Rightarrow x(x+2) - 1(x+2) = 0$$
$$\Rightarrow (x+2)(x-1) = 0 \quad \therefore x = -2, 1$$

So the area is,

$$A = \int_0^1 [(2-x) - x^2]\, dx$$
$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= 2.1 - \frac{1^2}{2} - \frac{1^3}{3}$$
$$= \frac{7}{6}$$



(d) $y = 3x - x^2$ —① $\quad y = x$ —②

From eqn ① & ②:

$3x - x^2 = x$

$\Rightarrow x^2 - 3x + x = 0.$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0 \quad \therefore x = 0, 2$

So the area is:

$$A = \int_0^2 (\text{upper function} - \text{lower function})\,dx$$

$$= \int_0^2 (3x - x^2 - x)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

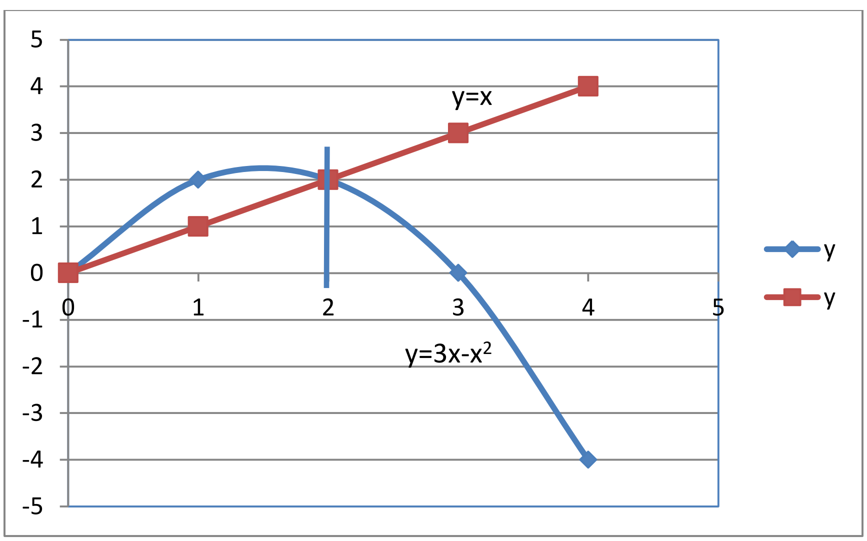
$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= 2^2 - \frac{2^3}{3}$$

$$= 4 - \frac{8}{3}$$

$$= \frac{12-8}{3}$$

$$= \frac{4}{3}$$



(e) $x = y^2$ — (1) & $y = x-2$ — (2)

$\therefore y = \sqrt{x}$ — (3)

From eqn (2) & (3)

$(x-2) = \sqrt{x}$

$\Rightarrow x^2 - 4x + 4 = x$

$\Rightarrow x^2 - 4x - x + 4 = 0,$

$\Rightarrow \cancel{x^2}\ x(x-4) - 1(x-4) = 0$

$\Rightarrow (x-4)(x-1) = 0 \quad \therefore x = 1, 4$

So the area is:

$$A = \int_1^4 (\sqrt{x} - x + 2)\,dx$$

$$= \int_1^4 (x^{1/2} - x + 2)\,dx$$

$$= \left[\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^2}{2} + 2x\right]_1^4$$

$$= \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right) - \frac{1}{2}\left(4^2 - 1^2\right) + 2(4-1)$$

$$= \frac{2}{3}(8-1) - \frac{1}{2}(16-1) + 2\cdot 3$$

$$= \frac{2}{3}\cdot 7 - \frac{1}{2}\cdot 15 + 6$$

$$= \frac{14}{3} - \frac{15}{2} + 6$$

$$= \frac{28 - 45 + 36}{6}$$

$$= \frac{64 - 45}{6}$$

$$= \frac{19}{6}$$