Exercise set 1.2.2

(a) $\int_1^3 (x^2\sqrt{x} + 2e^x + 1)\,dx$

$= \int_1^3 (x^2 x^{1/2} + 2e^x + 1)\,dx$

$= \int_1^3 (x^{2+\frac{1}{2}} + 2e^x + 1)\,dx$

$= \int_1^3 (x^{5/2} + 2e^x + 1)\,dx$

$= \left[\frac{x^{5/2+1}}{\frac{5}{2}+1} + 2e^x + x\right]_1^3$

$= \left[\frac{2}{7}x^{7/2} + 2e^x + x\right]_1^3$

$= \left[\frac{2}{7}3^{7/2} + 2e^3 + 3\right] - \left[\frac{2}{7} + 2e + 1\right]$

$= (13.36 + 40.17 + 3) - (0.28 + 5.43 + 1)$

$= 56.53 - 6.71$

$= 49.82$

(b) $\int_0^{\pi/2} (\sin 3x + \cos 3x)\,dx$

$= \left[-\frac{1}{3}\cos 3x + \frac{1}{3}\sin 3x\right]_0^{\pi/2}$

$= \frac{1}{3}\left[\left(-\cos 3.\frac{\pi}{2} + \sin 3.\frac{\pi}{2}\right) - \left(-\cos 3.0 + \sin 3.0\right)\right]$

$= \frac{1}{3}\left[-(-0.996) + 0.088) - (-1 + 0)\right]$

$= \frac{1}{3} \times 2.0843$

$= 0.694$

(c) $\int_1^2 \frac{(1+\ln x)^5}{x}\,dx$

$= \int_1^{1.693} \frac{u^5}{x} \cdot x\,du$

$= \left[\frac{u^6}{6}\right]_1^{1.693}$

$= \frac{1}{6}\left[(1.693)^6 - 1^6\right]$

$= \frac{1}{6}(23.55 - 1)$

$= \frac{22.559}{6}$

$= 3.759$

set, $u = 1+\ln x$

$du = \frac{1}{x}\,dx$

$\therefore dx = x\,du$

x	u
1	1
2	1.693

(d) $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$

$= \int_0^1 \frac{e^x}{1+(e^x)^2}\,dx$

$= \int_1^e \frac{u}{1+u^2} \cdot \frac{du}{e^x}$

$= \int_1^e \frac{u}{1+u^2} \frac{du}{u}$

$= \int_1^e \frac{du}{1+u^2}$

$= \left[\tan^{-1} u\right]_1^e = \tan^{-1} e - \tan^{-1} 1$

~~= 0 45~~ $= 69.8^\circ - 45^\circ$

$= 24.8^\circ$

set, $u = e^x$

$du = e^x\,dx$

$dx = \frac{du}{e^x}$

x	u
0	1
1	e

(e) $\int_0^5 \frac{dx}{25+x^2} = \int_0^5 \frac{dx}{5^2+x^2} = \left[\frac{1}{5}\tan^{-1}\frac{x}{5}\right]_0^5$

$= \frac{1}{5}\left[\tan^{-1}\frac{5}{5} - \tan^{-1}\frac{0}{5}\right]$

$= \frac{1}{5}\left[\tan^{-1}1 - \tan^{-1}0\right]$

$= \frac{1}{5} \cdot 45^\circ$

$= 9^\circ$

(f) $\int_{-1}^{2} \sqrt{3-x}\, dx$

$= \int_4^1 \sqrt{u}\,(-du)$

$= -\int_4^1 u^{1/2}\, du$

$= -\left[\frac{u^{3/2}}{\frac{3}{2}}\right]_4^1$

$= -\frac{2}{3}\left[1^{3/2} - 4^{3/2}\right]$

$= -\frac{2}{3}(1-8) = \frac{2}{3} \cdot 7 = \frac{14}{3}$

set, $u = 3-x$

$du = -\cancel{\ } dx$

$dx = -\frac{du}{\cancel{\ }}$

x	u
-1	4
2	1

(g) $\int_0^1 \frac{4(\arctan x)^3}{1+x^2}\,dx$

set,

$u = \tan^{-1} x$

$du = \frac{dx}{1+x^2}$

$dx = (1+x^2)\,du$

x	u
0	0
1	45

$= \int_0^1 \frac{4(\tan^{-1} x)^3}{1+x^2}\,dx$

$= \int_0^{45} \frac{4u^3}{(1+x^2)}(1+x^2)\,du$

$= 4\int_0^{45} u^3\,du = 4\left[\frac{u^4}{4}\right]_0^{45}$

$= \left[u^4\right]_0^{45} \cancel{= \left[\tan^{-1} x\right]_0^{45}}$

$= (45)^4 = 4100625$

(h) $\int_0^1 \frac{1}{\sqrt{64-x^2}}\,dx$

$= \int_0^1 \frac{dx}{\sqrt{8^2-x^2}}$

$= \left[\sin^{-1}\frac{x}{8}\right]_0^1$

$= \sin^{-1}\frac{1}{8} - \sin^{-1} 0$

$= 7.18^\circ$

(i) $\int_1^9 \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

$= \int_2^4 \frac{2\sqrt{x}\, du}{\sqrt{x}\;\; u^2}$

$= 2\int_2^4 \frac{du}{u^2}$

$= 2\int_2^4 u^{-2}\, du$

$= 2\left[\frac{u^{-1}}{-1}\right]_2^4$

$= -2\left[\frac{1}{u}\right]_2^4$

$= -2\left[\frac{1}{4} - \frac{1}{2}\right] = -2\left[\frac{1-2}{4}\right]$

$= -2\left(-\frac{1}{4}\right) = \frac{1}{2} = 0\cdot5$

set, $u = 1+\sqrt{x} = 1+x^{1/2}$

$du = \frac{1}{2}x^{1/2-1}\, dx$

$= \frac{1}{2}x^{-1/2}\, dx$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}\, dx$

$dx = 2\sqrt{x}\, du$

x	u
1	2
9	4