1. $\int x^2 \sin(2x)\,dx$

Here, $u = x^2$, $v = \sin 2x$

$$\therefore \int x^2 \sin(2x)\,dx = x^2 \int \sin(2x)\,dx - \int\left(\frac{d}{dx}x^2 \int \sin(2x)\,dx\right)dx$$

$$= x^2\left(-\frac{\cos 2x}{2}\right) - \int (2x)\left(-\frac{1}{2}\cos 2x\right)dx$$

$$= -\frac{1}{2}x^2\cos 2x + \int x\cos 2x\,dx$$

$$= -\frac{1}{2}x^2\cos 2x + x\int \cos 2x\,dx - \int\left(\frac{d}{dx}x \int \cos 2x\,dx\right)dx$$

$$= -\frac{1}{2}x^2\cos 2x + x\cdot\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x\,dx$$

$$= -\frac{1}{2}x^2\cos x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$$

2. $\int x \sin(2x+1)\,dx$

Here, $u = x$, $v = \sin(2x+1)$

$\therefore \int x\sin(2x+1)\,dx = x\int \sin(2x+1)\,dx - \int\left(\frac{d}{dx}x\int \sin(2x+1)\,dx\right)dx$

$= x\left(-\frac{1}{2}\cos(2x+1)\right) - \int 1\cdot\left(-\frac{1}{2}\cos(2x+1)\right)dx$

$= -\frac{1}{2}x\cos(2x+1) + \frac{1}{4}\sin(2x+1) + c$

3. $\int_0^{\pi} (2x^2+1)\cos 2x \, dx$

Here, $u = 2x^2+1$, $v = \cos 2x$

$\therefore \int_0^{\pi} (2x^2+1)\cos 2x\,dx = (2x^2+1)\int \cos 2x\,dx - \int\left(\frac{d}{dx}(2x^2+1)\int \cos 2x\,dx\right)dx$

$= (2x^2+1)\cdot\frac{1}{2}\sin 2x - \int 4x \cdot \frac{1}{2}\sin 2x\,dx$

$= \frac{1}{2}(2x^2+1)\sin 2x - 2\int x \sin 2x\,dx$

$= \frac{1}{2}(2x^2+1)\sin 2x - 2\left[x\int \sin 2x\,dx - \int\left(\frac{d}{dx}x\int \sin 2x\,dx\right)dx\right]$

$= \frac{1}{2}(2x^2+1)\sin 2x - 2\left[-\frac{1}{2}x\cos 2x + \int \frac{1}{2}\cancel{x}\cos \cancel{2}x\,dx\right]$

$= \frac{1}{2}(2x^2+1)\sin 2x + x\cos 2x - \int \cancel{x}\cos 2x\,dx$

$\Rightarrow \int (2x^2+1)\cos 2x\,dx = \frac{1}{2}(2x^2+1)\sin 2x + x\cos 2x - \frac{1}{2}\sin 2x + C$

$\Rightarrow \int_0^{\pi} (2x^2+1)\cos 2x\,dx = \frac{1}{2}\left[(2x^2+1)\sin 2x\right]_0^{\pi} + \left[x\cos 2x\right]_0^{\pi}$

$- \frac{1}{2}\left[\sin 2x\right]_0^{\pi}$

$= \frac{1}{2}\left[(2\pi^2+1)\sin 2\pi\right] + \left[\pi\cos 2\pi - 0\cos 0\right]$

$- \frac{1}{2}\left[\sin 2\pi\right]$

$\therefore \int_0^{\pi} (2x^2+1)\cos 2x\,dx = \pi$

1 $\int \sin 4x \cos 4x \, dx = \frac{1}{2} \int [\sin 8x + 0] \, dx$

$= -\frac{1}{16} \cos 8x + c$

$$2.\ \int \sin 3x \sin 2x \, dx = \frac{1}{2}\int [\cos x - \cos 5x]\, dx$$

$$= \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$$

$$3.\ \int_0^{\pi/6} \cos 4x \sin 2x \, dx = \frac{1}{2}\int_0^{\pi/6} [\sin 6x + \sin(-2x)]\, dx$$

$$= \frac{1}{2}\int_0^{\pi/6} \sin 6x - \frac{1}{2}\int_0^{\pi/6} \sin 2x \, dx$$

$$= -\frac{1}{12}\left[\cos 6x\right]_0^{\pi/6} + \frac{1}{4}\left[\cos 2x\right]_0^{\pi/6}$$

$$= -\frac{1}{2}\left[\cos 6\cdot\frac{\pi}{6} - \cos 0\right] + \frac{1}{4}\left[\cos 2\cdot\frac{\pi}{6} - \cos 0\right]$$

$$= -\frac{1}{2}\left[-1-1\right] + \frac{1}{4}\cdot\frac{\sqrt{3}}{2}$$

$$= 1 + \frac{\sqrt{3}}{8}$$

$$\therefore \int_0^{\pi/6} \cos 4x \sin 2x \, dx = 1{\cdot}216$$

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4. $\int_0^{\pi/4} \cos 4x \cos x \, dx = \frac{1}{2}\int_0^{\pi/4} [\cos 5x + \cos 3x] \, dx$

$= \frac{1}{10}[\sin 5x]_0^{\pi/4} + \frac{1}{3}[\sin 3x]_0^{\pi/4}$

$= \frac{1}{10}\left[\sin \frac{5\pi}{4}\right] + \frac{1}{3}\left[\sin \frac{3\pi}{4}\right]$

$= \frac{1}{10}\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{3} \cdot \frac{1}{\sqrt{2}}$

$= \frac{-3 + 10}{30\sqrt{2}}$

$\therefore \int_0^{\pi/4} \cos 4x \cos x \, dx = 0.165$

1. $\int \frac{1}{\sqrt{4-x^2}}\,dx$

Let, $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$$= \int \frac{2\cos\theta\, d\theta}{\sqrt{2^2 - 2^2\sin^2\theta}}$$

$$= \int \frac{2\cos\theta\, d\theta}{\sqrt{2^2(1-\sin^2\theta)}}$$

$$= \int \frac{2\cos\theta\, d\theta}{\sqrt{2^2 \cdot \cos^2\theta}}$$

$$= \int \frac{2\cos\theta\, d\theta}{2\cos\theta}$$

$$= \int d\theta$$

$$= \theta$$

$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

Since, $x = 2\sin\theta$

$\sin\theta = \frac{x}{2}$

$\therefore\ \theta = \sin^{-1}\left(\frac{x}{2}\right)$

2. $\int_0^1 x\sqrt{1-x^2}\,dx$

Let, $x = \sin\theta$

$dx = \cos\theta\, d\theta$

so $\int x\sqrt{1-x^2}\,dx = \int \sin\theta\sqrt{1-\sin^2\theta}\,.\cos\theta\, d\theta$

$= \int \sin\theta\,.\sqrt{\cos^2\theta}\,\cos\theta\, d\theta$

$= \int \sin\theta \cos^2\theta\, d\theta$

$= \int \sin\theta\,.u^2.\left(-\frac{du}{\sin\theta}\right)$

Again let,

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$d\theta = -\frac{du}{\sin\theta}$

$= -\int u^2\, du$

$= -\left[\frac{u^3}{3}\right] + c$

$= -\frac{1}{3}\cos^3\theta + c$

since, $x = \sin\theta$

$\therefore \theta = \sin^{-1}x$

$\Rightarrow \int x\sqrt{1-x^2}\,dx = -\frac{1}{3}\cos^3\sin^{-1}x + c$

$\Rightarrow \int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{3}\left[\cos^3\sin^{-1}x\right]_0^1$

$= -\frac{1}{3}\left[\cos^3\sin^{-1}1 - \cos^3\sin^{-1}0\right]$

$= -\frac{1}{3}\left[\cos^3 90^\circ - \cos^3 0^\circ\right]$

$\therefore \int_0^1 x\sqrt{1-x^2}\,dx = \frac{1}{3}$

3. $\int \frac{\sqrt{x^2-4}}{x}\,dx$

Let, $x = \cancel{a}\,2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$= \int \frac{\sqrt{2^2\sec^2\theta - 2^2}}{2\sec\theta} \cdot 2\sec\theta\tan\theta\,d\theta$ $\quad \cancel{\sin\theta}$

$= \int \frac{2\tan\theta}{2\sec\theta} \cdot 2\sec\theta\tan\theta\,d\theta$

$= 2\int \tan^2\theta\,d\theta$

$= 2\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$

$= 2\int \frac{1-\cos^2\theta}{\cos^2\theta}\,d\theta$

$= 2\int \left(\frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}\right)d\theta$

$= 2\int (\sec^2\theta - 1)\,d\theta$

$= 2(\tan\theta - \theta) + c$

$= 2\tan\theta - 2\theta + c$

$= 2\tan\sec^{-1}\left(\frac{x}{2}\right) - 2\sec^{-1}\left(\frac{x}{2}\right) + c$

Since,

$x = 2\sec\theta$

$\sec\theta = \frac{x}{2}$

$\therefore \theta = \sec^{-1}\left(\frac{x}{2}\right)$

1. $\int \sin^2 x \cos^2 x \, dx = \int (\sin x \cos x)^2 dx$

$= \int \left(\frac{1}{2} \sin 2x\right)^2 dx$

$\because \sin 2x = 2 \sin x \cos x$

$\Rightarrow \sin x \cos x = \frac{1}{2} \sin 2x$

$= \frac{1}{4} \int \sin^2 2x \, dx$

$= \frac{1}{4} \int \frac{1}{2} (1 - \cos 4x) dx$

$\because \cos 2\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow \sin^2\theta = \cos^2\theta - \cos 2\theta$

$\Rightarrow \sin^2\theta = 1 - \sin^2\theta - \cos 2\theta$

$\Rightarrow 2\sin^2\theta = 1 - \cos 2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$= \frac{1}{8} \int (1 - \cos 4x) dx$

$= \frac{1}{8} \left(x - \frac{1}{4} \sin 4x\right) + c$

$\cancel{= \frac{1}{32} \sin}$

$\therefore \int \sin^2 x \cos^2 x \, dx = \frac{1}{8} x - \frac{1}{32} \sin 4x + c$

2. $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$

so $\int \sin^3 x \cos^2 x \, dx = \int \sin x \sin^2 x \cos^2 x \, dx$

$= \int \sin x (1 - \cos^2 x) \cos^2 x \, dx$

Let, $u = \cos x$

$du = -\sin x \, dx$

$\therefore dx = -\frac{du}{\sin x}$

$= \int \sin x (1 - u^2) \, u^2 . \left(-\frac{du}{\sin x}\right)$

$= -\int (1 - u^2) \, u^2 \, du$

$= -\int (u^2 - u^4) \, du$

$= -\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + c$

$= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c$

$$\therefore \int_0^{\pi/2} \sin^3 x \cos^2 x \, dx = \left[-\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right]_0^{\pi/2}$$

$$= -\frac{1}{3}\left(\cos^3\frac{\pi}{2} - \cos^3 0\right) - \frac{1}{5}\left(\cos^5\frac{\pi}{2} - \cos^5 0\right)$$

$$= \frac{1}{3} + \frac{1}{5}$$

$$\therefore \int_0^{\pi/2} \sin^3 x \cos^2 x \, dx = \frac{8}{15}$$

3. $\int \sin^5 x \cos^3 x \, dx = \int \sin^5 x (1-\sin^2 x) \cos x \, dx$ let, $u = \sin x$

$du = \cos x \, dx$

$= \int \sin^5 x \cos x \, dx - \int \sin^7 x \cos x \, dx$ $\therefore dx = \frac{du}{\cos x}$

$= \int u^5 \cos x \frac{du}{\cos x} - \int u^7 \cos x \frac{du}{\cos x}$

$= \int u^5 du - \int u^7 du$

$= \frac{u^6}{6} - \frac{u^8}{8} + c$

$= \frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + c$

4. $\int_0^{\pi/6} \sin^2 3x \cos^3 3x \, dx$

Here $\int \sin^2 3x \cos^3 3x \, dx = \int \sin^2 3x \cos^2 3x \cos 3x \, dx$

Let, $u = \sin 3x$

$du = 3\cos 3x \, dx$

$dx = \frac{du}{3\cos 3x}$

$= \int \sin^2 3x (1 - \sin^2 3x) \cos 3x \, dx$

$= \int \sin^2 3x \cos 3x \, dx - \int \sin^4 3x \cos 3x \, dx$

$= \int u^2 \cos 3x \cdot \frac{du}{3\cos 3x} - \int u^4 \cos 3x \frac{du}{3\cos 3x}$

$= \frac{1}{3} \int u^2 du - \frac{1}{3} \int u^4 du$

$= \frac{1}{3} \frac{u^3}{3} - \frac{1}{3} \frac{u^5}{5} + c$

$\Rightarrow \int \sin^2 3x \cos^3 3x \, dx = \frac{1}{9} \sin^3 3x - \frac{1}{15} \sin^5 3x + c$

$\therefore \int_0^{\pi/6} \sin^2 3x \cos^3 3x \, dx = \frac{1}{9} \left[\sin^3 . 3 . \frac{\pi}{6} - 0\right] - \frac{1}{15} \left[\sin^5 3 . \frac{\pi}{6}\right]$

$= \frac{1}{9} \cdot 1 - \frac{1}{15} \cdot 1$

$= \frac{5-3}{45} = \frac{2}{45}$