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Chapter 15: Oscillations

15-2 ENERGY IN SIMPLE HARMONIC MOTION

In the case of a linear oscillator, the energy transfers back and forth between kinetic energy and potential energy, while the sum of the two—the mechanical energy E of the oscillator—remains constant.

That means,

$$\textit{Mechanical Energy} = \textit{Kinetic Energy} + \textit{Potential Energy} = \textit{constant}$$

$$\text{Or, } E = K + U = \textit{Constant}$$

Potential Energy

The potential energy of a linear oscillator like that of Fig.(a) is associated entirely with the spring. Its value depends on how much the spring is stretched or compressed—that is, on x(t).

We know,

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}k\ x^2_m \cos^2(\omega t + \varphi) \quad \ldots\ldots. (1)$$

$$[\text{ Since, } x(t) = x_m \cos(\omega t + \varphi)\ ]$$

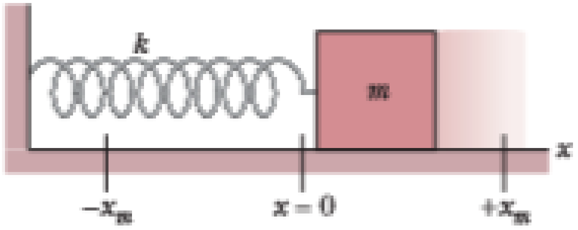

Figure (a) Linear Oscillator

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Kinetic Energy

The kinetic energy of the system of Figure (a) is associated entirely with the block. Its value depends on how fast the block is moving—that is, on $v(t)$.

We then find,

$$K(t) = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m\,\omega^2 x^2_m \sin^2(\omega t + \varphi) \quad \text{.........} \ (2) \qquad ; \quad [\ \text{Since}, \ \ v(t) = -\,\omega\, x_m \sin(\omega t + \varphi)\ ]$$

Again, we know, $\omega = \sqrt{\frac{k}{m}}$, **So,** $\omega^2 = \frac{k}{m}$.

Substituting the value of ω^2 in Eq (2) We get,

$$K(t) = \frac{1}{2} mv^2 = \frac{1}{2} k\, x^2_m \sin^2(\omega t + \varphi) \quad \text{.........} \ (3)$$

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Mechanical Energy

The mechanical energy follows from Eqs.1 and 3 and is

$$E = U + K$$

$$= \frac{1}{2} k\, x^2_m \cos^2 (\omega t + \varphi) + \frac{1}{2} k\, x^2_m \sin^2(\omega t + \varphi)$$

$$= \frac{1}{2} k\, x^2_m \,[\, \cos^2 (\omega t + \varphi) + \sin^2(\omega t + \varphi) \,]$$

For any angle α ,

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

Therefore,

$$E = U + K = \frac{1}{2} k\, x^2_m \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

The mechanical energy of a linear oscillator is indeed constant and independent of time.

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The potential energy and kinetic energy of a linear oscillator are shown as ***functions of time t*** in Fig. a

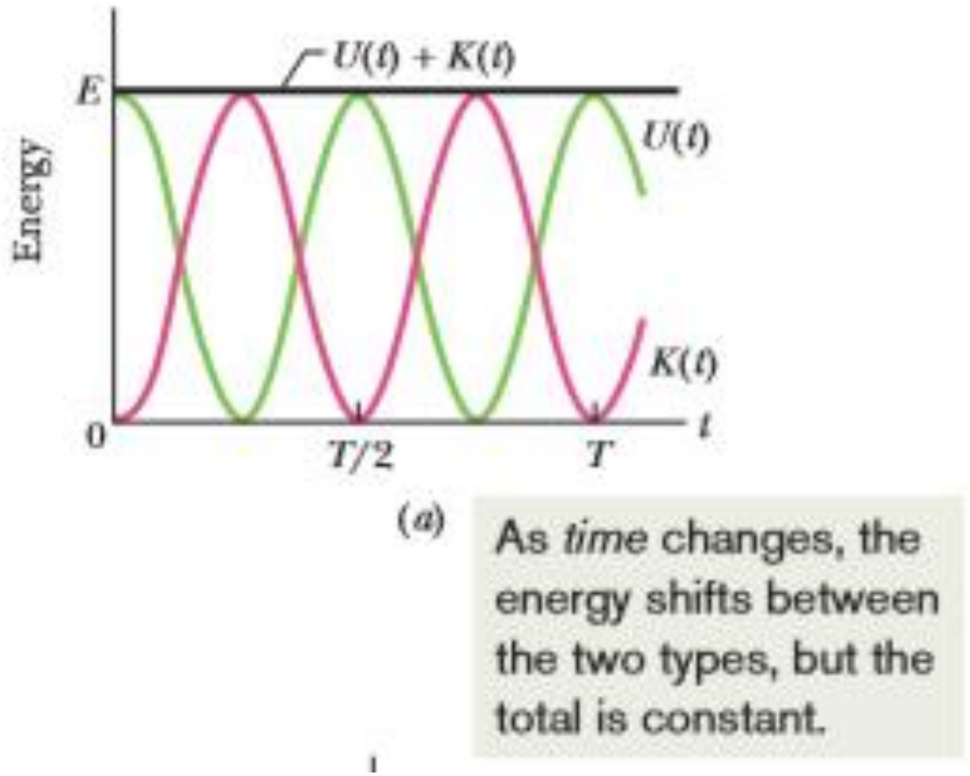

Figure : (a) Potential energy U(t), kinetic energy K(t),and mechanical energy E as functions of time t for a linear harmonic oscillator. Note that all energies are positive and that the potential energy and the kinetic energy peak twice during every period.

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The potential energy and kinetic energy of a linear oscillator are shown ***as functions of displacement x*** in Fig. b

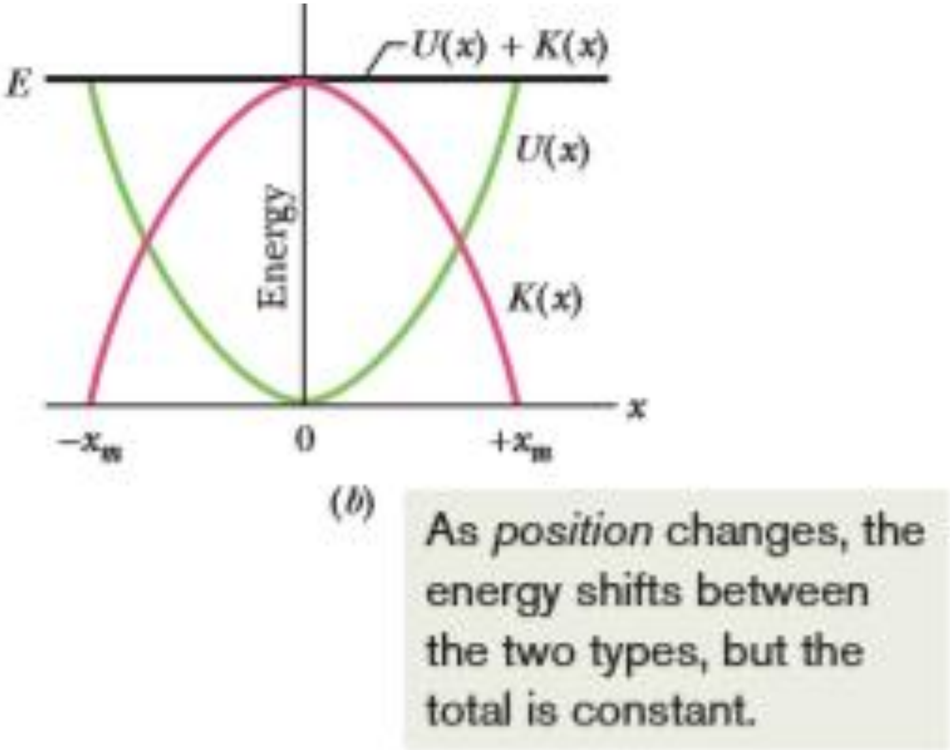

Figure (b) Potential energy U(x), kinetic energy K(x),and mechanical energy E as functions of position x for a linear harmonic oscillator with amplitude x_m. For x = 0 the energy is all kinetic, and for x = x_m it is all potential.

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Problem 30 : ***An oscillating block–spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s. Find (a) the spring constant,(b) the mass of the block, and (c) the frequency of oscillation.***

Solution : **(a)** The energy at the turning point is all potential energy: $\mathbf{E} = \frac{1}{2} k\, x_m^2$

where **E** = 1.00 J and x_m = 0.100 m.

$$\text{Thus, } k = \frac{2E}{x_m^2} = \frac{2 \times 1.00\, J}{(0.100\, m)^2} = 200 \text{ N/m}$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2} m v_m^{\,2}$$

$$\Rightarrow m = \frac{2E}{v_m^2} = \frac{2 \times 1.00\, J}{(1.20\, m/s)^2} = 1.39 \text{ kg}$$

(c) $\omega = 2\pi f$

Or, $f = \frac{1}{2\pi}\omega$; [Since $\omega = \sqrt{\frac{k}{m}}$]

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{200\ N/m}{1.39\ kg}} = 1.91\text{ Hz}$$

(Ans)

Problem 31 : A 5.00 kg object on a horizontal frictionless surface is attached to a spring with k 1000N/m. The object is displaced from equilibrium 50.0cm horizontally and given an initial velocity of 10.0 m/s back toward the equilibrium position. What are (a) the motion's frequency, (b) the initial potential energy of the block–spring system,(c) the initial kinetic energy, and (d) the motion's amplitude?

Solution : (a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\ N/m}{5.00\ kg}} = 2.25\text{ Hz}$

(b) the initial potential energy = ?

With $x_0 = 0.500\ m$, we have,

$$U = \frac{1}{2} k\, x_0^2 = \frac{1}{2} (1000\ N/m)\ (0.500\ m)^2 = 125\ \text{J}.$$

(c) **the initial kinetic energy = ?**

With v_0 = 10.0 m/s, the initial kinetic energy is

$$K = \frac{1}{2} m v_0^2 = 250\ \text{J}.$$

(d) **the motion's amplitude = ?**

Since the total energy $\mathbf{E = K_0 + U_0 = 375\ J}$ is conserved,

then consideration of the energy at the turning point leads to

$$E = \frac{1}{2} k\, x_m^2$$

$$\Rightarrow x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\,(375\ J)}{1000\ N/m}} = 0.866\ \text{m}$$

(Ans)

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Problem 36 : ***If the phase angle for a block–spring system in SHM is π/6 rad and the block's position is given by $x = x_m \cos(\omega t + \varphi)$, ,what is the ratio of the kinetic energy to the potential energy at time t = 0?***

Solution : Since the kinetic energy is $K = \frac{1}{2} m v^2$ and the potential energy is $U = \frac{1}{2} k x^2 = \frac{1}{2} m\, \omega^2 x^2$ **[k = ω^2 m]**

then the ratio of kinetic to potential energy is simply

$$\frac{K}{U} = \frac{\frac{1}{2} m v^2}{\frac{1}{2} m \omega^2 x^2} = \frac{v^2}{\omega^2 x^2} = \frac{(v/x)^2}{\omega^2} \quad \ldots\ldots.. \ (1)$$

Again, Given

$$x = x_m \cos(\omega t + \varphi)$$

Differentiating with respect to time we find; $v = \frac{dx}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \varphi)\right] = -\,\omega\, x_m \sin(\omega t + \varphi)$

Then, Dividing **v by x ;**

$$\frac{v}{x} = \frac{-\,\omega\, x_m \sin(\omega t + \varphi)}{x_m \cos(\omega t + \varphi)} = -\,\omega \tan(\omega t + \varphi) \quad \ldots\ldots. \ (2)$$

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Putting Eq (2) in Eq (1) we find,

$$\frac{K}{U} = \frac{(v/x)^2}{\omega^2} = \frac{\omega^2 \tan^2(\omega t + \varphi)}{\omega^2} = \tan^2(\omega t + \varphi)$$

Now, at $t = 0$ it becomes; $\dfrac{K}{U} = \tan^2 \varphi$

Since in this problem, $\varphi = \dfrac{\pi}{6}$;

then the ratio of kinetic to potential energy at $t = 0$ is ;

$$\frac{K}{U} = \tan^2(\pi/6) = 1/3.$$

(Ans)