Lesson - 19

The Principle of Superposition for Waves

- Suppose that two waves travel simultaneously along the same stretched string. Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone.

- The displacement of the string when the waves overlap is then the algebraic sum.

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

- Overlapping waves algebraically add to produce a resultant wave (or net wave)

- Overlapping waves do not in any way alter the travel of each other.

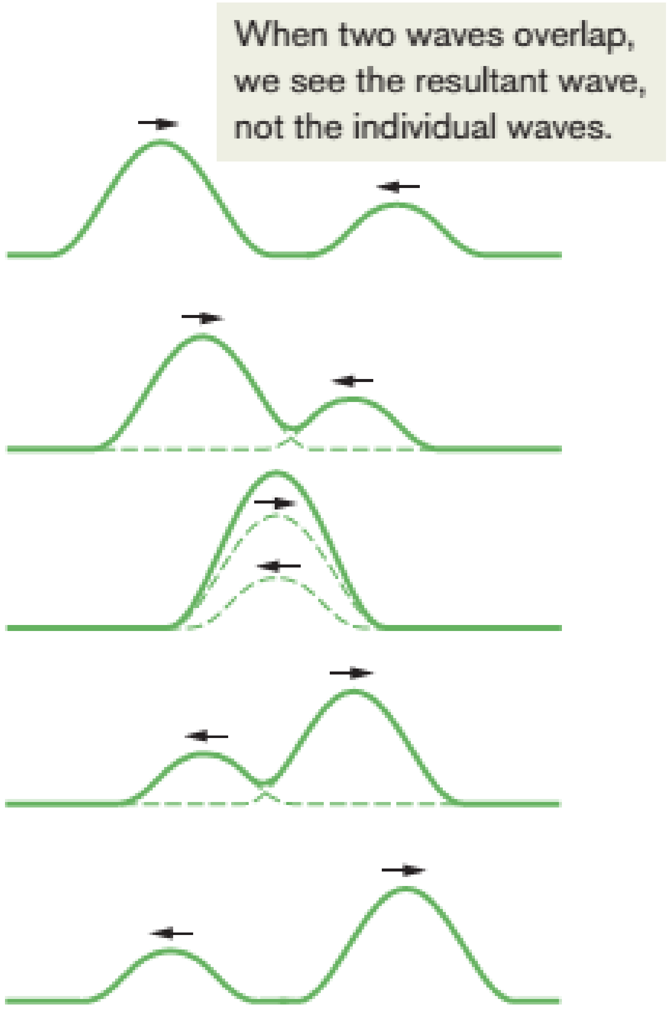


Figure 16-12 A series of snapshots that show two pulses traveling in opposite directions along a stretched string. The superposition principle applies as the pulses move through each other.

INTEREFERENCE :

Interference is a phenomenon in which two waves due to superposition form a resultant wave of greater, lower, or the same amplitude **of two waves.**

There are two types of interference:

Constructive interference

Constructive interference occurs whenever waves come together so that they are in phase with each other. This means that their oscillations at a given point are in the same direction and the resulting amplitude at that point being much larger than the amplitude of an individual wave.

Destructive Interference :

Destructive interference occurs when waves come together in such a way that they completely cancel each other out. When two waves interfere destructively, they must have the same amplitude in opposite directions. When the two waves are interfering; the net result, is that they all combine in some way to produce zero amplitude.

Interference of Waves

Suppose we send two sinusoidal waves of the same wavelength and amplitude in the same direction along a stretched string.

$$y_1(x, t) = y_m \sin(kx - \omega t)$$
$$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$$

$\omega\ (f),\ k\ (\lambda),\ y_m$, are same

Superposition principle, $y'(x, t) = y_1(x, t) + y_2(x, t)$

$$y'(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \varphi)$$

$$= y_m \{\sin(kx - \omega t) + \sin(kx - \omega t + \varphi)\}$$

$$= y_m \{2 \sin\left(\frac{kx - \omega t + kx - \omega t + \varphi}{2}\right) \cos\left(\frac{kx - \omega t - kx + \omega t - \varphi}{2}\right)\}$$

$$= 2y_m \sin\left\{\frac{2(kx - \omega t) + \varphi}{2}\right\} \cos\left(\frac{kx - \omega t - kx + \omega t - \varphi}{2}\right)$$

$$= 2y_m \sin\left\{(kx - \omega t) + \frac{\varphi}{2}\right\} \cos\left(\frac{-\varphi}{2}\right)$$

$$y'(x, t) = \left[2y_m \cos\left(\frac{\varphi}{2}\right)\right] \sin\left(kx - \omega t + \frac{\varphi}{2}\right)$$

Resultant displacement $= y'(x, t)$

Amplitude $= \left[2y_m \cos\left(\frac{\varphi}{2}\right)\right]$

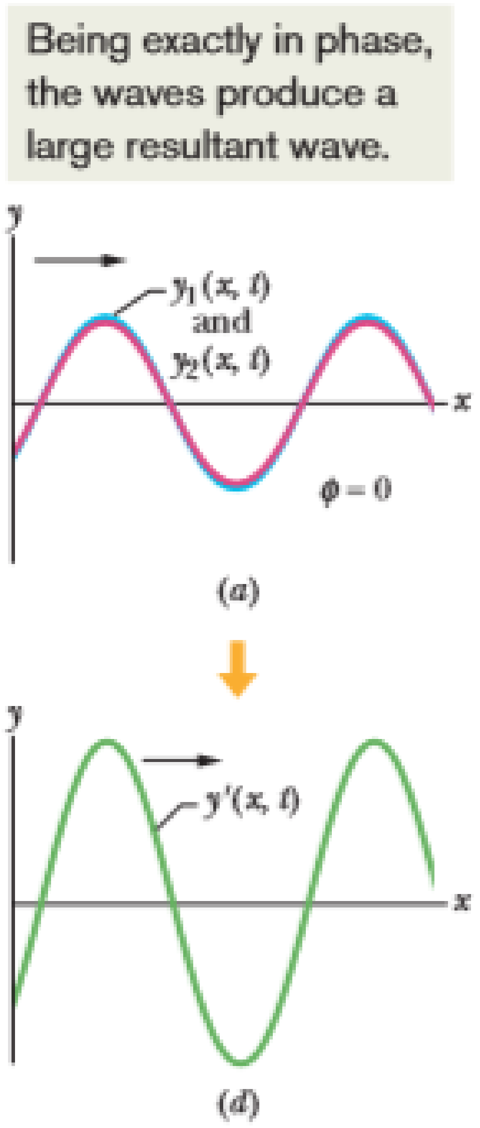
Oscillating term $= \sin\left(kx - \omega t + \frac{\varphi}{2}\right)$

If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

Interfering waves: $y_1(x, t) = y_m \sin(kx - \omega t)$

$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$

Resultant wave : $y'(x, t) = [2y_m \cos(\frac{\varphi}{2})] \sin(kx - \omega t + \frac{\varphi}{2})$



The resultant wave differs from the interfering waves in two respects:

(1) its phase constant is $\frac{\varphi}{2}$ and

(2) its amplitude is $y'_m = [2y_m \cos(\frac{\varphi}{2})]$

(1) If φ = 0 rad (0^0) : fully constructive interference

$$y'(x, t) = [2y_m \cos(\tfrac{0}{2})] \sin(kx - \omega t + \tfrac{0}{2}) \}$$

$$= [2y_m \cos 0] \sin(kx - \omega t)$$

$$y'(x, t) = 2y_m \sin(kx - \omega t)$$ [greatest amplitude]

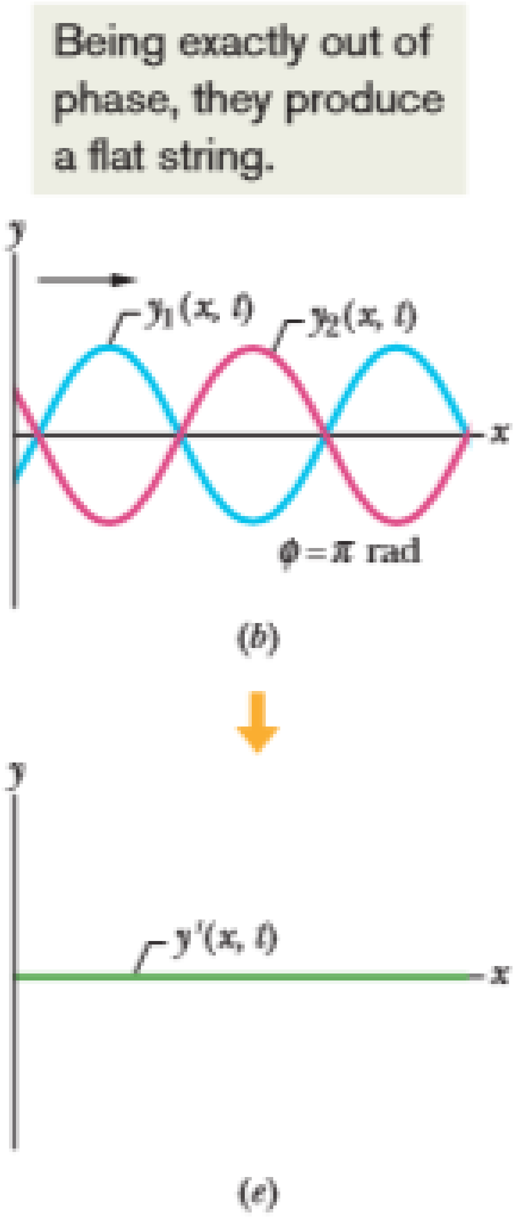
The resultant wave is plotted in Fig. d.

(2) If $\varphi = \pi$ rad (180^0): fully destructive interference

$$y'(x, t) = [2y_m \cos(\tfrac{\pi}{2})] \sin(kx - \omega t + \tfrac{\pi}{2})$$

$$= [2y_m (0)] \sin(kx - \omega t + \tfrac{\pi}{2})$$

$$y'(x, t) = 0$$



- The resultant wave is plotted in Fig. e. Although we sent two waves along the string, we see no motion of the string.

- This type of interference is called ***fully destructive interference.***

(3) If $\varphi = \frac{2\pi}{3}$ rad (120^0): intermediate interference

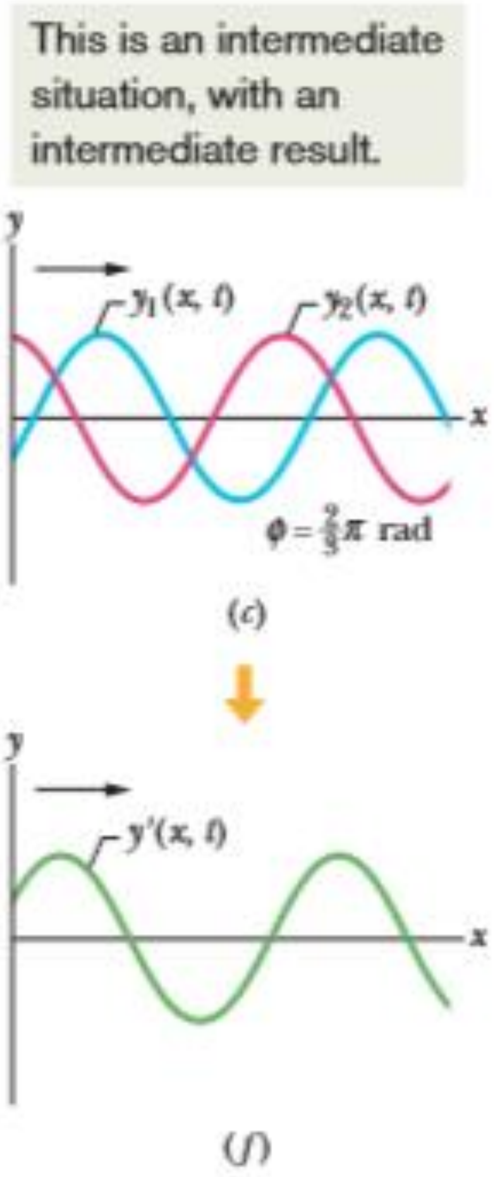
$$y'(x, t) = [2y_m \cos(\tfrac{\frac{2\pi}{3}}{2})] \sin\{kx - \omega t + (\tfrac{\frac{2\pi}{3}}{2})\}$$

$$= 2y_m \cos(\tfrac{\pi}{3}) \sin(kx - \omega t + \tfrac{\pi}{3})$$

$$= 2y_m (\tfrac{1}{2}) \sin(kx - \omega t + \tfrac{\pi}{3})$$

$$y'(x, t) = y_m \sin(kx - \omega t + \tfrac{\pi}{3})$$

- **W**hen interference is neither fully constructive nor fully destructive, it is called *intermediate interference*. The amplitude of the resultant wave is then intermediate between 0 and $2y_m$.



Problem 32. *What phase difference between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.*

Solution : $y_1(x, t) = y_m \sin(kx - \omega t)$

$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$

Superposition principle, $y'(x, t) = y_1(x, t) + y_2(x, t)$

$$y'(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \varphi)$$

$$= y_m \{\sin(kx - \omega t) + \sin(kx - \omega t + \varphi)\}$$

$$= y_m \{2 \sin\left(\frac{kx - \omega t + kx - \omega t + \varphi}{2}\right) \cos\left(\frac{kx - \omega t - kx + \omega t - \varphi}{2}\right)\}$$

$$= 2y_m \sin\left\{\frac{2(kx - \omega t) + \varphi}{2}\right\} \cos\left(\frac{kx - \omega t - kx + \omega t - \varphi}{2}\right)$$

$$= 2y_m \sin\left\{(kx - \omega t) + \frac{\varphi}{2}\right\} \cos\left(\frac{-\varphi}{2}\right)$$

$$y'(x, t) = \left[2y_m \cos\left(\frac{\varphi}{2}\right)\right] \sin\left(kx - \omega t + \frac{\varphi}{2}\right)$$ Traveling wave

(a) $\left[2y_m \cos\left(\frac{\varphi}{2}\right)\right] = 1.50\, y_m$

$$\cos\left(\frac{\varphi}{2}\right) = 1.50/2$$

$$\rightarrow \cos\left(\frac{\varphi}{2}\right) = 0.75$$

$$\rightarrow \frac{\varphi}{2} = cos^{-1}\ (0.75^0)$$

$$\rightarrow \frac{\varphi}{2} = 41.41$$

$$\rightarrow \varphi = 2(41.41)$$

$$\rightarrow \varphi = 82.82^0 \quad \textit{Ans.}$$

(b) $\frac{\varphi}{2} = cos^{-1}\ (0.75\ \text{rad}\ )$

$$\rightarrow \frac{\varphi}{2} = 0.7227\ \text{rad}$$

$$\rightarrow \varphi = 1.45\ \text{rad} \quad \text{Ans.}$$

(c) $2\pi\ \text{rad} = \lambda$

$$\rightarrow 1\ \text{rad} = \left(\frac{\lambda}{2\pi}\right)$$

$$\rightarrow 1.45\ \text{rad} = 1.45\left(\frac{\lambda}{2\pi}\right) = 0.23\lambda \quad \text{Ans.}$$