Chapter 8

System of Linear Differential Equations

Example 8.1 Find a general solution of the following system:

$$\frac{dx}{dt} = 2x - 2y \text{ .....(1)}$$

$$\frac{dy}{dt} = 2x + 2y \text{ .....(2)}$$

Solution: Using differential operator $D = \frac{d}{dt}$, we can write

$$Dx = 2x - 2y$$

$$Dy = 2x + 2y$$

This system can be written as:

$$(D-2)x + 2y = 0 \text{ ......(3)}$$

$$2x - (D-2)y = 0 \text{ ......(4)}$$

Multiplying eqn. (3) by 2 and operating on (4) with $(D-2)$ we get,

$$2(D-2)x + 4y = 0 \text{ ....(5)}$$

$$2(D-2)x - (D^2 - 4D + 4)y = 0 \text{ .....(6)}$$

Subtracting (6) from (5), we have

$$(D^2 - 4D + 8)y = 0$$

Auxiliary eqn. is $m^2 - 4m + 8 = 0$

and its solutions are $m = 2 \pm 2i$

Thus $y(t) = e^{2t}(A\cos 2t + B\sin 2t)$

Substituting y in (2), we get

$$\frac{d}{dt}e^{2t}(A\cos 2t + B\sin 2t) = 2x + 2e^{2t}(A\cos 2t + B\sin 2t)$$

$$\Rightarrow e^{2t}(-2A\sin 2t + 2B\cos 2t) + 2e^{2t}(A\cos 2t + B\sin 2t)$$
$$= 2x + 2e^{2t}(A\cos 2t + B\sin 2t)$$

$$\Rightarrow 2e^{2t}(B\cos 2t - A\sin 2t) = 2x$$
$$\Rightarrow x(t) = e^{2t}(B\cos 2t - A\sin 2t)$$

The required solutions are: $x(t) = e^{2t}(B\cos 2t - A\sin 2t)$

and $y(t) = e^{2t}(A\cos 2t + B\sin 2t)$.

Class Practice

Solve the following system of linear differential equations.

1. $$\frac{dx}{dt} = 2x + 2y$$
$$\frac{dy}{dt} = x + 3y$$

2. $$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = -4y$$

3. $$\frac{dx}{dt} = -x + y$$
$$\frac{dy}{dt} = -3x - 5y$$