1. $z = i$ or $(0,1)$

$z = i+2$ or $(2,1)$

Path equation $C \Rightarrow \frac{x-0}{0-2} = \frac{y-1}{1-1}$

$\Rightarrow -2y+2=0$

$\Rightarrow y = 1$

Here, $f(z) = z^2 = (x+iy)^2$

$= x^2 + i\,2x - 1$

$f(z) = Im(z^2) = 2x$

$z = x+iy$

$\therefore dz = dx$

$\therefore \int_C Im(z^2)\,dz = \int_0^2 \cancel{x^2 + i\,2x - 1\,dx}\; 2x\,dx$

$= \left[\frac{2x^2}{2}\right]_0^2$

$= 4$

2. $z = i$ or $(0,1)$

$z = i+1$ or $(1,1)$

Path equation $C \Rightarrow \dfrac{x-0}{0-1} = \dfrac{y-1}{1-1}$

$\Rightarrow -y+1=0$

$\Rightarrow y = 1$

Here, $f(z) = \sin z = \sin(x+iy)$

$= \sin(x+i)$

$z = x+iy = x+i$

$\therefore dz = dx$

$$\int_C \sin z\, dz = \int_0^1 \sin(x+i)\, dx$$

$$= \left[-\cos(x+i)\right]_0^1$$

$$= -\cos(1+i) - \cos i$$

3. $z=0$ or $(0,0)$

$z=3i$ or $(0,3)$

Path equation $C \Rightarrow \frac{x-0}{0-0} = \frac{y-0}{0-3}$

$\Rightarrow -3x=0$

$\Rightarrow x=0$

Here, $f(z) = e^{2z} = e^{2(x+iy)} = e^{2iy}$

$\therefore z = x+iy = iy$

$\Rightarrow dz = i\,dy$

$\therefore \int_C e^{2z}\,dz = \int_0^3 e^{2iy}\, i\,dy$

$= i\left[e^{2iy} \cdot \frac{1}{2i}\right]_0^3$

$= \frac{e^{i6}}{2} - \frac{1}{2}$

4. $z = 0$ or, $(0,0)$

$z = 2i$ or, $(0,2)$

Path equation $C \Rightarrow \frac{x-0}{0-0} = \frac{y-0}{0-2}$

$\Rightarrow -2x = 0$

$\Rightarrow x = 0$

Here, $f(z) = z^2 = (x+iy)^2 = x^2 + 2xi - y^2 = -y^2$

$z = x+iy = iy$

$\therefore dz = i\,dy$

$\therefore \int_C z^2\,dz = \int_0^2 -y^2\, i\,dy$

$= i\left[-\frac{y^3}{3}\right]_0^2$

$= -\frac{8}{3}i$

5. $|z| = 2$ or $z = 2e^{i\theta}$

$f(z) = 2\bar{z} = 2 \cdot 2e^{-i\theta} = 4e^{-i\theta}$

$z = re^{i\theta} = 2e^{i\theta}$

$\Rightarrow dz = 2i\, e^{i\theta} d\theta$

$$\therefore \int_C (2\bar{z})\, dz = \int_0^{2\pi} (4e^{-i\theta}) \cdot 2i e^{i\theta}\, d\theta$$

$$= 8i \int_0^{2\pi} 1\, d\theta$$

$$= 8i\, [\theta]_0^{2\pi}$$

$$= 16\pi i$$