1. $f(t) = e^{2t} \sinh 3t$

$$\mathcal{L}\, e^{2t} \sinh 3t = F(s-2)$$

$$F(s) = \mathcal{L} \sinh 3t$$

$$= \frac{3}{s^2 - 3^2}$$

$$= \frac{3}{s^2 - 9}$$

$$\mathcal{L}\, e^{2t} \sinh 3t = \frac{3}{(s-2)^2 - 9}$$

2. $f(t) = e^{-t} \sinh 4t$

$$\mathcal{L}\, e^{-t} \sinh 4t = F(s+1)$$

$$F(s) = \mathcal{L}\sinh 4t$$

$$= \frac{4}{s^2-4^2}$$

$$= \frac{4}{s^2-16}$$

$$\mathcal{L}\, e^{-t}\sinh 4t = \frac{4}{(s+1)^2-16}$$

3. $f(t) = e^{2t}\cos 3t$

$$\mathcal{L}\, e^{2t}\cos 3t = F(s-2)$$

$$F(s) = \mathcal{L}\cos 3t$$

$$= \frac{s}{s^2+3^2}$$

$$= \frac{s}{s^2+9}$$

$$\mathcal{L}\, e^{2t}\cos 3t = \frac{(s-2)}{(s-2)^2+9}$$

4. $f(t) = t^{10} e^{-7t}$

$$\mathcal{L}\, t^{10} e^{-7t} = F(s+7)$$

$$F(s) = \mathcal{L}\, t^{10}$$

$$= \frac{10!}{s^{10+1}}$$

$$= \frac{10!}{s^{11}}$$

$$\mathcal{L}\, t^{10} e^{-7t} = \frac{10!}{(s+7)^{11}}$$

5 $f(t) = e^{5t} \cosh 6t$

$\mathcal{L}\, e^{5t} \cosh 6t = F(s-5)$

$F(s) = \mathcal{L} \cosh 6t$

$= \dfrac{s}{s^2 - 6^2}$

$= \dfrac{s}{s^2 - 36}$

$\mathcal{L}\, e^{5t} \cosh 6t = \dfrac{(s-5)}{(s-5)^2 - 36}$

1. $f(t) = t \sin 2t$

$$\mathcal{L}\, t \sin 2t = (-1)^1 \frac{d}{ds} [F(s)]$$

$$= -\frac{d}{ds} [\mathcal{L} \sin 2t]$$

$$= -\frac{d}{ds} \left[\frac{2}{s^2+2^2}\right]$$

$$= -\frac{d}{ds} \left(\frac{2}{s^2+4}\right)$$

$$= -\frac{(s^2+4)\frac{d}{ds}(2) - 2 \cdot \frac{d}{ds}(s^2+4)}{(s^2+4)^2}$$

$$= \frac{2 \cdot 2s}{(s^2+4)^2}$$

$$= \frac{4s}{(s^2+4)^2}$$

2. $f(t) = t\cos bt$

$$\mathcal{L}\{t\cos bt\} = (-1)^1 \frac{d}{ds}[F(s)]$$

$$= (-1)\cdot \frac{d}{ds}[\mathcal{L}\{\cos bt\}]$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+b^2}\right)$$

$$= -\frac{(s^2+b^2)\frac{d}{ds}(s) - s\cdot\frac{d}{ds}(s^2+b^2)}{(s^2+b^2)^2}$$

$$= -\frac{(s^2+b^2) - s\cdot 2s}{s^2+b^2}$$

$$= -\left(\frac{s^2+b^2-2s^2}{s^2+b^2}\right)$$

$$= -\left(\frac{b^2-s^2}{s^2+b^2}\right)$$

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3. $f(t) = t^2 e^{-4t}$

$$\mathcal{L}\, t^2 e^{-4t} = (-1)^2 \cdot \frac{d^2}{ds^2} [F(s)]$$

$$= \frac{d^2}{ds^2} [\mathcal{L}\, e^{-4t}]$$

$$= \frac{d^2}{ds^2} \left(\frac{1}{s+4} \right)$$

$$= \frac{d}{ds} \left(\frac{(s+4) \frac{d}{ds}(1) - 1 \cdot \frac{d}{ds}(s+4)}{(s+4)^2} \right)$$

$$= \frac{d}{ds} \frac{-1}{(s+4)^2}$$

$$= (-1)(-2)(s+4)^{-3}$$

$$\cancel{= \frac{3}{(s+4)^3}} = \frac{2}{(s+4)^3}$$

9. $f(t) = t \sinh 3t$

$$\mathcal{L}\{t \sinh 3t\} = (-1)^1 \frac{d}{ds}[F(s)]$$

$$= -\frac{d}{ds}[\mathcal{L}\{\sinh 3t\}]$$

$$= -\frac{d}{ds}\left(\frac{3}{s^2-9}\right)$$

$$= -\frac{(s^2-9)\frac{d}{ds}(3) - 3 \cdot \frac{d}{ds}(s^2-9)}{(s^2-9)^2}$$

$$= -\frac{\cancel{s^2-9} - 3 \cdot 2s}{(s^2-9)^2}$$

$$= \frac{6s}{(s^2-9)^2}$$

5. $f(t) = t \cosh 2t$

$$\mathcal{L}\, t \cosh 2t = (-1)^1 \frac{d}{ds} [F(s)]$$

$$= -\frac{d}{ds} [\mathcal{L} \cosh 2t]$$

$$= -\frac{d}{ds} \left(\frac{s}{s^2 - 2^2} \right)$$

$$= -\frac{d}{ds} \left(\frac{s}{s^2 - 4} \right)$$

$$= -\frac{(s^2-4) \frac{d}{ds}(s) - s \cdot \frac{d}{ds}(s^2-4)}{(s^2-4)^2}$$

$$= -\frac{s^2 - 4 - 2s^2}{(s^2-4)^2}$$

$$= -\frac{-4 - s^2}{(s^2-4)^2}$$

$$= -\frac{-(s^2+4)}{(s^2-4)^2} = \frac{s^2+4}{(s^2-4)^2}$$