1. $f(t) = t\,u(t-1)$

$$= e^{-s}\, \mathcal{L}\{f(t+1)\}$$

$$= e^{-s}\, \mathcal{L}(t+1)$$

$$= e^{-s} \cdot \left(\frac{1!}{s^{1+1}} + \frac{1}{s}\right)$$

$$= e^{-s} \cdot \left(\frac{1}{s^2} + \frac{1}{s}\right)$$

2. $f(t) = (t-1)\,u(t-3)$

$$= e^{-3\times s}\, \mathcal{L}\, f(t+3)$$

$$= e^{-3s} \cdot \mathcal{L}(t+3-1)$$

$$= e^{-3s}\, \mathcal{L}(t+2)$$

$$= e^{-3s}\left(\frac{1!}{s^{1+1}} + \frac{2}{s}\right)$$

$$= e^{-3s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$$

3. $f(t) = (t+2)^2\, u(t-1)$

$$= e^{-s}\,\mathcal{L}\{f(t+1)\}$$

$$= e^{-s}\,\mathcal{L}\,(t+1+2)^2$$

$$= e^{-s}\,\mathcal{L}\,(t+3)^2$$

$$= e^{-s}\,\mathcal{L}\,(t^2+6t+9)$$

$$= e^{-s}\left(\frac{2!}{s^{2+1}} + 6\cdot\frac{1!}{s^{1+1}} + \frac{9}{s}\right)$$

$$= e^{-s}\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right)$$

4. $f(t) = e^{-2t}\, u(t-3)$

$$= e^{-3\times s}\, \mathcal{L}\, f(t+3)$$

$$= e^{-3s}\, \mathcal{L}\, e^{-2(t+3)}$$

$$\cancel{= e^{-3s}\, \mathcal{L}\, e^{-2t+6}}$$

$$= e^{-3s}\, \mathcal{L}\, e^{-2t} \cdot e^{-6}$$

$$= e^{-3s} \cdot e^{-6} \cdot \frac{1}{s+2}$$

$$= e^{-3(s+2)}\, \frac{1}{s+2}$$

5. $f(t) = 4\cos t\, u(t-\pi)$

$$= e^{-\pi \times s} \cdot \mathcal{L}\, f(t+\pi)$$

$$= e^{-\pi s} \cdot \mathcal{L}\, 4 \cdot \cos(t+\pi)$$

$$= 4 \cdot e^{-\pi s} \cdot \mathcal{L}\left(\cos t \cdot \cos\pi - \sin t \cdot \sin\pi\right)$$

$$= -4 \cdot e^{-\pi s} \cdot \frac{s}{s^2+1}$$

6. $f(t) = \begin{cases} t; & 0 < t < 1 \\ 2; & t > 1 \end{cases}$

$$f(t) = t\,u(t) - t\,u(t-1) + 2\,u(t-1)$$

$$F(s) = \mathcal{L}\, t\,u(t) - \mathcal{L}\, t u(t-1) + 2\,\mathcal{L}\, u(t-1)$$

Scanned with CamScanner

$$= e^{-0 \times s} \mathcal{L} f(t) - e^{-1 \times s} \mathcal{L} f(t+1) + 2 \frac{e^{-1 \times s}}{s}$$

$$= \mathcal{L}(t) - e^{-s} \mathcal{L}(t+1) + \frac{2 e^{-s}}{s}$$

$$= \frac{1!}{s^{1+1}} - e^{-s} \left(\frac{1!}{s^{1+1}} + \frac{1}{s} \right) + 2 . \frac{e^{-s}}{s}$$

$$= \frac{1}{s^2} - e^{-s} \left(\frac{1}{s^2} + \frac{1}{s} \right) + \frac{2 . e^{-s}}{s}$$

7. $f(t) = \begin{cases} t^2 ; & 0 \le t < 1 \\ t - 3 ; & t \geq 1 \end{cases}$

$\therefore f(t) = t^2 u(t) - t^2 u(t-1) + (t-3) u(t-1)$

Scanned with CamScanner

$$F(s) = \mathcal{L}\, t^2\, u(t) - \mathcal{L}\, t^2\, u(t-1) + \mathcal{L}(t-3)\, u(t-1)$$

$$= e^{-0 \times s}\, \mathcal{L}\, f(t) - e^{-1 \times s}\, \mathcal{L}\, f(t+1) + e^{-1 \times s}\, \mathcal{L}\, f(t+1)$$

$$= \mathcal{L}\,(t) - e^{-s}\, \mathcal{L}\,(t+1)^2 + e^{-s}\, \mathcal{L}(t+1-3)$$

$$= \frac{1!}{s^{1+1}} - e^{-s}\, \mathcal{L}\,(t^2+2t+1) + e^{-s}\, \mathcal{L}\,(t-2)$$

$$= \frac{1}{s^2} - e^{-s}\left(\frac{2!}{s^{2+1}} + 2 \cdot \frac{1!}{s^{1+1}} + \frac{1}{s}\right) + e^{-s}\left(\frac{1!}{s^{1+1}} - \frac{2}{s}\right)$$

$$= \frac{1}{s^2} - e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) + e^{-s}\left(\frac{1}{s^2} - \frac{2}{s}\right)$$