Name: Nafinur Leo

Id : 20-42195-1

Section: G

Slide-16

* $f(z) = \frac{1}{z}$

sigular point $z=0$, order $=1$

$$Res(z=0) = \lim_{z \to 0} \frac{1}{z} z$$
$$= 1$$

* $f(z) = \frac{1}{z^2-1} = \frac{1}{(z+1)(z-1)}$

Singular point $z = 1, -1$, order $= 1$

$$Res(z=1) = \lim_{z \to 1} \frac{1}{(z+1)(z-1)} (z-1)$$
$$= \frac{1}{2}$$

$$Res(z=-1) = \lim_{z \to -1} \frac{1}{(z+1)(z-1)} (z+1)$$
$$= -\frac{1}{2}$$

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* $f(z) = \frac{\sin z}{z}$

singular point $z = 0$, order $= 1$

$\therefore$ $Res\ (z = 0) = \lim_{z \to 0} \frac{\sin z}{z} \times z$

$= 0$

* $f(z) = \frac{z^2+1}{z^2+z^3}$

$= \frac{z^2+1}{z(z+1)}$

singular point $z = 0, -1$, order $= 1$

$Res\ (z=0) = \lim_{z \to 0} \frac{z^2+1}{z(z+1)} \times z$

$= 1$

$Res\ (z = -1) = \lim_{z \to -1} \frac{z^2+1}{z(z+1)} \times (z+1)$

$= -2$

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$f(z) = \frac{z+2}{z(z+1)}$

singular point $z = 0, -1$, order $= 1$

$\therefore \text{Res}(z=0) = \lim_{z\to 0} \frac{(z+2)}{z(z+1)} \times z$

$= 2$

$\text{Res}(z=-1) = \lim_{z\to -1} \frac{(z+2)}{z(z+1)} \times (z+1)$

$= -1$

$f(z) = \frac{2z}{(2z-i)^3}$

singular point $z = \frac{i}{2}$, order $= 3$

$\text{Res}(z = i/2) = \lim_{z\to i/2} \frac{1}{(3-1)!} \frac{d^{3-1}}{dz^{3-1}} \left\{ (z - i/2)^3 \frac{2z}{(2z-i)^3} \right\}$

$= \frac{1}{16} \lim_{z\to i/2} \frac{d^2}{dz^2} (2z)$

$= 0$

$f(z) = \frac{2z-1}{z^2-z} = \frac{2z-1}{z(z-1)}$

singular point $z = 0, 1$, order $= 1$

$\text{Res}(z=0) = \lim_{z\to 0} \frac{2z-1}{z(z-1)} \times z = 1$

$\text{Res}(z=1) = \lim_{z\to 1} \frac{2z-1}{z(z-1)} \times (z-1) = 1$

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$$* \oint \frac{e^{-z}}{z^2} \; ; \; |z|=2$$

singular point $z=0$, order $=2$ which is inside circle $|z|=2$

$$\text{Res}(z=0) = \lim_{z \to 0} \frac{1}{(2-1)!} \frac{d^{2-1}}{dz^{2-1}} \left(z^2 \times \frac{e^{-z}}{z^2} \right)$$

$$= \lim_{z \to 0} \frac{d}{dz} (e^{-z})$$

$$= -1$$

CRT: $\oint \frac{e^{-z}}{z^2} = 2\pi i \times (-1) = -2\pi i$

(Ans.)

$$* \oint \frac{dz}{z^2+4}$$

i) $|z+2i| = 1$.

singular point $z=-2i$, order $=1$ which is inside circle $|z+2i|=1$

$$\text{Res}(z=-2i) = \lim_{z \to 2i} (z+2i) \frac{1}{(z+2i)(z-2i)}$$

$$= -\frac{1}{4i}$$

CRT: $\oint \frac{dz}{z^2+4} = 2\pi i \times \left(-\frac{1}{4i}\right)$

$= -\frac{\pi}{2}$

ii) $|z-2i| = 1$

singular point $= 2i$, order $= 1$ which is inside circle $|z-2i|=1$

$\text{Res}(z=2i) = \lim_{z \to 2i} (z-2i) \times \frac{1}{(z-2i)(z+2i)}$

$= \frac{1}{4i}$

CRT: $\oint \frac{dz}{z^2+4} = 2\pi i \times \frac{1}{4i}$

$= \frac{\pi}{2}$

* $\oint \frac{\cos(\pi z^3)}{(z-1)(z-2)}$

i) $|z-3| = 4$

singular point $z = 1, 2$, order $= 1$ which is inside circle $|z-3|=4$

$\text{Res}(z=1) = \lim_{z \to 1} (z-1) \frac{\cos(\pi z^3)}{(z-2)(z-1)}$

$= \cos \frac{\pi}{-1} = 1$

$$\text{Res}(z=2) = \lim_{z\to 2} (z-2) \frac{\cos(\pi z^3)}{(z-1)(z-2)}$$

$$= \frac{\cos \pi 8}{2-1}$$

$$= 1$$

$$\therefore CRT = \oint \frac{\cos \pi z^3}{(z-2)(z-1)}$$

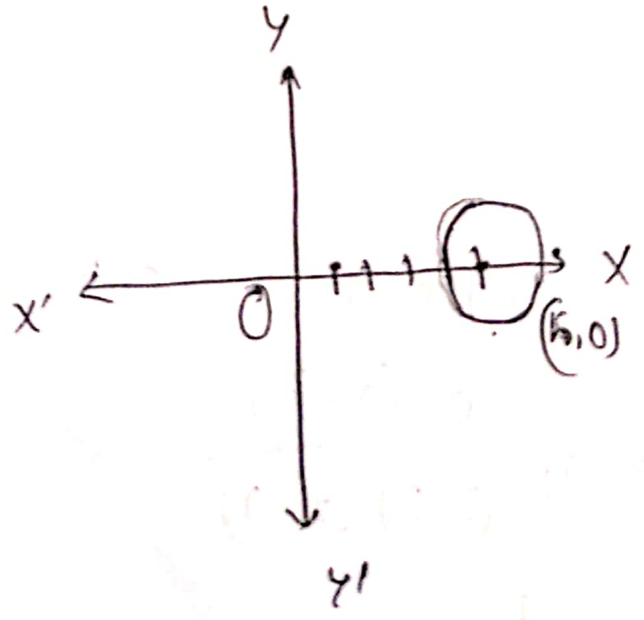
$$= 2\pi i \times (1+1)$$

$$= 4\pi i$$

ii) $|z-5| = 1$

singular point $z = 1, 2$, order $= 1$ which is outside the circle and total is zero.

$|z-5| = 1$, center $(5,0)$, radius $= 1$



iii) $|z|=1$

Sigular point $z=1,2$, order $=1$ where point 1 is inside the circle and point 2 is outside the circle.

$$Res(z=1) = \lim_{z \to 1} \frac{\cos \pi z^3}{(z-1)(z-2)}$$

$$= \frac{-1}{-1} = 1$$

$$CRT = \oint \frac{\cos(\pi z^3)}{(z-1)(z-2)}$$

$$= 2\pi i \times 1$$

$$= 2\pi i$$

* $\oint \frac{\sin 3z}{(z-\pi)^2}$

Singular point $z=\pi$, order $=2$ which is inside circle $|z|=4$

$$Res(z=\pi) = \lim_{z \to \pi} \frac{1}{(2-1)!} \frac{d^{2-1}}{dz^{2-1}} \left\{ (z-\pi)^2 \times \frac{\sin 3z}{(z-\pi)^2} \right\}$$

$$= \lim_{z \to \pi} 3\cos 3z$$

$$= -3$$

$$CRT = \oint \frac{\sin 3z}{(z-\pi)^2} = 2\pi i \times (-3)$$

$$= -6\pi i$$

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* $$\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+5}$$

Singular path $z = -1 \pm 2i$, order $= 1$.

$z_1 = -1+2i$ [interior]

$z_2 = -1-2i$ [exterior]

$Res(z=z_2) = 0$

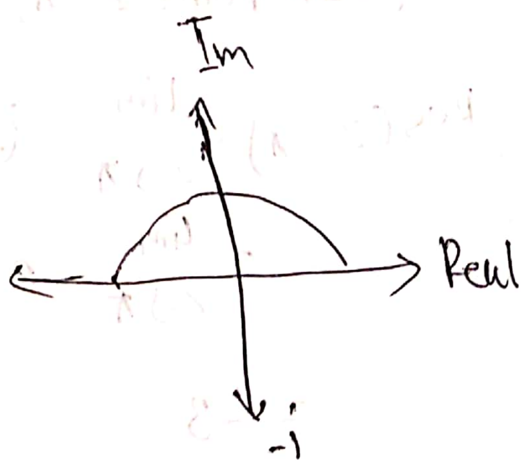
$$Res(z=z_1) = \lim_{z \to z_1} \frac{1}{(z-z_1)(z-z_2)} (z-z_1)$$

$$= \frac{1}{z_1 - z_2} = \frac{1}{-1+2i+1+2i}$$

$$= \frac{1}{4i}$$

$$CRT = 2\pi i \times \frac{1}{4i} = \frac{\pi}{2}$$

* $$\int_{0}^{\infty} \frac{dx}{x^2+1}$$



Singular point $z = \pm i$, order $= 1$

$z_1 = i$ [interior]

$z_2 = -i$ [exterior]

$Res\,(z=z_2)=0$

$$Res\,(z=z_1)=\lim_{z\to z_2}\left[\frac{1}{(z-z_1)(z-z_2)}(z-z_1)\right]$$

$$=\frac{1}{z_1-z_2}$$

$$=\frac{1}{1+i}=\frac{1}{2i}$$

$$\therefore CRT = 2\pi i \times \frac{1}{2i}$$

$$=\pi$$

$$* \int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+1)^2}$$

Singular point $z=\pm i$, order $=2$

$z_1=-i$ [exterior]

$z_2=i$ [interior]

$Res\,(z=z_1)=0$

$$Res\,(z=z_2)=\lim_{z\to i}\frac{d}{dz}\left[\frac{z^2}{(z-i)^2(z+i)^2}\times(z-i)^2\right]$$

$$=\frac{-8i+4i}{16\times 1}=-\frac{i}{4}$$

$$\therefore CRT = 2\pi i\times -\frac{i}{4}=\frac{\pi}{2}$$

* $\int_{-\infty}^{\infty} \frac{dx}{(x^2 - 2x+2)^2}$

Singular point $z = 1 \pm i$, Order $= 2$

$z_1 = 1+i$ [interior]

$z_2 = 1-i$ [exterior]

$Res(z = z_2) = 0$

$$Res(z = z_1) = \lim_{z \to 1+i} \frac{d}{dz} \left\{ \frac{1}{(z-z_1)^2 (z-z_2)^2} \times (z-z_1)^2 \right\}$$

$$= \lim_{z \to 1+i} \frac{(z-z_2)^2 \cdot 1 - 2 \cdot 1 \cdot (z-z_2)}{(z-z_2)^4}$$

$$= \frac{-4-4i}{16} = -\frac{1+i}{4}$$

$$\therefore CRT = 2\pi i \times \frac{-(1+i)}{4}$$

$$= \frac{-\pi i + \pi i^2}{2}$$

$$= \frac{-\pi i - \pi}{2}$$

$$= \frac{-\pi(1+i)}{2}$$

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* $f(z) = \frac{1}{z(z+3)}$

a) $0 < |z| < 3$

$|z| < 3$

$\Rightarrow \frac{|z|}{3} < 1$

$$\therefore f(z) = \frac{1}{z(z+\frac{z}{3})}$$

$$= \frac{1}{3z}\left(1+\frac{z}{3}\right)^{-1}$$

$$= \frac{1}{3z}\left(1-\frac{z}{3}+\frac{z^2}{9}-\frac{z^3}{27}+\cdots\right)$$

b) $|z| > 3$

$\Rightarrow \frac{3}{|z|} < 1$

$$\therefore f(z) = \frac{1}{z(z+3)}$$

$$= \frac{1}{z^2}\left(1+\frac{3}{z}\right)^{-1}$$

$$= \frac{1}{z^2}\left(1-\frac{3}{z}+\frac{9}{z^2}-\frac{27}{z^3}+\cdots\right)$$

* $f(z) = \frac{5z}{(z+1)(z+2)}$

a) $1 < |z| < 2$

$1 < |z|$

$\Rightarrow \frac{1}{|z|} < 1$

$|z| < 2$

$\Rightarrow \frac{|z|}{2} < 1$

$z = -1,$

$= \frac{5 \times (-1)}{-1+2} = -5$

$z = -2$

$= \frac{-10}{-2+1} = 10$

$\therefore \frac{-5}{z+1} + \frac{10}{z+2}$

$= \frac{-5}{z\left(1+\frac{1}{z}\right)} + \frac{10}{2\left(1+\frac{z}{2}\right)}$

$= -\frac{5}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} + \cdots\right) + 5\left(1 - \frac{z}{2} + \frac{z^2}{4} - \cdots\right)$

b) $|z| > 2$

$\frac{2}{|z|} < 1$

Here,

$$\frac{-5}{z+1} + \frac{10}{z+2}$$

$$= \frac{-5}{z+1} + \frac{5}{1+z/2}$$

$$= -5(1+z)^{-1} + 5\left(1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots\right)$$

c) $|z| < 1$

Here,

$$\frac{-5}{(z+1)} + \frac{10}{z+2}$$

$$= \frac{-5}{z(1+\frac{1}{z})} + \frac{10}{(z+2)}$$

$$= -\frac{5}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots\right) + 10(z+2)^{-1}$$

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* $f(z) = \dfrac{2}{(z-1)(3-z)}$

a) for $|z|<1$

$f(z) = \dfrac{1/2}{(z-1)} + \dfrac{3}{(3-z)}$

$= \dfrac{1}{2z}\left(1-\dfrac{1}{z}\right)^{-1} + \dfrac{3}{(3-z)}$

$= \dfrac{1}{2z}\left(1+\dfrac{1}{z}+\dfrac{1}{z^2}+\dfrac{1}{z^3}+\cdots\right) + \dfrac{3}{(3-z)}$

b) $1<|z|<3$

$\dfrac{1}{|z|}<3$, $\dfrac{|z|}{3}<1$

Here, $f(z) = \dfrac{1/2}{(z-1)} + \dfrac{3}{(3-z)}$

$= \dfrac{1}{2z}\left(1-\dfrac{1}{z}\right)^{-1} + \dfrac{3}{3}\left(1-z/3\right)^{-1}$

$= \dfrac{1}{2z}\left(1-\dfrac{1}{z}+\dfrac{1}{z^2}-\dfrac{1}{z^3}+\cdots\right) +$

$1\left(1-\dfrac{z}{3}+\dfrac{z^2}{9}-\dfrac{z^3}{27}+\cdots\right)$

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Slide-20

1. $Z\{\delta[n-m]\} \longleftrightarrow z^{-m}$

We know, $Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$\therefore Z\{\delta[n-m]\} = \sum_{n=-\infty}^{\infty} \delta[n-m] z^{n-m}$

$= \sum_{n=-\infty}^{-1} 0 \times z^{-n-m} + 1 \times z^{-0-m} + \sum_{n=1}^{\infty} 0 \times z^{-n-m}$

$= z^{-m}$

So, $Z^{-1}\{z^{-m}\} = \delta[n-m]$

2. $Z\{na^n u[n]\} \longleftrightarrow \frac{az}{(z-a)^2}$

We know, $Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$\therefore Z\{na^n u[n]\} = \sum_{n=-\infty}^{\infty} na^n u[n] z^{-n}$

$= \sum_{n=-\infty}^{-1} 0 \times z^{-n} + \sum_{n=0}^{\infty} na^n \times z^{-n}$

$$= 0 + az^{-1} + 2a^2 z^{-2} + 3a^3 z^{-3} + \cdots$$

$$= az^{-1}(1 + 2az^{-1} + 3a^2 z^{-2} + \cdots)$$

$$= az^{-1}(1 - \frac{a}{z})^{-2} \quad ; \quad |\frac{a}{z}| < 1$$

$$= \frac{az^{-1}}{(\frac{z-a}{z})^2} \quad ; \quad |a| < |z|$$

$$= \frac{az}{(z-a)^2} \quad ; \quad |z| > a$$

So, $z^{-1}\left\{\frac{az}{(z-a)^2}\right\} = na^n u[n]$

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$* \; z\{(3^n + 2) u[n]\}$

$$= z\{3^n u[n] + 2u[n]\}$$

$$= \frac{z}{z-3} + \frac{2z}{z-1} \quad \text{ROC}: |z| > 3$$

$$= \frac{z(3z-7)}{(z-3)(z-1)}$$

* $Z\{((-1)^n+1)u[n]\}$

$= \frac{z}{z+1} + \frac{z}{z-1}$ ROC: $|z|>1$

$= \frac{2z^2}{z^2-1}$

* $Z\{(-2)^n u[n]\}$

$= \frac{z}{z+2}$ ROC: $|z|>2$

* $Z\{(\frac{2}{3})^n u[n]\}$

$= \frac{z}{z-\frac{2}{3}}$

$= \frac{3z}{3z-2}$ ROC: $|z| > \frac{2}{3}$

* $Z\{\delta[n+4]\}$

$= z^4$

* $Z\{-3\}$

$$= \frac{-3}{1-z^{-1}}$$

$$= \frac{-3z}{z-1}$$

* $Z^{-1}\left\{\frac{1}{1+\frac{2}{5}z^{-1}}\right\}; \; |z| > \frac{2}{5}$

$$= \left(-\frac{2}{5}\right)^n u[n]$$

* $z^{-1}\left\{\frac{1}{2-4z^{-1}}\right\}; \; |z|>2$

$$= z^{-1}\left(\frac{1}{2(1-2z^{-1})}\right)$$

$$= \frac{1}{2} \times 2^n \, u[n]$$

* $z^{-1}\left\{\frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{9}z^{-2}}\right\}; \; |z|>1$

$$= Z^{-1}\left\{\frac{\cancel{\left(1+\frac{1}{3}z^{-1}\right)}}{\cancel{\left(1+\frac{1}{3}z^{-1}\right)}\left(1-\frac{1}{3}z^{-1}\right)}\right\}$$

$= z^{-1}\left(\frac{1}{1-\frac{1}{3}z^{-1}}\right)$

$= \left(\frac{1}{3}\right)^n u[n]$

* $z^{-1}\left\{\frac{1+2z^{-1}}{1-2z^{-1}}\right\}$; $|z| > 2$

$= z^{-1}\left\{\frac{-(1-2z^{-1})+2}{1-2z^{-1}}\right\}$

$= z^{-1}\left\{(-1)+\frac{2}{1-2z^{-1}}\right\}$

$= -\delta[n] + 2 \times 2^n u[n]$

$= \delta[n] + 2^{n+1} u[n]$

* $z^{-1}\left\{\frac{1-z^{-1}}{1-az^{-1}}\right\}$; $|z| > |a|$

$= z^{-1}\left\{\frac{1}{1-az^{-1}} - \frac{az^{-1}}{a(1-az^{-1})}\right\}$

$= a^n u[n] - \frac{1}{a} n a^n u[n]$

$= a^n u[n] - n a^{n-1} u[n]$

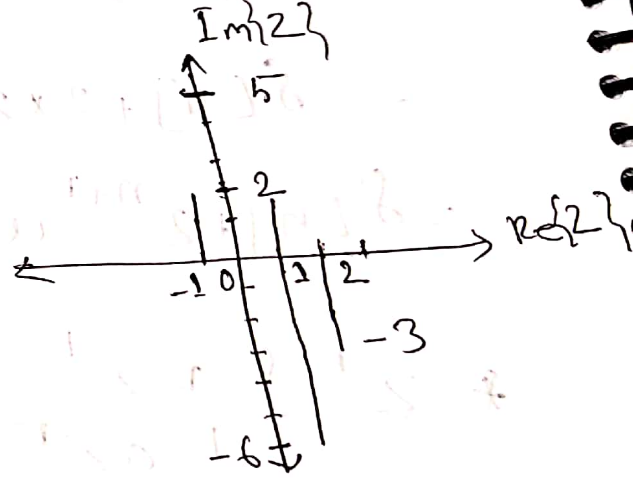
$$X(z) = (1+2z)(1+3z^{-1})(1-z^{-1})$$

$$\Rightarrow X(z) = (1+2z)(1+2z^{-1}-3z^{-2})$$

$$\Rightarrow X(z) = (5+2z^{-1}-3z^{-2}+2z-6z^{-1})$$

$$\Rightarrow z^{-1}\{X(z)\} = z^{-1}(5+2z^{-1}-3z^{-2}+2z-6z^{-1})$$

$$\Rightarrow x[n] = 5\delta[n] + 2\delta[n-1] - 3\delta[n-2] + 2\delta[n+1] - 6\delta[n-1])$$



$$x[n] = \begin{cases} 5, & n=0 \\ 2, & n=1 \\ -3, & n=2 \\ 2, & n=-1 \\ -6, & n=1 \\ 0, & \text{otherwise} \end{cases}$$

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