Introduction to Electrical Circuits

Mid Term
Lecture – 6

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W3	MC6	Chapter 8	8.9 NODAL ANALYSIS (Either General or Format Approach)	See the circuit given	32(a, b), 33(a, b), 35(I, II), 36(I)

Nodal Analysis

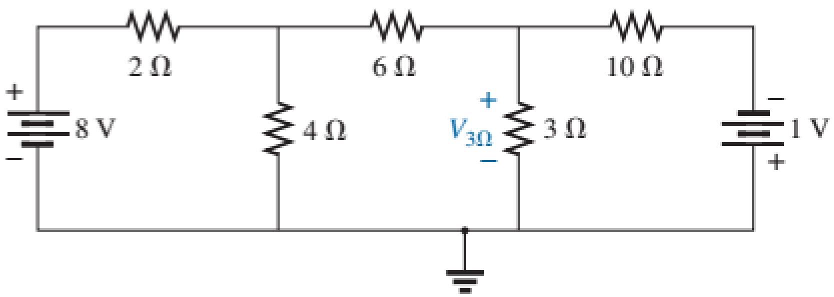
1. Determine the number of nodes within the network.

2. Pick a reference node, and label each remaining node with a subscripted value of voltage: $V1$, $V2$, and so on.

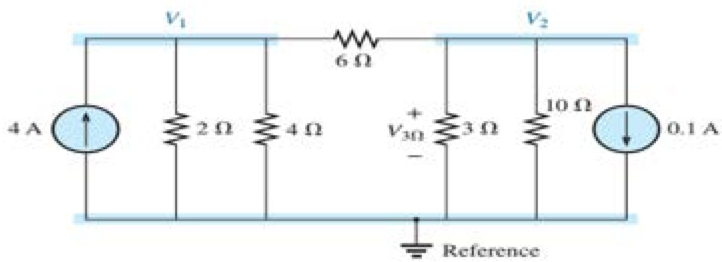
3. Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.

4. Solve the resulting equations for the nodal voltages.

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.



Solution: Converting sources and choosing nodes (Fig. 8.62), we have



$$\left.\begin{aligned}\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2 &= +4\text{ A}\\ \left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1 &= -0.1\text{ A}\end{aligned}\right\}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

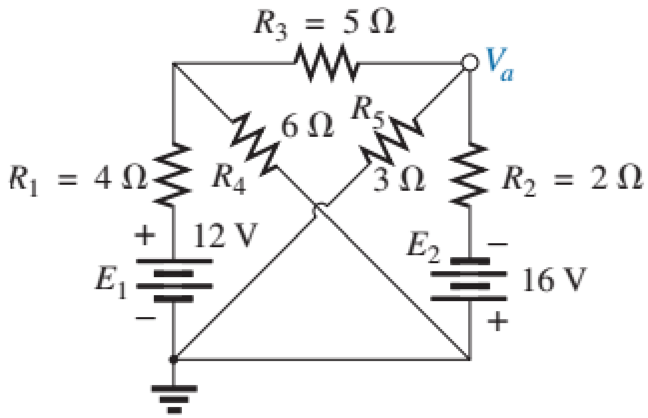
$$11V_1-2V_2=+48$$

$$-5V_1+18V_2=-3$$

$$V_2=V_{3\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.10\text{ V}}$$

Exercise Problems

32. Using mesh analysis, determine $I_{5\Omega}$ and V_a for the network in Fig. 8.121(b).



(b)

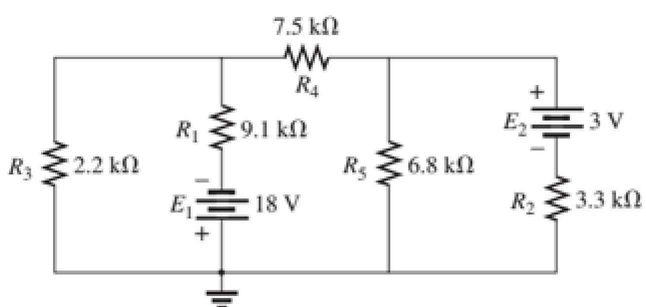
Solution:

From Sol. 24(b)
I_1 I_2 I_3

$$I_1(6+4) - 4I_2 = -12$$
$$I_2(4+5+2) - 4I_1 - 2I_3 = 12 + 16$$
$$I_3(2+3) - 2I_2 = -16$$

$I_{5\Omega} = I_2 = \mathbf{1.95\ A}$
$I_3 = -2.42\ \text{A},\ \therefore\ V_a = (I_3)(3\ \Omega) = (-2.42\ \text{A})(3\ \Omega) = \mathbf{-7.26\ V}$

33. Using mesh analysis, determine the mesh currents for the networks in Fig. 8.122.



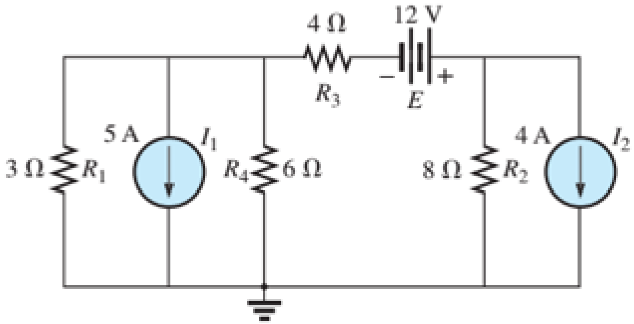
(I)

(I): I_1 I_2 I_3

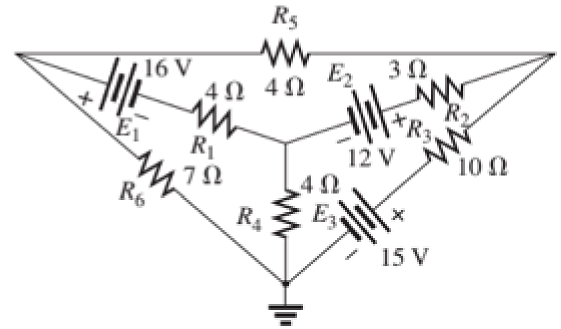
$$(2.2\ \text{k}\Omega + 9.1\ \text{k}\Omega)I_1 - 9.1\ \text{k}\Omega I_2 = 18$$
$$(9.1\ \text{k}\Omega + 7.5\ \text{k}\Omega + 6.8\ \text{k}\Omega)I_2 - 9.1\ \text{k}\Omega\ I_1 - 6.8\ \text{k}\Omega I_3 = -18$$
$$(6.8\ \text{k}\Omega + 3.3\ \text{k}\Omega)I_3 - 6.8\ \text{k}\Omega I_2 = -3$$

$I_1 = \mathbf{1.21\ mA},\ I_2 = \mathbf{-0.48\ mA},\ I_3 = \mathbf{-0.62\ mA}$

36. **a.** Write the nodal equations for the networks in Fig. 8.126.
b. Using determinants, solve for the nodal voltages.
c. Determine the magnitude and polarity of the voltage across each resistor.



(I)



(II)

Solution:

(I): V_1 V_2

$$V_1\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{4}\right]-\frac{1}{4}V_2 = -5-3$$

$$V_2\left[\frac{1}{8}+\frac{1}{4}\right]-\frac{1}{4}V_1 = 3-4$$

$V_1 = \mathbf{-14.86\ V}$, $V_2 = \mathbf{-12.57\ V}$

$V_{R_1} = V_{R_4} = \mathbf{-14.86\ V}$

$V_{R_2} = \mathbf{-12.57\ V}$

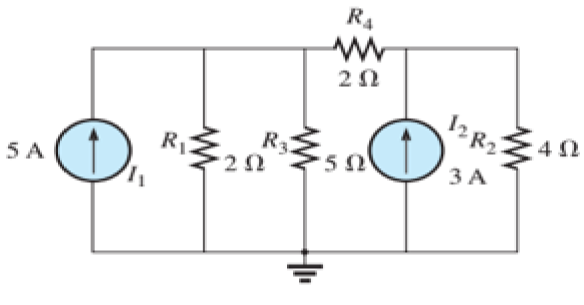
${}^{+}V_{R_3}{}^{-} = 12\text{ V} + 12.57\text{ V} - 14.86\text{ V} = \mathbf{9.71\ V}$

(II):

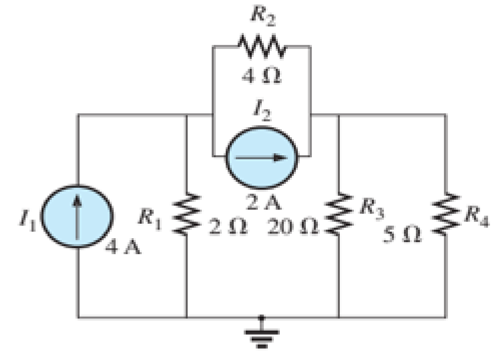
$$(4\,\Omega+4\,\Omega+3\,\Omega)I_1-3\,\Omega\,I_2-4\,\Omega\,I_3 = 16-12$$
$$(4\,\Omega+3\,\Omega+10\,\Omega)I_2-3I_1-4\,\Omega\,I_3 = 12-15$$
$$(4\,\Omega+4\,\Omega+7\,\Omega)I_3-4I_1-4I_2 = -16$$

$I_1 = \mathbf{-0.24\ A}$, $I_2 = \mathbf{-0.52\ A}$, $I_3 = \mathbf{-1.28\ A}$

35. Write the nodal equations for the networks in Fig. 8.125. Using determinants, solve for the nodal voltages. Is symmetry present?



(a)



(b)

a. V_1 V_2

$$V_1\left[\frac{1}{2}+\frac{1}{5}+\frac{1}{2}\right]-\frac{1}{2}V_2=5$$

$$V_2\left[\frac{1}{2}+\frac{1}{4}\right]-\frac{1}{2}V_1=3$$

$V_1 = \mathbf{8.08\ V}$

$V_2 = \mathbf{9.39\ V}$

Symmetry is present

b. V_1 V_2

$$V_1\left[\frac{1}{2}+\frac{1}{4}\right]-\frac{1}{4}V_2=4-2$$

$$V_2\left[\frac{1}{4}+\frac{1}{20}+\frac{1}{5}\right]-\frac{1}{4}V_1=2$$

$V_1 = \mathbf{4.80\ V}$

$V_2 = \mathbf{6.40\ V}$

Symmetry is present


END