Introduction to Electrical Circuits

Mid Term
Lecture – 10

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W5	MC10	Chapter 11	11.5 R-L TRANSIENTS: THE STORAGE PHASE	11.3	12, 19
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11.5 *R-L* TRANSIENTS: THE STORAGE PHASE

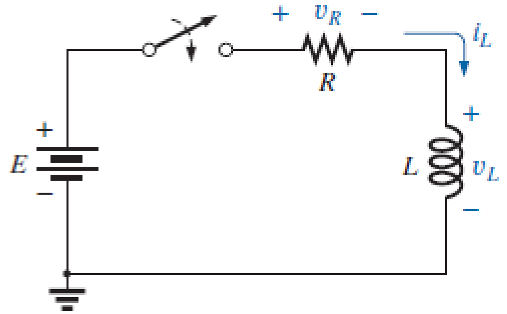
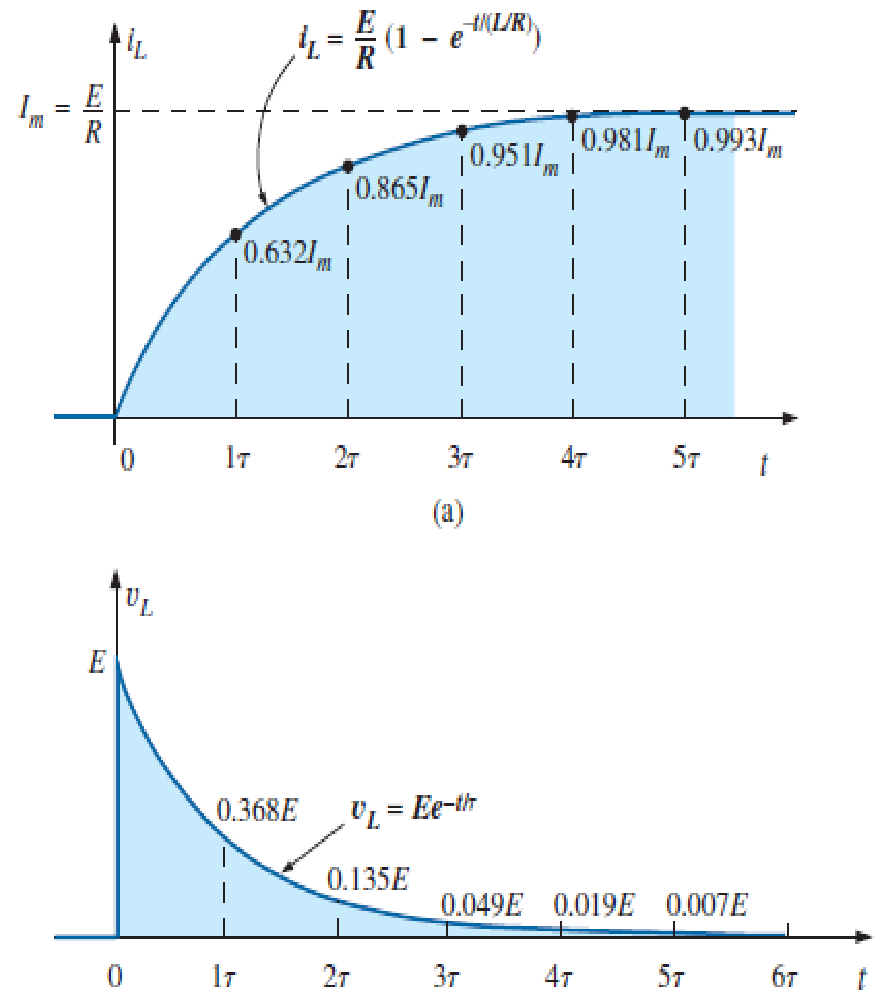


FIG. 11.31
Basic R-L transient network.

$$\boxed{i_L = \frac{E}{R}(1 - e^{-t/\tau})} \qquad \text{(amperes, A)} \qquad \textbf{(11.13)}$$

$$\boxed{\tau = \frac{L}{R}} \qquad \text{(seconds, s)} \qquad \textbf{(11.14)}$$

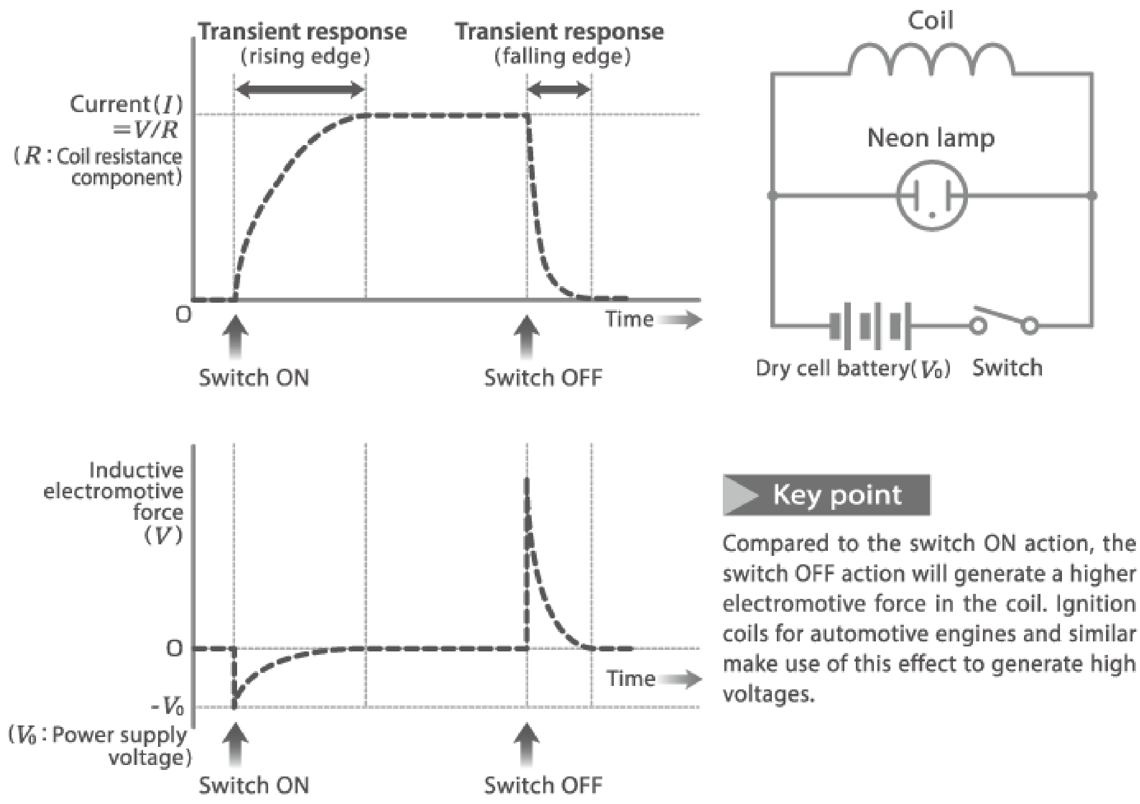
$$\boxed{v_L = Ee^{-t/\tau}} \qquad \text{(volts, V)} \qquad \textbf{(11.15)}$$



(a)

(b)

Transient Response of Inductor



EXAMPLE 11.3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0$ s. Sketch the resulting curves.

Solution: First, the time constant is determined:

$$\tau = \frac{L}{R_1} = \frac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$$

Then the maximum or steady-state current is

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$$

Substituting into Eq. (11.13):

$$\mathbf{i_L = 25\text{ mA}\,(1 - e^{-t/2\text{ms}})}$$

Using Eq. (11.15):

$$\mathbf{v_L = 50\text{ V}e^{-t/2\text{ms}}}$$

The resulting waveforms appear in Fig. 11.37.

FIG. 11.35
Circuit in Fig. 11.31 under steady-state conditions.

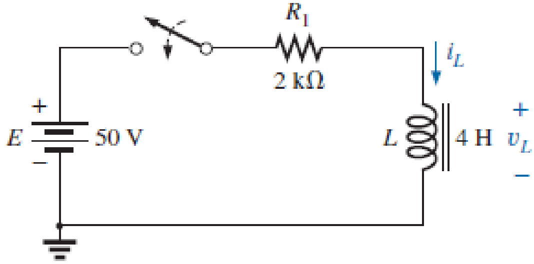


FIG. 11.36
Series R-L circuit for Example 11.3.

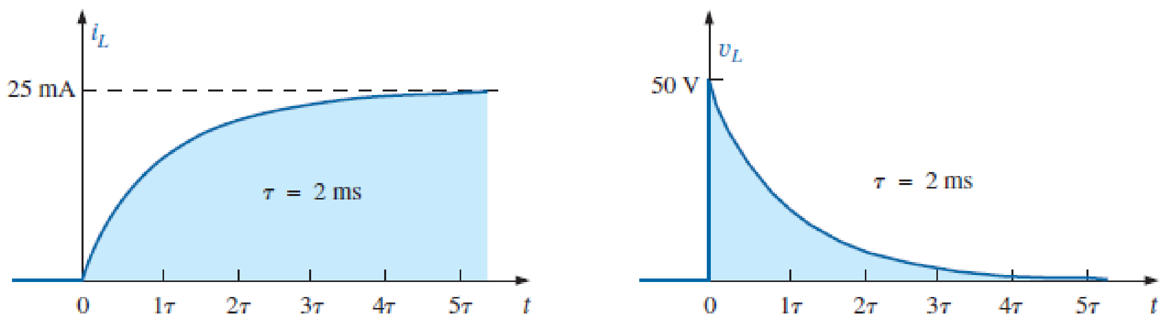


FIG. 11.37
i_L and v_L for the network in Fig. 11.36.

11.7 *R-L* TRANSIENTS: THE RELEASE PHASE

R-L TRANSIENTS: DECAY PHASE

- In the analysis of *R-C* circuits, we found that the capacitor could hold its charge and store energy in the form of an electric field for a period of time determined by the leakage factors.
- In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil.
- Unlike the capacitor, however, an isolated inductor cannot continue to store energy since the absence of a closed path would cause the current to drop to zero, releasing the energy stored in the form of a magnetic field.
- If the series *R-L* circuit of Fig. 12.26 had reached steady-state conditions and the switch were quickly opened, a spark would probably occur across the contacts due to the rapid change in current from a maximum of *E*/*R* to zero amperes. The change in current *di*/*dt* of the equation *vL* _ *L*(*di*/*dt*) would establish a high voltage *vL* across the coil that in conjunction with the applied voltage *E* appears across the points of the switch.

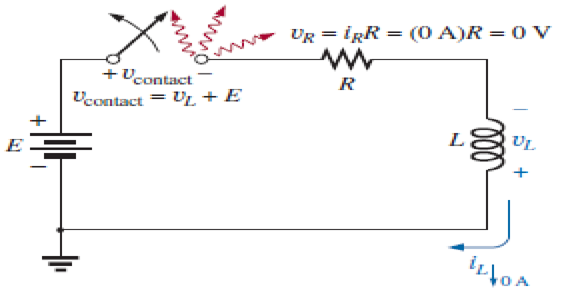


FIG. 11.41
Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

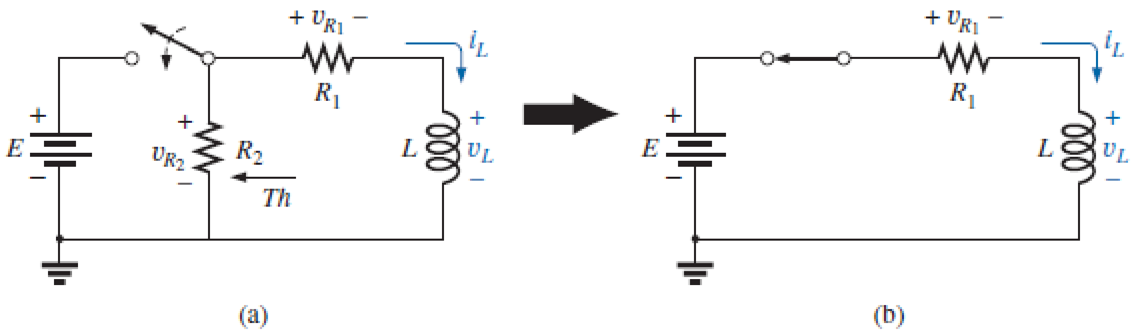


FIG. 11.42

Initiating the storage phase for an inductor by closing the switch.

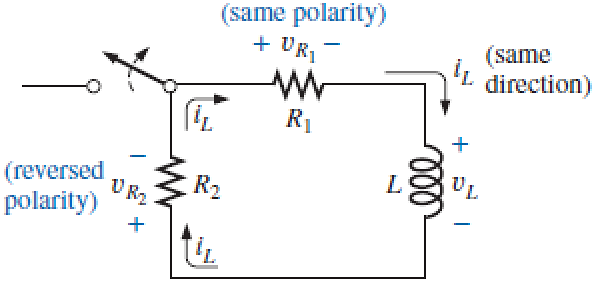


FIG. 11.43

Network in Fig. 11.42 the instant the switch is opened.

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to resistor R_2, which provides a complete path for the current i_L. In fact, for clarity the discharge path is isolated in Fig. 11.43. The voltage v_L across the inductor reverses polarity and has a magnitude determined by

$$\boxed{v_L = -(v_{R_1} + v_{R_2})}$$

$$\begin{aligned} v_L &= -(v_{R_1} + v_{R_2}) = -(i_1R_1 + i_2R_2) \\ &= -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2) = -\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)E \end{aligned}$$

and

$$\boxed{v_L = -\left(1 + \frac{R_2}{R_1}\right)E} \quad \text{switch opened} \qquad (11.19)$$

As an inductor releases its stored energy, the voltage across the coil decays to zero in the following manner:

$$\boxed{v_L = -V_i e^{-t/\tau'}} \qquad (11.20)$$

with $$V_i = \left(1 + \frac{R_2}{R_1}\right)E$$

and $$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2}$$

The current decays from a maximum of $I_m = E/R_1$ to zero.

Using Eq. (11.17):

$$I_i = \frac{E}{R_1} \quad \text{and} \quad I_f = 0 \text{ A}$$

so that $$i_L = I_f + (I_i - I_f)e^{-t/\tau'} = 0 \text{ A} + \left(\frac{E}{R_1} - 0 \text{ A}\right)e^{-t/\tau'}$$

and $$\boxed{i_L = \frac{E}{R_1}e^{-t/\tau'}} \qquad (11.21)$$

with $$\tau' = \frac{L}{R_1 + R_2}$$

EXAMPLE 11.5 Resistor R_2 was added to the network in Fig. 11.36 as shown in Fig. 11.44.

a. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Fig. 11.43.

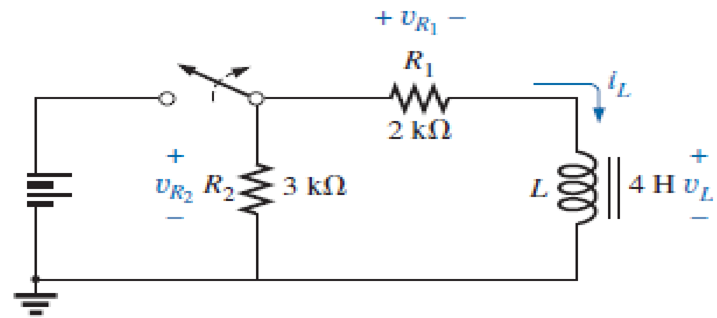


FIG. 11.44
ined polarities for v_{R_1}, v_{R_2}, v_L, and current direction for i_L Example 11.5.

a. From Example 11.3:

$$i_L = 25\text{ mA}(1 - e^{-t/2\text{ms}})$$
$$v_L = 50\text{ V}e^{-t/2\text{ms}}$$
$$v_{R_1} = i_{R_1}R_1 = i_L R_1$$
$$= \left[\frac{E}{R_1}(1 - e^{-t/\tau})\right]R_1$$
$$= E(1 - e^{-t/\tau})$$

and
$$v_{R_1} = 50\text{ V}(1 - e^{-t/2\text{ms}})$$
$$v_{R_2} = E = 50\text{ V}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \dfrac{4\text{ H}}{5 \times 10^3\ \Omega}$

$= 0.8 \times 10^{-3}\text{ s} = 0.8\text{ ms}$

By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and $$\upsilon_L = -V_i e^{-t/\tau'} = -\mathbf{125\ V}e^{-t/-0.8\text{ms}}$$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and $$i_L = I_m e^{-t/\tau'} = \mathbf{25\ mA}e^{-t/0.8\text{ms}}$$

By Eq. (11.22):

$$\upsilon_{R_1} = Ee^{-t/\tau'} = \mathbf{50\ V}e^{-t/0.8\text{ms}}$$

By Eq. (11.23):

$$\upsilon_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = -\mathbf{75\ V}e^{-t/0.8\text{ ms}}$$

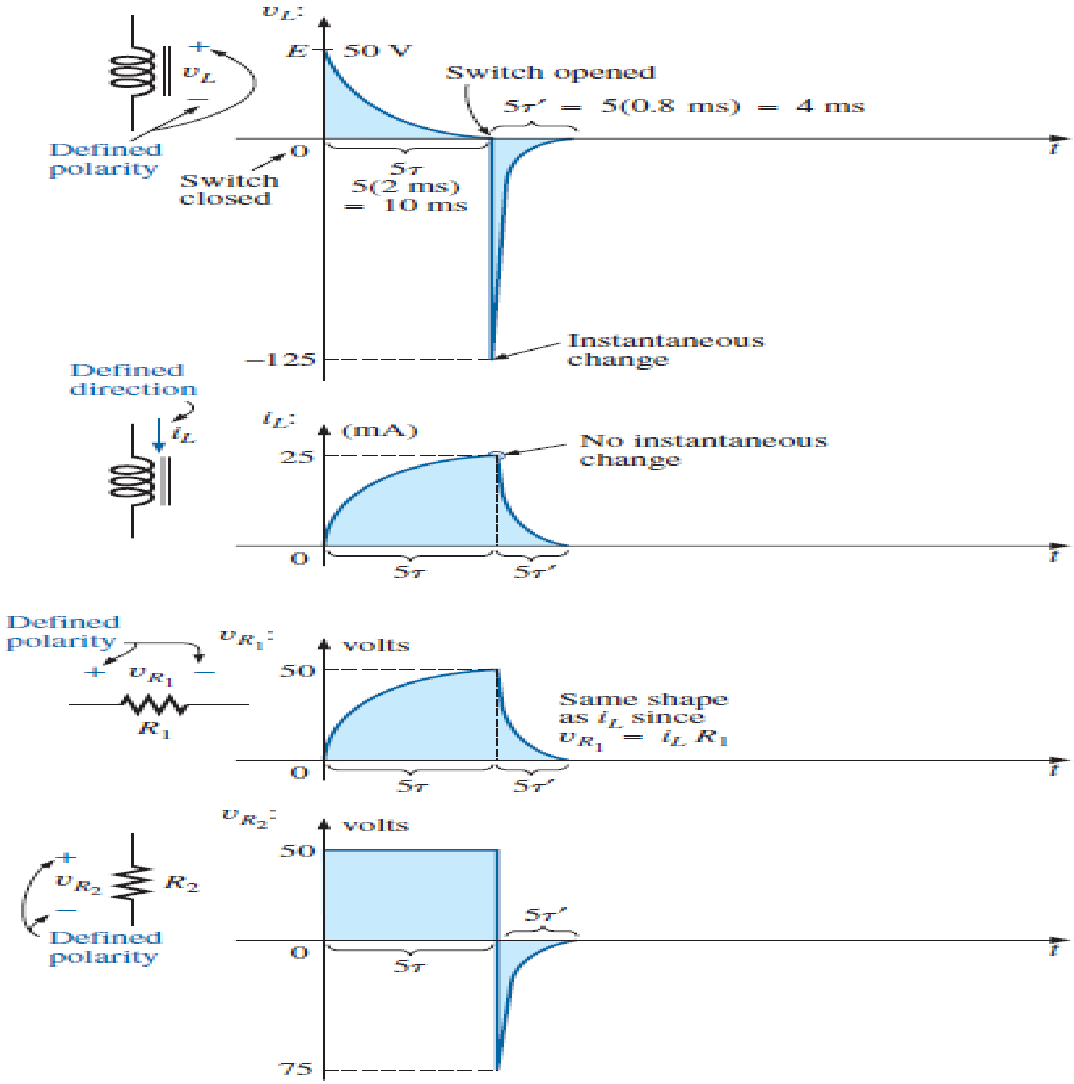


FIG. 11.45

The various voltages and the current for the network in Fig. 11.44.

Exercise Problems

12. For the circuit in Fig. 11.84:

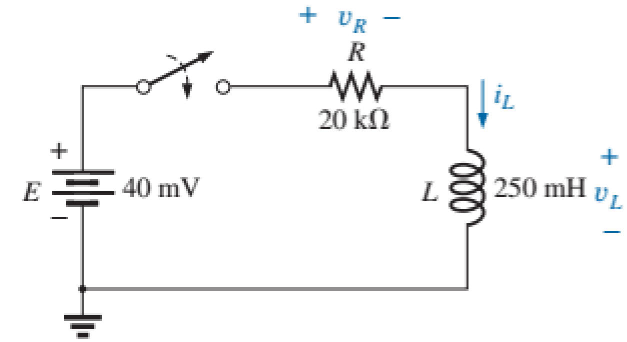
a. Determine the time constant.

b. Write the mathematical expression for the current i_L after the switch is closed.

c. Repeat part (b) for υ_L and υ_R.

d. Determine i_L and υ_L at one, three, and five time constants.

e. Sketch the waveforms of i_L, υ_L, and υ_R.



Solution:

a. $\tau = \frac{L}{R} = \frac{250\text{ mH}}{20\text{ k}\Omega} = \mathbf{12.5\ \mu s}$

b. $i_L = \frac{E}{R}(1 - e^{-t/\tau}) = \frac{40\text{ mV}}{20\text{ k}\Omega}(1 - e^{-t/\tau})$

$= \mathbf{2\ \mu A(1 - e^{-t/12.5\mu s})}$

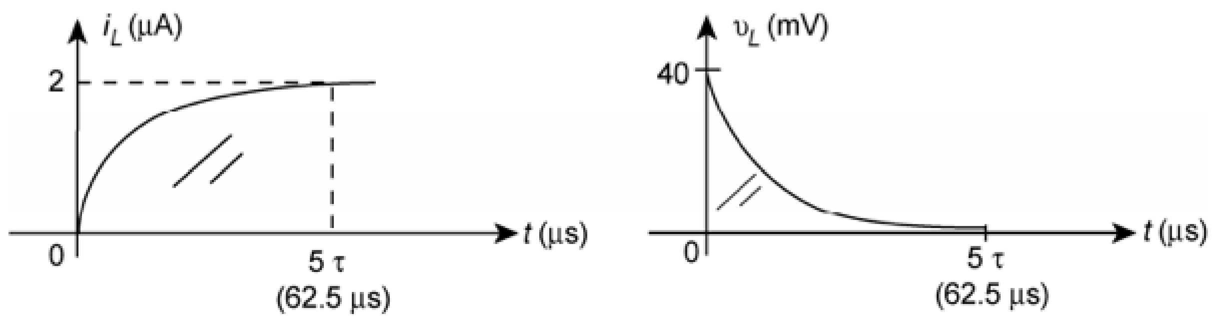
c. $\upsilon_L = Ee^{-t/\tau} = \mathbf{40\ mV}e^{-t/12.5\ \mu s}$

$\upsilon_R = i_R R = i_L R = E(1 - e^{-t/\tau}) = \mathbf{40\ mV(1 - e^{-t/12.5\ \mu s})}$

d. i_L: $1\tau = \mathbf{1.26\ \mu A}$, , $3\tau = \mathbf{1.9\ \mu A}$, $5\tau = \mathbf{1.99\ \mu A}$

υ_L: $1\tau = \mathbf{14.72\ V}$, $3\tau = \mathbf{1.99\ V}$, $5\tau = \mathbf{0.27\ V}$

e.

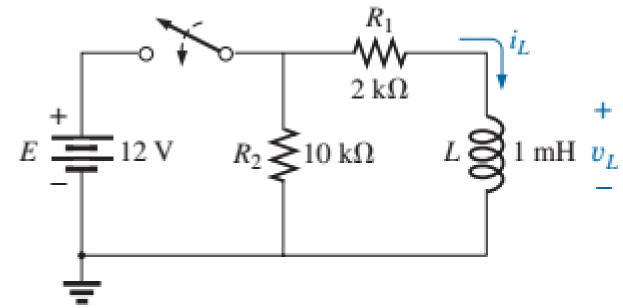


*20. For the network in Fig. 11.92:

a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.

b. Repeat part (a) if the switch is opened at $t = 1\ \mu s$.

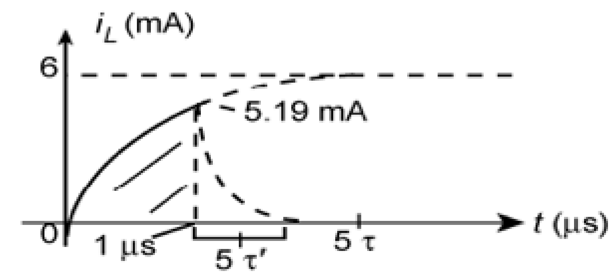
c. Sketch the waveforms of parts (a) and (b) on the same set of axes.



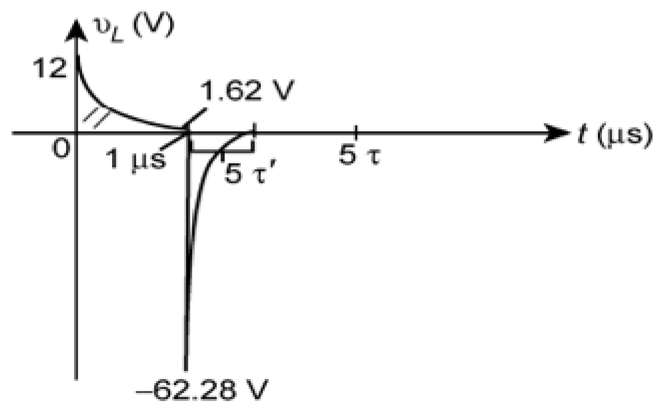
Solution:

a.
$$\tau = \frac{L}{R} = \frac{1\text{ mH}}{2\text{ k}\Omega} = 0.5\ \mu s$$
$$i_L = \frac{E}{R}(1 - e^{-t/\tau}) = \frac{12\text{ V}}{2\text{ k}\Omega}(1 - e^{-t/\tau}) = \mathbf{6\ mA(1 - e^{-t/0.5\mu s})}$$
$$v_L = Ee^{-t/\tau} = \mathbf{12\ V\ e^{-t/0.5\mu s}}$$

c.



b.
$$i_L = 6\text{ mA}(1 - e^{-t/0.5\mu s}) = 6\text{ mA}(1 - e^{-1\mu s/0.5\mu s})$$
$$= 6\text{ mA}(1 - e^{-2}) = 5.19\text{ mA}$$
$$i_L = I'_m e^{-t/\tau'} \qquad \tau' = \frac{L}{R} = \frac{1\text{ mH}}{12\text{ k}\Omega} = 83.3\text{ ns}$$
$$i_L = \mathbf{5.19\ mAe^{-t/83.3ns}}$$
$$t = 1\ \mu s: \ v_L = 12\text{ V}e^{-t/0.5\mu s} = 12\text{ V}e^{-2} = 12\text{ V}(0.1353) = 1.62\text{ V}$$
$$V'_L = (5.19\text{ mA})(12\text{ k}\Omega) = 62.28\text{ V}$$
$$v_L = \mathbf{-62.28\ Ve^{-t/83.3ns}}$$

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