Introduction to Electrical Circuits

Final Term
Lecture - 04

Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Series Parallel Network Analysis

- In general, when working with series-parallel ac networks, consider the following approach:
  - *Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.*
  - *Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts.*
  - *After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations.*
  - *When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit.*

Example: For the network in following Fig. 16.1:

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.
e. Compute the power delivered.
f. Find F_p of the network.

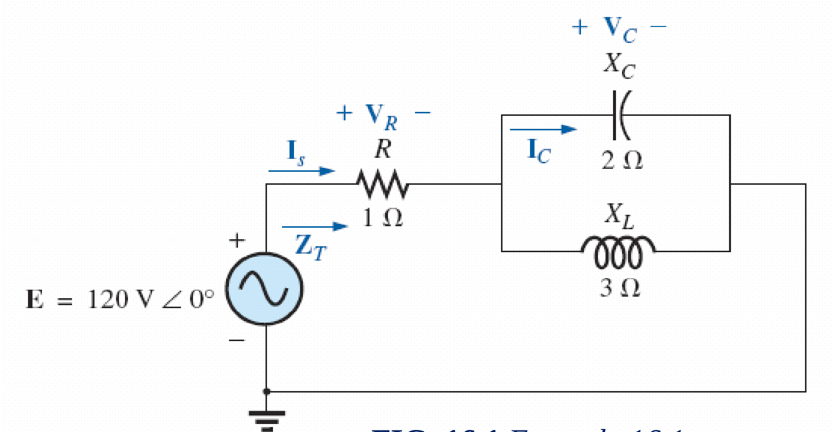


FIG. 16.1 *Example 16.1.*

Solutions:

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\mathbf{Z}_1 = R \angle 0° = 1\ \Omega \angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j X_C + j X_L} = \frac{(2\ \Omega \angle -90°)(3\ \Omega \angle 90°)}{-j\, 2\ \Omega + j\, 3\ \Omega}$$

$$= \frac{6\ \Omega \angle 0°}{j\, 1} = \frac{6\ \Omega \angle 0°}{1 \angle 90°} = 6\ \Omega \angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\, 6\ \Omega = \mathbf{6.08\ \Omega \angle -80.54°}$$

b. $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\text{ V} \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54°}$$

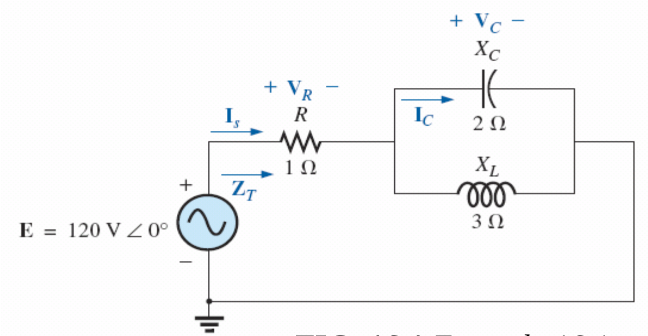


FIG. 16.1 *Example 16.1.*

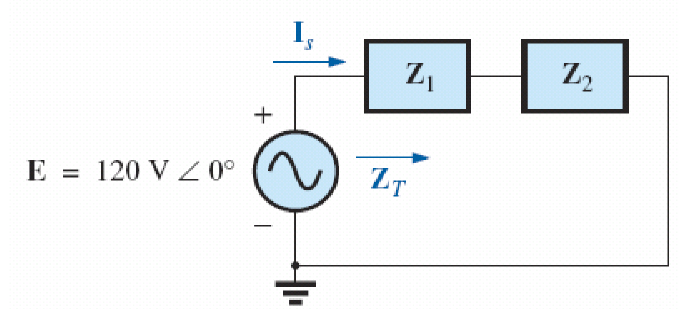


FIG. 16.2 Network in Fig. 16.1 after assigning the block impedances.

c. Referring to Fig. 16.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74 \text{ A} \angle 80.54°)(1 \ \Omega \angle 0°) = \mathbf{19.74 \text{ V} \angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74 \text{ A} \angle 80.54°)(6 \ \Omega \angle -90°)$$
$$= \mathbf{118.44 \text{ V} \angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44 \text{ V} \angle -9.46°}{2 \ \Omega \angle -90°} = \mathbf{59.22 \text{ A} \angle 80.54°}$$

e. $P_{\text{del}} = I_s^2 R = (19.74 \text{ A})^2(1 \ \Omega) = \mathbf{389.67 \text{ W}}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164 \text{ leading}}$

Example: For the network in following Fig. 16.5:

a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.

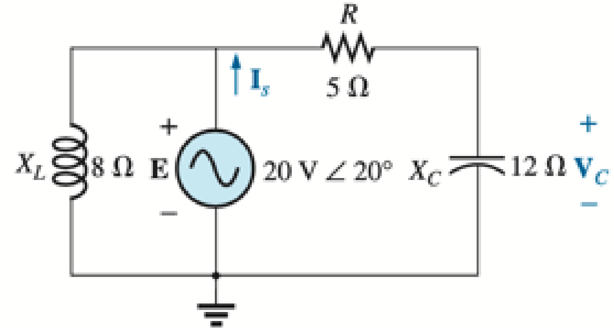


FIG. 16.5
Example 16.3.

Solutions:

a. The network is redrawn as shown in Fig. 16.6, with

$$\mathbf{Z}_1 = 5\ \Omega = 5\ \Omega \angle 0^\circ$$
$$\mathbf{Z}_2 = -j\,12\ \Omega = 12\ \Omega \angle -90^\circ$$
$$\mathbf{Z}_3 = +j\,8\ \Omega = 8\ \Omega \angle 90^\circ$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega \angle -90^\circ)(20\ \text{V} \angle 20^\circ)}{5\ \Omega - j\,12\ \Omega} = \frac{240\ \text{V} \angle -70^\circ}{13 \angle -67.38^\circ}$$
$$= \mathbf{18.46\ V \angle -2.62^\circ}$$

b. $$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\ \text{V} \angle 20^\circ}{8\ \Omega \angle 90^\circ} = 2.5\ \text{A} \angle -70^\circ$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V} \angle 20^\circ}{13\ \Omega \angle -67.38^\circ} = 1.54\ \text{A} \angle 87.38^\circ$$

and

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2$$
$$= 2.5\ \text{A} \angle -70^\circ + 1.54\ \text{A} \angle 87.38^\circ$$
$$= (0.86 - j\,2.35) + (0.07 + j\,1.54)$$
$$\mathbf{I}_s = 0.93 - j\,0.81 = \mathbf{1.23\ A \angle -41.05^\circ}$$

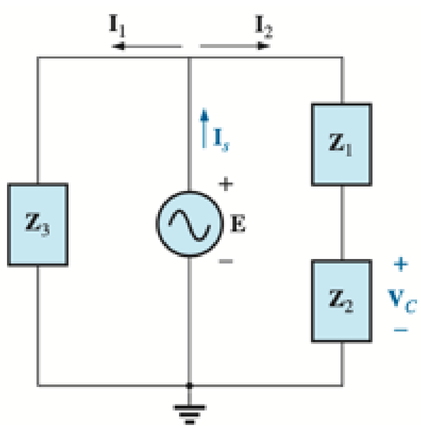


FIG. 16.6
Network in Fig. 16.5 after assigning the block impedances.

Example: For the network in following Figure:

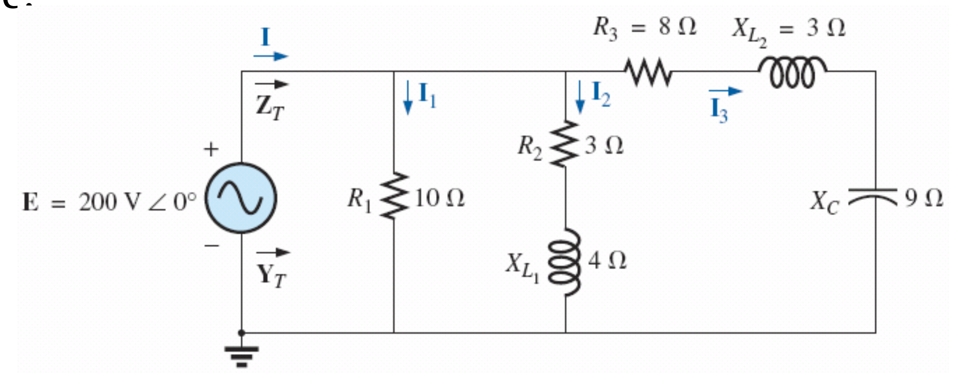
a. Compute **I.**

b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

c. Verify Kirchhoff's current law by showing that

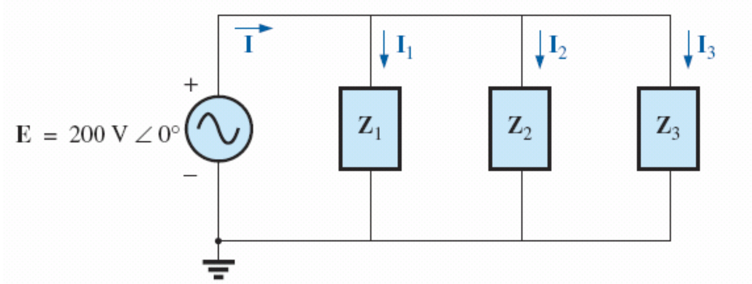
$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.



Solutions:

a. Redrawing the circuit as in Fig. 17.15 reveals a strictly parallel network where



$$\mathbf{Z}_1 = R_1 = 10\ \Omega \angle 0^\circ$$
$$\mathbf{Z}_2 = R_2 + jX_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + jX_{L_2} - jX_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$

The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\
&= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega} \\
&= 0.1\ \text{S} + \frac{1}{5\ \Omega \angle 53.13^\circ} + \frac{1}{10\ \Omega \angle -36.87^\circ} \\
&= 0.1\ \text{S} + 0.2\ \text{S} \angle -53.13^\circ + 0.1\ \text{S} \angle 36.87^\circ \\
&= 0.1\ \text{S} + 0.12\ \text{S} - j\,0.16\ \text{S} + 0.08\ \text{S} + j\,0.06\ \text{S} \\
&= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S} \angle -18.435^\circ
\end{aligned}$$

The current **I** is given by

$$\begin{aligned}
\mathbf{I} &= \mathbf{E}\mathbf{Y}_T = (200\ \text{V} \angle 0^\circ)(0.316\ \text{S} \angle -18.435^\circ) \\
&= \mathbf{63.2\ A \angle -18.44^\circ}
\end{aligned}$$

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle 0^\circ} = \mathbf{20\ A\ \angle 0^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{40\ A\ \angle -53.13^\circ}$$

$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle -36.87^\circ} = \mathbf{20\ A\ \angle +36.87^\circ}$$

c.

$$\begin{aligned} \mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 \\ 60 - j\,20 &= 20\,\angle 0^\circ + 40\,\angle -53.13^\circ + 20\,\angle +36.87^\circ \\ &= (20 + j\,0) + (24 - j\,32) + (16 + j\,12) \\ 60 - j\,20 &= 60 - j\,20 \qquad \text{(checks)} \end{aligned}$$

d.

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316\ \text{S}\ \angle -18.435^\circ} = \mathbf{3.17\ \Omega\ \angle 18.44^\circ}$$

Thank You