Introduction to Electrical Circuits

Final Term
Lecture - 08

Reference Book:

[1] A Textbook of Electrical Technology , Volume- II, - B.L. Theraja, A.K. Theraja

[2] Principles of Electrical Machines -V.K. Mehta, Rohit Mehta

CONTENT

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.
W11	FC8		**Transformer:** Definition, Basic working principle and construction, classification, emf equation, voltage transformation ratio (ex 32.1), losses and efficiency (ex 32.59), applications, cooling of transformers. **Alternator:** Basic principles and construction, Armature windings: Single layer and double layer winding, short-pitch winding, (ex 35.1) Distribution winding factor (35.2, 35.4), Equation of induced EMF (35.6), Factors affecting terminal voltage from induced voltage. Equivalent circuit and phasor diagram of an alternator.	32.1, 32.59, 35.1, 35.2, 35.4, 35.6

Transformer

Definition: *A transformer is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.*

Construction: It consists of primary and secondary windings, as shown in Fig. 1. The winding connected to the a.c. source is called primary and the one connected to load is called secondary. The alternating voltage V_1 is applied to the primary. Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2 and V_2 will appear across the load.

If $V_2 > V_1$, it is called a step up-transformer.
If $V_2 < V_1$, it is called a step down-transformer.

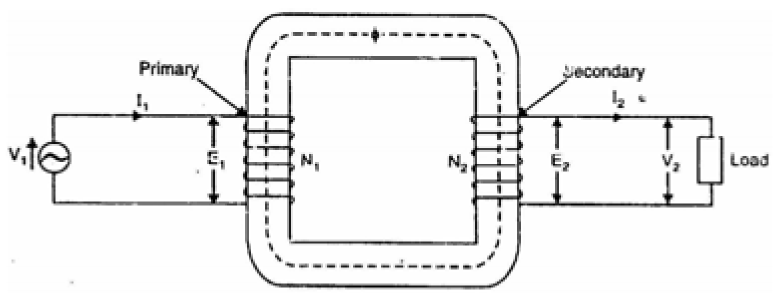
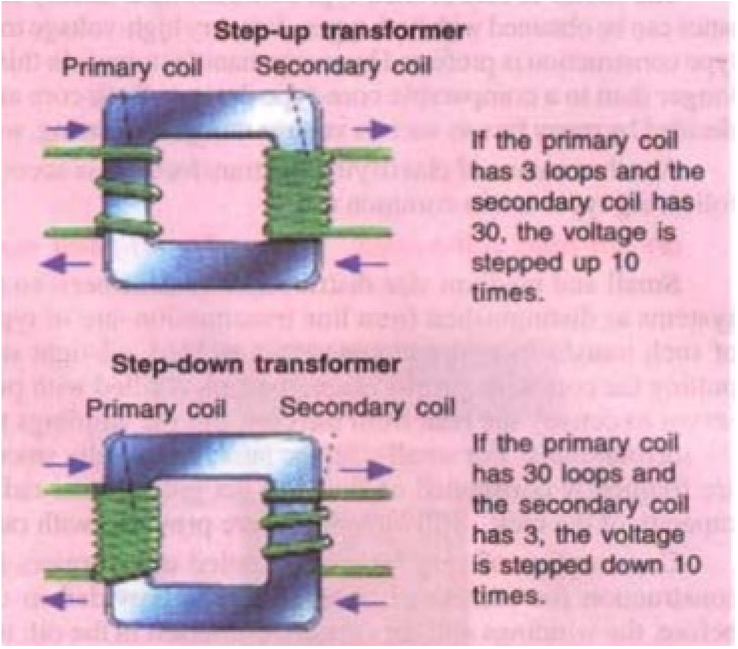
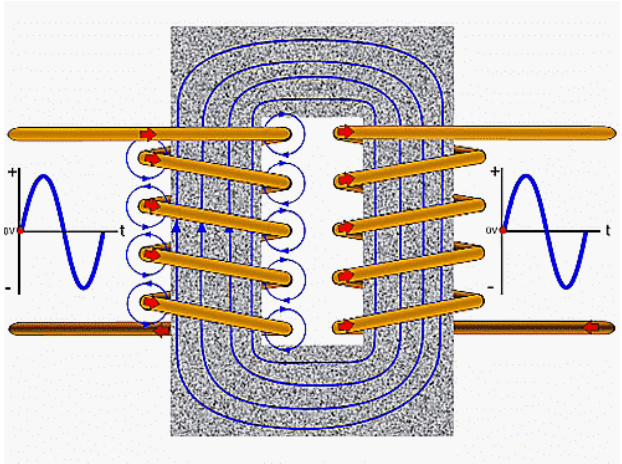


Figure 1



Working Principle of Transformer

- The working principle of a transformer is very simple. Mutual induction between two or more windings (also known as coils) allows for electrical energy to be transferred between circuits.
- Say you have one winding (also known as a coil) which is supplied by an alternating electrical source. The alternating current through the winding produces a continually changing and alternating flux that surrounds the winding.
- If another winding is brought close to this winding, some portion of this alternating flux will link with the second winding. As this flux is continually changing in its amplitude and direction, there must be a changing flux linkage in the second winding or coil.
- According to Faraday's Law of Electromagnetic Induction, there will be an EMF induced in the second winding. If the circuit of this secondary winding is closed, then a current will flow through it. This is the basic working principle of a transformer.

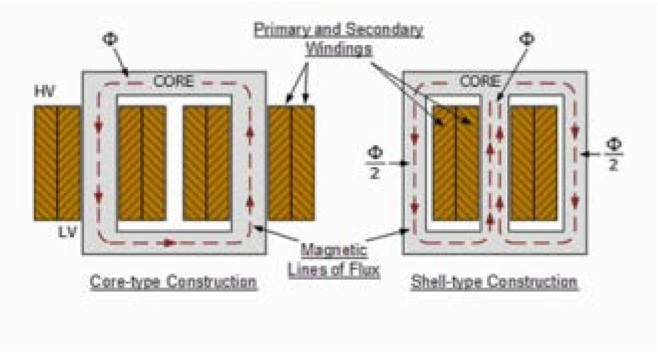


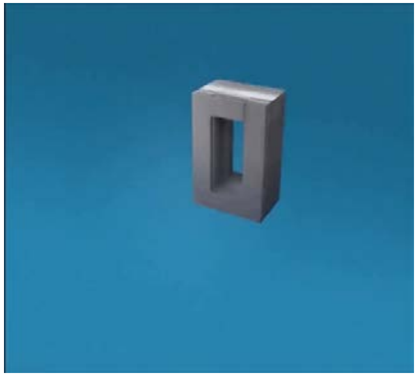
Classification of Transformer

The transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core.

The two types are known as (i) Core type and (ii) Shell type.

Another recent development is spiral-core or wound-core type transformer, the trade name being Spirakore transformer.





Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use :

(*a*) *oil-filled self-cooled*

(*b*) *oil-filled water-cooled*

(*c*) *air-blast type*

See the video in the link: https://www.youtube.com/watch?v=XrIXioEn3yQ

E.M.F. Equation of a Transformer

Let N_1 = No. of turns in primary

N_2 = No. of turns in secondary

Φ_m = Maximum flux in core in webers

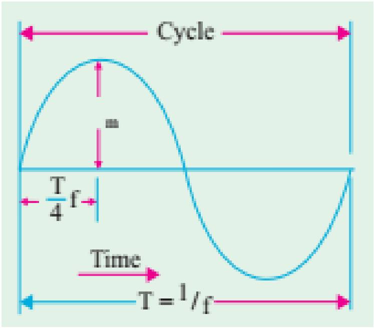
$= B_m \times A$

f = Frequency of a.c. input in Hz

As shown in Fig. 32.14, flux increases from its zero value to maximum value Φ_m in one quarter of the cycle *i.e.* in 1/4 f second.

$$\therefore \quad \text{Average rate of change of flux} = \frac{\Phi_m}{1/4f}$$

$$= 4 f \Phi_m \text{ Wb/s or volt}$$



Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\therefore \quad \text{Average e.m.f./turn} = 4 f \Phi_m \text{ volt}$$

If flux Φ varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

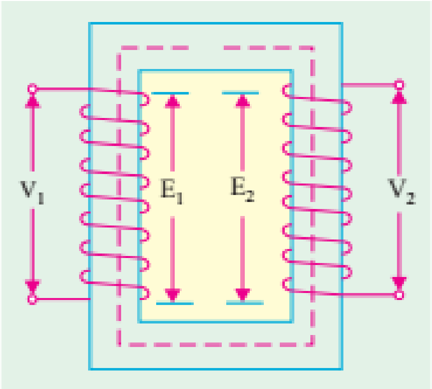
$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$$\therefore \quad \text{r.m.s. value of e.m.f./turn} = 1.11 \times 4 f \Phi_m = 4.44 f \Phi_m \text{ volt}$$

Now, r.m.s. value of the induced e.m.f. in the whole of primary winding

$$= (\text{induced e.m.f/turn}) \times \text{No. of primary turns}$$

$$E_1 = 4.44 f N_1 \Phi_m = 4.44 f N_1 B_m A \qquad \text{...}(i)$$



Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E_2 = 4.44 f N_2 \Phi_m = 4.44 f N_2 B_m A \qquad \text{...}(ii)$$

It is seen from (*i*) and (*ii*) that $E_1/N_1 = E_2/N_2 = 4.44 f \Phi_m$. It means that e.m.f./turn is the *same* in both the primary and secondary windings.

In an ideal transformer on no-load, $V_1 = E_1$ and $E_2 = V_2$ where V_2 is the terminal voltage

Voltage Transformation Ratio

From equations (*i*) and (*ii*), we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

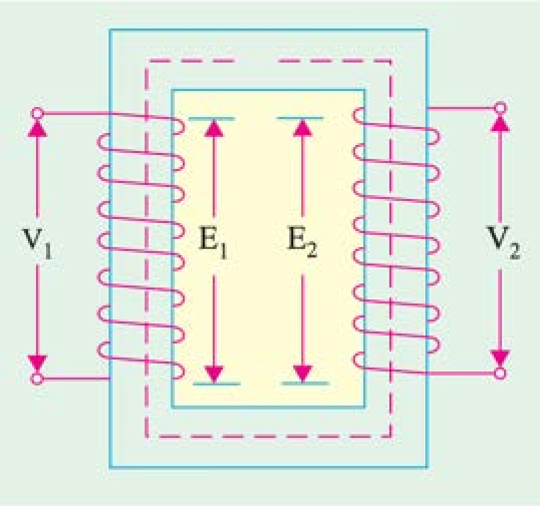
This constant K is known as voltage transformation ratio.

(*i*) If $N_2 > N_1$ *i.e.* $K > 1$, then transformer is called ***step-up*** transformer.

(*ii*) If $N_2 < N_1$ *i.e.* $K < 1$, then transformer is known as ***step-down*** transformer.

Again, for an *ideal* transformer, input VA = output VA.

$$V_1 I_1 = V_2 I_2 \text{ or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$



Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Example 32.1. *The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m². If the e.m.f. per turn is 8 volt, determine*

(*i*) *primary and secondary turns* (*ii*) *area of the core.*

Solution. (*i*) $E_1 = N_1 \times \text{e.m.f. induced/turn}$

$N_1 = 250/8 = \mathbf{32}$; $N_2 = 3000/8 = \mathbf{375}$

(*ii*) We may use $E_2 = -4.44 f N_2 B_m A$

$\therefore$ $3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$; $\mathbf{A = 0.03 m^2}$.

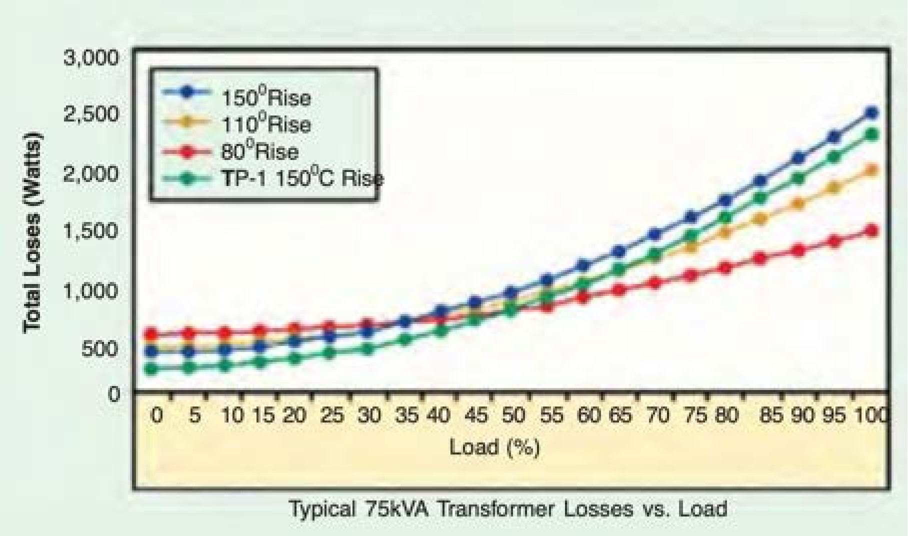
Losses in a Transformer

In a static transformer, there are no friction or windage losses. Hence, the only losses occurring are :

(*i*) Core or Iron Loss: It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load). The core loss is practically the same at all loads.

Hysteresis loss $W_h = \eta B^{1.6}_{max} f V$ watt; eddy current loss $W_e = PB^2_{max} f^2 t^2$ watt

Iron or core loss is found from the *O.C.* test. ***The input of the transformer when on no- load measures the core loss.***



(*ii*) Cu Loss: This loss is due to the ohmic resistance of the transformer windings. Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} + I_2^2 R_{02}$.

Cu loss is proportional to $(\text{current})^2$ or kVA^2. In other words, Cu loss at half the full-load is one-fourth of that at full-load.

Efficiency of Transformer

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input–the two being measured in the same units (either watts or kilowatts).

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

But a transformer being a highly efficient piece of equipment, has very small loss, hence it is impractical to try to measure transformer, efficiency by measuring input and output. These quantities are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from ;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{\text{Output}}{\text{Output} + \text{Cu loss} + \text{iron loss}}$$

or

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$$

Example Problem

Example 32.59. *In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on (i) full load (ii) half full-load.* **(Elect. Engg. & Electronic, Bangalore Univ. 1990 and Similar example in U.P. Technical University 2001)**

Solution. (*i*) Full-load Unity p.f.

Total loss $= 350 + 400 = 750$ W

F.L. output at u.p.f. $= 25 \times 1 = 25$ kW ; $\eta = 25/25.75 = 0.97$ or **97%**

(*ii*) Half F.L. Unity p.f.

Cu loss $= 400 \times (1/2)^2 = 100$ W. Iron loss remains constant at 350 W, Total loss $= 100 + 350 = 450$ W.

Half-load output at u.p.f. $= 12.5$ kW

$\therefore \quad \eta = 12.5/(12.5 + 0.45) =$ **96.52%**

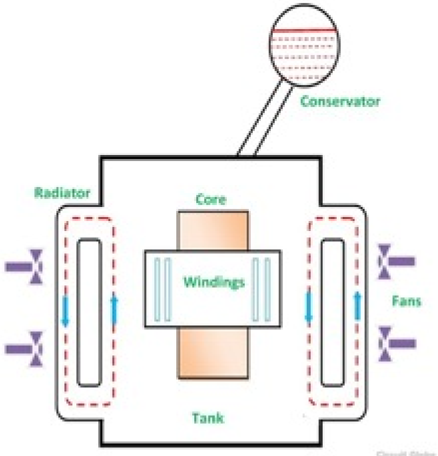
Cooling of Transformer

Heat is produced in a transformer by the iron losses in the core and I^2R loss in the windings. To prevent undue temperature rise, this heat is removed by cooling.

(i) In small transformers (below 50 kVA), natural air cooling is employed i.e., the heat produced is carried away by the surrounding air.

(ii) Medium size power or distribution transformers are generally cooled by housing them in tanks filled with oil. The oil serves a double purpose, carrying the heat from the windings to the surface of the tank and insulating the primary from the secondary.

(iii) For large transformers, external radiators are added to increase the cooling surface of the oil filled tank. The oil circulates around the transformer and moves through the radiators where the heat is released to surrounding air. Sometimes cooling fans blow air over the radiators to accelerate the cooling process.



Applications of Transformer

There are four principal applications of transformers viz.

(i) power transformers
(ii) distribution transformers
(iii) autotransformers
(iv) instrument transformers

- Transformers are used for impedance matching.
- To isolate two circuits electrically.
- It is used to increase or decrease the alternating voltages in electric power applications.
- In voltmeter, ammeters, protective relay etc.
- For stepping up low voltage in case of measurement.
- For stepping down high voltage for safety.
- It is used in rectifier.
- It is used in voltage regulators, voltage stabilizers, power supplies etc.

Alternator

An alternating-current generator is frequently referred to as an alternator. Alternator is also called the synchronous generator because it should be operated (by prime mover) in a speed (synchronous speed) in order to get an induced EMF of the required frequency (infinite bus-bar frequency).

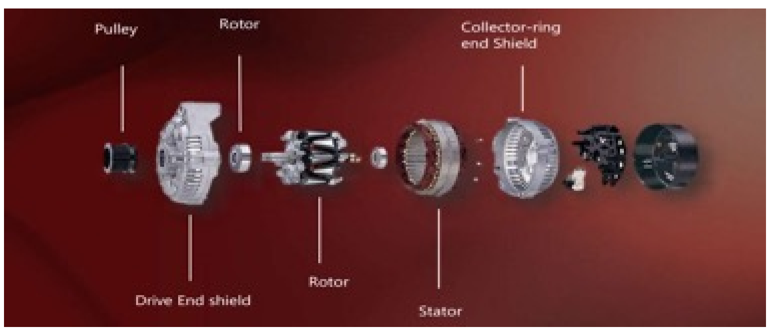
Basic Principle and Construction:

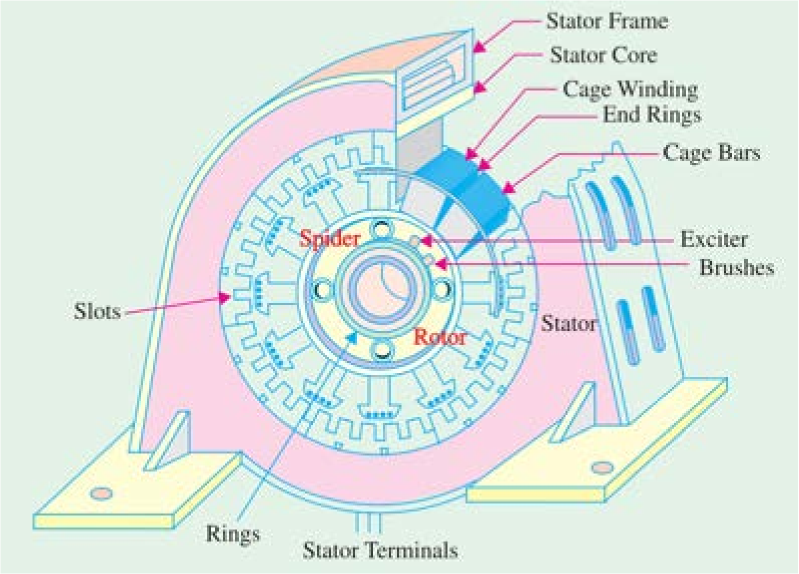
AC generator or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as DC generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in DC generators, the armature rotates and the field system is stationary, the arrangement in alternators just the reverse of it.

Construction:

The construction consists of

I. armature winding mounted on a stationary element called *stator*
II. field windings on a rotating element called *rotor*





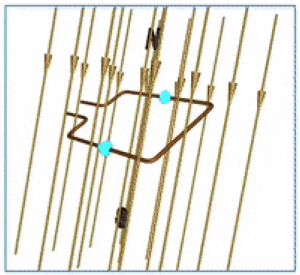
- The **stator** consist of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors.

- The **rotor** is like a flywheel having alternate *N* and *S* poles fixed to its outer rim.

- The **magnetic poles** are excited (or magnetized) from direct current supply by a DC source at 125 to 600 volts

- When the rotor rotates, the stator conductors (being stationary) are cut by the magnetic flux, hence they have induced emf produced in them. Because the magnetic poles are alternately *N* and *S*, they induce an emf and hence current in armature conductors, which first flows in one direction and then in the other.

- Hence, an alternating emf is produced in the stator conductors (*i*) whose frequency depends on the number of *N* and *S* poles moving past a conductor in a one second, and (*ii*) whose direction is given by Fleming's Right-hand rule.



Armature **W**indings

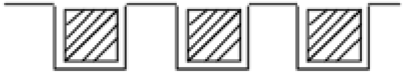
The two types of armature windings most commonly used for 3-phase alternators are:

(i) single-layer winding, and

(ii) double-layer winding.

❖ ***Single- layer winding***

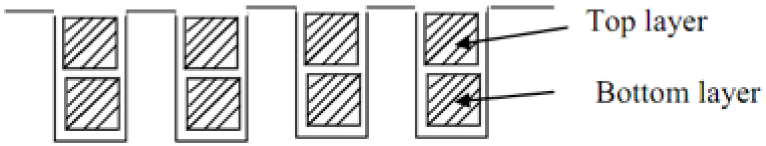
- One coil-side occupies the total slot area
- Used only in small ac machines



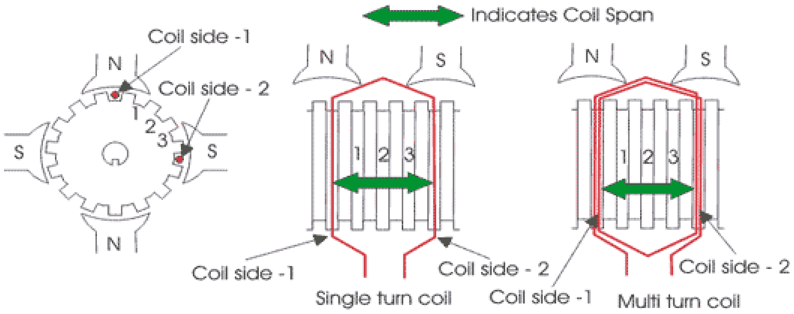
one coil-side per slot

❖ ***Double- layer winding***

- Slot contains even number (may be 2,4,6 etc.) of coil-sides in two layers
- Double-layer winding is more common above about 5kW machines



Wave winding.

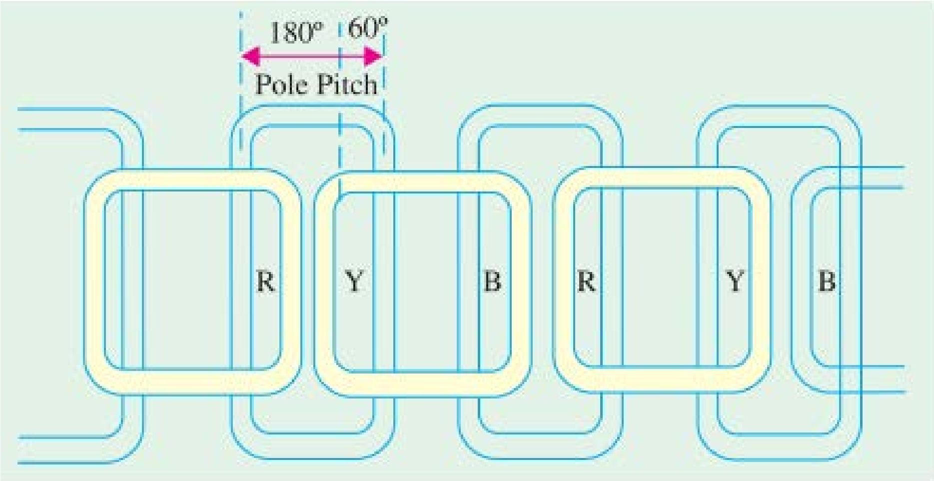


Wave winding star connection.

Concentric or Chain Windings

For this type of winding, the number of slots is equal to twice the number of coils or equal to the number of *coil* sides. In Figure below is shown a concentric winding for 3-phase alternator. It has one coil per pair of poles per phase.

Pole – pitch. A pole pitch is defined as the peripheral distance between identical points on two adjacent poles. Pole pitch is always equal to 180 degree electrical.



Short-pitch Winding : Pitch factor/chording factor

If the coil-span (or coil-pitch) is equal to the pole-pitch, then the coil is termed a full-pitch coil. In case the coil pitch is less than pole pitch, then it is called chorded, short-pitch or fractional-pitch coil

The pitch factor or coil-span factor k_p or k_c is defined as

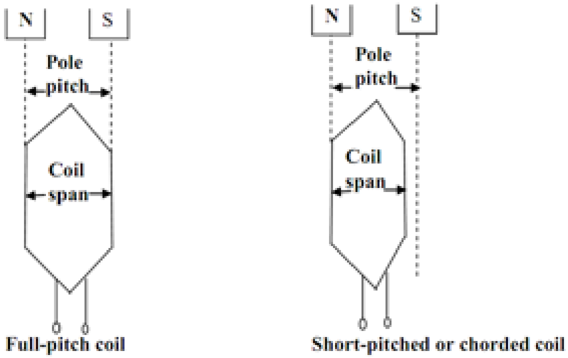
$$= \frac{\text{vector sum of the induced e.m.fs. per coil}}{\text{arithmetic sum of the induced e.m.fs. per coil}}$$

For a coil having a span equal to 5/6 of a pole-pitch. It falls short by 1/6 pole-pitch or by 180°/6 = 30°.

$$\therefore \quad E = 2\,E_S \cos 30°/2 = 2E_S \cos 15°$$

$$k_c = \frac{\text{vector sum}}{\text{arithmetic sum}} = \frac{E}{2\,E_S} = \frac{2\,E_S \cos 15°}{2\,E_S} = \cos 15° = 0.966$$

Hence, pitch factor, $k_c = 0.966$.



It is always less than unity. In general, if the coil span falls short of full-pitch by an angle α (electrical)*, then $k_c = \cos \alpha/2$.

Note. The value of α will usually be given in the question, if not, then assume $k_c = 1$.

Example 37.1. *Calculate the pitch factor for the under-given windings : **(a)** 36 stator slots, 4-poles, coil-span, 1 to 8 **(b)** 72 stator slots, 6 poles, coils span 1 to 10 and **(c)** 96 stator slots, 6 poles, coil span 1 to 12. Sketch the three coil spans.*

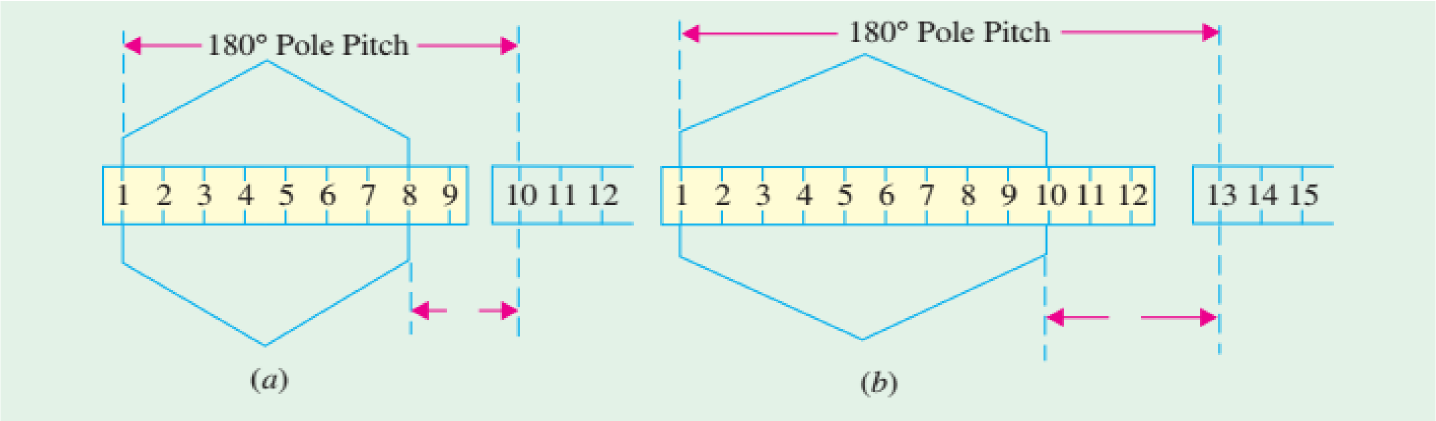


Fig. 37.18

Solution. (*a*) Here, the coil span falls short by $(2/9) \times 180° = 40°$

$$\alpha = 40°$$

$\therefore$ $k_c = \cos 40°/2 = \cos 20° = \mathbf{0.94}$

(*b*) Here $\alpha = (3/12) \times 180° = 45°$ $\quad\therefore$ $k_c = \cos 45°/2 = \cos 22.5° = \mathbf{0.924}$

(*c*) Here $\alpha = (5/16) \times 180° = 56° \, 16'$ $\quad\therefore$ $k_c = \cos 28°8' = \mathbf{0.882}$

The coil spans have been shown in Fig. 37.18.

Distribution or Breadth Factor or Winding Factor or Spread Factor:

All the winding turns are arranged in several full-pitch or fractional-pitch coils
These coils are then housed in the slots spread around the air-gap periphery to form phase or commutator winding
Examples of distributed winding are

- Stator and rotor of induction machines
- The armatures of both synchronous and D.C. machines

$$\begin{aligned} E &= E_S \cos 20° + E_S + E_S \cos 20° \\ &= 2\,E_S \cos 20° + E_S \\ &= 2\,E_S \times 0.9397 + E_S = 2.88\,E_S \end{aligned}$$

The distribution factor (k_d) is defined as

$$= \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

In the present case

$$k_d = \frac{\text{e.m.f. with winding in 3 slots/pole/phase}}{\text{e.m.f. with winding in 1 slots/pole/phase}} = \frac{E}{3\,E_S} = \frac{2.88\,E_S}{3\,E_S} = 0.96$$

General case,

$$k_d = \frac{\text{vector sum of coils e.m.fs.}}{\text{arithmetic sum of coil e.m.fs.}} = \frac{\sin m\,\beta/2}{m \sin \beta/2}$$

Where, β is angular displacement between the slots and

$$\beta = \frac{180°}{\text{No. of slots/pole}} = \frac{180°}{n}$$

Let m = No. of slots/phase/pole
$m\beta$ = phase spread angle

Example 37.2. *Calculate the distribution factor for a 36-slots, 4-pole, single-layer three-phase winding.* **(Elect. Machine-I Nagpur Univ. 1993)**

Solution.

$$n = 36/4 = 9;\ \beta = 180°/9 = 20°;\ m = 36/4 \times 3 = 3$$

$$kd = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3 \times 20°/2}{3 \sin 20°/2} = \mathbf{0.96}$$

Example 37.4. *Find the value of k_d for an alternator with 9 slots per pole for the following cases :*

(i) *One winding in all the slots* ***(ii)*** *one winding using only the first 2/3 of the slots/pole (iii) three equal windings placed sequentially in 60° group.*

Solution. Here, $\beta = 180°/9 = 20°$ and values of *m i.e.* number of slots in a group are 9, 6 and 3 respectively.

(*i*) $m = 9, \quad \beta = 20°, \quad k_d = \dfrac{\sin 9 \times 20°/2}{9 \sin 20°/2} = \mathbf{0.64} \left[\text{or } k_d = \dfrac{\sin \pi/2}{\pi/2} = 0.637\right]$

(*ii*) $m = 6, \quad \beta = 20°, \quad k_d = \dfrac{\sin 6 \times 20°/2}{6 \sin 20°/2} = \mathbf{0.83} \left[\text{or } k_d = \dfrac{\sin \pi/3}{\pi/3} = 0.827\right]$

(*iii*) $m = 3, \quad \beta = 20°, \quad k_d = \dfrac{\sin 3 \times 20°/2}{3 \sin 20°/2} = \mathbf{0.96} \left[\text{or } k_d = \dfrac{\sin \pi/6}{\pi/6} = 0.955\right]$

Equation of Induced E.M.F.

Let Z = No. of conductors or coil sides in series/phase

$= 2T$ — where T is the No. of coils or turns per phase (remember one turn or coil has two sides)

P = No. of poles

f = frequency of induced e.m.f. in Hz

Φ = flux/pole in webers

k_d = distribution factor $= \dfrac{\sin m\beta/2}{m \sin \beta/2}$

k_c or k_p = pitch or coil span factor $= \cos \alpha/2$

k_f = from factor = **1.11** —if e.m.f. is assumed sinusoidal

N = rotor r.p.m.

In one revolution of the rotor (*i.e.* in 60./N second) each stator conductor is cut by a flux of ΦP webers.

$\therefore \quad d\Phi = \Phi P$ and $dt = 60/N$ second

$$\therefore \quad \text{Average e.m.f. induced per conductor} = \frac{d\Phi}{dt} = \frac{\Phi P}{60/N} = \frac{\Phi NP}{60}$$

Now, we know that $f = PN/120$ or $N = 120 f/P$

Substituting this value of N above, we get

$$\text{Average e.m.f. per conductor} = \frac{\Phi P}{60} \times \frac{120 f}{P} = 2 f \Phi \text{ volt}$$

If there are Z conductors in series/phase, then Average e.m.f./phase $= 2f\Phi Z$ volt $= 4 f \Phi T$ volt

R.M.S. value of e.m.f./phase $= 1.11 \times 4f\Phi T = 4.44 f\Phi T$ volt*.

This would have been the actual value of the induced voltage if all the coils in a phase were ***(i)*** full-pitched and ***(ii)*** concentrated or bunched in one slot (instead of being distributed in several slots under poles). But this not being so, the actually available voltage is reduced in the ratio of these two factors.

$\therefore$ Actually available voltage/phase $= 4.44\, k_c k_d f \Phi T = 4\, k_f k_c k_d f \Phi T$ volt.

If the alternator is star-connected (as is usually the case) then the line voltage is $\sqrt{3}$ times the phase voltage (as found from the above formula).

Example 37.6. *A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency rpm and the phase and line e.m.f. Assume full-pitched coil.*

(Elect. Machines, AMIE Sec. B, 1991)

Solution. $f = PN/120 = 16 \times 375/120 = \mathbf{50\ Hz}$

Since k_c is not given, it would be taken as unity.

$$n = 144/16 = 9; \beta = 180°/9 = 20°; m = 144/16 \times 3 = 3$$

$$k_d = \sin 3 \times (20°/2)/3 \sin (20°/2) = 0.96$$

$$Z = 144 \times 10 / 3 = 480; T = 480/2 = 240 / \text{phase}$$

$$E_{ph} = 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 240 = \mathbf{15.34\ V}$$

Line voltage, $E_L = \sqrt{3}\ E_{ph} = \sqrt{3} \times 1534 = \mathbf{2658\ V}$

Factors Affecting Terminal Voltage from Induced Voltage:

As the load on an alternator is varied, its terminal voltage is also found to vary as in D.C. generators. This variation in terminal voltage V is due to the following reasons:

1. Voltage drop due to armature resistance R_a (IR_a): The armature resistance / phase R_a causes a voltage drop / phase of IR_a which is in phase with the armature current I. However, this voltage drop is practically negligible.

2. Voltage drop due to armature leakage reactance X_L (IX_L): When current flows through the armature conductors, fluxes are set up which do not cross the air-gap but take different paths. Such fluxes are known as *leakage fluxes*.

3. Voltage drop due to armature reaction: As in D.C. generators, armature reaction is the effect of armature flux on the main field flux.


THE END