Introduction to Electrical Circuits

Mid Term
Lecture – 7

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W4	MC7	Chapter 9	9.2 SUPERPOSITION THEOREM	See the circuit given	1,2,3,4
			9.3 THÉVENIN'S THEOREM	Try Exercise.	6, 7, 8, 9, 10, 13, 14, 15,

SUPERPOSITION THEOREM

Find the solution to networks with two or more sources that are not in series or parallel.

Does not require the use of a mathematical technique such as determinants to find the required voltages or currents.

The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

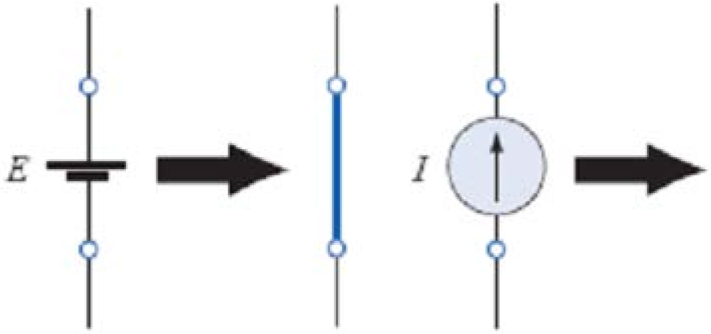
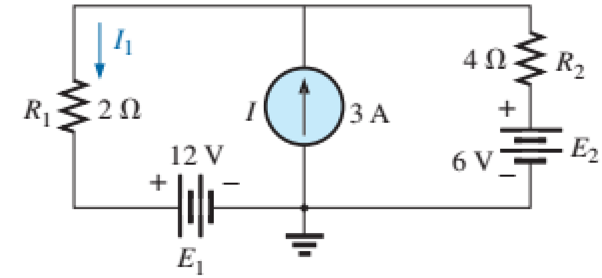


FIG. 9.1 *Removing the effects of ideal sources.*

9.2 SUPERPOSITION THEOREM

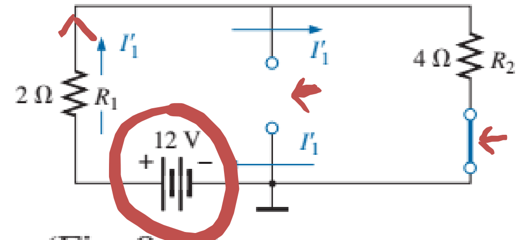
EXAMPLE 9.5 Find the current through the 2 Ω resistor of the network in Fig. 9.18. The presence of three sources results in three different networks to be analyzed.



Solution: **Step 1:**

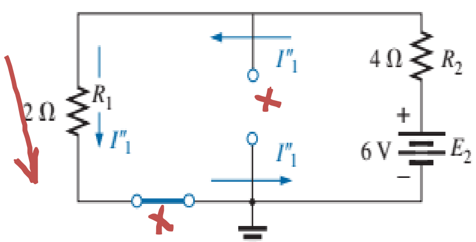
Considering the effect of the 12 V source (Fig. 9.19):

$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$



Step 2:

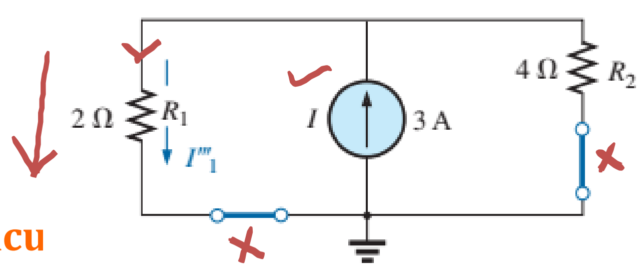
Considering the effect of the 6 V source (Fig. 9.20):



$$= \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

Step 3:

$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{12\text{ A}}{6} = 2\text{ A}$$



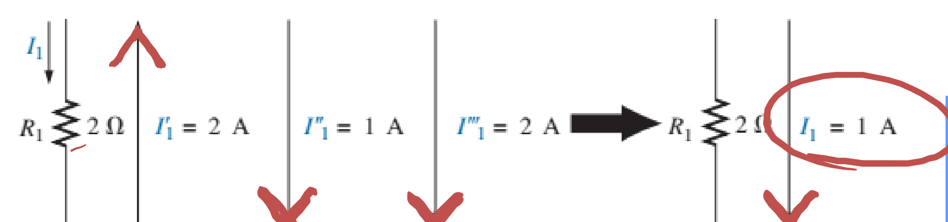
Step 4:

The total current through the 2 Ω resistor appears in Fig. 9.22 and

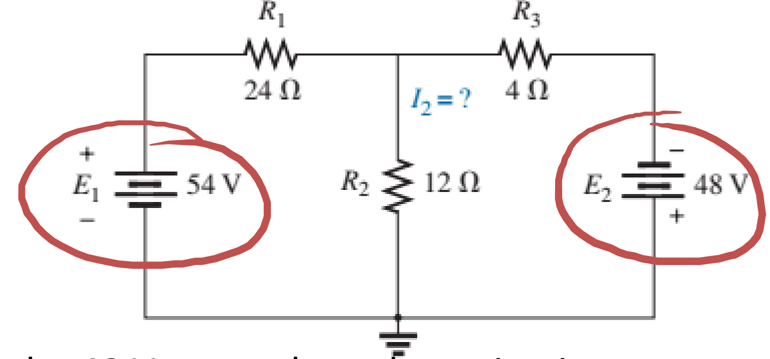
Same direction as I_1 in Fig. 9.18

Opposite direction to I_1 in Fig. 9.18

$$I_1 = \overbrace{I''_1 + I'''_1} - I'_1$$
$$= 1\text{ A} + 2\text{ A} - 2\text{ A} = 1\text{ A}$$



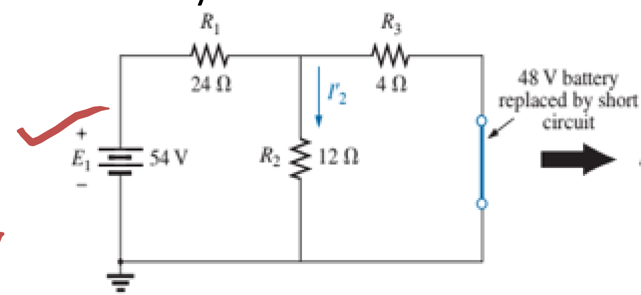
EXAMPLE 9.2 Using the superposition theorem, determine the current through the 12 Ω resistor in Fig. 9.5. Note that this is a two-source network of the type examined in the previous chapter when we applied branch-current analysis and mesh analysis.



Solution:

Step 1:

Considering the effects of the 54 V source requires replacing the 48 V source by a short-circuit equivalent.

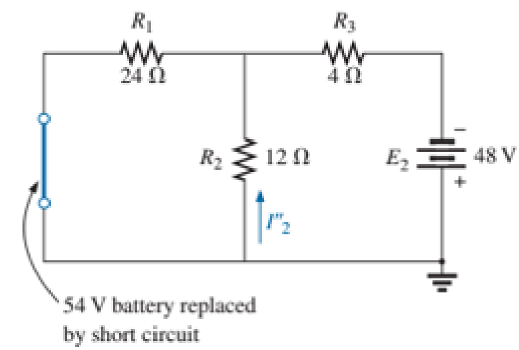


$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A} \qquad I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

Step 2:

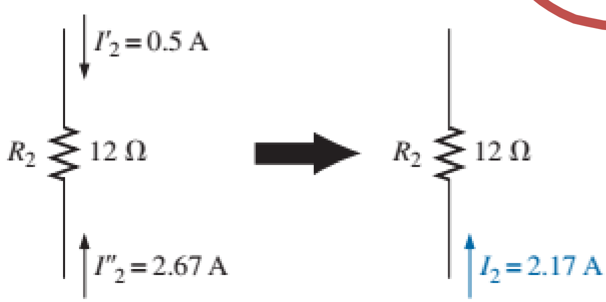
If we now replace the 54 V source by a short-circuit equivalent, the network in Fig. 9.7 results.



$$R_T = R_3 + R_2 \| R_1 = 4\ \Omega + 12\ \Omega \| 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

$$I_s = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A} \qquad I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\ \text{A})}{24\ \Omega + 12\ \Omega} = 2.67\ \text{A}$$

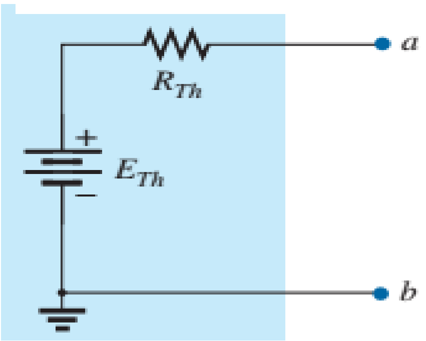
Step 3: $I_2 = I''_2 - I'_2 = 2.67\ \text{A} - 0.5\ \text{A} = \mathbf{2.17\ A}$



9.3 THÉVENIN'S THEOREM

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in the figure. This is called Thevenin Equivalent Circuit.

Thévenin equivalent circuit.



Steps:

1. Remove that portion of the network where the Thévenin equivalent circuit is found. In this circuit the load resistor R_L be temporarily removed from the network. Marked the terminals (a and b) across which thevenin is going to be applied.

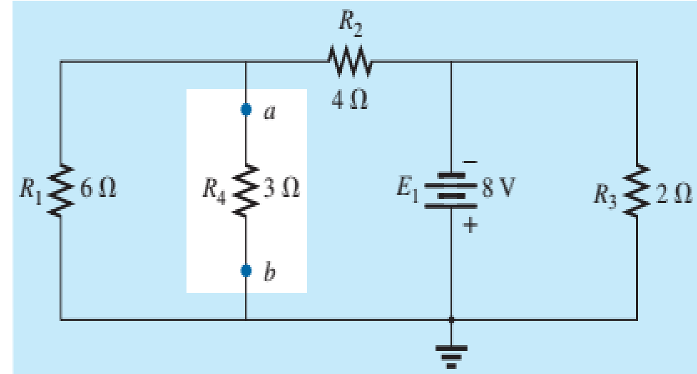
2. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits).

3. Consider an imaginary current is entering through the marked terminal 'a'. Calculate R_{Th}.

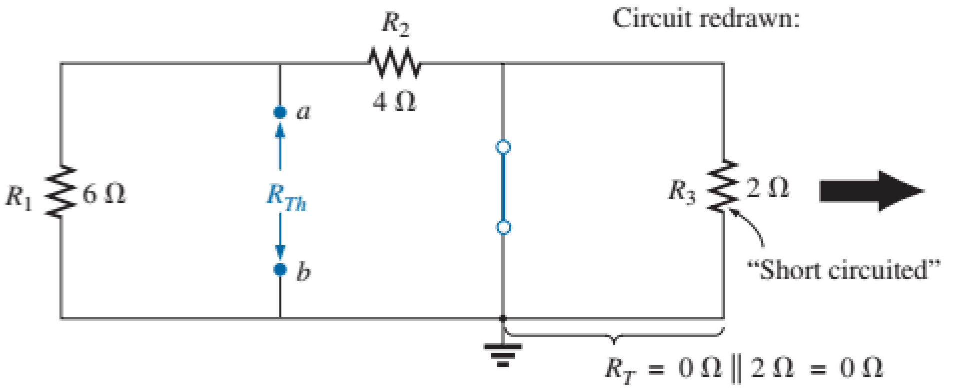
4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage (V_{ab}) between the marked terminals a and b.

5. Draw the Thévenin equivalent circuit.

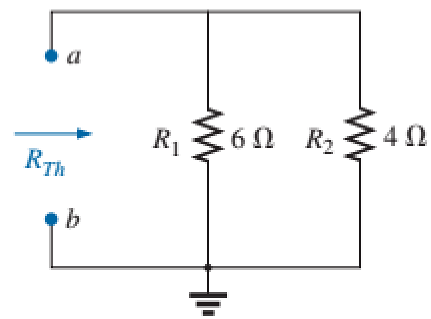
EXAMPLE 9.8 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.37. Note in this example that there is no need for the section of the network to be preserved to be at the "end" of the configuration.



Solution:

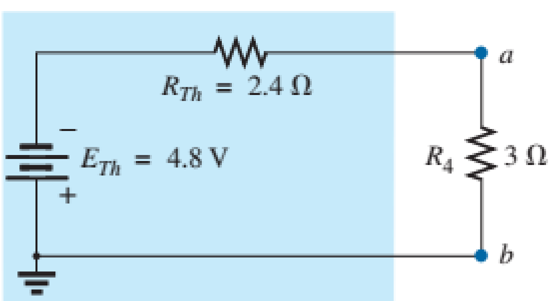


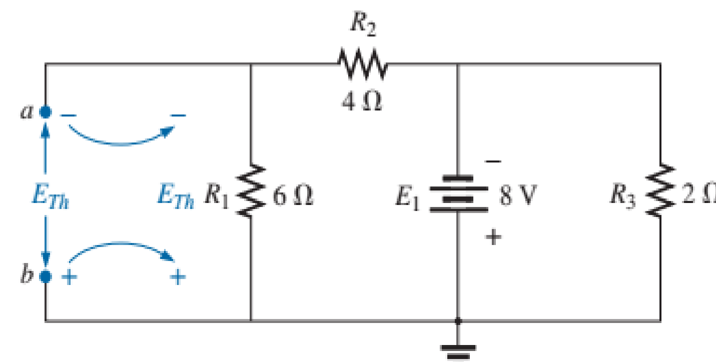
Circuit redrawn:



$$R_{Th} = R_1 \| R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

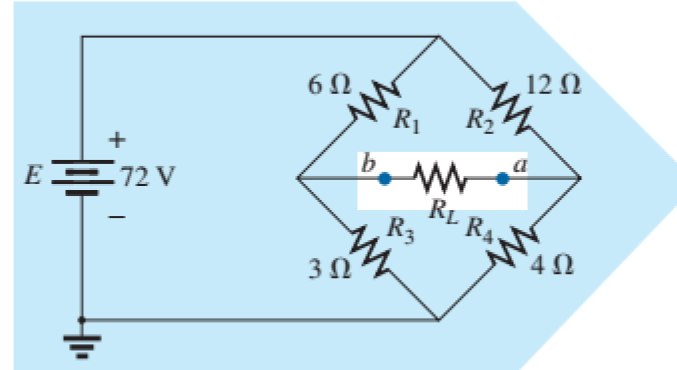
Resultant Thevenin Equivalent Circuit:



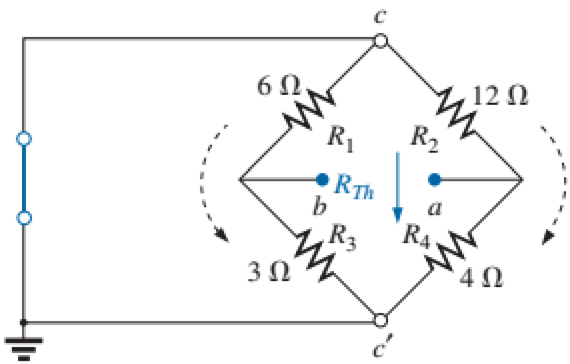


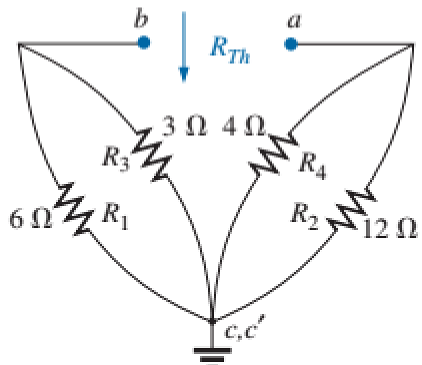
$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network in Fig. 9.43.



Solution:

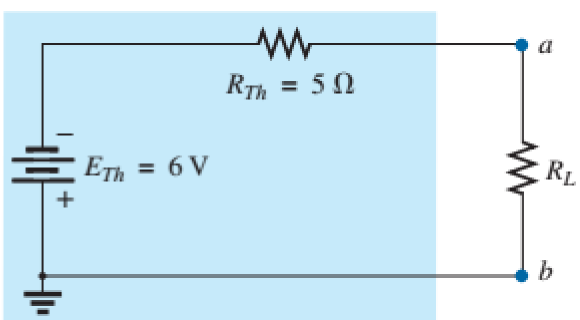


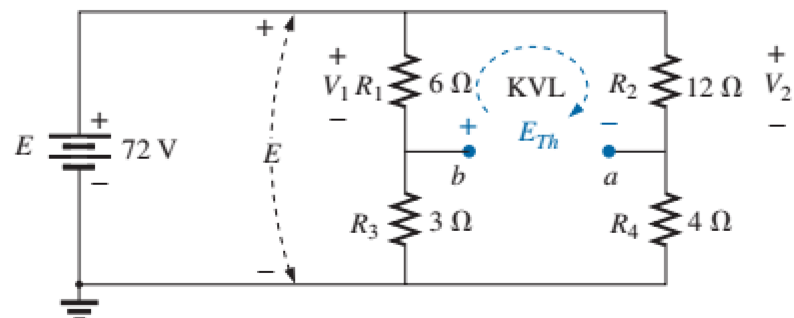


$$V_1 = \frac{R_1E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

$$\begin{aligned} R_{Th} = R_{a-b} &= R_1 \parallel R_3 + R_2 \parallel R_4 \\ &= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega \\ &= 2\ \Omega + 3\ \Omega = \mathbf{5\ \Omega} \end{aligned}$$

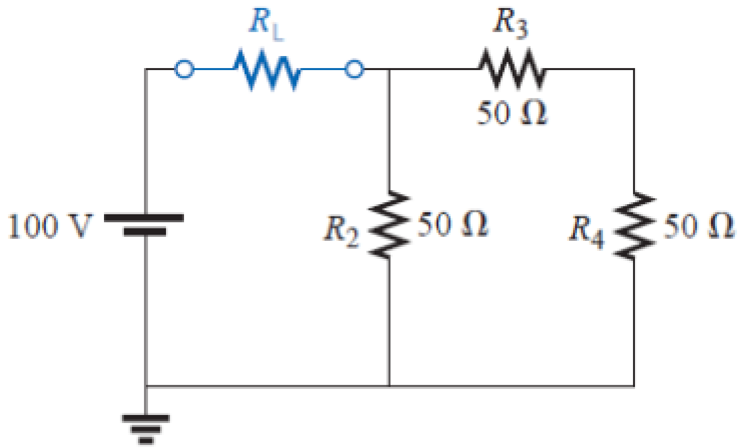


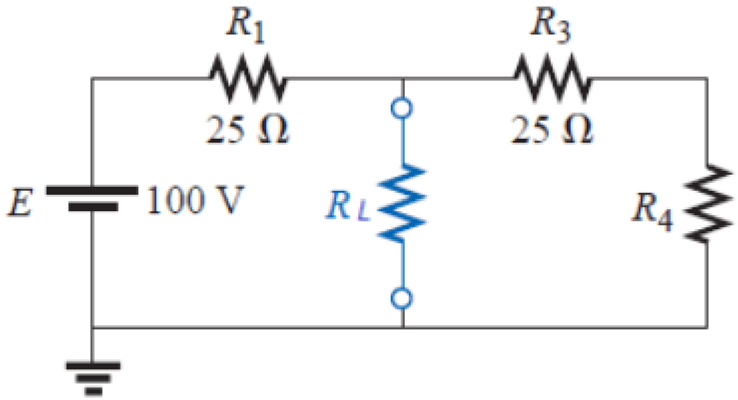


$$\Sigma_{C}\, V = +E_{Th} + V_1 - V_2 = 0$$

$$E_{Th} = V_2 - V_1 = 54\ \text{V} - 48\ \text{V} = \mathbf{6\ V}$$

Class Practice





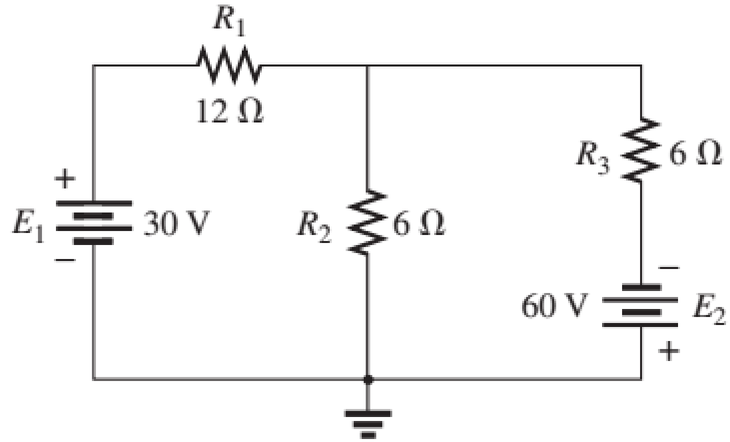
Exercise Problems

1. a. Using superposition, find the current through each resistor of the network in Fig. 9.119.

b. Find the power delivered to R_1 for each source.

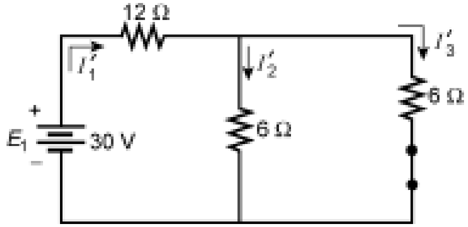
c. Find the power delivered to R_1 using the total current through R_1.

d. Does superposition apply to power effects? Explain.

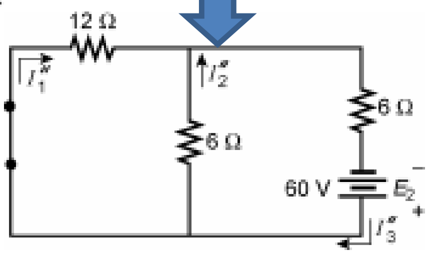


Solution:

a. E_1:



E_2:



$$I'_1 = \frac{30\text{ V}}{12\,\Omega + 6\,\Omega \parallel 6\,\Omega} = \frac{30\text{ V}}{12\,\Omega + 3\,\Omega} = 2\text{ A}$$

$$I'_2 = I'_3 = \frac{I'_1}{2} = 1\text{ A}$$

$$I''_3 = \frac{60\text{ V}}{6\,\Omega + 6\,\Omega \parallel 12\,\Omega} = \frac{60\text{ V}}{6\,\Omega + 4\,\Omega} = 6\text{ A}$$

$$I''_1 = \frac{6\,\Omega(I''_3)}{6\,\Omega + 12\,\Omega} = 2\text{ A}$$

$$I''_2 = \frac{12\,\Omega(I''_3)}{12\,\Omega + 6\,\Omega} = 4\text{ A}$$

$I_1 = I'_1 + I''_1 = 2\text{ A} + 2\text{ A} = \mathbf{4\text{ A}}$ (dir. of I'_1)

$I_2 = I''_2 - I'_2 = 4\text{ A} - 1\text{ A} = \mathbf{3\text{ A}}$ (dir. of I''_2)

$I_3 = I'_3 + I''_3 = 1\text{ A} + 6\text{ A} = \mathbf{7\text{ A}}$ (dir. of I'_3)

b. E_1: $P'_1 = I_1'^2 R_1 = (2\text{ A})^2\, 12\,\Omega = \mathbf{48\text{ W}}$

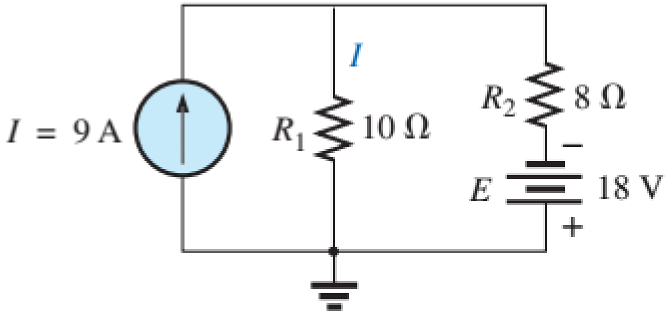
E_2: $P''_1 = I_1''^2 R_1 1'' R_1 = (2\text{ A})^2\, 12\,\Omega = \mathbf{48\text{ W}}$

c. $P_1 = I_1^2 R_1 = (4\text{ A})^2\, 12\,\Omega = \mathbf{192\text{ W}}$

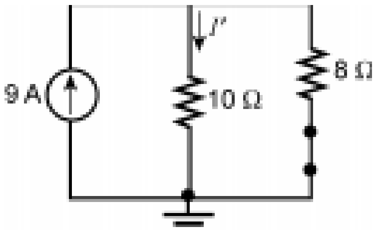
d. $P'_1 + P''_1 = 48\text{ W} + 48\text{ W} = 96\text{ W} \neq 192\text{ W} = P_1$

2. Using superposition, find the current I through the 10 Ω resistor for the network in Fig. 9.120.

Solution:

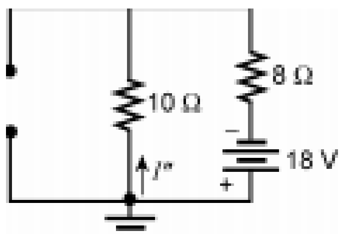


I:



$$I' = \frac{8\,\Omega(9\text{ A})}{8\,\Omega + 10\,\Omega} = 4\text{ A}$$

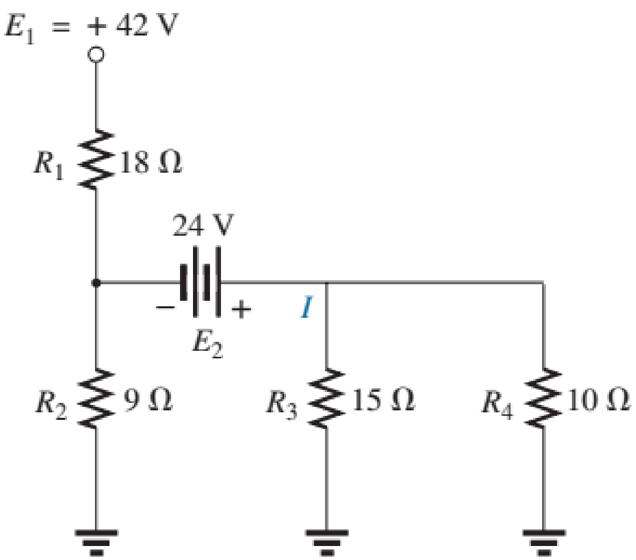
E:



$$I'' = \frac{18\text{ V}}{10\,\Omega + 8\,\Omega} = 1\text{ A}$$

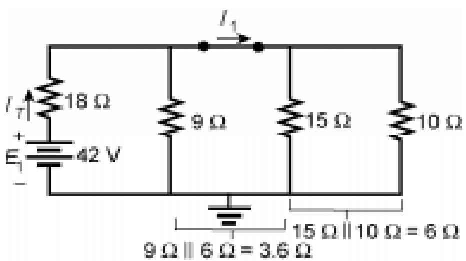
$I = I' - I'' = 4\text{ A} - 1\text{ A} = \mathbf{3\text{ A}}$ **(dir of I')**

3. Using superposition, find the current I through the 24 V source in Fig. 9.121.



Solution:

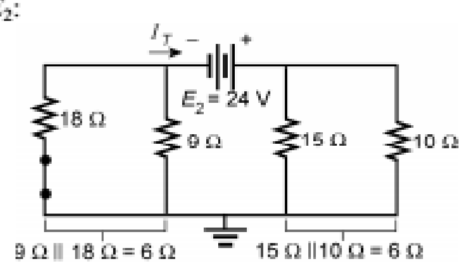
E_1:



$$I_T = \frac{42\text{ V}}{18\,\Omega + 3.6\,\Omega} = 1.944\text{ A}$$

$$I_1 = \frac{9\,\Omega(I_T)}{9\,\Omega + 6\,\Omega} = \frac{9\,\Omega(1.944\text{ A})}{15\,\Omega} = 1.17\text{ A}$$

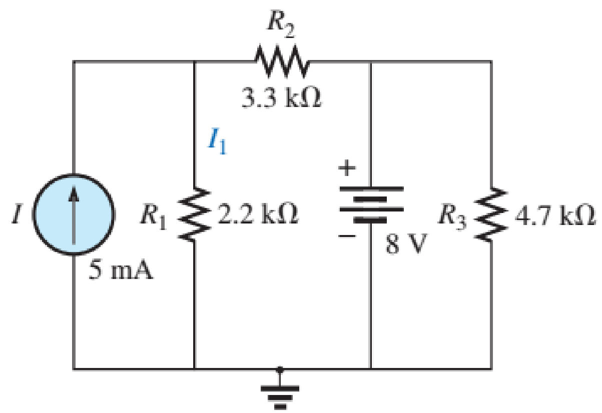
E_2:



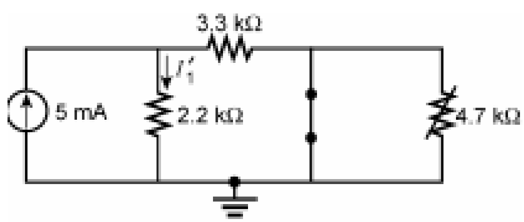
$$I_T = \frac{E_2}{R_T} = \frac{24\text{ V}}{12\,\Omega} = 2\text{ A}$$

$I_{24V} = I_T + I_1 = 2\text{ A} + 1.17\text{ A} = \mathbf{3.17\text{ A}}$ **(dir. of I_1)**

*4. Using superposition, find the current through R_1 for the network in Fig. 9.122.

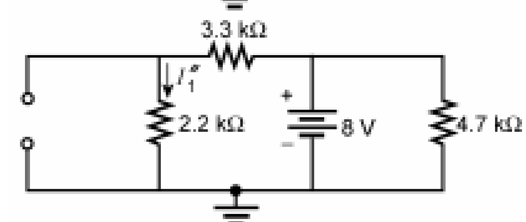


Solution:



$$I_1' = \frac{3.3\,\text{k}\Omega(5\text{ mA})}{2.2\,\text{k}\Omega + 3.3\,\text{k}\Omega} = \frac{16.5\text{ mA}}{5.5}$$

$$= 3\text{ mA}$$

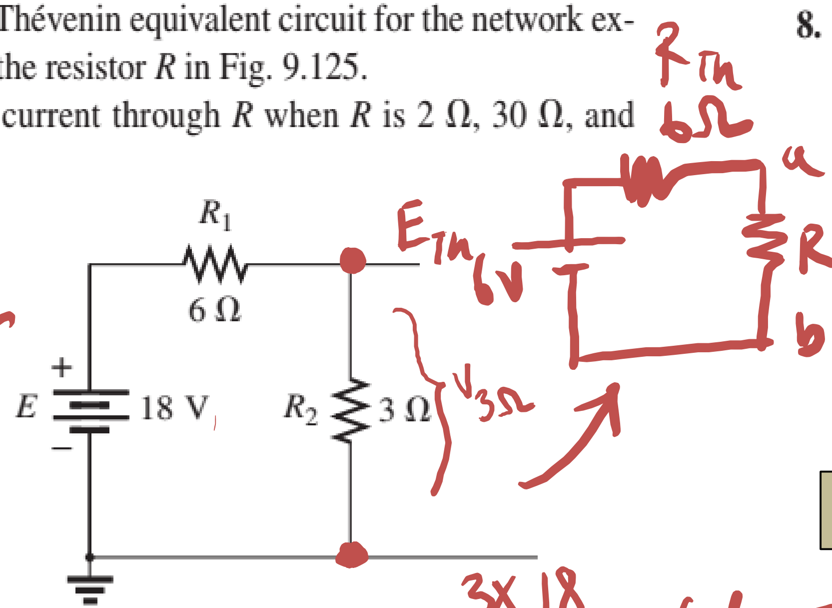


$$I''_1 = \frac{8\text{ V}}{3.3\,\text{k}\Omega + 2.2\,\text{k}\Omega} = \frac{8\text{ V}}{5.5\,\text{k}\Omega}$$

$$= 1.45\text{ mA}$$

$I_1 = I'_1 + I''_1 = 3\text{ mA} + 1.45\text{ mA} = \mathbf{4.45\text{ mA}}$

7. **a.** Find the Thévenin equivalent circuit for the network external to the resistor R in Fig. 9.125.
b. Find the current through R when R is 2 Ω, 30 Ω, and 100 Ω.



$$V_{3\Omega} = \frac{3 \times 18}{3+6} = 6V = E_{Th}$$

Solution:

a. $R_{Th} = R_3 + R_1 \parallel R_2 = 4\ \Omega + 6\ \Omega \parallel 3\ \Omega = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$

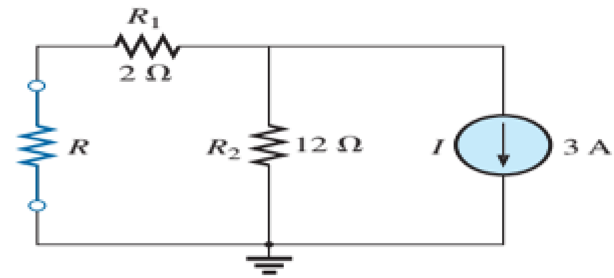
$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{3\ \Omega(18\ \text{V})}{3\ \Omega + 6\ \Omega} = 6\ \text{V}$$

b. $$I_1 = \frac{E_{Th}}{R_{Th} + R} = \frac{6\ \text{V}}{6\ \Omega + 2\ \Omega} = \mathbf{0.75\ A}$$

$$I_2 = \frac{6\ \text{V}}{6\ \Omega + 30\ \Omega} = \mathbf{166.67\ mA}$$

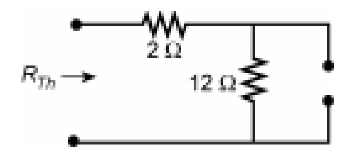
$$I_3 = \frac{6\ \text{V}}{6\ \Omega + 100\ \Omega} = \mathbf{56.60\ mA}$$

8. **a.** Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.126.
b. Find the power delivered to R when R is 2 Ω and 100 Ω.

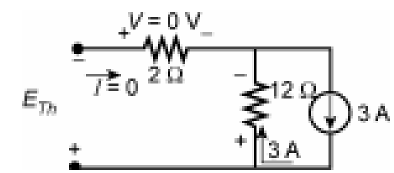


Solution:

a.



$R_{Th} = 2\ \Omega + 12\ \Omega = \mathbf{14\ \Omega}$



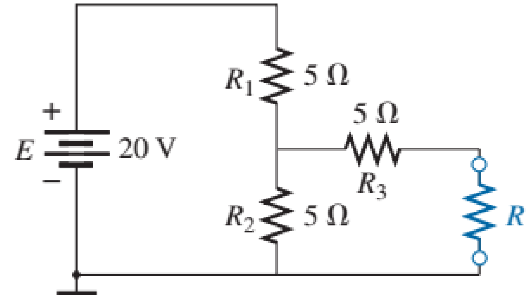
$E_{Th} = IR = (3\ \text{A})(12\ \Omega) = \mathbf{36\ V}$

b. $$R = 2\ \Omega:\ P = \left(\frac{E_{Th}}{R_{Th} + R}\right)^2 R = \left(\frac{36\ \text{V}}{14\ \Omega + 2\ \Omega}\right)^2 2\ \Omega = \mathbf{10.13\ W}$$

$$R = 100\ \Omega:\ P = \left(\frac{36\ \text{V}}{14\ \Omega + 100\ \Omega}\right)^2 100\ \Omega = \mathbf{9.97\ W}$$

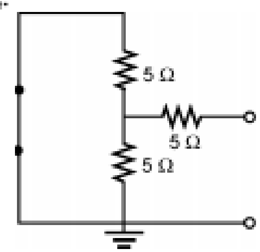
9. a. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.127.

b. Find the power delivered to R when R is 2 Ω and 100 Ω.



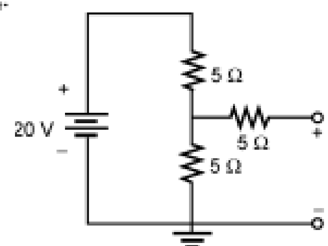
Solution:

a. R_{Th}:



$\leftarrow R_{Th} = 5\,\Omega + 5\,\Omega \parallel 5\,\Omega = \mathbf{7.5\,\Omega}$

E_{Th}:

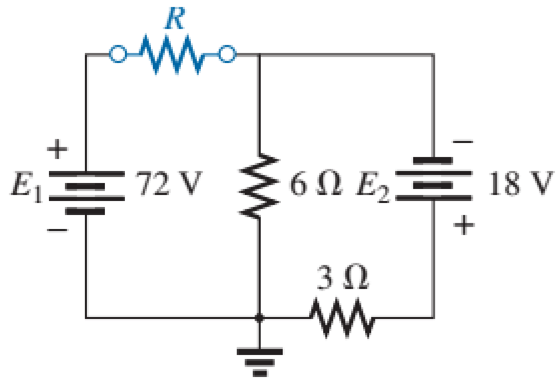


$$E_{Th} = \frac{20\text{ V}}{2} = \mathbf{10\text{ V}}$$

b. $R = 2\,\Omega$: $P = \left(\dfrac{E_{Th}}{R_{Th} + R}\right)^2 R = \left(\dfrac{10\text{ V}}{7.5\,\Omega + 2\,\Omega}\right)^2 2\,\Omega = \mathbf{2.22\text{ W}}$

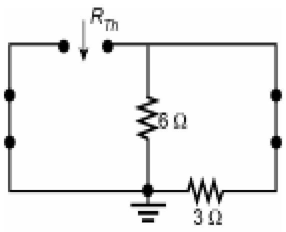
$R = 100\,\Omega$: $P = \left(\dfrac{10\text{ V}}{7.5\,\Omega + 100\,\Omega}\right)^2 100\,\Omega = \mathbf{0.87\text{ W}}$

10. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. 9.128.



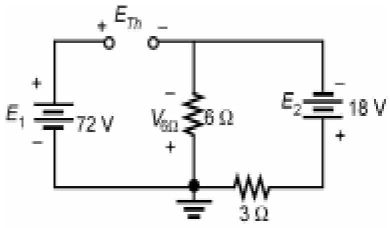
Solution:

R_{Th}:



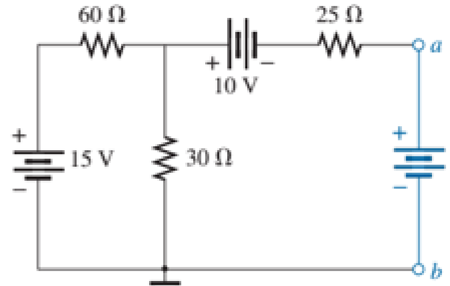
$R_{Th} = 6\ \Omega \parallel 3\ \Omega = \mathbf{2\ \Omega}$

E_{Th}:

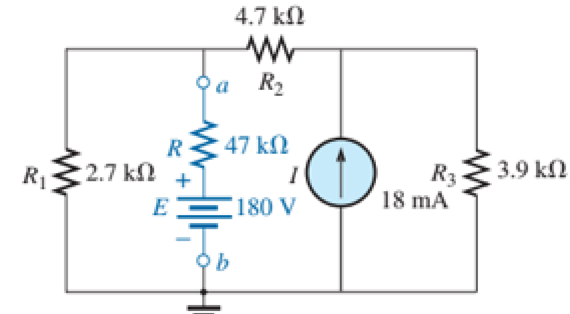


$$V_{6\Omega} = \frac{6\,\Omega(18\text{ V})}{6\,\Omega + 3\,\Omega} = 12\text{ V}$$
$$E_{Th} = 72\text{ V} + 12\text{ V} = \mathbf{84\text{ V}}$$

***13.** Find the Thévenin equivalent circuit for the portions of the networks in Fig. 9.131 external to points a and b.



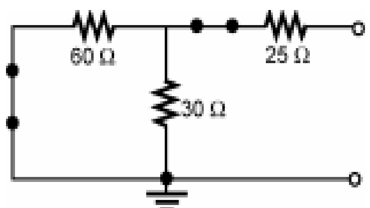
(I)



(II)

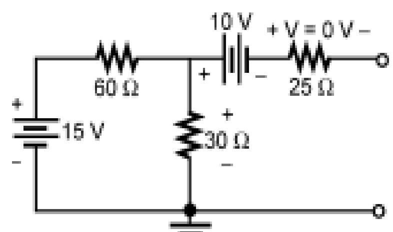
Solution:

(I): R_{Th}:



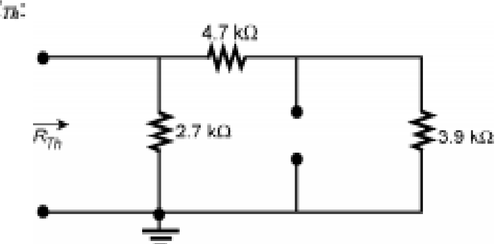
$\leftarrow R_{Th} = 25\ \Omega + 60\ \Omega \parallel 30\ \Omega = \mathbf{45\ \Omega}$

E_{Th}:



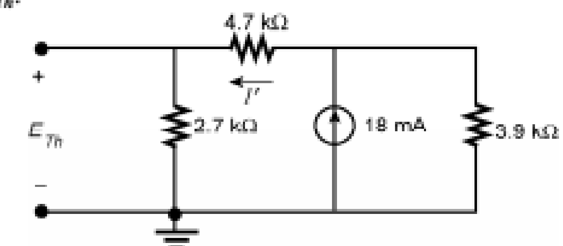
$$\begin{aligned} E_{Th} &= V_{30\Omega} - 10\text{ V} - 0 \\ &= \frac{30\ \Omega(15\text{ V})}{30\ \Omega + 60\ \Omega} - 10\text{ V} \\ &= 5\text{ V} - 10\text{ V} = \mathbf{-5\ V} \end{aligned}$$

(II): R_{Th}:



$R_{Th} = 2.7\text{ k}\Omega \parallel (4.7\text{ k}\Omega + 3.9\text{ k}\Omega) = 2.7\text{ k}\Omega \parallel 8.6\text{ k}\Omega = \mathbf{2.06\ k\Omega}$

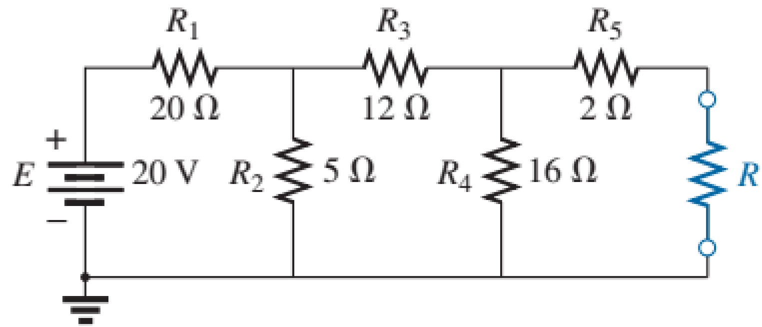
E_{Th}:



$$I' = \frac{3.9\text{ k}\Omega(18\text{ mA})}{3.9\text{ k}\Omega + 7.4\text{ k}\Omega} = 6.21\text{ mA}$$

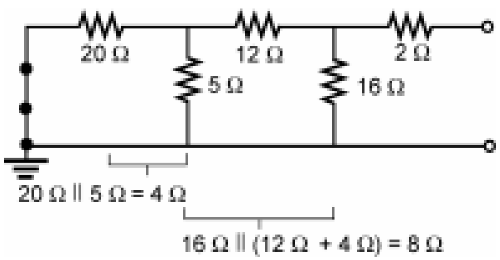
$$E_{Th} = I'(2.7\text{ k}\Omega) = (6.21\text{ mA})(2.7\text{ k}\Omega) = \mathbf{16.77\ V}$$

***14.** Determine the Thévenin equivalent circuit for the network external to the resistor R in both networks in Fig. 9.132.



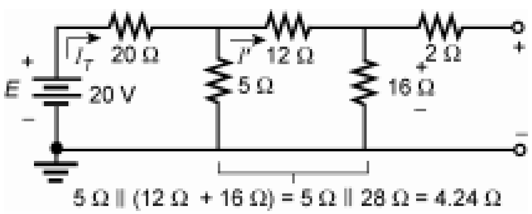
Solution:

(I): R_{Th}:



$\leftarrow R_{Th} = 2\ \Omega + 8\ \Omega = \mathbf{10\ \Omega}$

E_{Th}:



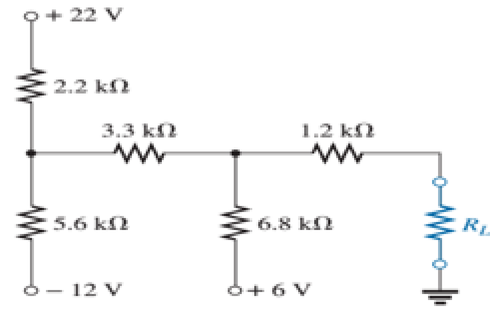
$E_{Th} = V_{16\Omega}$

$$I_T = \frac{20\ \text{V}}{20\ \Omega + 4.24\ \Omega} = 825.08\ \text{mA}$$

$$I' = \frac{5\ \Omega(I_T)}{5\ \Omega + 28\ \Omega} = \frac{5\ \Omega(825.08\ \text{mA})}{33\ \Omega} = 125.01\ \text{mA}$$

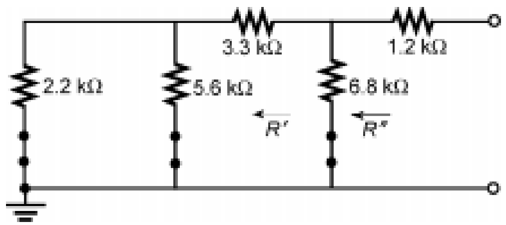
$$E_{Th} = V_{16\Omega} = (I')(16\ \Omega) = (125.01\ \text{mA})(16\ \Omega) = \mathbf{2\ V}$$

*15. For the network in Fig. 9.133, find the Thévenin equivalent circuit for the network external to the load resistor R_L.



Solution:

R_{Th}:



$R_{Th} = 2.2\text{ k}\Omega \parallel 5.6\text{ k}\Omega = 1.58\text{ k}\Omega$

$R' = 1.58\text{ k}\Omega + 3.3\text{ k}\Omega$
$= 4.88\text{ k}\Omega$

$R'' = 4.88\text{ k}\Omega \parallel 6.8\text{ k}\Omega = 2.84\text{ k}\Omega$

$R_{Th} = 1.2\text{ k}\Omega + R'' = 1.2\text{ k}\Omega + 2.84\text{ k}\Omega = \mathbf{4.04\text{ k}\Omega}$

E_{Th}: Source conversions:

$$I_1 = \frac{22\text{ V}}{2.2\text{ k}\Omega} = 10\text{ mA},\ R_s = 2.2\text{ k}\Omega$$

$$I_2 = \frac{12\text{ V}}{5.6\text{ k}\Omega} = 2.14\text{ mA},\ R_s = 5.6\text{ k}\Omega$$

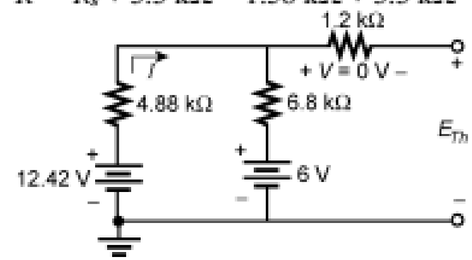
Combining parallel current sources: $I'_T = I_1 - I_2 = 10\text{ mA} - 2.14\text{ mA} = 7.86\text{ mA}$

$2.2\text{ k}\Omega \parallel 5.6\text{ k}\Omega = 1.58\text{ k}\Omega$

Source conversion:

$E = (7.86\text{ mA})(1.58\text{ k}\Omega) = 12.42\text{ V}$

$R' = R_s + 3.3\text{ k}\Omega = 1.58\text{ k}\Omega + 3.3\text{ k}\Omega = 4.88\text{ k}\Omega$



$$I = \frac{12.42\text{ V} - 6\text{ V}}{4.88\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{6.42\text{ V}}{11.68\text{ k}\Omega} = 549.66\ \mu\text{A}$$

$$V_{6.8k\Omega} = I(6.8\text{ k}\Omega) = (549.66\ \mu\text{A})(6.8\text{ k}\Omega) = 3.74\text{ V}$$

$$E_{Th} = 6\text{ V} + V_{6.8k\Omega} = 6\text{ V} + 3.74\text{ V} = \mathbf{9.74\text{ V}}$$


END