Introduction to Electrical Circuits

Final Term
Lecture - 03

Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11^{th} Edition

Parallel Circuits

Admittance

The reciprocal of impedance is called admittance.

The unit of admittance is Siemens [S].

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = Y\angle\theta_y = G + jB = g + jb = (\textbf{Conducatcne}) + j(\textbf{Susceptance}) \quad [S]$$

$$\textbf{Conducatcne}: G = g = Y\cos\theta_y \quad [S] \qquad \textbf{Susceptance}: B = b = Y\sin\theta_y \quad [S]$$

G or g is called conductance. The unit of conductance is mho or Siemens. Conductance (G or g) is the reciprocal of resistance (R).

B or b is called susceptance. The unit of susceptance is Siemens. Susceptance (B or b) is the reciprocal of reactance (X).

$$G = g = \frac{1}{R} \quad [S] \qquad B = b = \frac{1}{X} \quad [S] \qquad B_L = b_L = \frac{1}{X_L} \quad [S] \qquad B_C = b_C = \frac{1}{X_C} \quad [S]$$

B_L or b_L is called inductive susceptance.
B_C or b_C is called capacitive susceptance.

Admittance of a Resistance, Inductance and Capacitance

Admittance of a resistance: $$Y_R = \frac{1}{Z_R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ \quad [S]$$

Admittance of an inductance: $$Y_L = \frac{1}{Z_L} = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ \quad [S]$$

Admittance of a capacitance: $$Y_C = \frac{1}{Z_C} = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ \quad [S]$$

Admittance of a *RL* series branch: $$Y_{RL} = \frac{1}{Z_{RL}} = \frac{1}{R\angle 0^\circ + X_L\angle 90^\circ} = \frac{1}{R + jX_L} \quad [S]$$

Admittance of a *RC* series branch: $$Y_{RC} = \frac{1}{Z_{RC}} = \frac{1}{R\angle 0^\circ + X_C\angle -90^\circ} = \frac{1}{R - jX_C} \quad [S]$$

Admittance of a *RLC* series branch:

$$Y_{RLC} = \frac{1}{Z_{RLC}} = \frac{1}{R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ} = \frac{1}{R + jX_L - jX_C} \quad [S]$$

For ac parallel networks, the total admittance is simply the sum of the admittance levels of all the parallel branches.

$$\boxed{\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N}$$

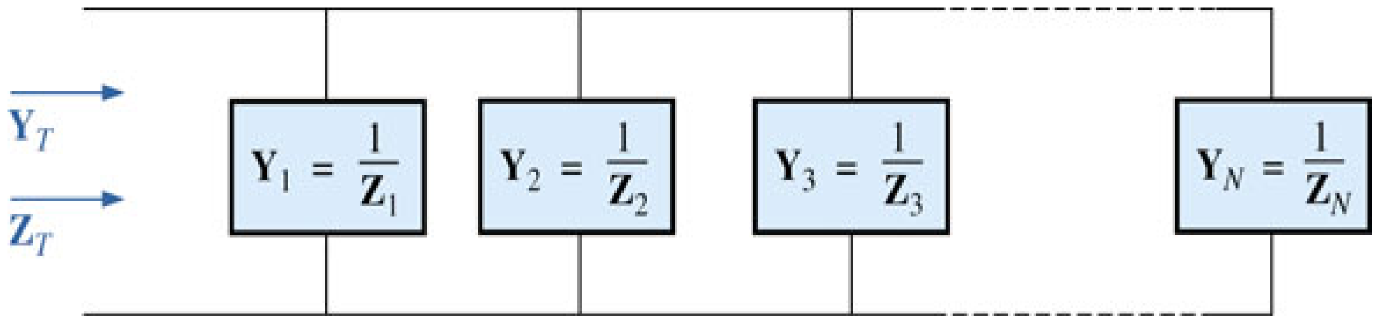


FIG. 15.58 *Parallel ac network.*

$$\boxed{\mathbf{Y}_T = \frac{1}{\mathbf{Z}_T}}$$

$$\boxed{\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}}$$

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

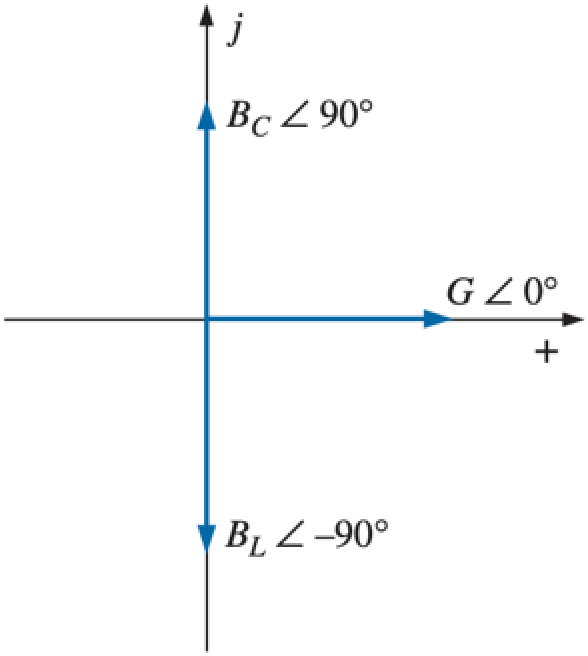


FIG. 16.8
Admittance diagram.

Example: For the parallel *R-L* network of Fig. 16.3:

a. Determine the input impedance.
b. Draw the impedance diagram.
c. Find the admittance of each parallel element.
d. Calculate the total admittance of the network.
e. Sketch the admittance diagram.

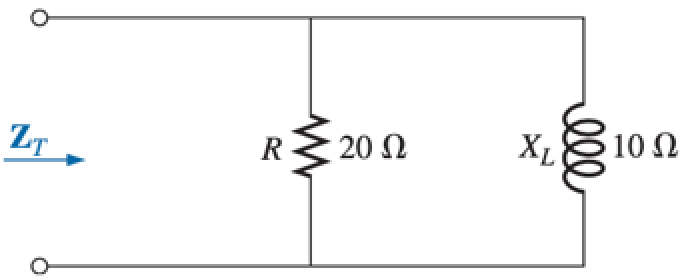


FIG. 16.3
Example 16.1.

Solutions:

a. $$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega\ \angle 0^\circ)(10\ \Omega\ \angle 90^\circ)}{20\ \Omega + j\,10\ \Omega}$$
$$= \frac{200\ \Omega\ \angle 90^\circ}{22.361\ \angle 26.57^\circ} = \mathbf{8.93\ \Omega\ \angle 63.43^\circ}$$
$$= \mathbf{4.00\ \Omega} + \boldsymbol{j\,7.95\ \Omega} = R_T + jX_L = \mathbf{8.93\Omega\ \angle 63.43^\circ}$$

b. The impedance diagram appears in Fig. 16.4.

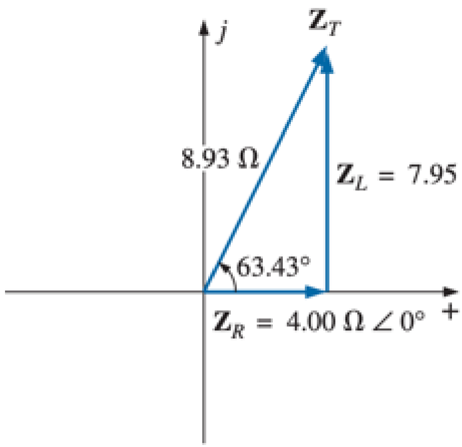


FIG. 16.4
Impedance diagram for the network in Fig. 16.3.

c. $\mathbf{Y}_R = G \angle 0° = \dfrac{1}{R} \angle 0° = \dfrac{1}{20\ \Omega} \angle 0° = \mathbf{0.05\ S \angle 0°}$

$= \mathbf{0.05\ S + j\,0}$

$\mathbf{Y}_L = B_L \angle -90° = \dfrac{1}{X_L} \angle -90° = \dfrac{1}{10\ \Omega} \angle -90°$

$= \mathbf{0.1\ S \angle -90° = 0 - j\,0.1\ S}$

d. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ S + j0) + (0 - j0.1\ S)$

$= \mathbf{0.05\ S - j\ 0.1\ S} = G - jB_L = \mathbf{0.112\ S \angle -63.43°}$

e. The admittance diagram appears in Fig. 16.9.

$\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.112\ S \angle -63.43°}$

$= \mathbf{8.93\ \Omega \angle 63.43°}$—a perfect match

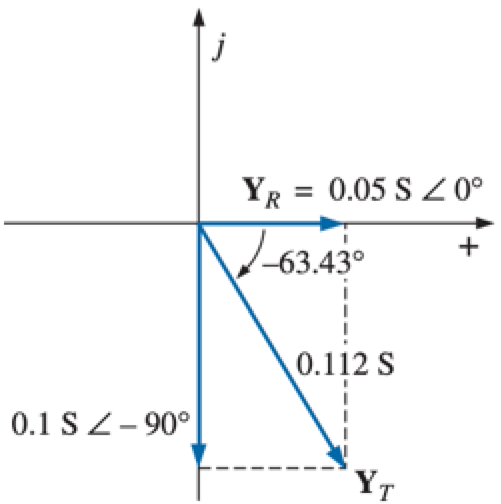


FIG. 16.9
Admittance diagram for the network in Fig. 16.3.

Example: For the network of Fig 16.5:

a. Determine the total impedance.
b. Sketch the impedance diagram.
c. Find the admittance for each parallel branch.
d. Calculate the total admittance of the network.
e. Sketch the admittance diagram.

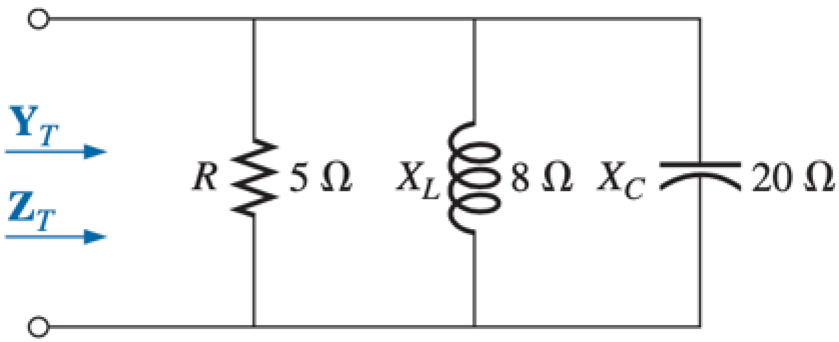


FIG. 16.5
Example 16.2.

Solutions:

$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_R} + \dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_C}}$$

$$= \frac{1}{\dfrac{1}{5\ \Omega\ \angle 0^\circ} + \dfrac{1}{8\ \Omega\ \angle 90^\circ} + \dfrac{1}{20\ \Omega\ \angle -90^\circ}}$$

$$= \frac{1}{0.2\ \text{S}\ \angle 0^\circ + 0.125\ \text{S}\ \angle -90^\circ + 0.05\ \text{S}\ \angle 90^\circ}$$

$$= \frac{1}{0.2\ \text{S} - j\,0.075\ \text{S}} = \frac{1}{0.2136\ \text{S}\ \angle -20.56^\circ}$$

$$= \mathbf{4.68\ \Omega\ \angle 20.56^\circ}$$

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$

$$= \frac{(5\ \Omega\ \angle 0^\circ)(8\ \Omega\ \angle 90^\circ)(20\ \Omega\ \angle -90^\circ)}{(5\ \Omega\ \angle 0^\circ)(8\ \Omega\ \angle 90^\circ) + (8\ \Omega\ \angle 90^\circ)(20\ \Omega\ \angle -90^\circ) + (5\ \Omega\ \angle 0^\circ)(20\ \Omega\ \angle -90^\circ)}$$

$$= \frac{800\ \Omega\ \angle 0^\circ}{40\ \angle 90^\circ + 160\ \angle 0^\circ + 100\ \angle -90^\circ}$$

$$= \frac{800\ \Omega}{160 + j\,40 - j\,100} = \frac{800\ \Omega}{160 - j\,60}$$

$$= \frac{800\ \Omega}{170.88\ \angle -20.56^\circ}$$

$$= \mathbf{4.68\ \Omega\ \angle 20.56^\circ = 4.38\ \Omega + j\,1.64\ \Omega}$$

b. The impedance diagram appears in Fig. 16.6.

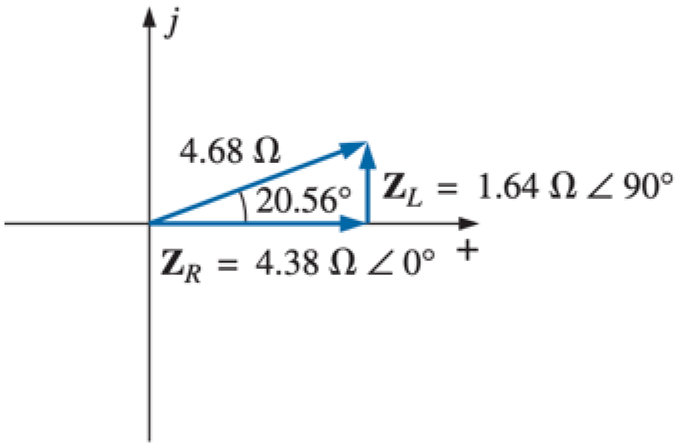


FIG. 16.6
Impedance diagram for the network in Fig. 16.5.

c. $\mathbf{Y}_R = G\,\angle 0^\circ = \dfrac{1}{R}\angle 0^\circ = \dfrac{1}{5\,\Omega}\angle 0^\circ$

$= \mathbf{0.2\,S\,\angle 0^\circ = 0.2\,S + j\,0}$

$\mathbf{Y}_L = B_L\angle -90^\circ = \dfrac{1}{X_L}\angle -90^\circ = \dfrac{1}{8\,\Omega}\angle -90^\circ$

$= \mathbf{0.125\,S\,\angle -90^\circ = 0 - j\,0.125\,S}$

$\mathbf{Y}_C = B_C\angle 90^\circ = \dfrac{1}{X_C}\angle 90^\circ = \dfrac{1}{20\,\Omega}\angle 90^\circ$

$= \mathbf{0.05\,S\,\angle +90^\circ = 0 + j\,0.05\,S}$

d. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$

$= (0.2\,S + j\,0) + (0 - j\,0.125\,S) + (0 + j\,0.05\,S)$

$= 0.2\,S - j\,0.075\,S = \mathbf{0.214\,S\,\angle -20.56^\circ}$

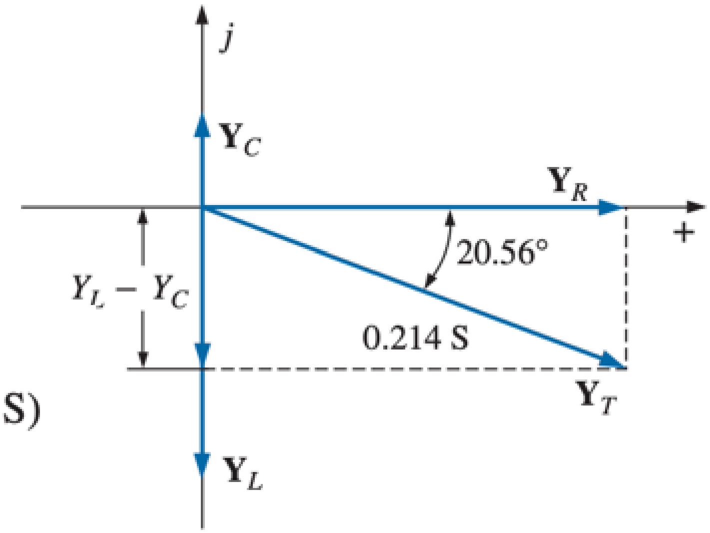


FIG. 16.10
Admittance diagram for the network in Fig. 16.5.

e. The admittance diagram appears in Fig. 16.10.

$\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.214\,S\,\angle -20.56^\circ}$

$= \mathbf{4.68\,\Omega\,\angle 20.56^\circ}$—a perfect match

Example

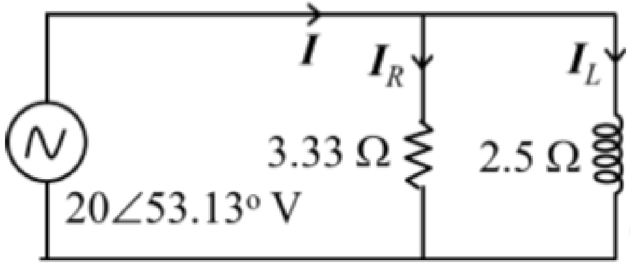
Calculate the total admittance and current for the following circuit.

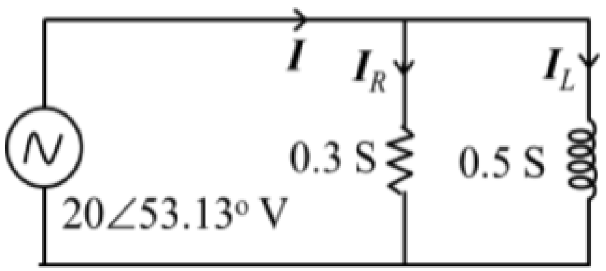
$$G = \frac{1}{3.33} = 0.3\ [S] \qquad B_L = \frac{1}{2.5} = 0.4\ [S]$$

$$\boldsymbol{Y}_R = 0.3\angle 0^\circ = 0.3\ [S] \qquad \boldsymbol{Y}_L = 0.4\angle -90^\circ = -j0.4\ [S]$$

$$\boldsymbol{Y} = 0.3\angle 0^\circ + 0.4\angle -90^\circ = 0.3 - j0.4 = 0.5\angle -53.13^\circ\ [S]$$

$$\boldsymbol{Z} = \frac{1}{\boldsymbol{Y}} = \frac{1}{0.5\angle -53.13^\circ} = 2\angle 53.13^\circ\ [\Omega]$$





$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \boldsymbol{YV} = (0.5\angle -53.13^\circ)(20\angle 53.13^\circ) = 10\angle 0^\circ\ [\mathrm{A}]$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{V}}{\boldsymbol{Z}_R} = \boldsymbol{Y}_R\boldsymbol{V} = (0.3\angle 0^\circ)(20\angle 53.13^\circ) = 6\angle 53.13^\circ\ [\mathrm{A}]$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{V}}{\boldsymbol{Z}_L} = \boldsymbol{Y}_L\boldsymbol{V} = (0.4\angle -90^\circ)(20\angle 53.13^\circ) = 8\angle -36.87^\circ [\mathrm{A}]$$

$$\theta = \theta_v - \theta_i = 53.13° - 0° = 53.13°$$

$$pf = \cos(53.13°) = 0.6 \text{ lagging}$$

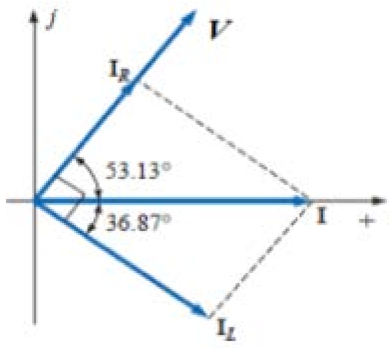
$$rf = \sin(53.13°) = 0.8$$

$$S = VI = 20 \times 10 = 200 \text{ VA}$$

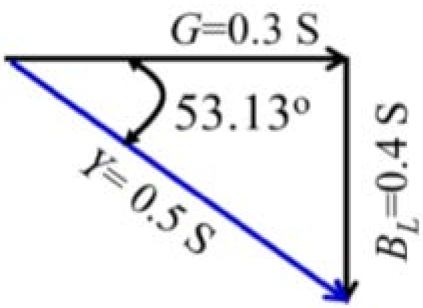
$$P = VI\cos\theta = 20 \times 10 \times 0.6 = 120 \text{ W}$$

$$Q = VI\sin\theta = 20 \times 10 \times 0.8 = 160 \text{ VAR}$$

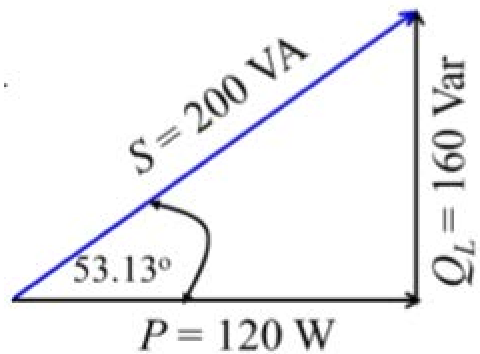
$$p(t) = 120(1 - \cos 2\omega t) + 160\sin 2\omega t$$



$$Q = I_L^2 X_L = 8^2 \times 2.5 = 160 \text{ VAR}$$



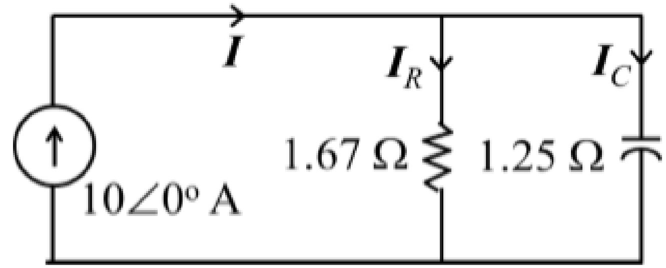
Admittance Diagram



Power Triangle

Example

Calculate the total admittance and current for the following circuit.

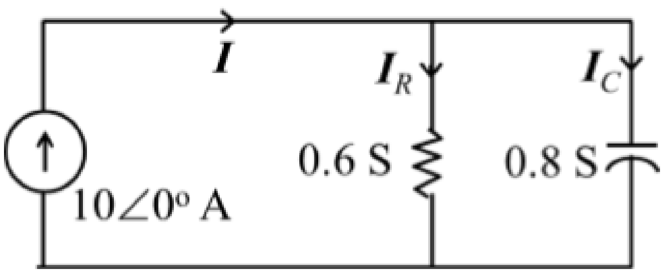


$$G = \frac{1}{1.67} = 0.6\ [S] \qquad B_C = \frac{1}{1.25} = 0.8\ [S]$$

$$Y_R = 0.6\angle 0^\circ = 0.6\ [S] \qquad Y_C = 0.8\angle 90^\circ = j0.8\ [S]$$

$$Y = 0.6\angle 0^\circ + 0.8\angle 90^\circ = 0.6 + j0.8 = 1.0\angle 53.13^\circ\ [S]$$

$$Z = \frac{1}{Y} = \frac{1}{1\angle 53.13^\circ} = 1\angle -53.13^\circ\ [\Omega]$$



$$V = IZ = \frac{I}{Y} = \frac{10\angle 0^\circ}{1\angle 53.13^\circ} = 10\angle -53.13^\circ\ [V]$$

$$I_R = Y_R V = (0.6\angle 0^\circ)(10\angle -53.13^\circ) = 6\angle -53.13^\circ\ [A]$$

$$I_C = Y_C V = (0.8\angle 90^\circ)(10\angle -53.13^\circ) = 8\angle 36.87^\circ [A]$$

$$\theta = \theta_v - \theta_i = -53.13° - 0° = -53.13°$$

$$pf = \cos(-53.13°) = 0.6 \text{ leading}$$

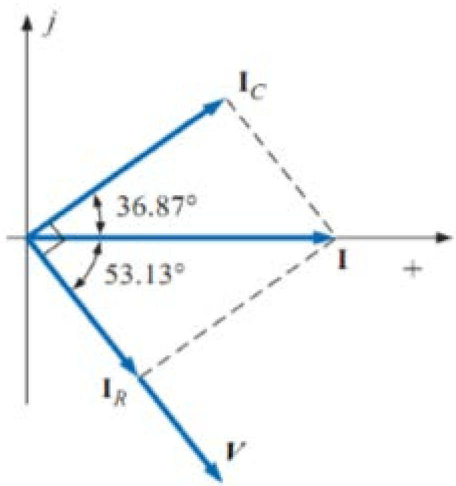
$$rf = \sin(-53.13°) = -0.8$$

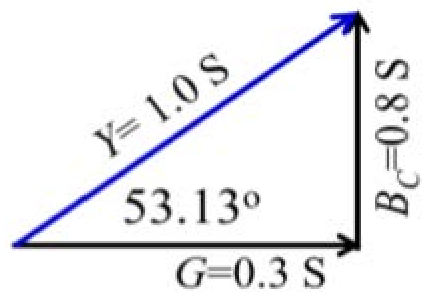
$$Q = I_C^2 X_C = 8^2 \times 1.25 = 80 \text{ VAR}$$

$$S = VI = 10 \times 10 = 100 \text{ VA}$$

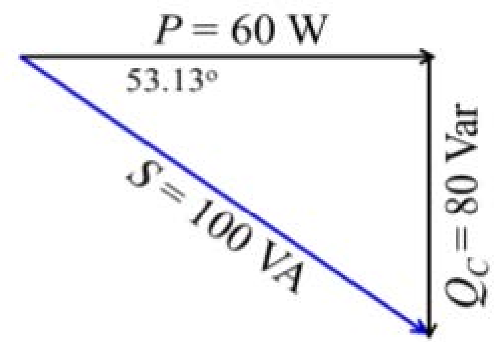
$$P = VI\cos\theta = 10 \times 10 \times 0.6 = 60 \text{ W}$$

$$Q = VI\sin\theta = 10 \times 10 \times -0.8 = -80 \text{ VAR}$$





Admittance Diagram



Power Triangle

Current Divider Rule

The current (I_x) flows one or more elements in parallel that have total admittance Y_x or impedance Z_x, can be given by:

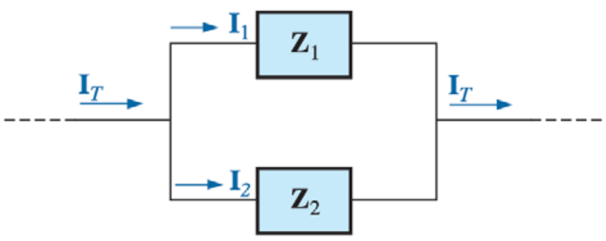
$$I_x = \frac{Y_x}{Y_T} I = \frac{Z_T}{Z_x} I$$

where, I is the total current flows the parallel circuit, and Y_T is the total admittance of the parallel circuit.

Current Divider Rule only for two branch circuit:

$$I_1 = \frac{Y_1}{Y_T} I_T = \frac{Z_T}{Z_1} I_T = \frac{Z_2}{Z_1 + Z_2} I_T$$

$$I_2 = \frac{Y_2}{Y_T} I_T = \frac{Z_T}{Z_2} I_T = \frac{Z_1}{Z_1 + Z_2} I_T$$



Example: Using the current divider rule, find the current through each impedance in Fig. 16.28.

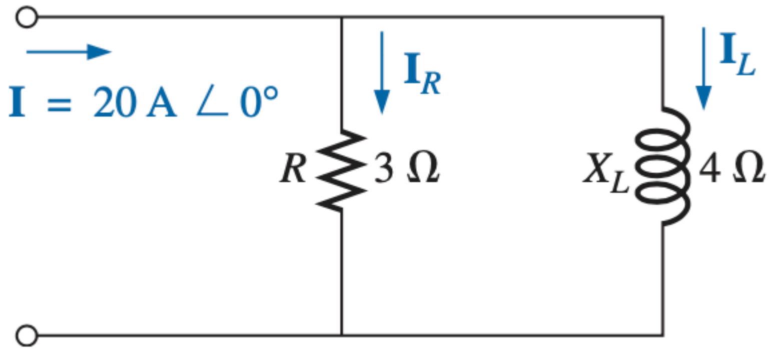


FIG. 16.28
Example 16.6.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\ \angle 90^\circ)(20\ \text{A}\ \angle 0^\circ)}{3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ} = \frac{80\ \text{A}\ \angle 90^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{16\ A\ \angle 36.87^\circ}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\ \angle 0^\circ)(20\ \text{A}\ \angle 0^\circ)}{5\ \Omega\ \angle 53.13^\circ} = \frac{60\ \text{A}\ \angle 0^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{12\ A\ \angle -53.13^\circ}$$

Example: Using the current divider rule, find the current through each parallel branch in Fig. 16.29.

$$Z_{RL} = 1 + j8 \ \Omega$$

$$Z_C = -j2 = 2\angle -90° \ \Omega$$

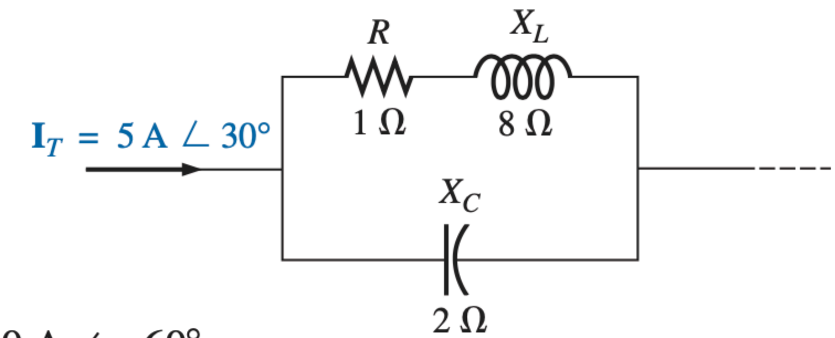


FIG. 16.29
Example 16.7.

Solution:

$$\mathbf{I}_{R-L} = \frac{\mathbf{Z}_C \mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R-L}} = \frac{(2\ \Omega \angle -90°)(5\text{ A} \angle 30°)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\text{ A} \angle -60°}{1 + j\,6}$$

$$= \frac{10\text{ A} \angle -60°}{6.083 \angle 80.54°} \cong \mathbf{1.64\ A \angle -140.54°}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R-L} \mathbf{I}_T}{\mathbf{Z}_{R-L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\text{ A} \angle 30°)}{6.08\ \Omega \angle 80.54°}$$

$$= \frac{(8.06 \angle 82.87°)(5\text{ A} \angle 30°)}{6.08 \angle 80.54°} = \frac{40.30\text{ A} \angle 112.87°}{6.083 \angle 80.54°}$$

$$= \mathbf{6.63\ A \angle 32.33°}$$

Thank You