Introduction to Electrical Circuits

Mid Term
Lecture - 11

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

W6	C11	Chapter 13	13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS	13.1		Quiz/ Presentation
			13.5 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT	13.8-13.11		
			13.6 PHASE RELATIONS	13.12		
			13.7 AVERAGE VALUE	13.14-13.16		
			13.8 EFFECTIVE (rms) VALUES	13.20-13.24		
	C12	Chapter 14	14.6 COMPLEX NUMBERS	14.16-14.18		

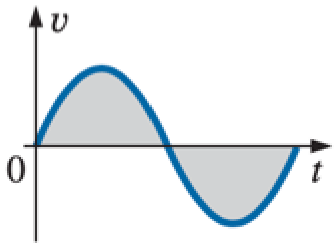
OBJECTIVES

- ✫ Become familiar with the characteristics of a sinusoidal waveform, including its general format, average value, and effective value.
- ✫ Be able to determine the phase relationship between two sinusoidal waveforms of the same frequency.
- ✫ Understand how to calculate the average and effective values of any waveform.

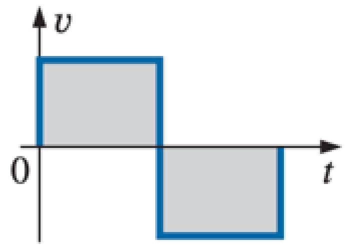
Introduction

Alternating Waveform

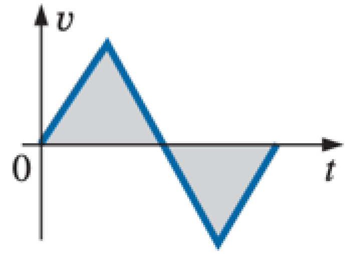
- *Alternating* **-->>** waveform alternates between two prescribed levels in a set time sequence.



Sinusoidal



Square wave



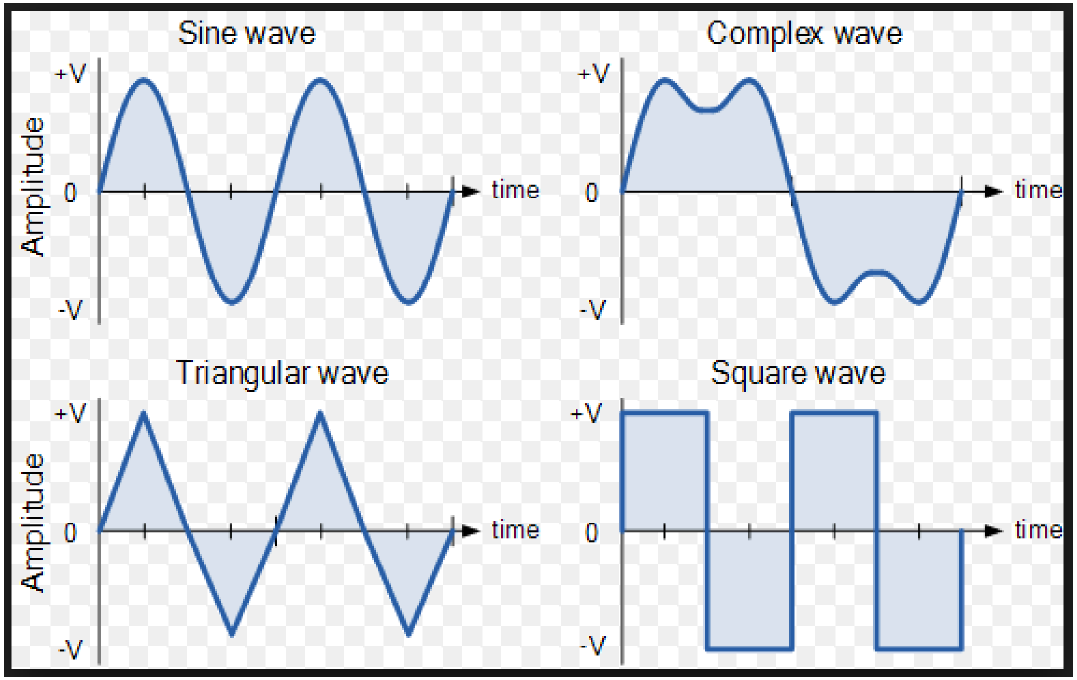
Triangular wave

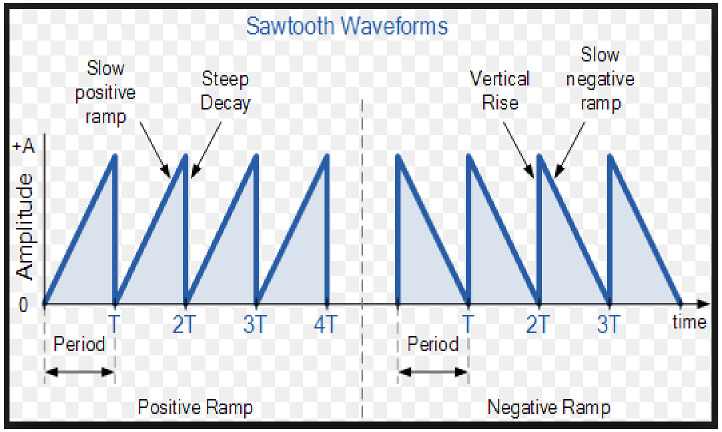
Definitions

Waveform: The graph of instantaneous values of an alternating quantity plotted against time is called waveform.

Periodic Waveform: A waveform that continually repeats itself after the same time interval.

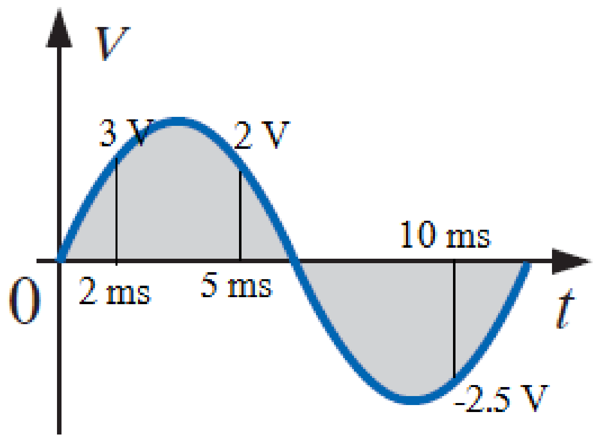
Different Types of Waveform

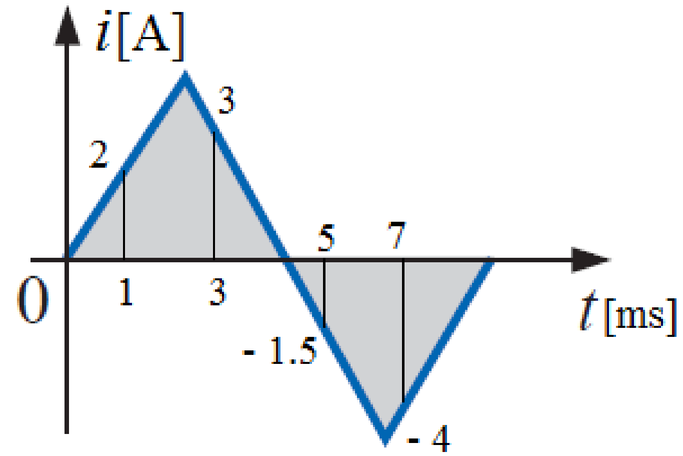




Definitions

Instantaneous Value: The value of an alternating quantity at a particular instant or moment of time is known as its instantaneous value.





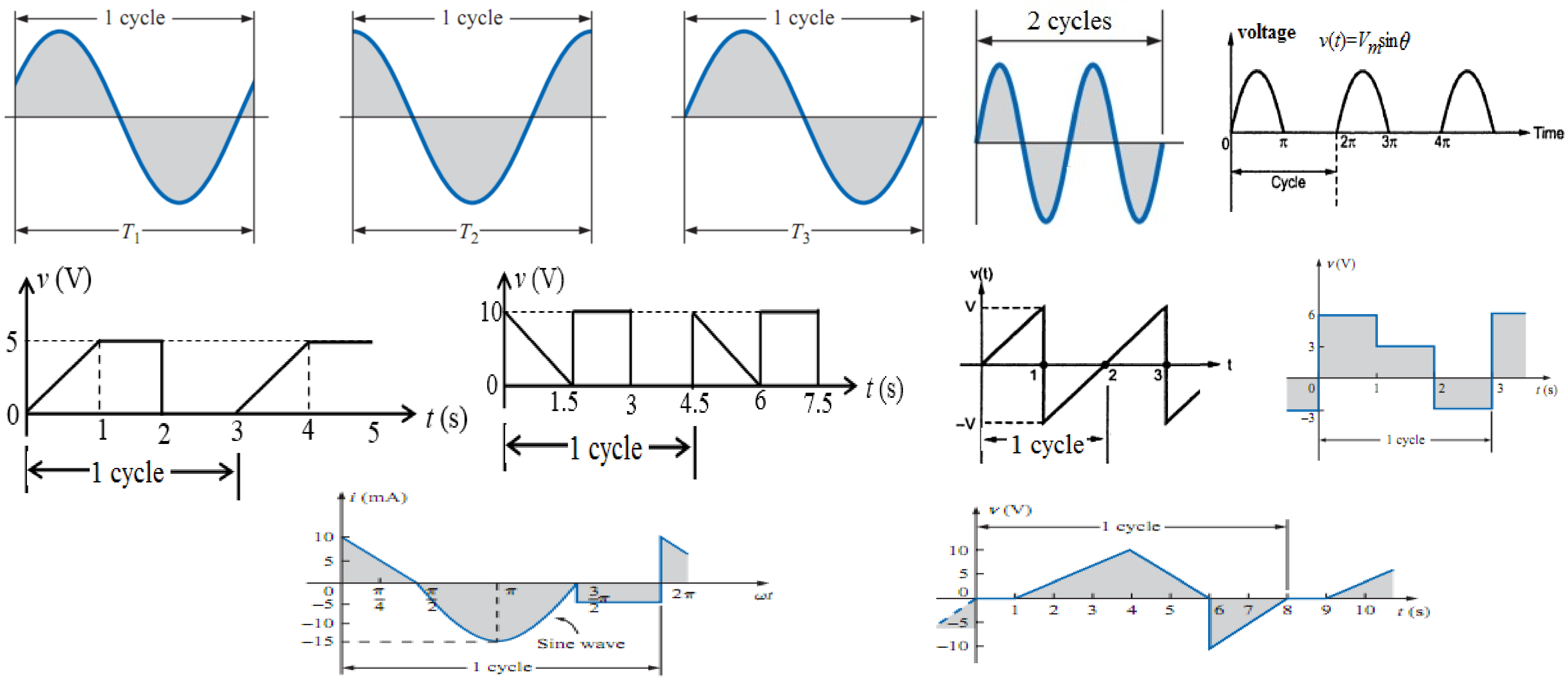
At t= 2 ms the voltage is 3 V
At t= 5 ms the voltage is 2 V
At t= 10 ms the voltage is -2.5 V

At t= 1 ms the current is 2 A
At t= 3 ms the current is 3 A
At t= 5 ms the current is -1.5 A
At t= 7 ms the current is -4 A

Definitions

Cycle: One complete set of positive and negative values of alternating quantity is called cycle.

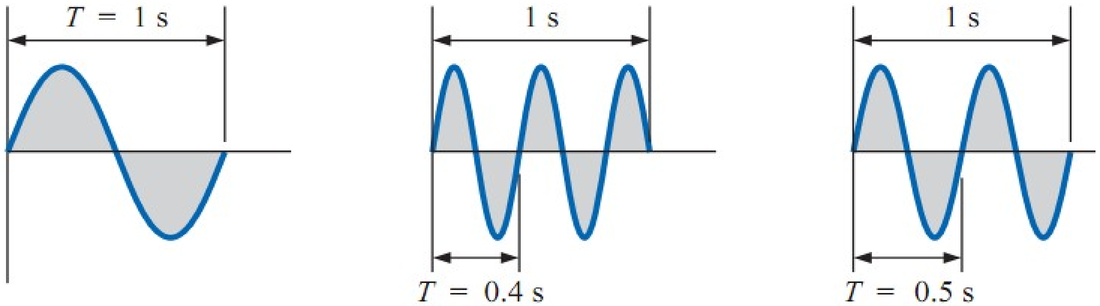
A **cycle** can also be defined as that interval of time during which complete set of non-repeating events or wave form variations occur (containing positive as well as negative loops). One such cycle of the alternating quantity is shown in the following Figure.

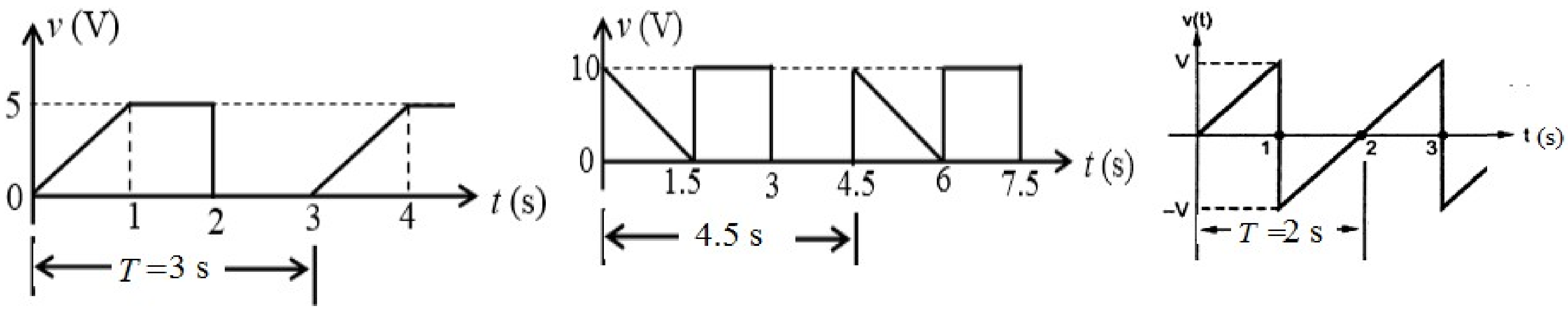


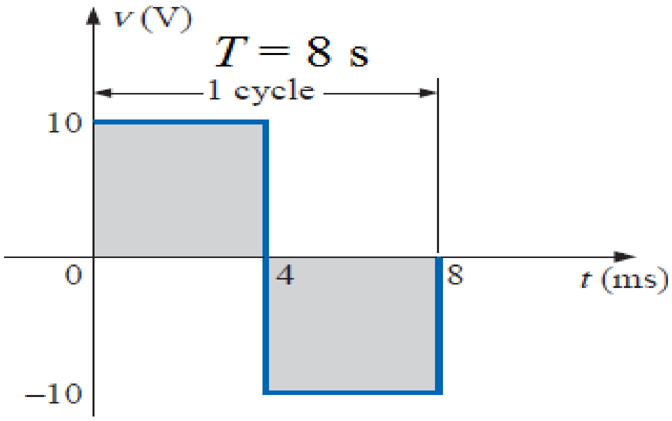
Definitions

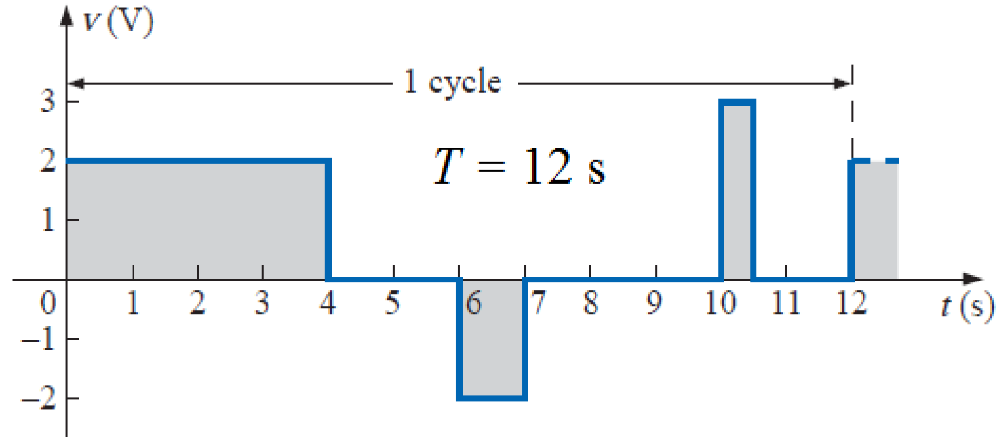
Time Period (T): The time taken by an alternating quantity to complete its one cycle is known as time period which is denoted by T seconds.

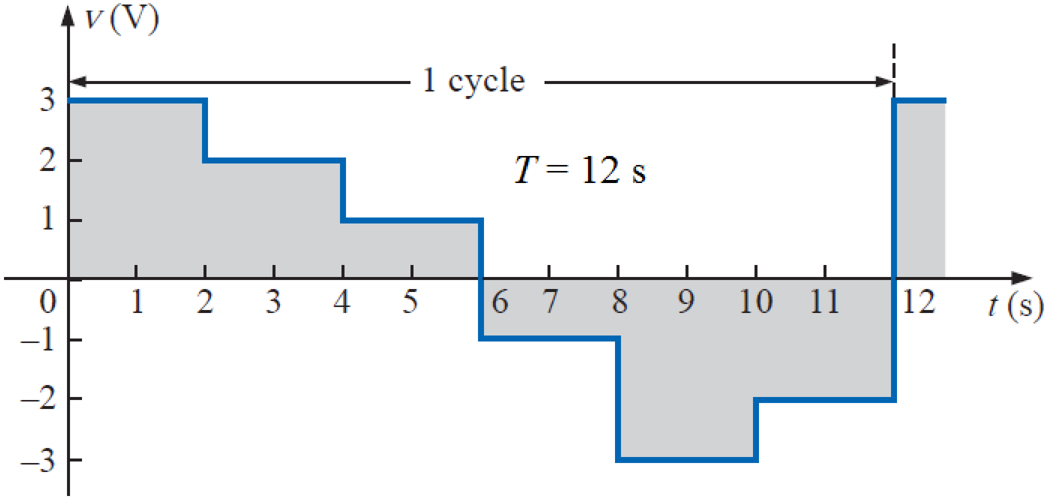
- After every seconds, the cycle of an alternating quantity repeats.
- The period of an alternating current or voltage is the smallest value of time which separates recurring values of the alternating quantity.






v (V)
T = 8 s
1 cycle
10
0
4
8
t (ms)
–10


v (V)
1 cycle
3
2
1
T = 12 s
0
1
2
3
4
5
6
7
8
9
10
11
12
t (s)
–1
–2


v (V)
1 cycle
3
2
1
T = 12 s
0
1
2
3
4
5
6
7
8
9
10
11
12
t (s)
–1
–2
–3

Definitions

Frequency (*f*): The number of cycles completed by an alternating quantity per second is known as frequency. It is denoted by *f* and it is measured in **cycle/second** which is known as **Hertz**, denoted by **Hz**.

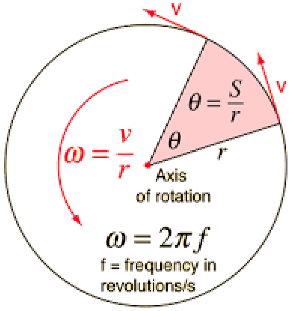
As time period (*T*) is time for one cycle i.e. seconds/cycle and frequency is cycles/second, it can be said that ***frequency is the reciprocal of the time period***.

$$\boxed{f = \frac{1}{T} \quad \text{Hz}}$$

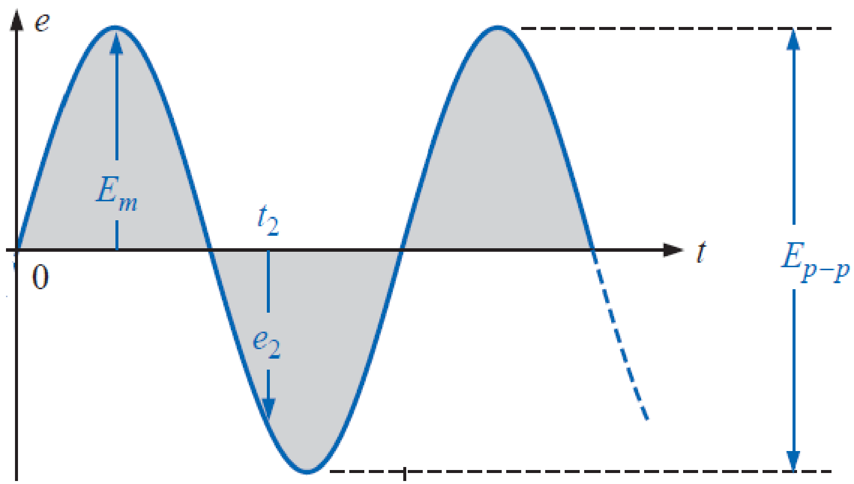
Audio Frequencies	: 15 Hz ~ 20 kHz
Radio Frequencies	: 3 kHz ~ 300 GHz
Infrared Frequencies	: More than 300 GHz

Angular Frequency [or Velocity] (ω): The rate of change of angular position is called angular frequency or angular velocity.

$$\boxed{\omega = \frac{d\theta}{dt} = \frac{2\pi}{T} = 2\pi f \ \ \text{rad/s}}$$



Peak or Amplitude or Maximum Value: The maximum instantaneous value attained by an alternating quantity during positive and negative half-cycle is called its amplitude or peak value. In the following Figure E_m or E_p or V_m or V_p is the peak value.



Peak-to-Peak Value: The full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks is called Peak-to-peak value which is denoted by $E_{p\text{-}p}$ or $V_{p\text{-}p}$.

$$\boxed{V_{p-p} = 2V_p = 2V_m}$$

Sinusoidal ac Voltage Characteristics and Definitions

EXAMPLE 13.1 For the sinusoidal waveform in Fig. 13.7:

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

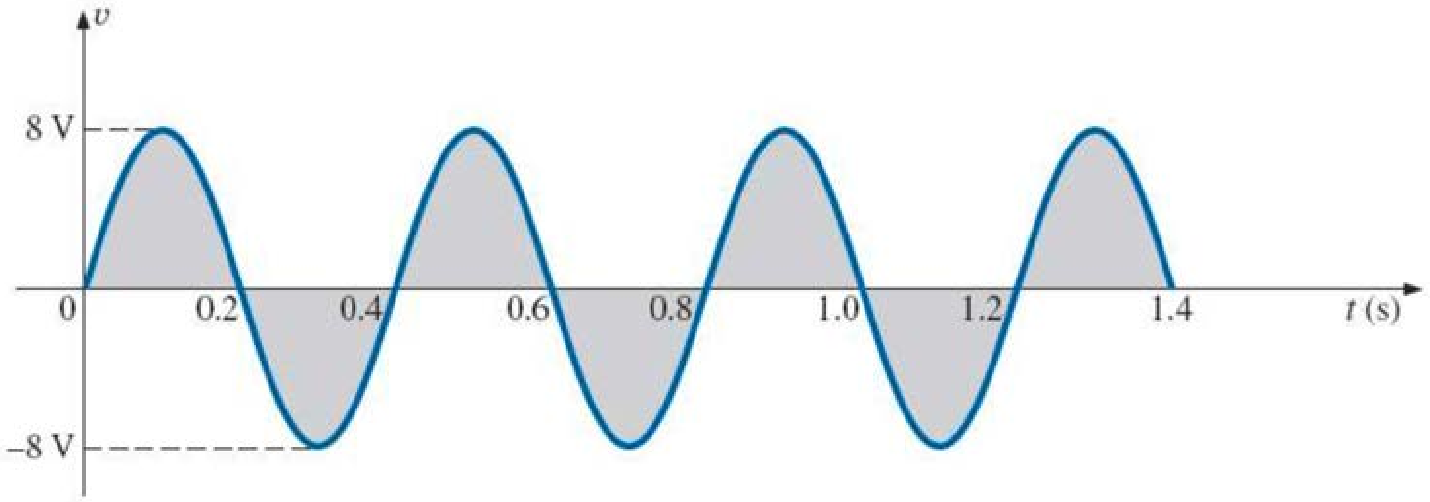


FIG. 13.7 *Example 13.1.*

General Format for the Sinusoidal Voltage or Current

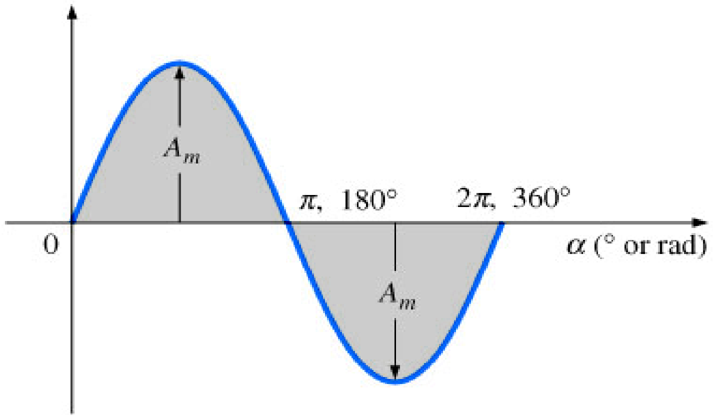
- The basic mathematical format for the sinusoidal waveform is:

$$\boxed{A_m \sin \alpha}$$

where:

A_m is the peak value of the waveform

α is the unit of measure for the horizontal axis



- The general format of a sine wave can also be as:

$$\boxed{A_m \sin \omega t}$$

where:

The angular velocity (ω) is:

$$\omega = \frac{\alpha}{t}$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

General Format for the Sinusoidal Voltage or Current

For electrical quantities such as current and voltage, the general format is:

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

where:

the capital letters with the subscript *m* represent the amplitude, and

the lower case letters *i* and *e* represent the instantaneous value of current and voltage, respectively, at any time *t*.

EXAMPLE 13.8 Given $e = 5\sin\alpha$, determine e at $\alpha = 40^\circ$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40^\circ$,

$$e = 5\sin 40^\circ = 5(0.6428) = \mathbf{3.21\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha(^\circ) = \frac{180^\circ}{\pi}(0.8\,\pi) = 144^\circ$$

and $$e = 5\sin 144^\circ = 5(0.5878) = \mathbf{2.94\ V}$$

EXAMPLE 13.11 Given $i = 6 \times 10^{-3}\sin 1000t$, determine i at $t = 2$ ms.

Solution:

$$\alpha = \omega t = 1000t = (1000\ \text{rad/s})(2 \times 10^{-3}\ \text{s}) = 2\ \text{rad}$$

$$\alpha(^\circ) = \frac{180^\circ}{\pi\ \text{rad}}(2\ \text{rad}) = 114.59^\circ$$

$$i = (6 \times 10^{-3})(\sin 114.59^\circ) = (6\ \text{mA})(0.9093) = \mathbf{5.46\ mA}$$

Phase Relations

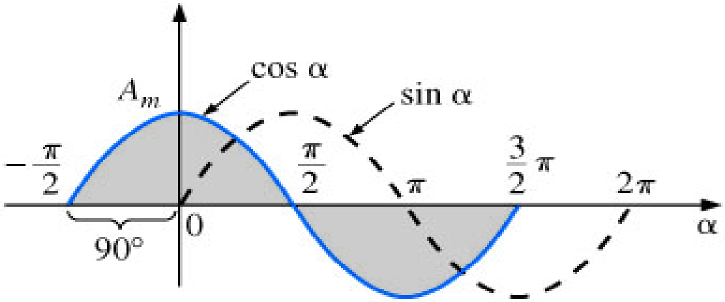
- If the waveform is shifted to the right or left of 0°, the expression becomes:

$$\boxed{A_m \sin(\omega t \pm \theta)}$$

where: θ is the angle (in degrees or radians) that the waveform has been shifted

- The **phase relationship** between two waveforms indicates which one leads or lags the other, and by how many degrees or radians.

- The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes



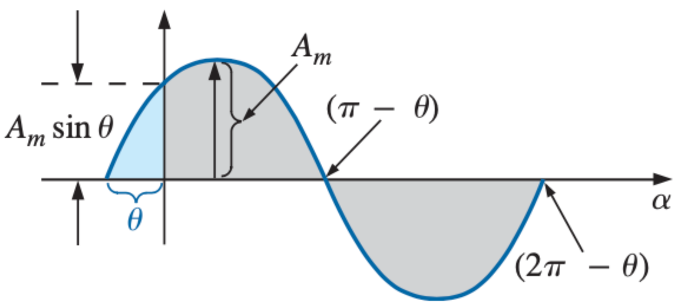
Phase Relations

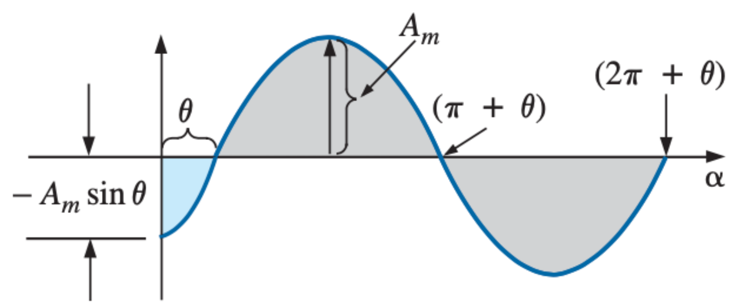
- If the waveform passes through the horizontal axis with a positive-going (increasing with the time) slope before 0° :

$$\boxed{A_m \sin(\omega t + \theta)}$$

- If the waveform passes through the horizontal axis with a positive-going slope *after* 0° :

$$\boxed{A_m \sin(\omega t - \theta)}$$





Phase Relations

$$\cos\alpha = \sin(\alpha + 90°)$$
$$\sin\alpha = \cos(\alpha - 90°)$$
$$-\sin\alpha = \sin(\alpha \pm 180°)$$
$$-\cos\alpha = \sin(\alpha + 270°) = \sin(\alpha - 90°)$$
etc.

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

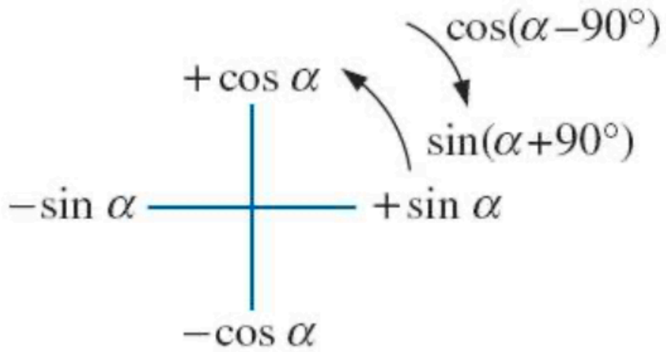


FIG. 13.30 *Graphic tool for finding the relationship between specific sine and cosine functions.*

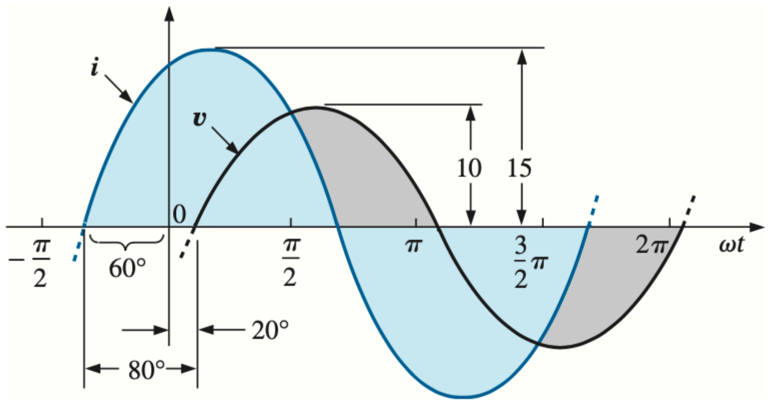
Phase Relations

Example 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$

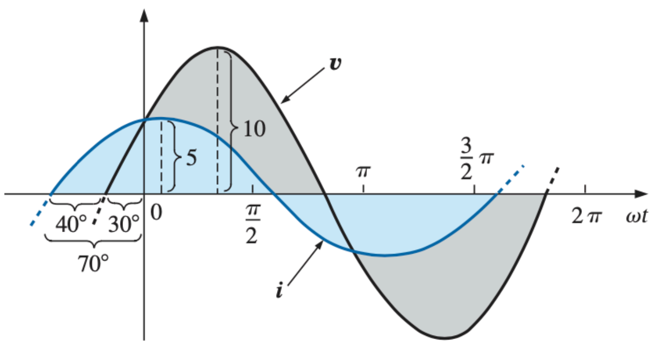
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$

c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$

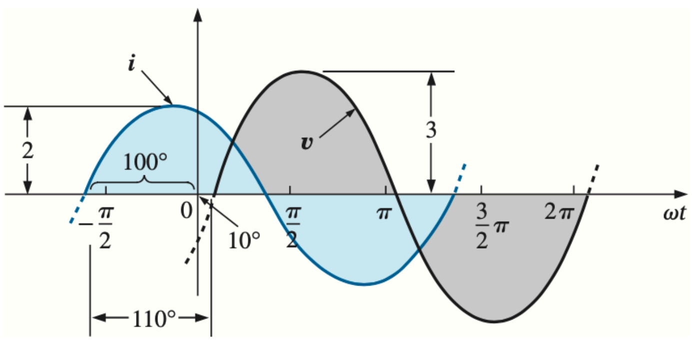


b. ***i* leads *v* by 80º, or *v* lags *i* by 80º.**

Solution:



a. ***i* leads *v* by 40º, or *v* lags *i* by 40º.**



c. ***i* leads *v* by 110º, or *v* lags *i* by 110º.**

$$i = 2 \cos(\omega t + 10°) = 2 \sin(\omega t + 10° + 90°)$$
$$= 2 \sin(\omega t + 100°)$$

Phase Relations

d. $i = -\sin(\omega t + 30°)$
$\quad v = 2\sin(\omega t + 10°)$

e. $i = -2\cos(\omega t - 60°)$
$\quad v = 3\sin(\omega t - 150°)$

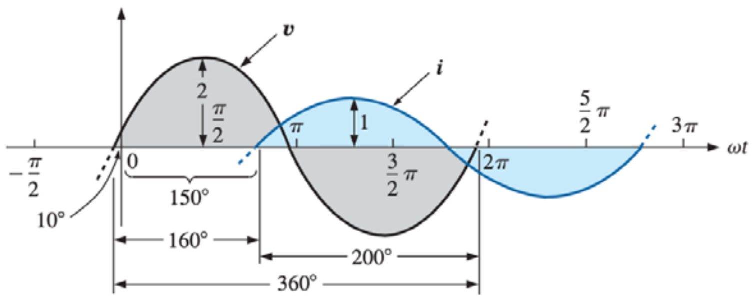


FIG. 13.34
Example 13.12(d): v leads i by 160°.

d. See Fig. 13.34.

$$\begin{aligned} -\sin(\omega t + 30°) &= \sin(\omega t + 30° \overset{\swarrow\text{Note}}{-} 180°) \\ &= \sin(\omega t - 150°) \end{aligned}$$

***v* leads *i* by 160°, or *i* lags *v* by 160°.**

Or using

$$\begin{aligned} -\sin(\omega t + 30°) &= \sin(\omega t + 30° \overset{\swarrow\text{Note}}{+} 180°) \\ &= \sin(\omega t + 210°) \end{aligned}$$

***i* leads *v* by 200°, or *v* lags *i* by 200°.**

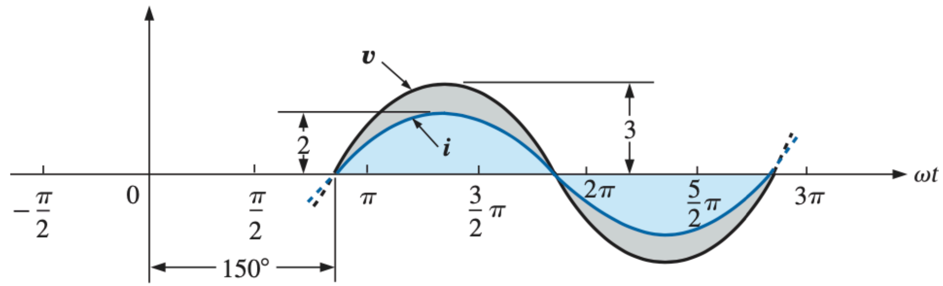


FIG. 13.35
Example 13.12(e): v and i are in phase.

e. See Fig. 13.35.

$$\begin{aligned} i = -2\cos(\omega t - 60°) &= 2\cos(\omega t - 60° \overset{\swarrow\text{By choice}}{-} 180°) \\ &= 2\cos(\omega t - 240°) \end{aligned}$$

However, $\quad \cos\alpha = \sin(\alpha + 90°)$

so that $\quad$ $$\begin{aligned} 2\cos(\omega t - 240°) &= 2\sin(\omega t - 240° + 90°) \\ &= 2\sin(\omega t - 150°) \end{aligned}$$

***v* and *i* are in phase.**

Average Value

The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of **half-cycle for a symmetrical waveform** and **full-cycle for a asymmetrical waveform**.

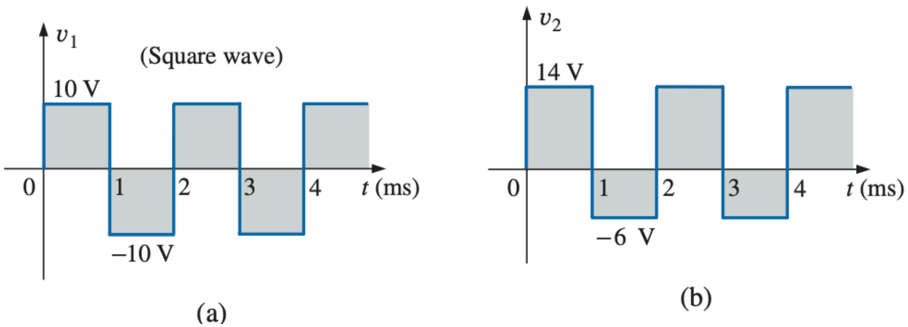
Average value can be calculated by the following methods:

- ❑ **Graphical Method**
- ❑ **Analytical Method**

The average value can be obtained by taking ratio of area under curve over to length of the base of curve.

$$G\,(\text{average value}) = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

EXAMPLE 13.14 Determine the average value of the waveforms in Fig. 13.44.



Solutions:

a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using Eq. (13.26) gives

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b. Using Eq. (13.26) gives

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

as shown in Fig. 13.45.

In reality, the waveform in Fig. 13.44(b) is simply the square wave in Fig. 13.44(b) with a dc shift of 4 V; that is,

$$v_2 = v_1 + 4\text{ V}$$

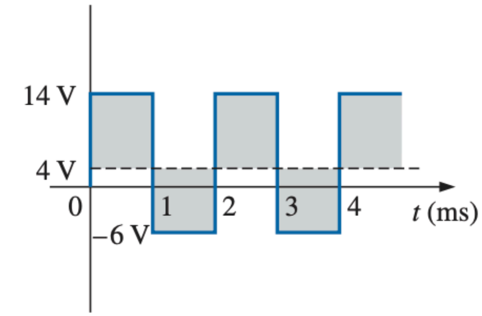
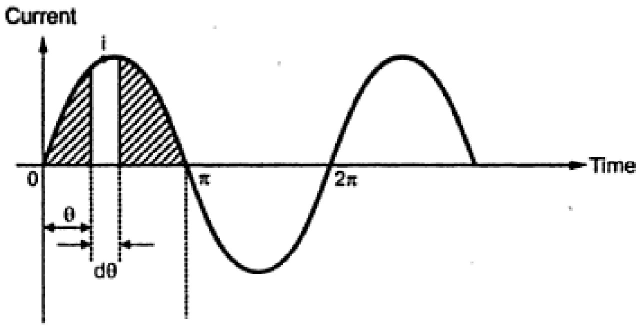


FIG. 13.45
Defining the average value for the waveform in Fig. 13.44(b).

Average Value Calculation Using Analytical Method

Consider sinusoidally varying current: $i(t) = I_m \sin\theta$

Consider elementary interval of instant '$d\theta$' as shown in the following Figure. The average value of current can be calculated by:



$$\boxed{I_{ave} = \frac{2}{T}\int_0^{T/2} i(t)dt = \frac{1}{\pi}\int_0^{\pi} i(\theta)d\theta}$$

$$I_{ave} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta d\theta = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta d\theta = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{2}{\pi}I_m = 0.637 I_m$$

The average value of a sinusoidal quantity is 63.7% of its maximum value. The average is are denoted by upper-case letter.

$$V_{ave} = \frac{2}{\pi}V_m = 0.637 V_m$$

EXAMPLE 13.16 Determine the average value of the sinusoidal waveform in Fig. 13.53.

Solution: By inspection it is fairly obvious that

the average value of a pure sinusoidal waveform over one full cycle is zero.

Eq. (13.26):

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

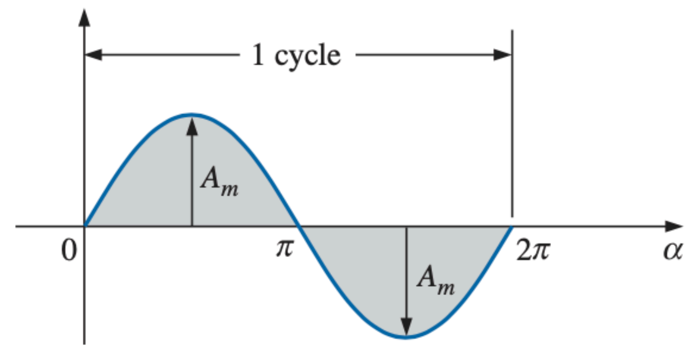


FIG. 13.53
Example 13.16.

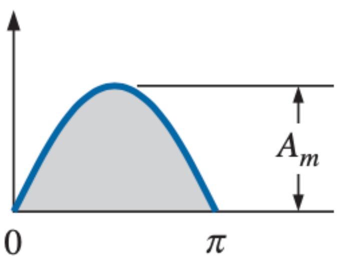
Explanation:

$$\text{Area} = \int_0^{\pi} A_m \sin\alpha \, d\alpha$$

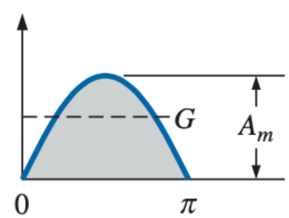
$$\begin{aligned}\text{Area} &= A_m[-\cos\alpha]_0^{\pi} \\ &= -A_m(\cos\pi - \cos 0°) \\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$G = \frac{2A_m}{\pi}$$

$$\boxed{\text{Area} = 2A_m}$$



$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m}$$



Thank You