Introduction to Electrical Circuits

Mid Term
Lecture – 4

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Reference Book:

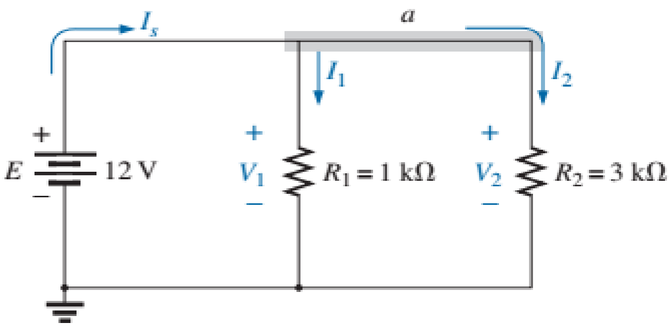
Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W2	MC4	Chapter 6	6.4 POWER DISTRIBUTION IN A PARALLEL CIRCUIT	6.15	19, 21,32
			6.5 KIRCHHOFF'S CURRENT LAW	6.17,6.18, 6.19	
			6.6 CURRENT DIVIDER RULE	6.21	
			6.7 VOLTAGE SOURCES IN PARALLEL		
		Chapter 7	Introduce how to analyze series-parallel circuits (REDUCE AND RETURN APPROACH or BLOCK DIAGRAM APPROACH)	7.3, 7.7, 7.9	8, 10, 11, 16, 17, 18.

6.3 PARALLEL CIRCUITS

(Equations associated to analyze series network)



1. The voltage is always the same across parallel elements. $\boxed{V_1 = V_2 = E}$

2. Power delivered by the source

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)}$$

3. Power consumed by the load

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}}$$

4. Branch Currents:

<u>Current divider approach:</u>

$$I_1 = \frac{R_2}{R_1 + R_2} \times I_S$$

$$I_2 = \frac{R_1}{R_1 + R_2} \times I_S$$

General formula for more than two branches

$$\boxed{I_x = \frac{R_T}{R_x} I_T}$$

5. KIRCHHOFF'S CURRENT LAW

$$\boxed{\Sigma I_i = \Sigma I_o}$$

EXAMPLE 6.16 Determine currents I_3 and I_4 in Fig. 6.33 using Kirchhoff's current law.

Solution: There are two junctions or nodes in Fig. 6.33. Node *a* has only one unknown, while node *b* has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node *a* first.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node *b,* using the result just obtained:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 6.33, the width of the blue shaded regions matches the magnitude of the current in that region.

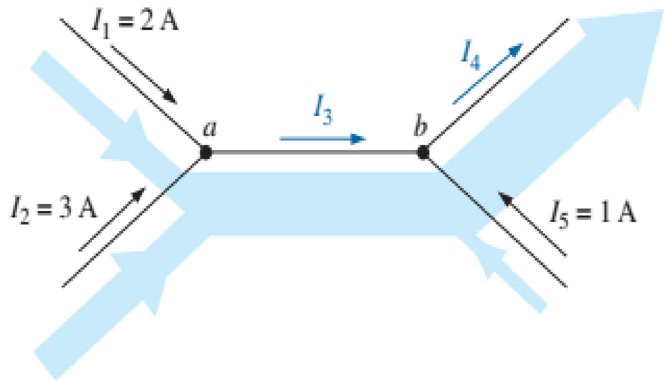


FIG. 6.33
Two-node configuration for Example 6.16.

EXAMPLE 6.18 Determine currents I_3 and I_5 in Fig. 6.35 through applications of Kirchhoff's current law.

Solution: Note first that since node b has two unknown quantities (I_3 and I_5), and node a has only one, Kirchhoff's current law must first be applied to node a. The result is then applied to node b.

At node a:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$4\text{ A} + 3\text{ A} = I_3 = \mathbf{7\text{ A}}$$

At node b:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 = I_4 + I_5$$
$$7\text{ A} = 1\text{ A} + I_5$$

and
$$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}$$

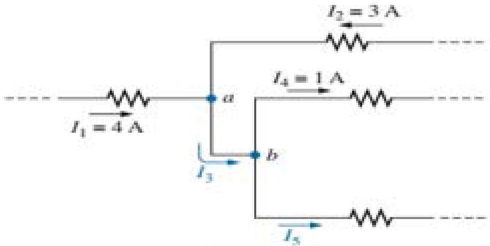
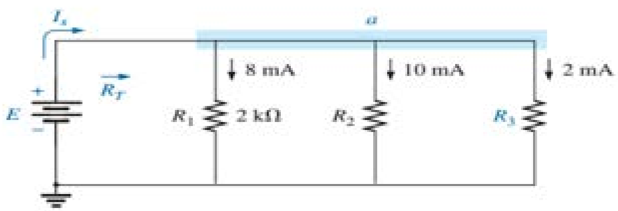


FIG. 6.35
Network for Example 6.18.

EXAMPLE 6.19 For the parallel dc network in Fig. 6.36.

a. Determine the source current I_s.
b. Find the source voltage E.



c. Determine R_3.
d. Calculate R_T.

Solutions:

a. First apply Eq. (6.13) at node a. Although node a in Fig. 6.36 may not initially appear as a single junction, it can be redrawn as shown in Fig. 6.37, where it is clearly a common point for all the branches.

   The result is

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$

Substituting values: $I_s = 8\text{ mA} + 10\text{ mA} + 2\text{ mA} = \mathbf{20\text{ mA}}$

   Note in this solution that you do not need to know the resistor values or the voltage applied. The solution is determined solely by the current levels.

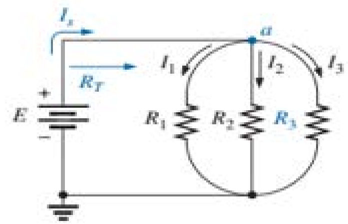


FIG. 6.37
Redrawn network in Fig. 6.36.

b. Applying Ohm's law:

$$E = V_1 = I_1R_1 = (8\text{ mA})(2\text{ k}\Omega) = \mathbf{16\text{ V}}$$

c. Applying Ohm's law in a different form:

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16\text{ V}}{2\text{ mA}} = \mathbf{8\text{ k}\Omega}$$

d. Applying Ohm's law again:

$$R_T = \frac{E}{I_s} = \frac{16\text{ V}}{20\text{ mA}} = \mathbf{0.8\text{ k}\Omega}$$

19. For the configuration in Fig. 6.89:

a. Find the total resistance and the current through each branch.

b. Find the power delivered to each resistor.

c. Calculate the power delivered by the source.

d. Compare the power delivered by the source to the sum of the powers delivered to the resistors.

e. Which resistor received the most power? Why?

Solution:

a. $$R_T = \frac{1}{\frac{1}{1\,\text{k}\Omega} + \frac{1}{33\,\text{k}\Omega} + \frac{1}{8.2\,\text{k}\Omega}} = \frac{1}{1000\times10^{-6}\text{S} + 30.303\times10^{-6}\text{S} + 121.951\times10^{-6}\text{S}}$$

$$= \frac{1}{1.152\times10^{-3}\text{S}} = \mathbf{867.86\ \Omega}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{100\text{ V}}{1\,\text{k}\Omega} = \mathbf{100\ mA},\quad I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{100\text{ V}}{33\,\text{k}\Omega} = \mathbf{3.03\ mA}$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{100\text{ V}}{8.2\,\text{k}\Omega} = \mathbf{12.2\ mA}$$

b. $$P_{R_1} = V_{R_1}\cdot I_{R_1} = (100\text{ V})(100\text{ mA}) = \mathbf{10\ W}$$

$$P_{R_2} = V_{R_2}\cdot I_{R_2} = (100\text{ V})(3.03\text{ mA}) = \mathbf{0.30\ W}$$

$$P_{R_3} = V_{R_3}\cdot I_{R_3} = (100\text{ V})(12.2\text{ mA}) = \mathbf{1.22\ W}$$

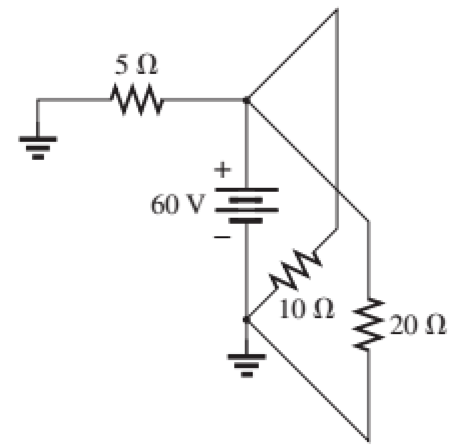
c. $$I_s = \frac{E}{R_T} = \frac{100\text{ V}}{867.86\ \Omega} = 115.23\text{ mA}$$

$$P_s = E_s I_s = (100\text{ V})(115.23\text{ mA}) = \mathbf{11.52\ W}$$

d. $P_s = \mathbf{11.52\ W} = 10\text{ W} + 0.30\text{ W} + 1.22\text{ W} = \mathbf{11.52\ W}$ (checks)

e. R_1 = the smallest parallel resistor

21. Determine the power delivered by the dc battery in Fig. 6.91.



Solution:

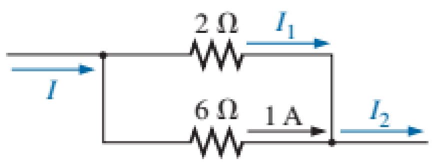
$$R_T = \frac{1}{\frac{1}{5\,\Omega}+\frac{1}{10\,\Omega}+\frac{1}{20\,\Omega}} = \frac{1}{200\times10^{-3}\text{S}+100\times10^{-3}\text{S}+50\times10^{-3}\text{S}}$$

$$= \frac{1}{350\times10^{-3}\text{S}} = 2.86\,\Omega$$

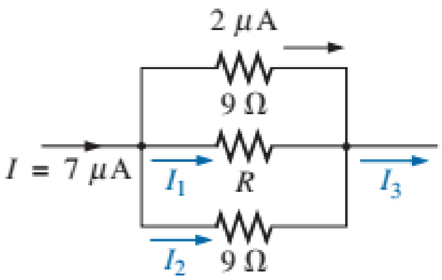
$$I_s = \frac{E}{R_T} = \frac{60\text{ V}}{2.86\,\Omega} = 20.98\text{ A}$$

$$P = E\cdot I_s = (60\text{ V})(20.98\text{ A}) = \mathbf{1.26\ kW}$$

32. Find the unknown quantities for the networks in Fig. 6.102 using the information provided.



(a)



(b)

Solution:

a. CDR: $I_{6\Omega} = \dfrac{2\ \Omega\, I}{2\ \Omega + 6\ \Omega} = 1\text{ A}$

$$I = \frac{1\text{ A}(8\ \Omega)}{2\ \Omega} = \mathbf{4\ A} = I_2$$

$$I_1 = I - 1\text{ A} = 3\text{ A}$$

b. $I_3 = I = \mathbf{7\ \mu A}$

By inspection: $I_2 = \mathbf{2\ \mu A}$

$$I_1 = I - 2(2\ \mu\text{A}) = 7\ \mu\text{A} - 4\ \mu\text{A} = \mathbf{3\ \mu A}$$

$V_R = (2\ \mu\text{A})(9\ \Omega) = 18\ \mu\text{V}$

$$R = \frac{V_R}{I_R} = \frac{18\ \mu\text{V}}{3\ \mu\text{A}} = \mathbf{6\ \Omega}$$

Voltage Sources in Parallel

Voltage sources can be placed in parallel only if they have the same voltage.

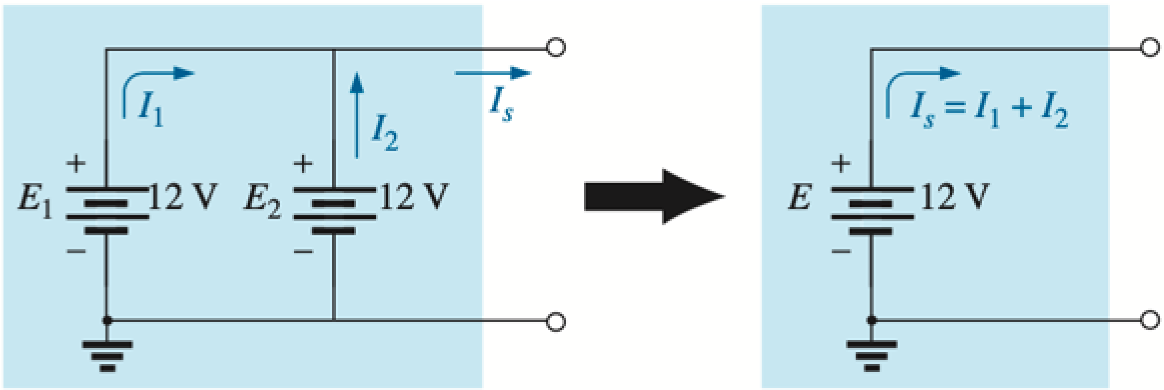
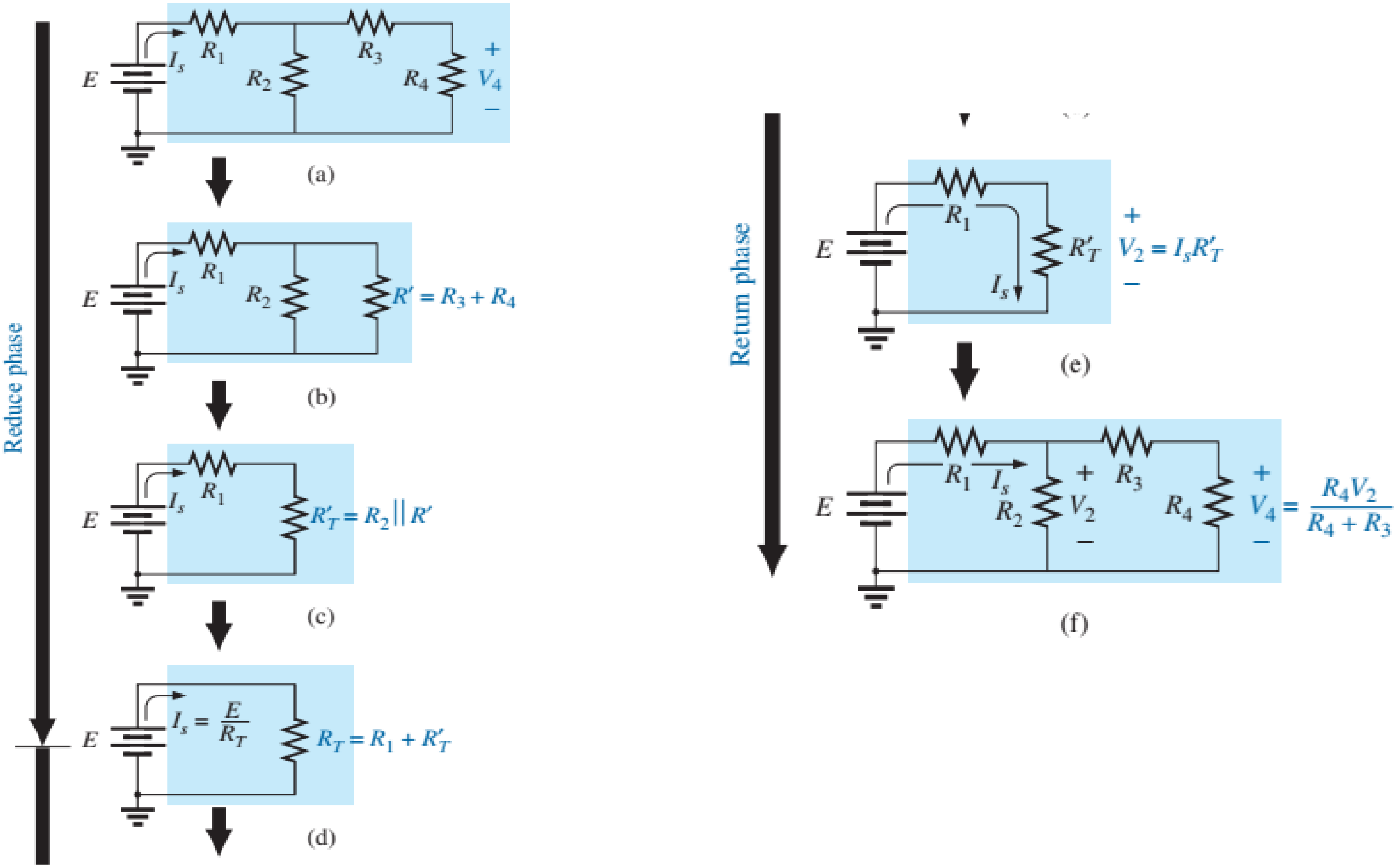


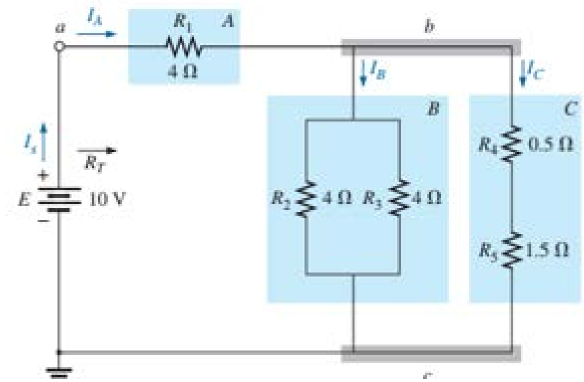
FIG. 6.47

Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.


Reduce phase
E
I_s
R_1
R_2
R_3
R_4
V_4
(a)
$R' = R_3 + R_4$
(b)
$R'_T = R_2 \| R'$
(c)
$I_s = \frac{E}{R_T}$
$R_T = R_1 + R'_T$
(d)
Return phase
$V_2 = I_s R'_T$
(e)
$V_4 = \frac{R_4 V_2}{R_4 + R_3}$
(f)

7.4 BLOCK DIAGRAM APPROACH

EXAMPLE 7.3 Determine all the currents and voltages of the network in Fig. 7.10.



Solution: Blocks A, B, and C have the same relative position, but the internal components are different. Note that blocks B and C are still in parallel and block A is in series with the parallel combination. First, reduce each block into a single element and proceed as described for Example 7.1.

In this case:

$$A{:}\ R_A = 4\ \Omega$$

$$B{:}\ R_B = R_2 \parallel R_3 = R_{2\|3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C{:}\ R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

Blocks B and C are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

with

$$R_T = R_A + R_{B\|C}$$
$$= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

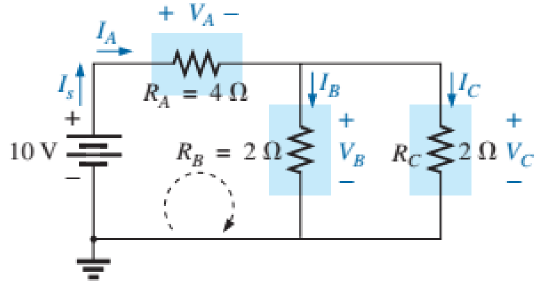
and

$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

$$I_A = I_s = \mathbf{2\ A}$$

and

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$



Returning to the network in Fig. 7.10, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. we obtain

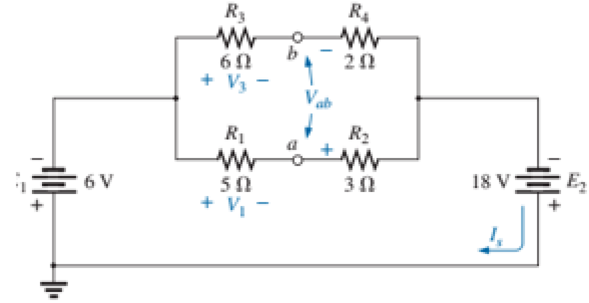
$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$

or

$$10\ \text{V} = 10\ \text{V} \quad \text{(checks)}$$

EXAMPLE 7.7

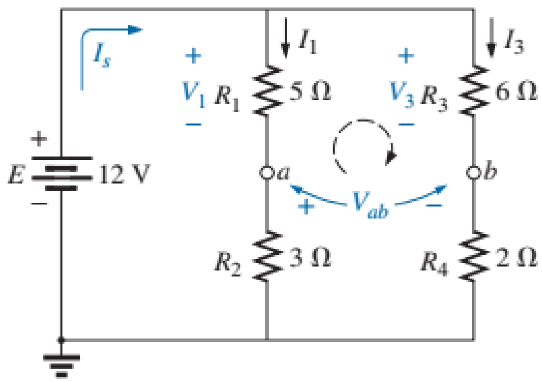
a. Find the voltages V_1, V_3, and V_{ab} for the network in Fig. 7.20.
b. Calculate the source current I_s.



Solutions:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$



$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

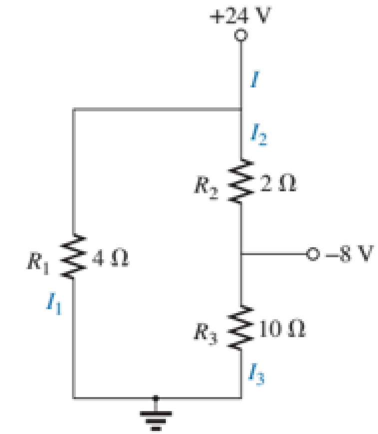
$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

10. **a.** Find the magnitude and direction of the currents I, I_1, I_2, and I_3 for the network in Fig. 7.70.

b. Indicate their direction on Fig. 7.70.



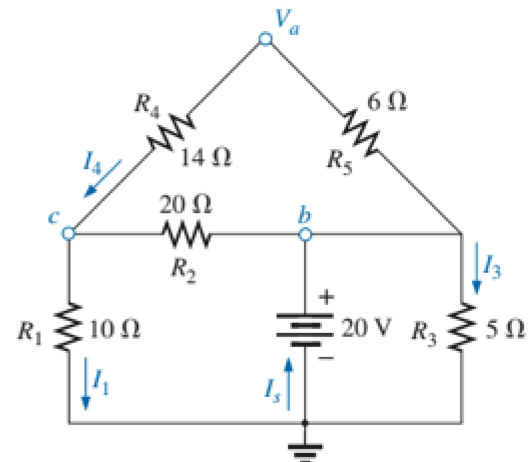
Solution:

a, b.
$$I_1 = \frac{24\text{ V}}{4\,\Omega} = \mathbf{6\text{ A}}\downarrow,\ I_3 = \frac{8\text{ V}}{10\,\Omega} = \mathbf{0.8\text{ A}}\uparrow$$
$$I_2 = \frac{24\text{ V} + 8\text{ V}}{2\,\Omega} = \frac{32\text{ V}}{2\,\Omega} = \mathbf{16\text{ A}}$$
$$I = I_1 + I_2 = 6\text{ A} + 16\text{ A} = \mathbf{22\text{ A}}\downarrow$$

***11.** For the network in Fig. 7.71:

a. Determine the currents I_s, I_1, I_3, and I_4.

b. Calculate V_a and V_{bc}.



Solution:

a.
$$R' = R_4 + R_5 = 14\,\Omega + 6\,\Omega = 20\,\Omega$$
$$R'' = R_2 \parallel R' = 20\,\Omega \parallel 20\,\Omega = 10\,\Omega$$
$$R''' = R'' + R_1 = 10\,\Omega + 10\,\Omega = 20\,\Omega$$
$$R_T = R_3 \parallel R''' = 5\,\Omega \parallel 20\,\Omega = \mathbf{4\,\Omega}$$
$$I_s = \frac{E}{R_T} = \frac{20\text{ V}}{4\,\Omega} = \mathbf{5\text{ A}}$$
$$I_1 = \frac{20\text{ V}}{R_1 + R''} = \frac{20\text{ V}}{10\,\Omega + 10\,\Omega} = \frac{20\text{ V}}{20\,\Omega} = \mathbf{1\text{ A}}$$
$$I_3 = \frac{20\text{ V}}{5\,\Omega} = \mathbf{4\text{ A}}$$
$$I_4 = \frac{I_1}{2} = (\text{since } R' = R_2) = \frac{1\text{ A}}{2} = \mathbf{0.5\text{ A}}$$

b.
$$V_a = I_3R_3 - I_4R_5 = (4\text{ A})(5\,\Omega) - (0.5\text{ A})(6\,\Omega) = 20\text{ V} - 3\text{ V} = \mathbf{17\text{ V}}$$
$$V_{bc} = \left(\frac{I_1}{2}\right)R_2 = (0.5\text{ A})(20\,\Omega) = \mathbf{10\text{ V}}$$

Exercise Problems

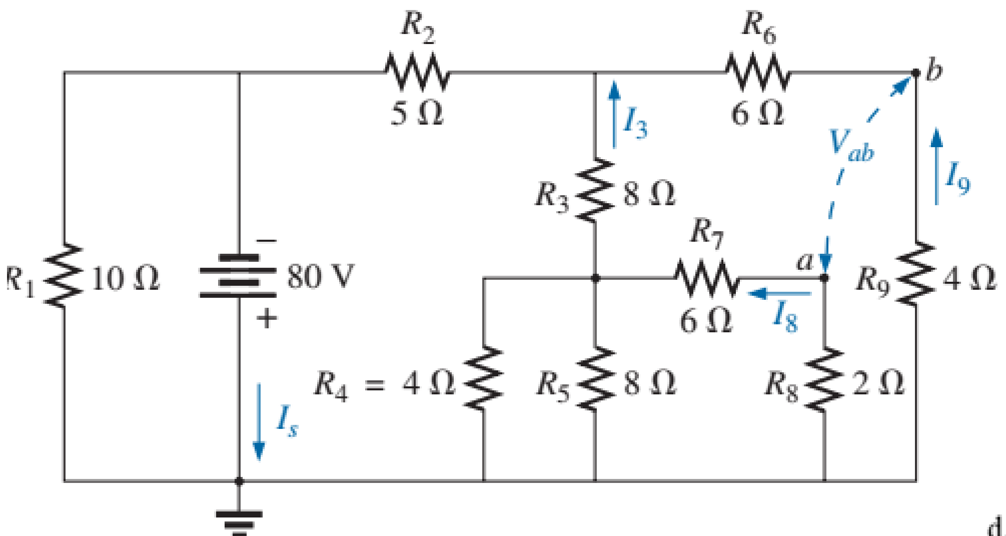
*8. For the series-parallel configuration in Fig. 7.68:

a. Find the source current I_s.

b. Find currents I_3 and I_9.

c. Find current I_8.

d. Find voltage V_x.



Solution:

a. $R' = R_4 \parallel R_5 \parallel (R_7 + R_8) = 4\,\Omega \parallel 8\,\Omega \parallel (6\,\Omega + 2\,\Omega) = 4\,\Omega \parallel 8\,\Omega \parallel 8\,\Omega$

$= 4\,\Omega \parallel 4\,\Omega = 2\,\Omega$

$R'' = (R_3 + R') \parallel (R_6 + R_9) = (8\,\Omega + 2\,\Omega) \parallel (6\,\Omega + 4\,\Omega)$

$= 10\,\Omega \parallel 10\,\Omega = 5\,\Omega$

$R_T = R_1 \parallel (R_2 + R'') = 10\,\Omega \parallel (5\,\Omega + 5\,\Omega) = 10\,\Omega \parallel 10\,\Omega = \mathbf{5\,\Omega}$

$$I = \frac{E}{R_T} = \frac{80\text{ V}}{5\,\Omega} = \mathbf{16\text{ A}}$$

b. $$I_{R_2} = \frac{I}{2} = \frac{16\text{ A}}{2} = \mathbf{8\text{ A}}$$

$$I_3 = I_9 = \frac{8\text{ A}}{2} = \mathbf{4\text{ A}}$$

c. $$I_8 = \frac{(R_4 \parallel R_5)(I_3)}{(R_4 \parallel R_5) + (R_7 + R_8)}$$

$$= \frac{(4\,\Omega \parallel 8\,\Omega)(4\text{ A})}{(4\,\Omega \parallel 8\,\Omega) + (6\,\Omega + 2\,\Omega)}$$

$$= \frac{(2.67)(4\text{ A})}{2.67\,\Omega + 8\,\Omega} = \mathbf{1\text{ A}}$$

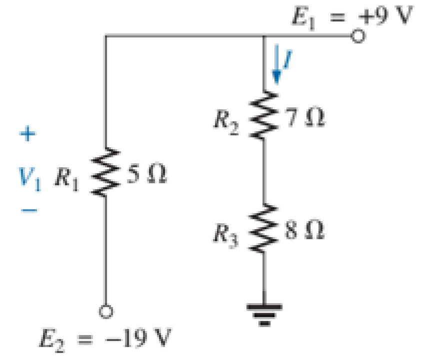
d. $-I_8R_8 - V_x + I_9R_9 = 0$

$V_x = I_9R_9 - I_8R_8 = (4\text{ A})(4\,\Omega) - (1\text{ A})(2\,\Omega) = 16\text{ V} - 2\text{ V} = \mathbf{14\text{ V}}$

16. For the network in Fig. 7.76:

a. Determine the current I.

b. Find V_1.



Solution:

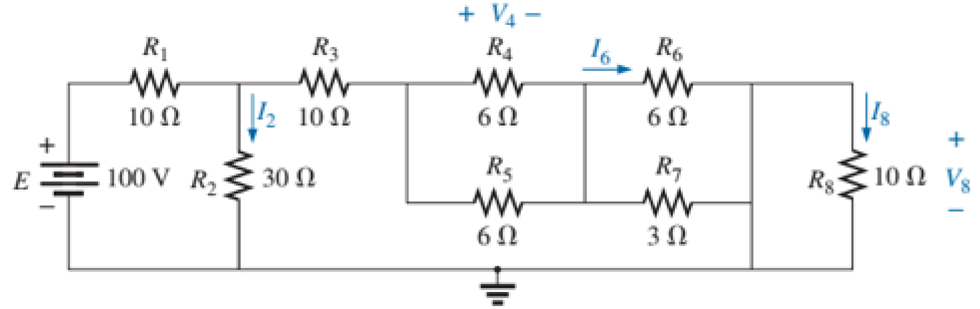
a. $$I = \frac{E_1}{R_2 + R_3} = \frac{9\text{ V}}{7\,\Omega + 8\,\Omega} = \mathbf{0.6\text{ A}}$$

b. $$E_1 - V_1 + E_2 = 0$$
$$V_1 = E_1 + E_2 = 9\text{ V} + 19\text{ V} = \mathbf{28\text{ V}}$$

17. For the configuration in Fig. 7.77:

a. Find the currents I_2, I_6, and I_8.

b. Find the voltages V_4 and V_8.



Solution:

a. R_8 "shorted out"

$$R' = R_3 + R_4 \parallel R_5 + R_6 \parallel R_7$$
$$= 10\ \Omega + 6\ \Omega \parallel 6\ \Omega + 6\ \Omega \parallel 3\ \Omega$$
$$= 10\ \Omega + 3\ \Omega + 2\ \Omega$$
$$= 15\ \Omega$$

$$R_T = R_1 + R_2 \parallel R'$$
$$= 10\ \Omega + 30\ \Omega \parallel 15\ \Omega = 10\ \Omega + 10\ \Omega$$
$$= 20\ \Omega$$

$$I = \frac{E}{R_T} = \frac{100\text{ V}}{20\ \Omega} = 5\text{ A}$$

$$I_2 = \frac{R'(I)}{R' + R_2} = \frac{(15\ \Omega)(5\text{ A})}{15\ \Omega + 30\ \Omega} = \mathbf{1.67\text{ A}}$$

$$I_3 = I - I_2 = 5\text{ A} - 1\frac{2}{3}\text{ A} = 3\frac{1}{3}\text{ A}$$

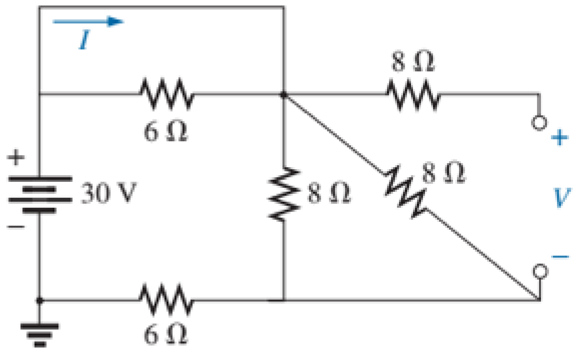
$$I_6 = \frac{R_7 I_3}{R_7 + R_6} = \frac{3\ \Omega\left(\frac{10}{3}\text{ A}\right)}{3\ \Omega + 6\ \Omega} = \mathbf{1.11\text{ A}}$$

$$I_8 = \mathbf{0\text{ A}}$$

b. $$V_4 = I_3(R_4 \parallel R_5) = \left(\frac{10}{3}\text{ A}\right)(3\ \Omega) = \mathbf{10\text{ V}}$$

$$V_8 = \mathbf{0\text{ V}}$$

18. Determine the voltage V and the current I for the network in Fig. 7.78.



Solution:

$$8\ \Omega \parallel 8\ \Omega = 4\ \Omega$$

$$I = \frac{30\text{ V}}{4\ \Omega + 6\ \Omega} = \frac{30\text{ V}}{10\ \Omega} = \mathbf{3\ A}$$

$$V = I(8\ \Omega \parallel 8\ \Omega) = (3\text{ A})(4\ \Omega) = \mathbf{12\ V}$$


END