Introduction to Electrical Circuits

Final Term
Lecture - 05

Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

W10	FC5	Chapter 18	18.2 SUPERPOSITION THEOREM	18.1, 18.2
			18.3 THÉVENIN'S THEOREM	18.8
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Superposition Theorem

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

- *Select a single source acting alone. Short the other voltage sources and open the current sources, of internal impedances are not known. If known, replace them by their internal impedances.*

- *Find the current through or the voltage across the required element, due to the source under consideration, using a suitable simplification technique.*

- *Repeat the above two steps for all the sources.*

- *Add all the individual effects produced by individual sources, to obtained the total current in or voltage across the element.*

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4 Ω reactance (X_{L_2}) in Fig. 18.1.

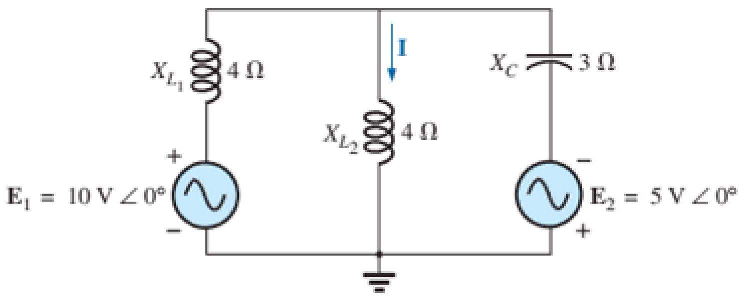


FIG. 18.1
Example 18.1.

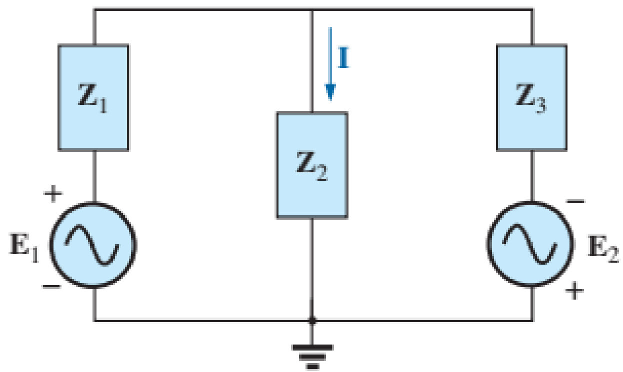


Fig. 18.2

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$

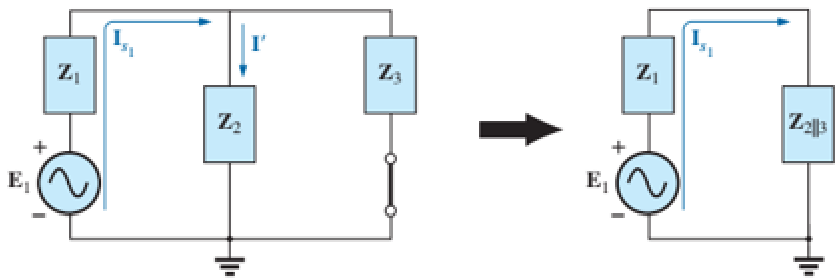


Fig. 18.3

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega\,\angle -90^\circ$$

$$\mathbf{I}_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V}\,\angle 0^\circ}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V}\,\angle 0^\circ}{8\,\Omega\,\angle -90^\circ}$$
$$= 1.25\text{ A}\,\angle 90^\circ$$

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\,\Omega)(j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A}\,\angle -90^\circ$$

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have

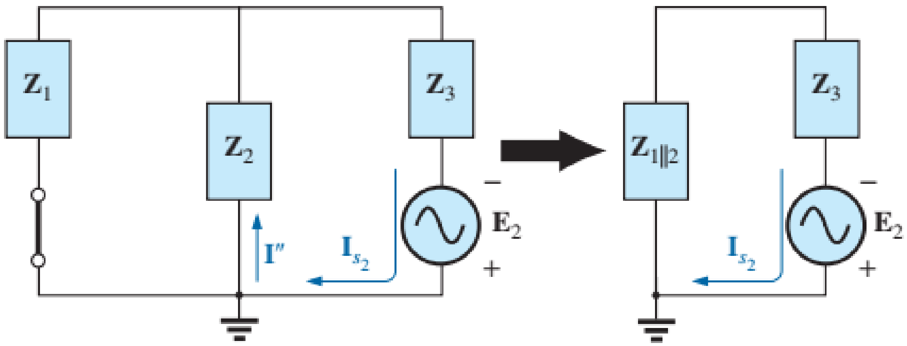


Fig. 18.4

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\,\Omega}{2} = j\,2\,\Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\text{ V}\angle 0^\circ}{j\,2\,\Omega - j\,3\,\Omega} = \frac{5\text{ V}\angle 0^\circ}{1\,\Omega\angle -90^\circ} = 5\text{ A}\angle 90^\circ$$

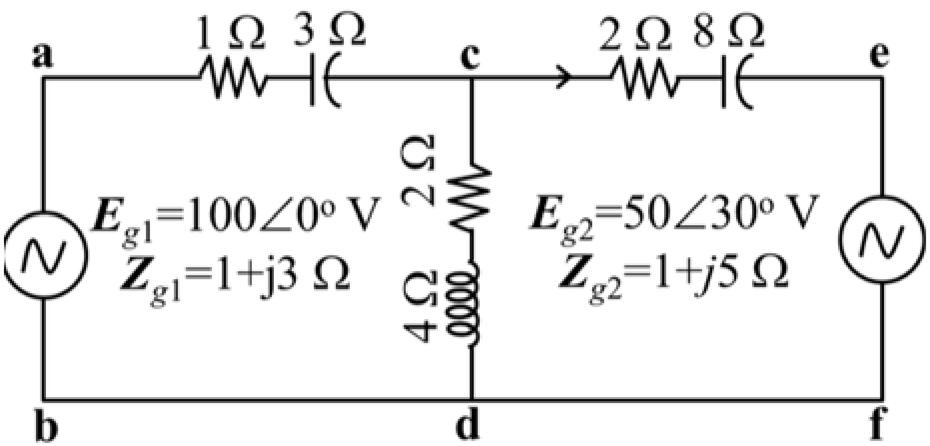
and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\text{ A}\angle 90^\circ$$

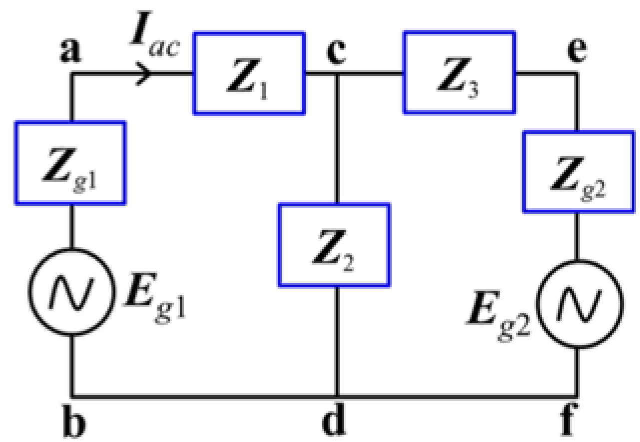
The resultant current through the 4 Ω reactance X_{L_2} (Fig. 18.5) is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}' - \mathbf{I}''\\ &= 3.75\text{ A}\angle -90^\circ - 2.50\text{ A}\angle 90^\circ = -j\,3.75\text{ A} - j\,2.50\text{ A}\\ &= -j\,6.25\text{ A}\\ \mathbf{I} &= \mathbf{6.25\text{ A}\angle -90^\circ}\end{aligned}$$

Example

Calculate the current (using Superposition Theorem) in branch *ac* for the following network.

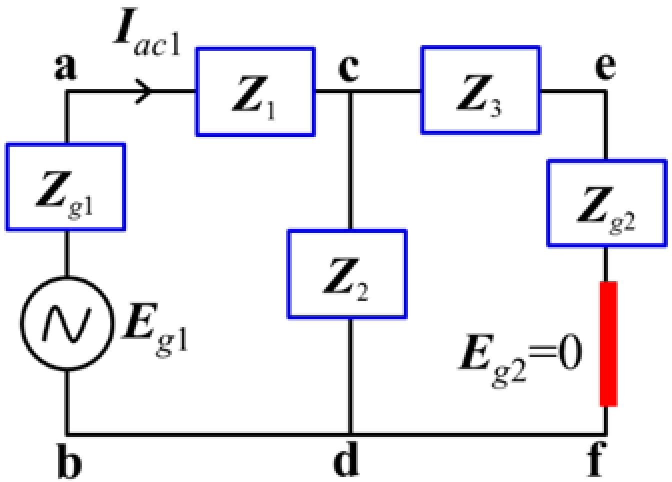


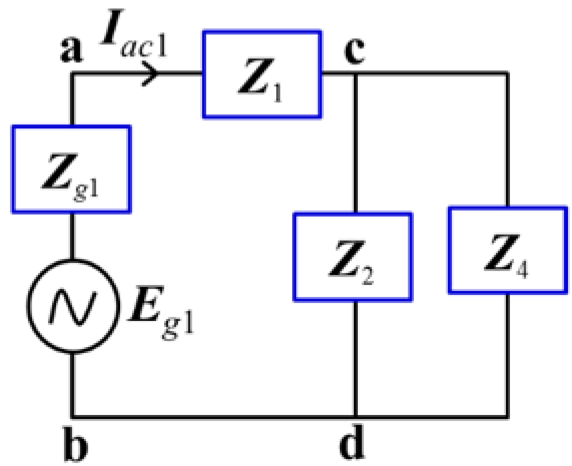


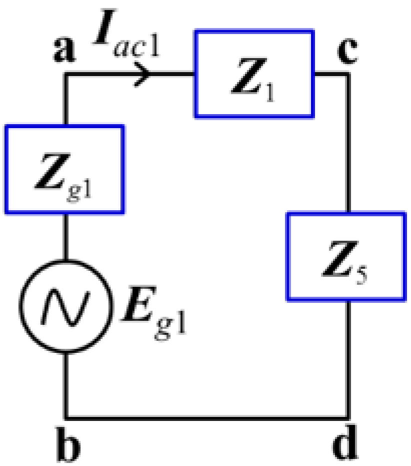
$$Z_1 = 1 - j3\ \Omega$$
$$Z_2 = 2 + j4\ \Omega$$
$$Z_3 = 2 - j8\ \Omega$$

$$Z_{g1} = 1 + j3\ \Omega$$
$$Z_{g2} = 1 + j5\ \Omega$$

Consider $E_{g1}=100\angle 0^o$ while set $E_{g1}=0$ i.e. short circuit.







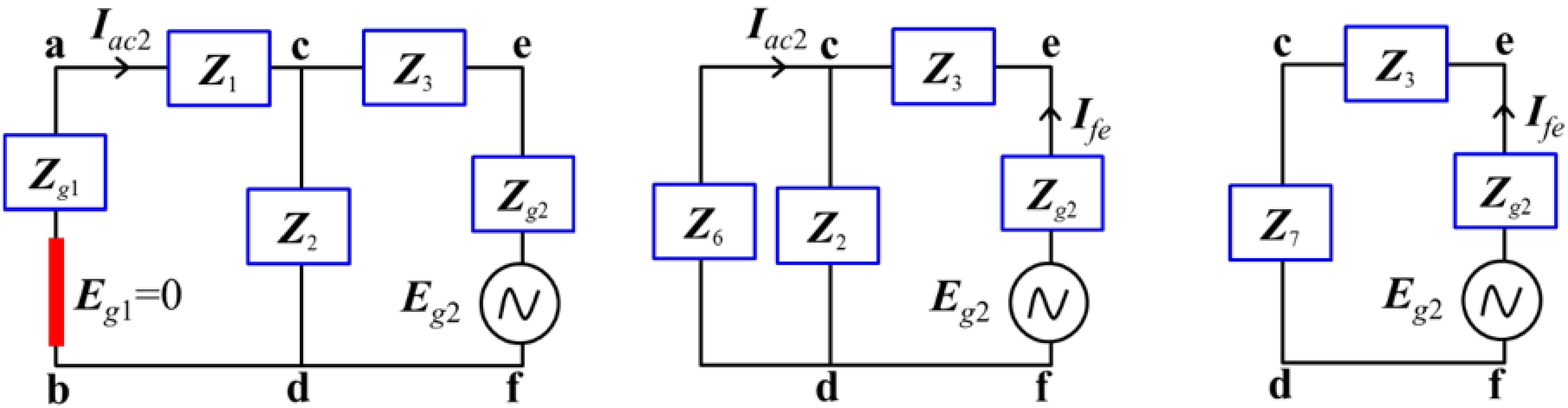
$$Z_4 = Z_{g2} + Z_3 = 3 - j3\ \Omega$$

$$Z_5 = \frac{Z_2 Z_4}{Z_2 + Z_4} = 3.7 + j0.46\ \Omega$$

$$Z_{T1} = Z_{g1} + Z_1 + Z_5 = 5.7 + j0.46\ \Omega$$

$$I_{ac1} = \frac{E_{g1}}{Z_{T1}} = 17.45 - j1.41\ \text{A}$$

Consider E_{g2}=50∠30° while set E_{g2}=0 i.e. short circuit.



$$Z_6 = Z_{g1} + Z_1 = 2\ \Omega \qquad Z_7 = \frac{Z_2 Z_6}{Z_2 + Z_6} = 1.5 + j0.5\ \Omega$$

$$Z_{T2} = Z_{g2} + Z_3 + Z_7 = 4.5 - j2.5\ \Omega$$

$$I_{fe} = \frac{E_{g2}}{Z_{T2}} = 4.99 + j8.33\ \text{A} \qquad I_{ac2} = -\frac{Z_2}{Z_2 + Z_6} I_{fe} = -1.66 - j7.5\ \text{A}$$

$$I_{ac} = I_{ac1} + I_{ac2} = 15.79 - j8.91\ \text{A}$$

EXAMPLE 18.2 Using superposition, find the current **I** through the 6 Ω resistor in Fig. 18.6.

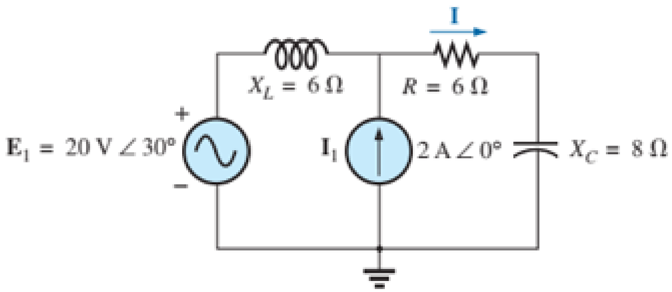


Fig. 18.6

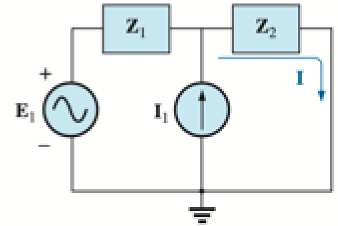


Fig. 18.7

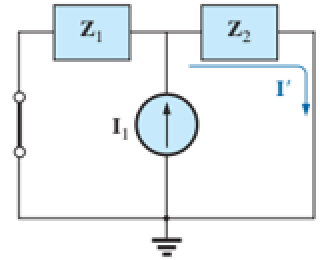


Fig. 18.8

Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\ \Omega \quad \mathbf{Z}_2 = 6\ \Omega - j\,8\ \Omega$$

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\ \Omega)(2\ \text{A})}{j\,6\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{j\,12\ \text{A}}{6 - j\,2}$$

$$= \frac{12\ \text{A}\ \angle 90°}{6.32\ \angle -18.43°}$$

$$\mathbf{I}' = 1.9\ \text{A}\ \angle 108.43°$$

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V}\ \angle 30°}{6.32\ \Omega\ \angle -18.43°}$$

$$= 3.16\ \text{A}\ \angle 48.43°$$

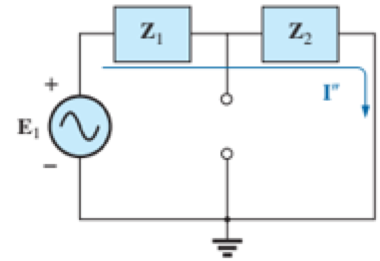
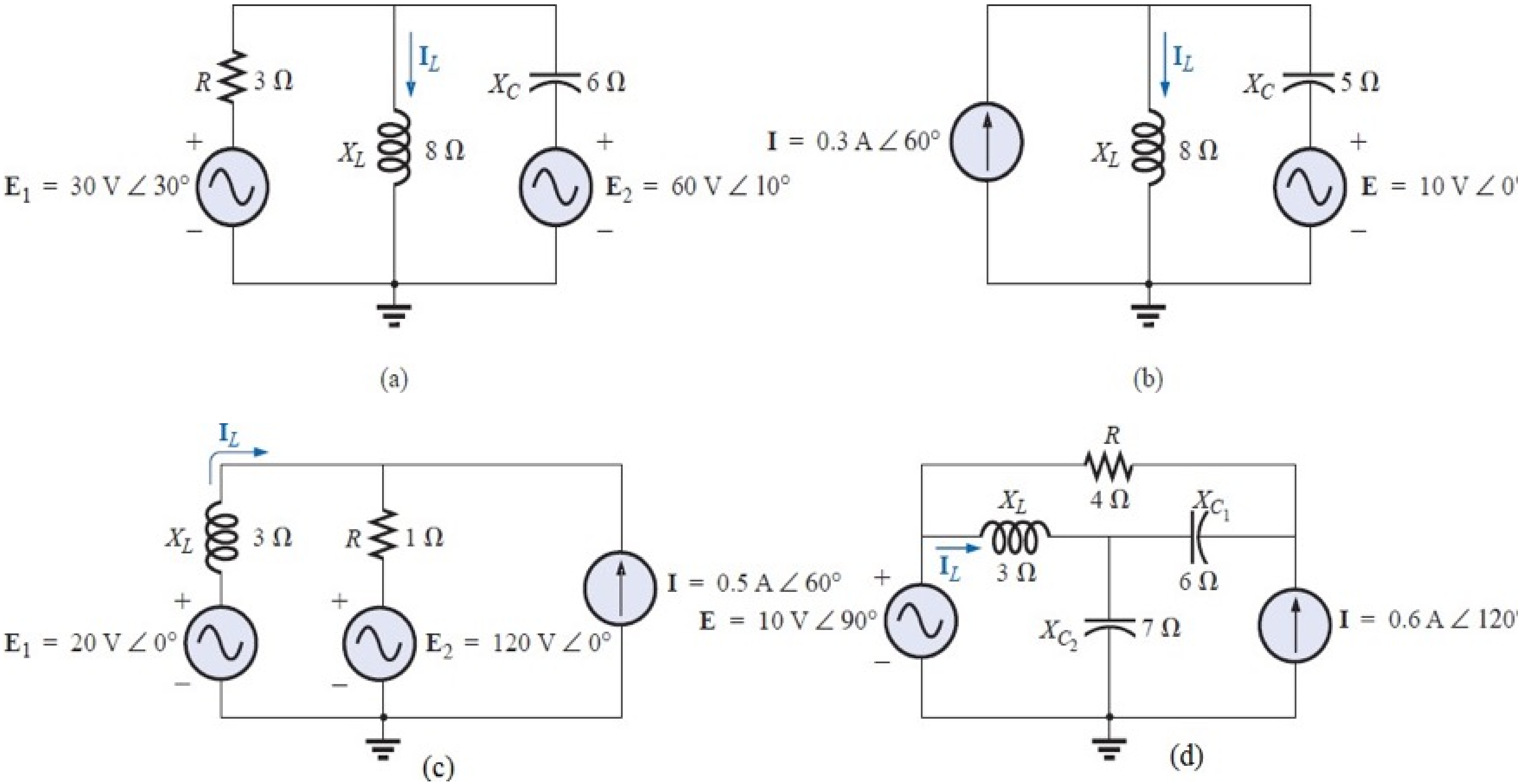


Fig. 18.9

The total current through the 6 Ω resistor (Fig. 18.10) is

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}''$$
$$= 1.9\ \text{A}\ \angle 108.43° + 3.16\ \text{A}\ \angle 48.43°$$
$$= (-0.60\ \text{A} + j\,1.80\ \text{A}) + (2.10\ \text{A} + j\,2.36\ \text{A})$$
$$= 1.50\ \text{A} + j\,4.16\ \text{A}$$
$$\mathbf{I} = \mathbf{4.42\ A\ \angle 70.2°}$$

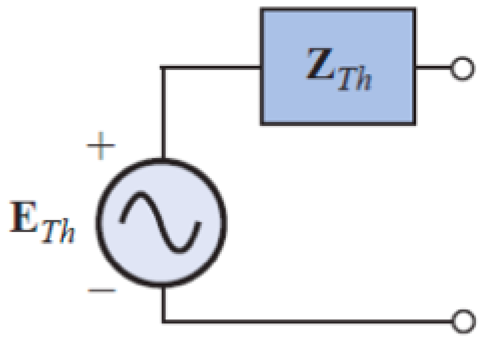
Problem: Using superposition, determine the current I_L for each network as shown in the following figure.

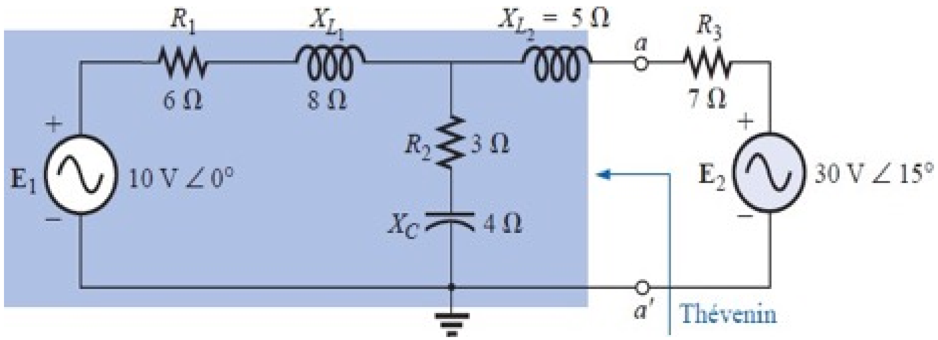


Thevenin’s Theorem

Thévenin’s theorem states the following:

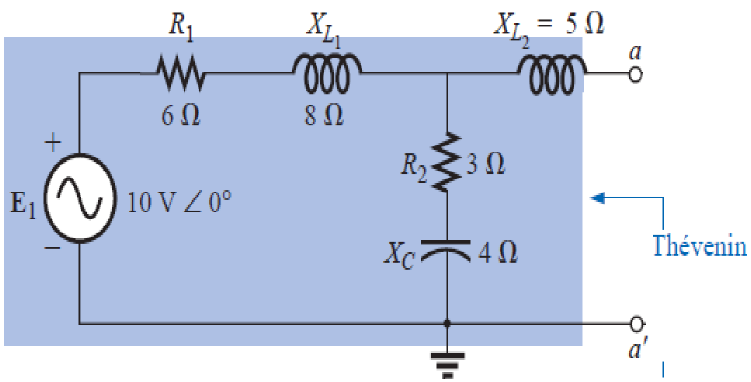
Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a voltage source and a series impedance, as shown in the following figure.



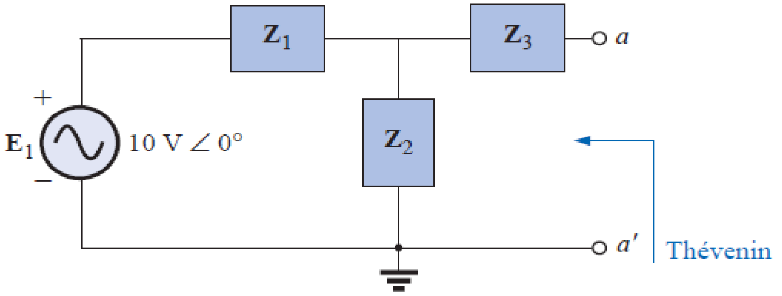


Calculate the Thevenin's Impedance (Z_{th}):

Step 1: Remove that portion of the network across which the Thévenin equivalent circuit is to be found.



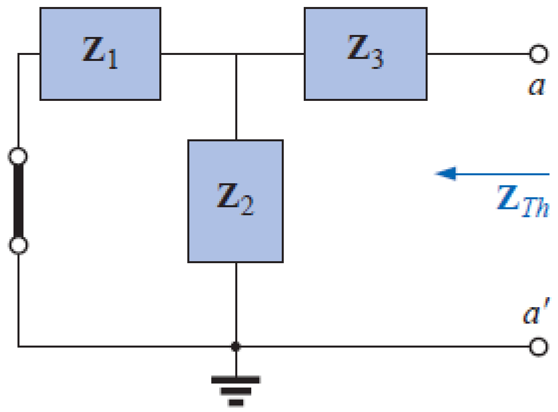
Step 2: Mark the terminals of the remaining two-terminal network.



$$Z_1 = R_1 + jX_{L1} = 6 + j8\ \Omega = 10\angle 53.13^\circ\ \Omega$$

$$Z_2 = R_2 - jX_C = 3 - j4\Omega = 5\angle -53.13^\circ\Omega$$

$$Z_3 = jX_{L2} = j5\ \Omega = 5\angle 90^\circ\ \Omega$$



Step 3: **Set all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits). If the internal impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.**

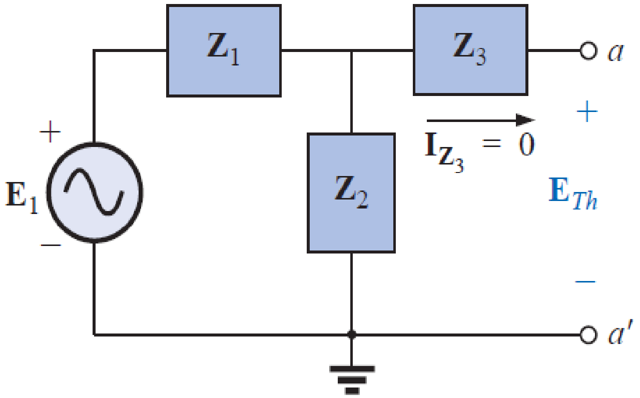
Step 4: **Find the impedance between the two marked terminals.**

$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = j5 + \frac{(6+j8)(3-j4)}{6+j8+3-j4}$$

$$= 4.64 + j2.94\ \Omega = 5.49\angle 32.36^\circ$$

Calculate the Thevenin's Voltage (E_{th}):

Step 1: Remove that portion of the network across which the Thévenin equivalent circuit is to be found.

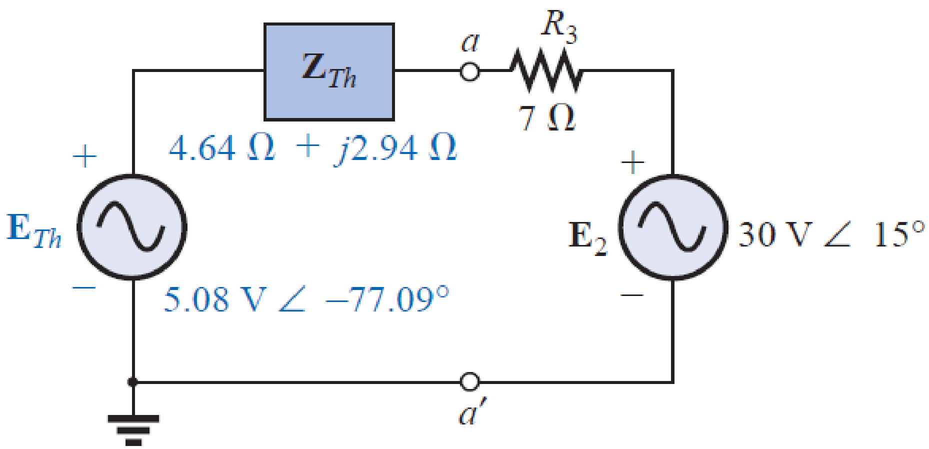


Step 2: Calculate the voltage drop across terminals of the remaining two-terminal network.

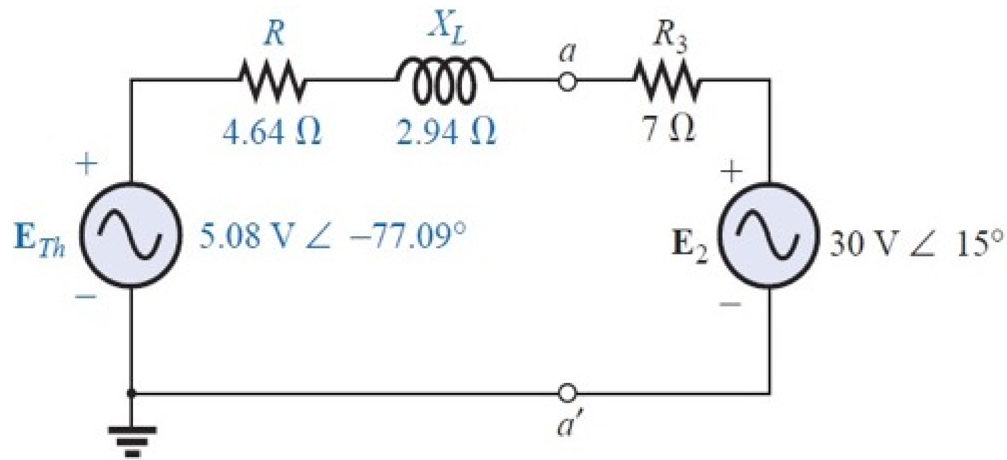
Since a-a' is an open circuit, $\boldsymbol{I}_{Z3} = 0$. Then $\boldsymbol{E}_{Th}$ is the voltage drop across $\boldsymbol{Z}_2$:

$$\boldsymbol{E}_{Th} = \frac{\boldsymbol{Z}_2 \boldsymbol{E}}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} \text{(VDR)} = \frac{(3 - j4)(10 + j0)}{6 + j8 + 3 - j4}$$

$$= 5.08\angle -77.09°$$

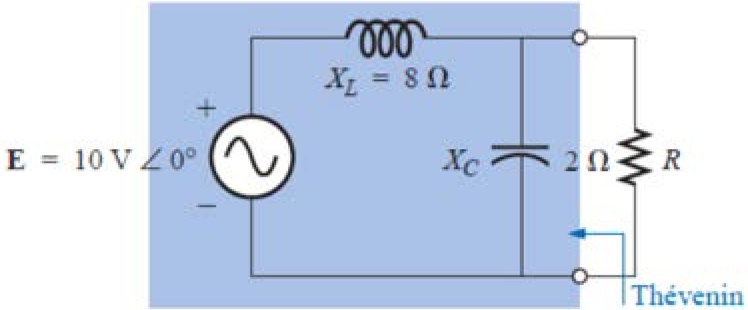


Finally draw the Thevenin's equivalent circuit by connecting the removed part.



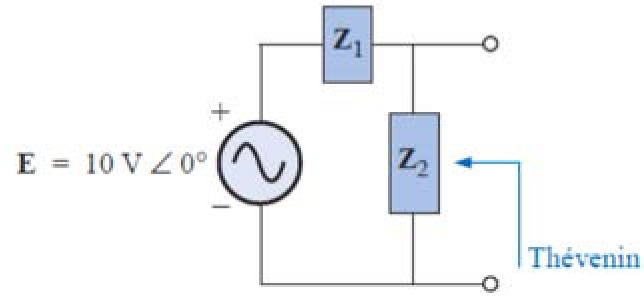
Example

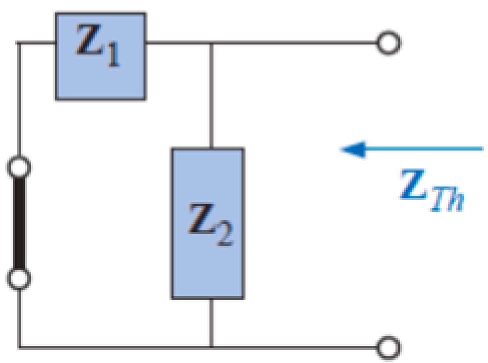
Determine the Thevenin equivalent circuit for the network external to the *R* resistor of following figure.



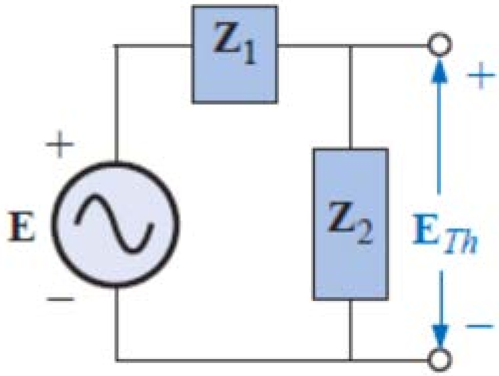
$$Z_1 = j8\ \Omega$$

$$Z_2 = -j2\ \Omega$$



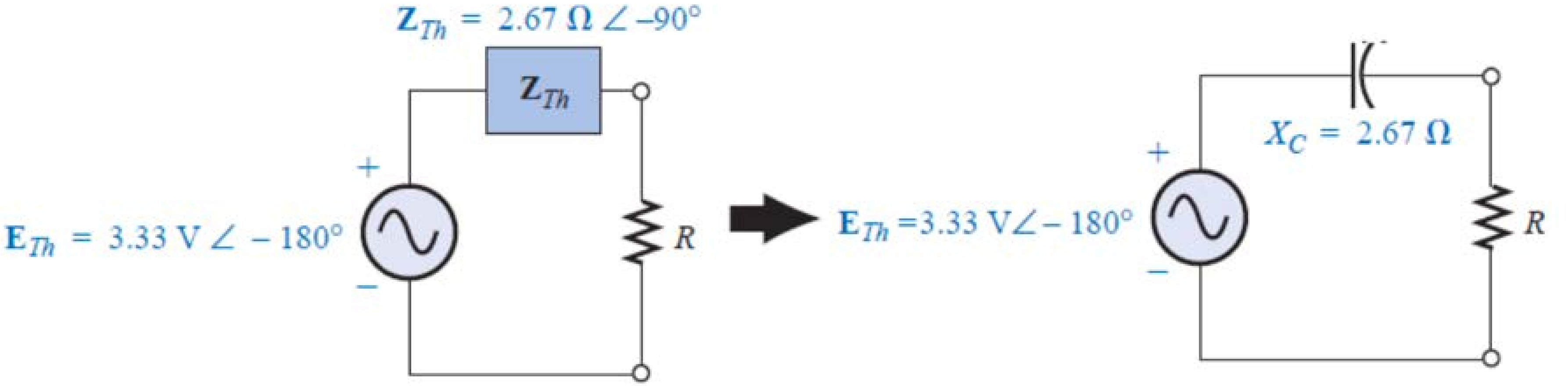


$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = 2.67\angle -90°$$

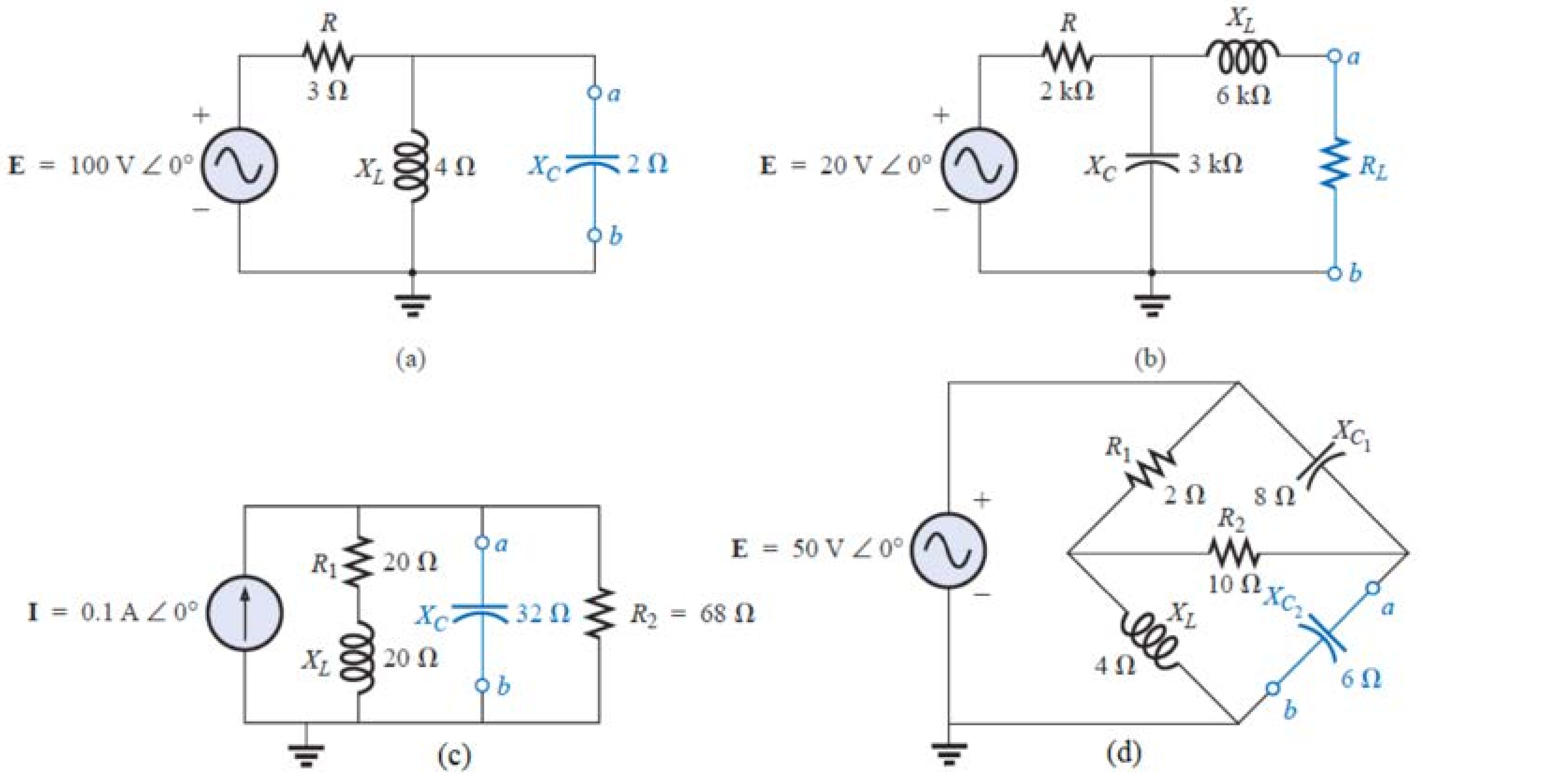


$$E_{Th} = \frac{Z_2 E}{Z_1 + Z_2}\,(\text{VDR}) = 3.33\angle -180°$$

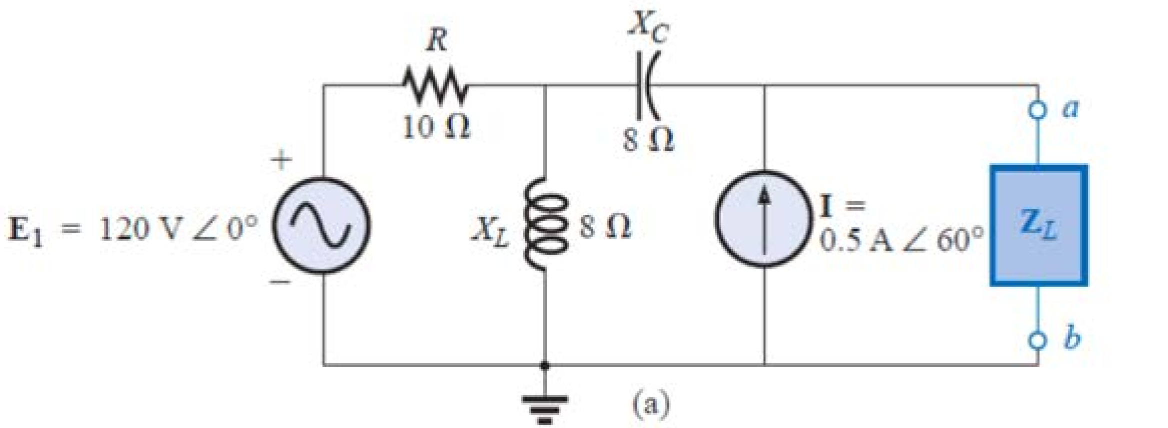
Thevnin equivalent circuit shown in the following figure.



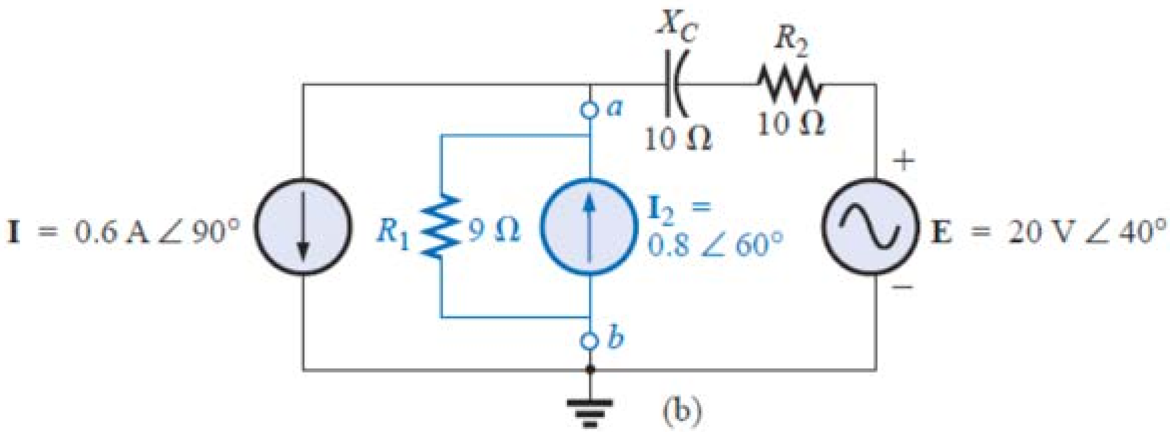
Problem 1: Find the Thévenin equivalent circuit for the portions of the networks of following figures external to the elements between points a and b.



Problem 2: Find the Thévenin equivalent circuit for the portions of the networks of following figures external to the elements between points a and b.



(a)



(b)

Thank You