Introduction to Electrical Circuits

Mid Term
Lecture – 3

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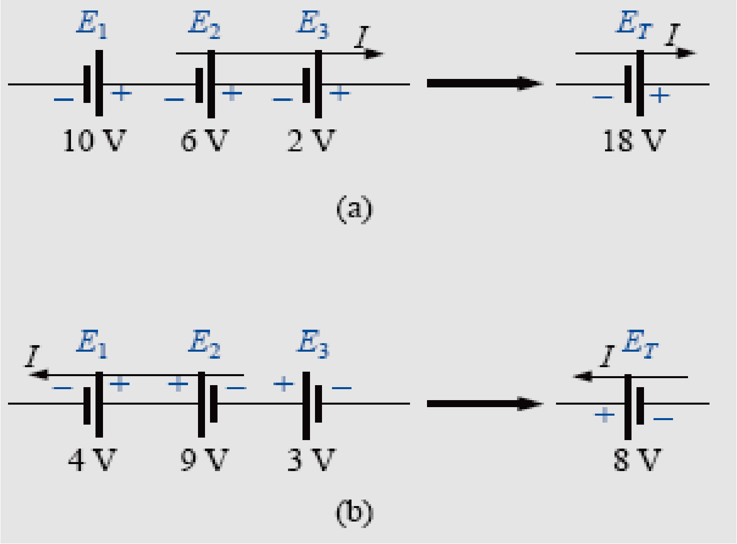
Reference Book:

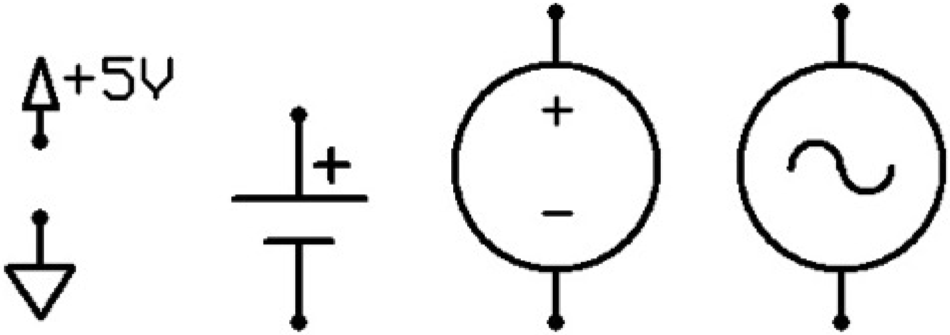
Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W2	MC3	Chapter 5	5.7 VOLTAGE DIVISION IN A SERIES CIRCUIT (Voltage Divider Rule only)	5.15	26
		Chapter 6	6.2 PARALLEL RESISTORS	6.1, 6.2	12, 16, 17.
			6.3 PARALLEL CIRCUITS	6.11	

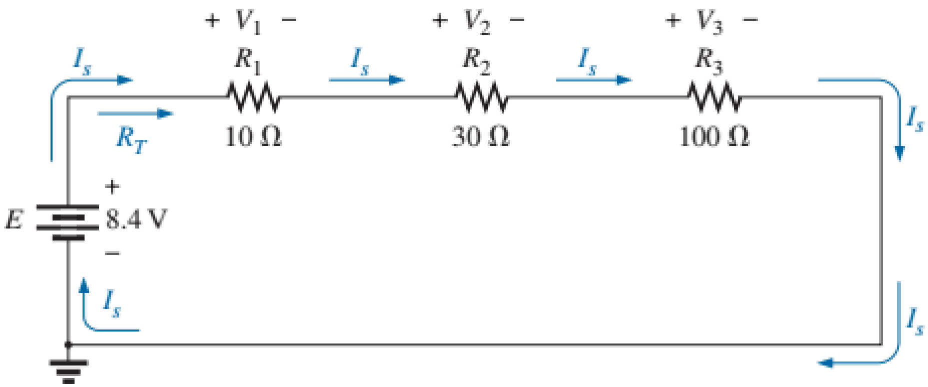
Voltage Sources in Series





Voltage Division in a Series Circuit

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.



Voltage divider approach:

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} \times E$$

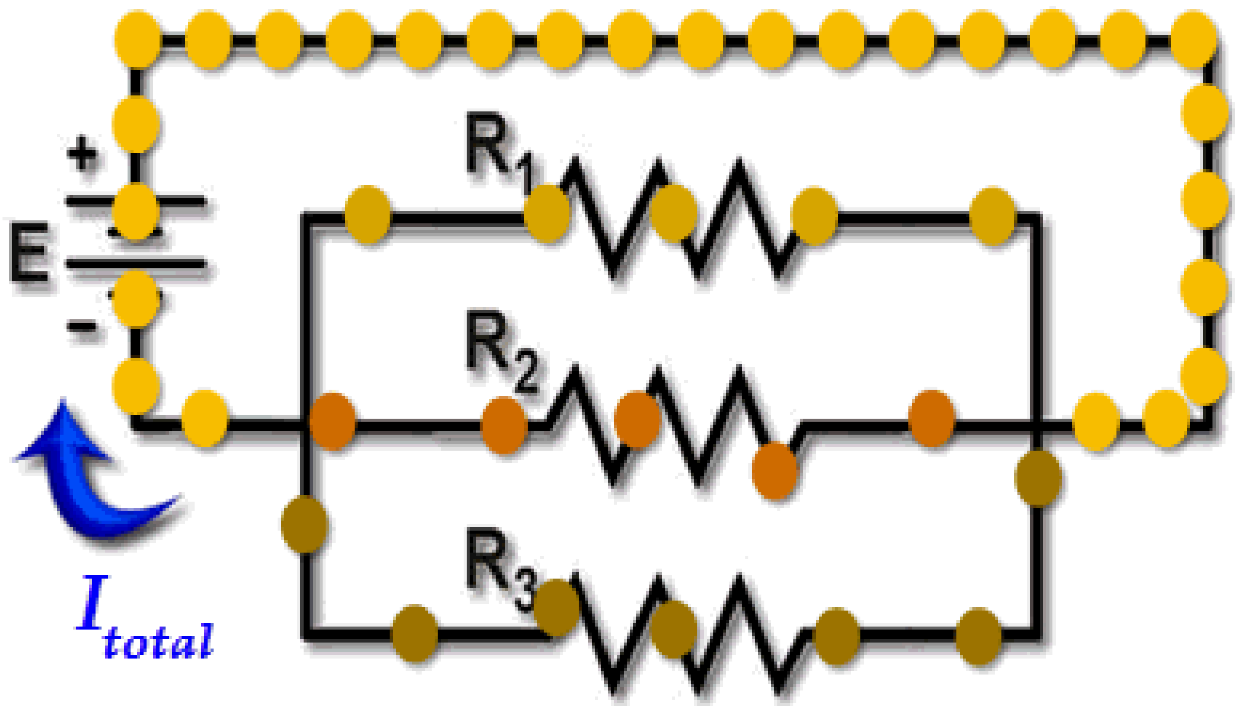
$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} \times E$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} \times E$$

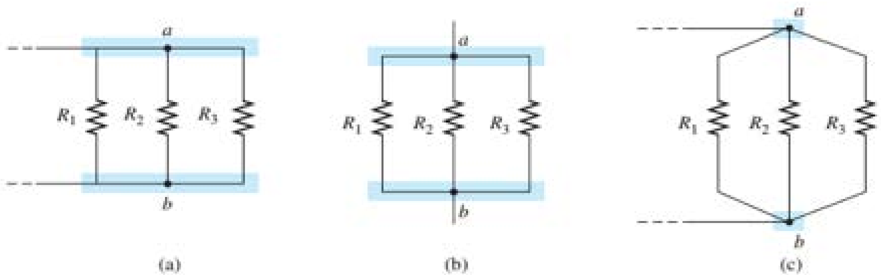
General formula:

$$\boxed{V_x = R_x \frac{E}{R_T}}$$ (voltage divider rule)

Parallel Circuits



Two elements, branches, or circuits are in parallel if they have two points in common. The total resistance of parallel resistors is always less than the value of the smallest resistor. The total resistance of parallel resistors will always drop as new resistors are added in parallel, irrespective of their value.



$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

Since $G = 1/R$,

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \qquad \text{(siemens, S)}$$

In general, however, when the total resistance is desired, the following format is applied:

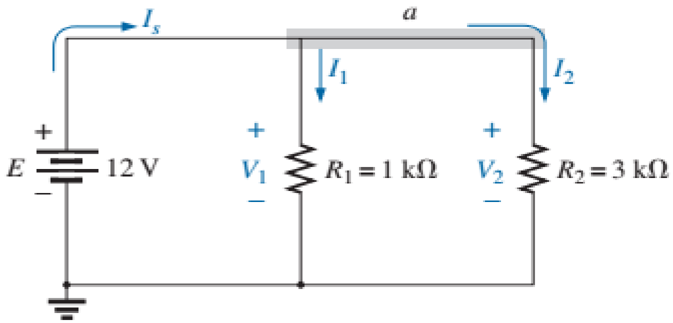
$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \qquad \textbf{(6.3)}$$

Special Case: Two Parallel Resistors

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

6.3 PARALLEL CIRCUITS

(Equations associated to analyze series network)



1. The voltage is always the same across parallel elements.

$$\boxed{V_1 = V_2 = E}$$

2. Total Resistance:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

3. Total Current:

$$\boxed{I_s = \frac{E}{R_T}}$$

4. Branch Currents: <u>**Ohm's Law approach:**</u>

$$\boxed{I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}}$$

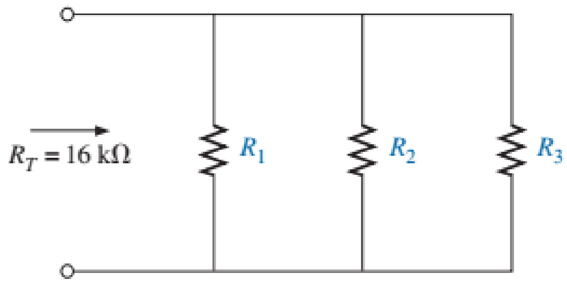


FIG. 6.16
Parallel network for Example 6.11.

EXAMPLE 6.11 Determine the values of R_1, R_2 and R_3 in Fig. 6.16 if $R_2 = 2R_1$, $R_3 = 2R_2$, and the total resistance is 16 kΩ.

Solution: Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $\quad R_2 = 2R_1 \quad \text{and} \quad R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ \text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

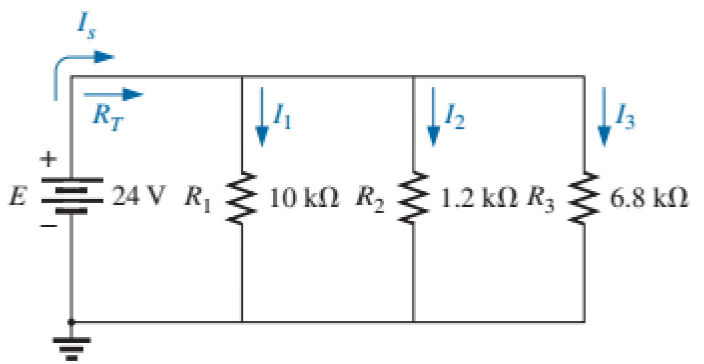
resulting in $\quad R_1 = 1.75(16\ \text{k}\Omega) = \mathbf{28\ k\Omega}$

so that $\quad R_2 = 2R_1 = 2(28\ \text{k}\Omega) = \mathbf{56\ k\Omega}$

and $\quad R_3 = 2R_2 = 2(56\ \text{k}\Omega) = \mathbf{112\ k\Omega}$

Exercise Problems

12. Repeat the analysis of Problem 10 for the network in Fig. 6.83, constructed of standard value resistors.



Solution:

a. $$R_T = \frac{1}{\frac{1}{10\text{ k}\Omega}+\frac{1}{1.2\text{ k}\Omega}+\frac{1}{6.8\text{ k}\Omega}} = \frac{1}{100\times10^{-6}\text{S}+833.333\times10^{-6}\text{S}+147.059\times10^{-6}\text{S}}$$

$$= \frac{1}{1.080\times10^{-3}\text{S}} = \mathbf{925.93\ \Omega}$$

b. $V_{R_1} = V_{R_2} = V_{R_3} = \mathbf{24\ V}$

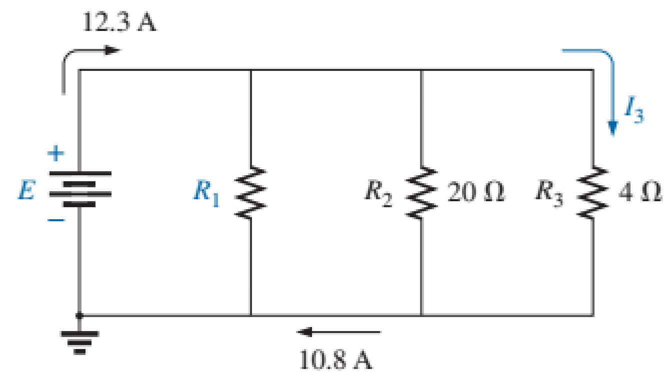
c. $$I_s = \frac{E}{R_T} = \frac{24\text{ V}}{925.93\ \Omega} = \mathbf{25.92\ mA}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{24\text{ V}}{10\text{ k}\Omega} = \mathbf{2.4\ mA},\ I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{24\text{ V}}{1.2\text{ k}\Omega} = \mathbf{20\ mA},$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{24\text{ V}}{6.8\text{ k}\Omega} = \mathbf{3.53\ mA}$$

d. $I_T = \mathbf{25.92\ mA} = 2.4\text{ mA} + 20\text{ mA} + 3.53\text{ mA} = \mathbf{25.93\ mA}$ (checks)

16. Given the information provided in Fig. 6.86, find the unknown quantities: E, R_1, and I_3.



Solution:

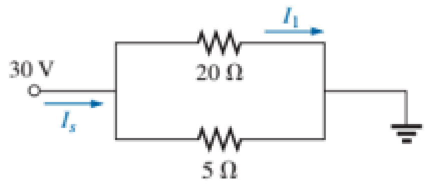
$$I_3 = \frac{(20\ \Omega)(10.8\text{ A})}{20\ \Omega + 4\ \Omega} = \mathbf{9\ A}$$

$$E = V_{R_3} = I_3R_3 = (9\text{ A})(4\ \Omega) = \mathbf{36\ V}$$

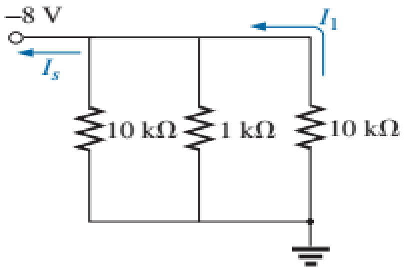
$$I_{R_1} = 12.3\text{ A} - 10.8\text{ A} = 1.5\text{ A}$$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{36\text{ V}}{1.5\text{ A}} = \mathbf{24\ \Omega}$$

17. Determine the currents I_1 and I_s for the networks in Fig. 6.87.



(a)



(b)

Solution:

a. $R_T = 20\ \Omega \parallel 5\ \Omega = 4\ \Omega$

$$I_s = \frac{E}{R_T} = \frac{30\text{ V}}{4\ \Omega} = \mathbf{7.5\ A}$$

$$\text{CDR: } I_1 = \frac{5\ \Omega\, I_s}{5\ \Omega + 20\ \Omega} = \frac{1}{5}(7.5\text{ A}) = \mathbf{1.5\ A}$$

b. $10\text{ k}\Omega \parallel 10\text{ k}\Omega = 5\text{ k}\Omega$

$R_T = 1\text{ k}\Omega \parallel 5\text{ k}\Omega = 0.833\text{ k}\Omega$

$$I_s = \frac{E}{R_T} = \frac{8\text{ V}}{0.833\text{ k}\Omega} = \mathbf{9.6\ mA}$$

$R'_T = 10\text{ k}\Omega \parallel 1\text{ k}\Omega = 0.9091\text{ k}\Omega$

$$I_1 = \frac{R'_T I_s}{R'_T + 10\text{ k}\Omega} = \frac{(0.9091\text{ k}\Omega)(9.6\text{ mA})}{0.9091\text{ k}\Omega + 10\text{ k}\Omega} = \frac{8.727\text{ mA}}{10.9091} = \mathbf{0.8\ mA}$$


END