Introduction to Electrical Circuits

Mid Term
Lecture – 9

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

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10.5 TRANSIENTS IN CAPACITIVE NETWORKS: THE CHARGING PHASE

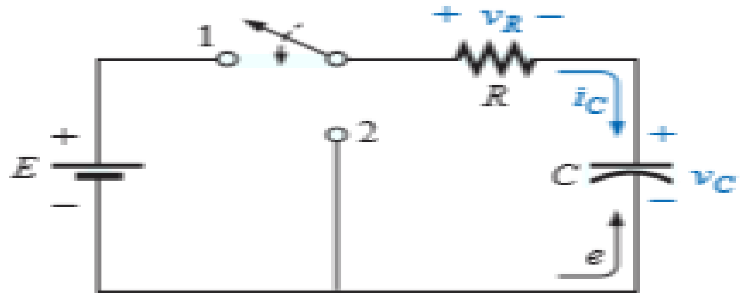
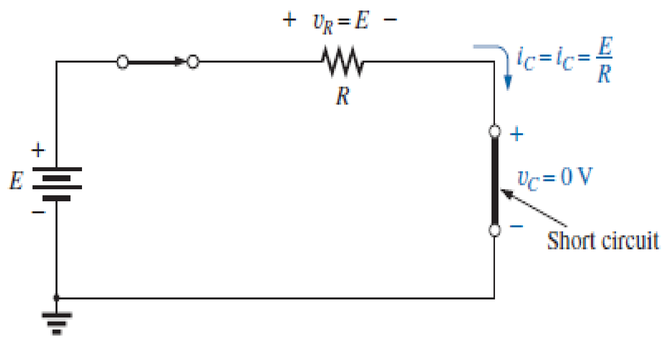
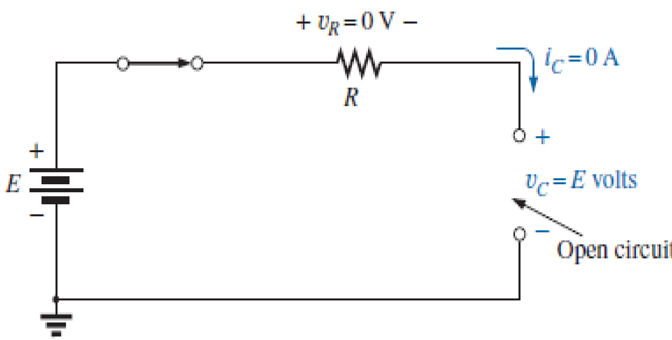


FIG. 10.24
Basic charging network.

a capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.



A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.



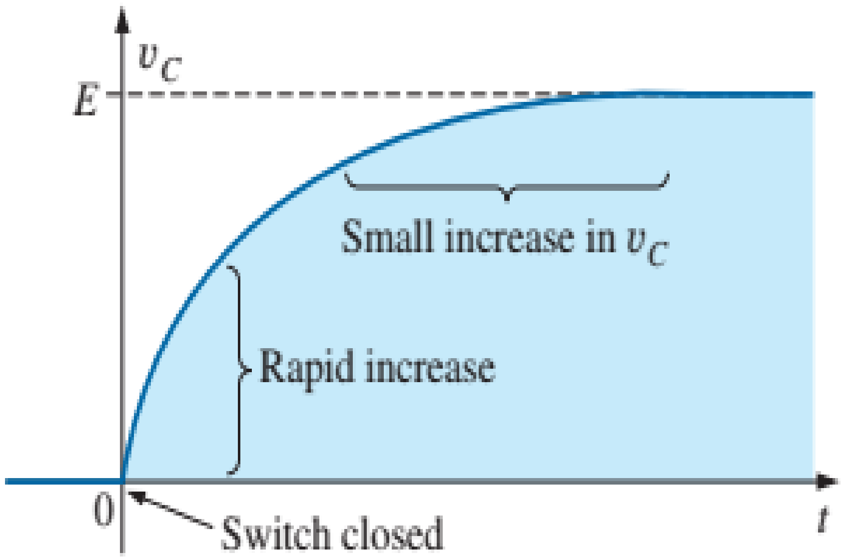
Equations:

$$\boxed{v_C = E(1 - e^{-t/\tau})}_{\text{charging}} \qquad \text{(volts, V)}$$

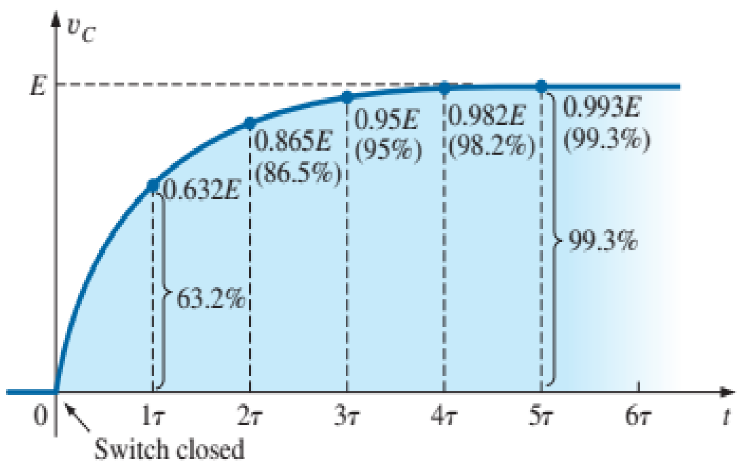
$$\boxed{\tau = RC} \qquad \text{(time, s)} \qquad \textbf{(10.14)}$$

The factor τ, called the **time constant** of the network, has the units of time as shown below using some of the basic equations introduced earlier in this text:

$$\tau = RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{\cancel{V}}{\cancel{Q}/t}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right) = t \text{ (seconds)}$$



v_C *during the charging phase.*



t	$e^{-t/\tau}$	$(1 - e^{-t/\tau})$	$E \times (1 - e^{-t/\tau})$
0	1	0	0
1 τ	0.368	0.632	0.632E
2 τ	0.135	0.865	0.865E
3 τ	0.05	0.95	0.95E
4 τ	0.018	0.98	0.98E
5 τ	0.0067	0.99	0.99E

Equations:

$$\boxed{i_C = \frac{E}{R}e^{-t/\tau}}_{\text{charging}} \qquad \text{(amperes, A)}$$

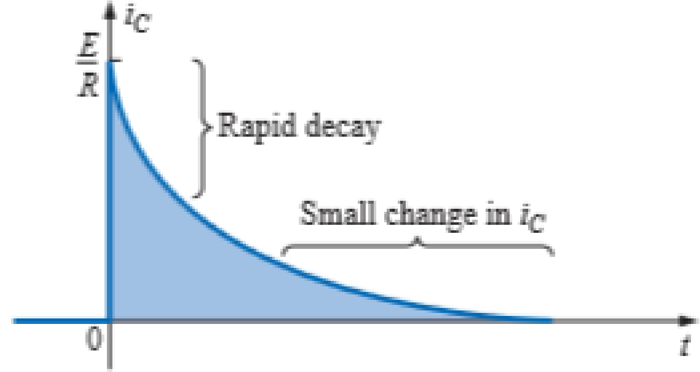
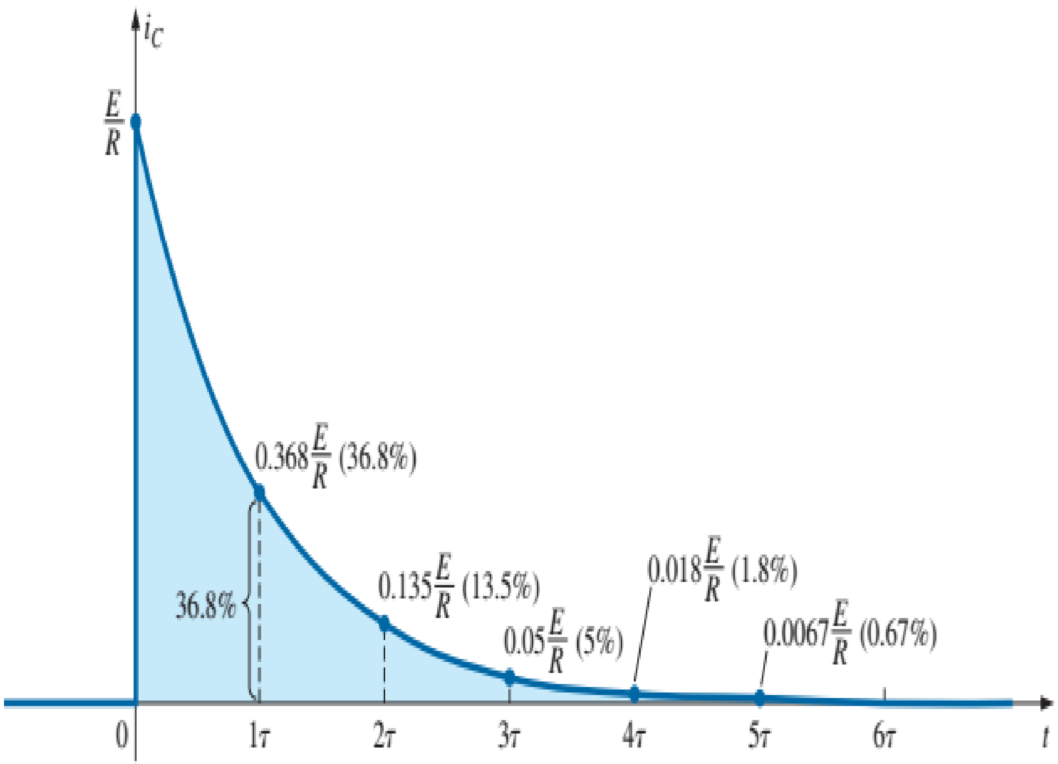


FIG. 10.25
i_C during the charging phase.



t	$e^{-t/\tau}$	$\frac{E}{R} \times e^{-t/\tau}$
0	1	$1\,\frac{E}{R}$
1 τ	0.368	$0.368\frac{E}{R}$
2 τ	0.135	$0.135\frac{E}{R}$
3 τ	0.05	$0.05\frac{E}{R}$
4 τ	0.018	$0.018\frac{E}{R}$
5 τ	0.0067	$0.0067\frac{E}{R}$

EXAMPLE 10.6 After v_C in Example 10.5 has reached its final value of 40 V, the switch is thrown into position 2, as shown in Fig. 10.40. Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R after the closing of the switch. Plot the curves for v_C, i_C, and v_R using the defined directions and polarities of Fig. 10.35. Assume that $t = 0$ when the switch is moved to position 2.

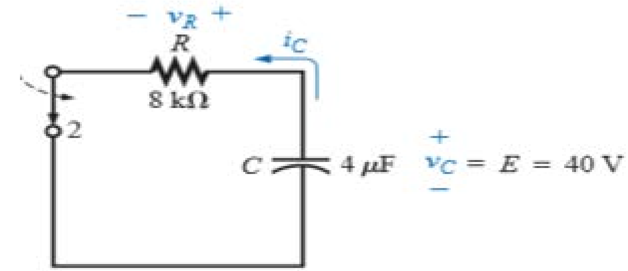


FIG. 10.40
Example 10.6.

Solution:

$$\tau = 32 \text{ ms}$$

By Eq. (10.18),

$$v_C = Ee^{-t/\tau} = \mathbf{40}e^{-t/(32\times10^{-3})}$$

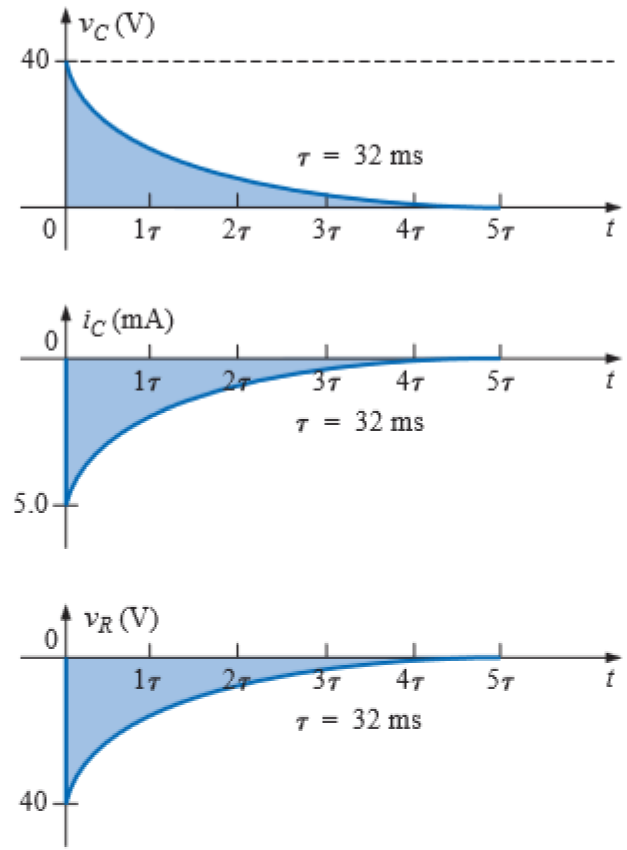
By Eq. (10.19),

$$i_C = -\frac{E}{R}e^{-t/\tau} = \mathbf{-(5 \times 10^{-3})}e^{-t/(32\times10^{-3})}$$

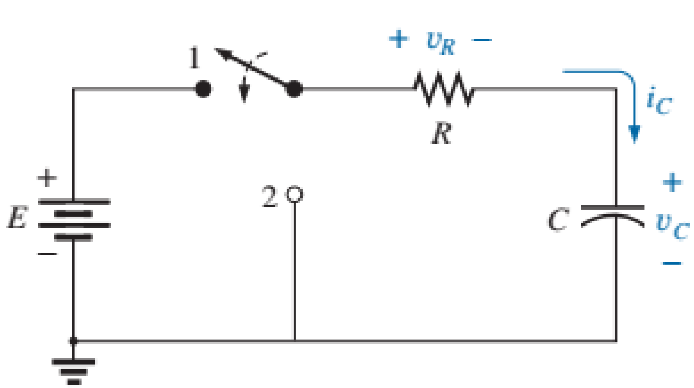
By Eq. (10.20),

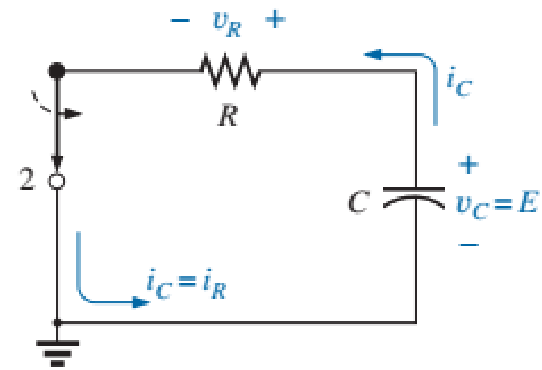
$$v_R = -Ee^{-t/\tau} = \mathbf{-40}e^{-t/(32\times10^{-3})}$$

The curves appear in Fig. 10.41.



10.6 TRANSIENTS IN CAPACITIVE NETWORKS: THE DISCHARGING PHASE

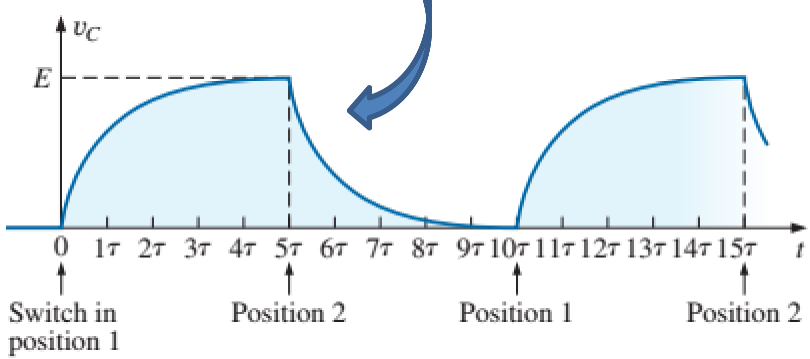


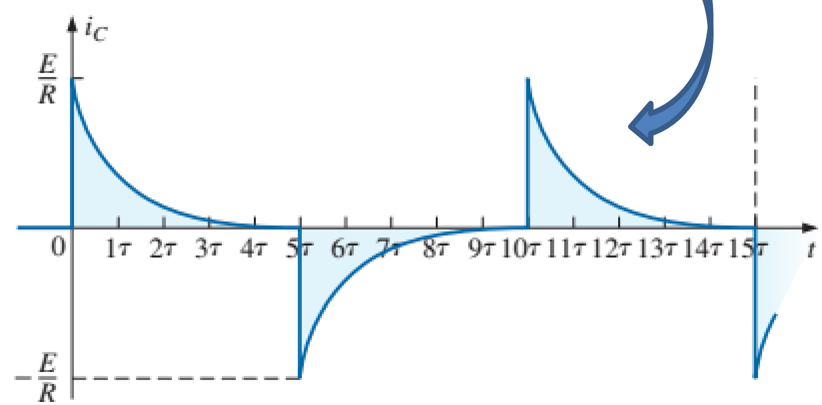


$$\boxed{v_C = Ee^{-t/\tau}}$$ discharging

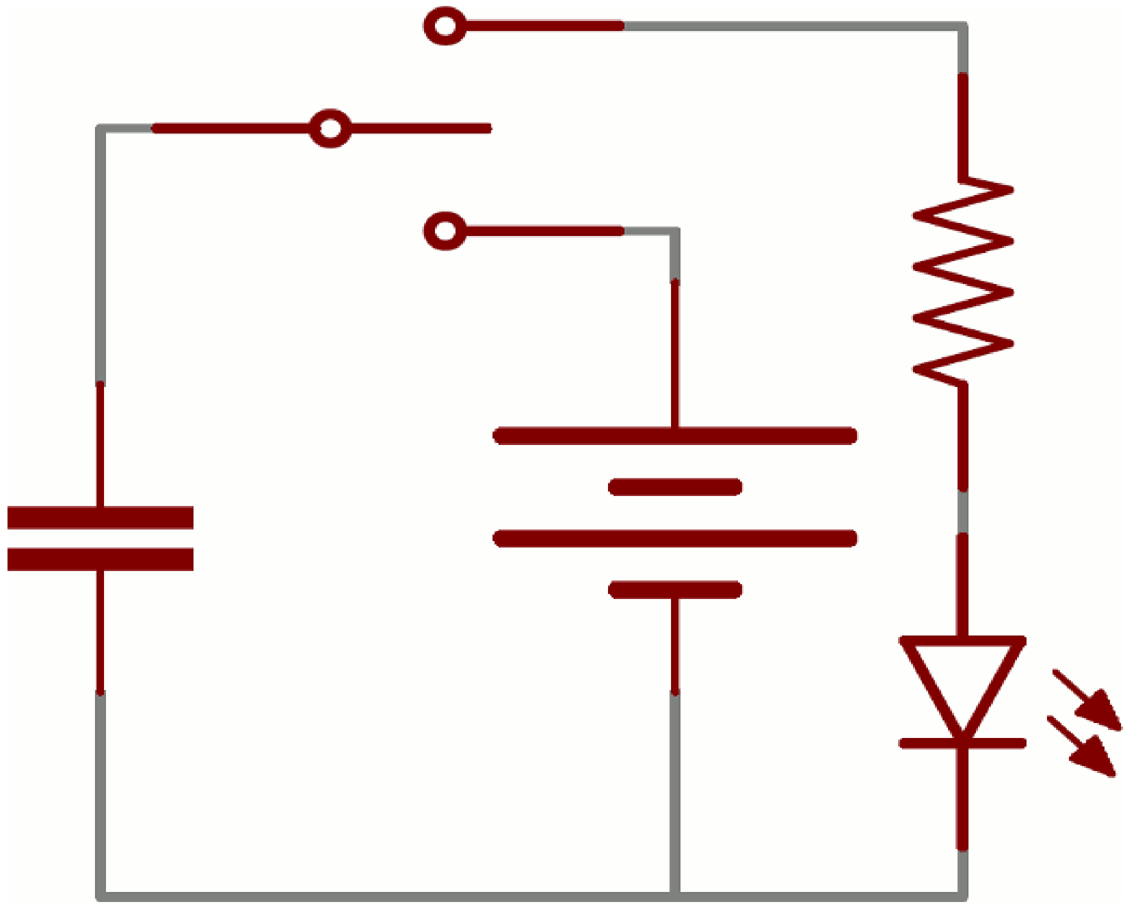
$$\boxed{\tau = RC}$$ discharging

$$\boxed{i_C = \frac{E}{R}e^{-t/\tau}}$$ discharging





Capacitor Charging and Discharging



EXAMPLE 10.7

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor of Fig. 10.42 if the switch is thrown into position 1 at $t = 0$ s.

b. Repeat part (a) for i_C.

c. Find the mathematical expressions for the response of v_C and i_C if the switch is thrown into position 2 at 30 ms (assuming that the leakage resistance of the capacitor is infinite ohms).

d. Find the mathematical expressions for the voltage v_C and current i_C if the switch is thrown into position 3 at $t = 48$ ms.

e. Plot the waveforms obtained in parts (a) through (d) on the same time axis for the voltage v_C and the current i_C using the defined polarity and current direction of Fig. 10.42.

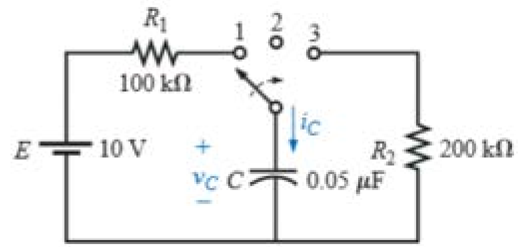


FIG. 10.42
Example 10.7.

Solutions:

a. *Charging phase:*

$$v_C = E(1 - e^{-t/\tau})$$

$$\tau = R_1C = (100 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 5 \times 10^{-3}\ \text{s} = 5\ \text{ms}$$

$$\mathbf{v_C = 10(1 - e^{-t/(5\times10^{-3})})}$$

b. $$i_C = \frac{E}{R_1}e^{-t/\tau}$$

$$= \frac{10\ \text{V}}{100 \times 10^3\ \Omega}e^{-t/(5\times10^{-3})}$$

$$\mathbf{i_C = (0.1 \times 10^{-3})e^{-t/(5\times10^{-3})}}$$

c. *Storage phase:*

$$\mathbf{v_C = E = 10\ V}$$
$$\mathbf{i_C = 0\ A}$$

d. *Discharge phase* (starting at 48 ms with $t = 0$ s for the following equations):

$$v_C = Ee^{-t/\tau'}$$

$$\tau' = R_2C = (200 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 10 \times 10^{-3}\ \text{s} = 10\ \text{ms}$$

$$\mathbf{v_C = 10e^{-t/(10\times10^{-3})}}$$

$$i_C = -\frac{E}{R_2}e^{-t/\tau'}$$

$$= -\frac{10\ \text{V}}{200 \times 10^3\ \Omega}e^{-t/(10\times10^{-3})}$$

$$\mathbf{i_C = -(0.05 \times 10^{-3})e^{-t/(10\times10^{-3})}}$$

e. See Fig. 10.43.

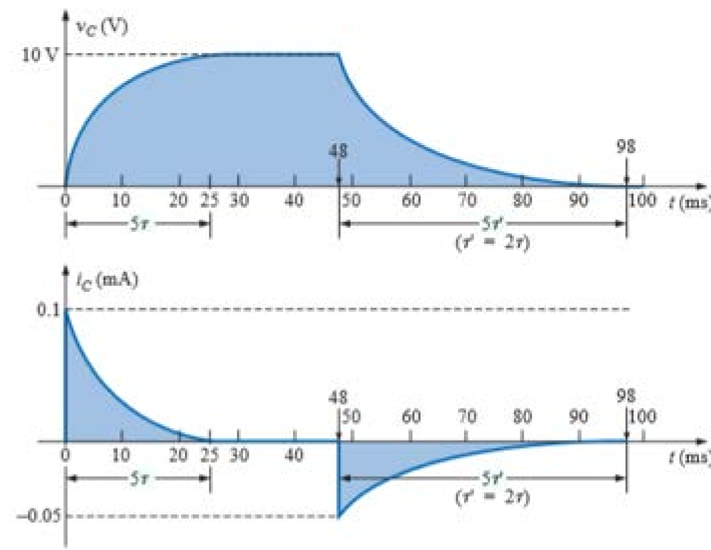


FIG. 10.43
The waveforms for the network of Fig. 10.42.

Exercise Problems

27. For the R-C circuit in Fig. 10.90, composed of standard values:

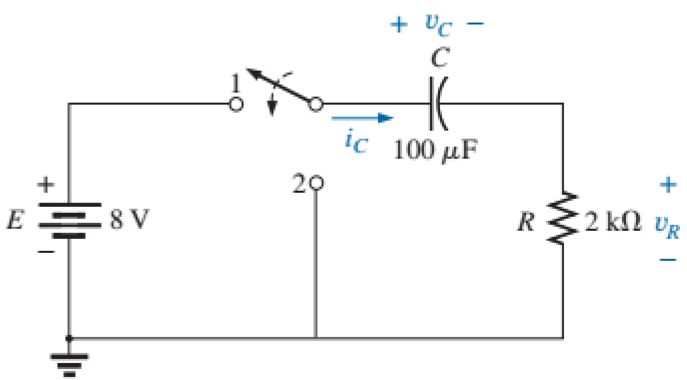
a. Determine the time constant of the circuit when the switch is thrown into position 1.

b. Find the mathematical expression for the voltage across the capacitor and the current after the switch is thrown into position 1.

c. Determine the magnitude of the voltage v_C and the current i_C the instant the switch is thrown into position 2 at $t = 1$ s.

d. Determine the mathematical expression for the voltage v_C and the current i_C for the discharge phase.

e. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.



Solution:

a. $\tau = RC = (2\ \text{k}\Omega)(100\ \mu\text{F}) = \mathbf{200\ ms}$

b. $v_C = E(1 - e^{-t/\tau}) = \mathbf{8\ V(1 - e^{-t/200ms})}$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{8\ \text{V}}{2\ \text{k}\Omega}e^{-t/200\text{ms}} = \mathbf{4\ mAe^{-t/200ms}}$$

c. $v_C(1\ \text{s}) = 8\ \text{V}(1 - e^{-1\text{s}/200\text{ms}}) = 8\ \text{V}(1 - e^{-5})$

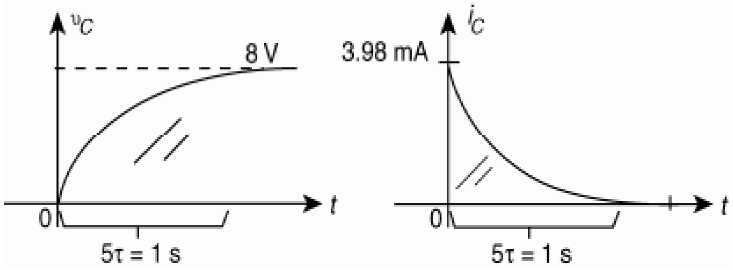
$= 8\ \text{V}(1 - 6.738 \times 10^{-3}) = 8\ \text{V}(0.9933) = \mathbf{7.95\ V}$

$i_C\ (1\ \text{s}) = 4\ \text{mA}e^{-5} = 4\ \text{mA}(6.738 \times 10^{-3}) = \mathbf{26.95\ \mu A}$

d. $v_C = \mathbf{7.95\ Ve^{-t/200ms}}$

$$i_C = \frac{7.95\ \text{V}}{2\ \text{k}\Omega}e^{-t/200\text{ms}} = \mathbf{3.98\ mAe^{-t/200ms}}$$

e.

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