Introduction to Electrical Circuits

Final Term
Lecture - 09

Reference Book:

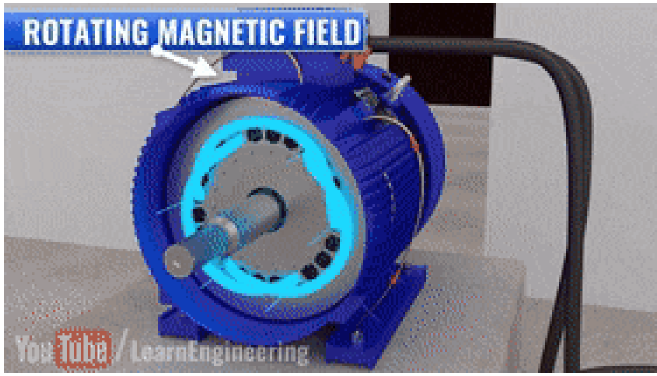
[1] Principles of Electrical Machines -V.K. Mehta, Rohit Mehta

[2] A Textbook of Electrical Technology , Volume- II, - B.L. Theraja, A.K. Theraja

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.
W12	FC9		**Introduction to Induction Motor:** Basic principles of an induction motor, double field and cross field theory, making self-starting technique, difference between capacitor start and capacitor run motor with respect to applications (prob. 6.1, 6.2, 36.3). Draw the circuit and vector diagram.	36.1, 36.2, 36.3.

Induction motor: Basic Principle

- The ***Induction Motor***, the most versatile of the AC motors, has truly emerged as the prime mover in industrial applications.
- In A.C. motors, the rotor does receive electric power by ***induction*** in the same way as the secondary of a 2-winding transformer receives its power from the primary.
- They can be treated as ***rotating transformer*** i.e. one in which the primary winding is stationary, but the secondary is free to rotate.

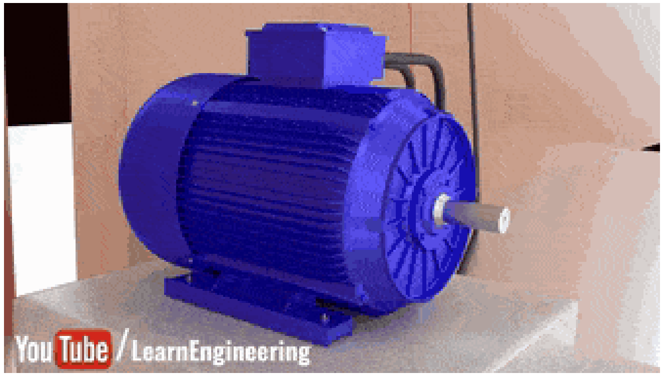


An induction motor has two main parts -

(i) Stator - The stationary section that contain the windings (magnetic field).

(ii) Rotor - The rotating section that contains the conductors.

The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.



Single-Phase Induction Motor

A single-phase induction motor is very similar to a 3-phase squirrel cage induction motor. It has -

(i) a squirrel-cage rotor identical to a 3-phase motor and

(ii) a single-phase winding on the stator.

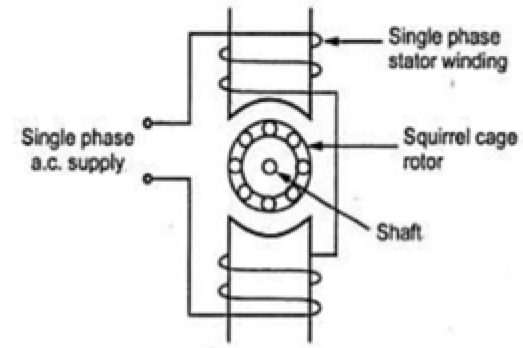


Fig: Single-phase induction motor

- A single-phase induction motor is not ***self-starting***, it does not inherently develop any starting torque.
- However, if the rotor is started by auxiliary means, it will continue to run in the direction of rotation and quickly accelerates until it reaches a speed slightly below the synchronous speed.
- This strange behavior of the single-phase motor can be explained in two ways:

(i) by double-field revolving theory, and

(ii) by cross-field theory.

Double-Field Revolving Theory

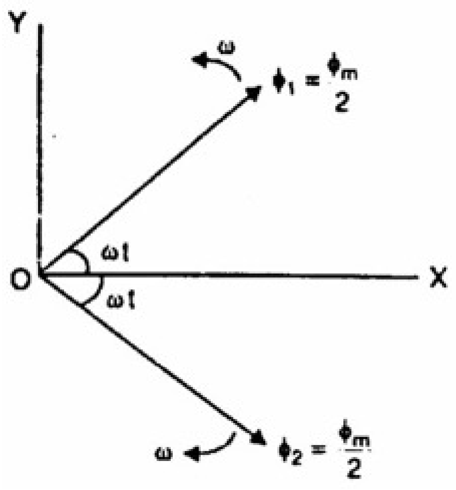
This theory based on the fact that the **alternating sinusoidal flux** produced by the stator winding can be represented by **two revolving fluxes**.

Each equal to one-half of the maximum value of alternating flux (i.e., Φ_m / 2) and each rotating at synchronous speed (N_s = 120 f / P, $\omega = 2\pi f$) in opposite directions.

The instantaneous value of flux due to the stator current of a single-phase induction motor is given by;

$$\Phi = \Phi_m \cos \omega t$$

Consider two rotating magnetic fluxes, each be equal to $\Phi_m/2$ start revolving from OX axis at t = 0 in anti-clockwise and clockwise directions respectively, with angular velocity ω.



Double-Field Revolving Theory

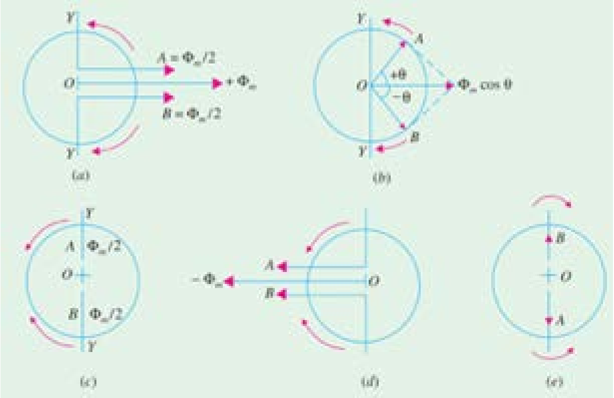


Fig. 1

- After time t seconds, the angle through which the flux vectors have rotated is $\boldsymbol{\theta = \omega t}$. Resolving the flux vectors along-X-axis and Y-axis, we have,

$$\text{Total X-component} = \frac{\phi_m}{2}\cos\omega t + \frac{\phi_m}{2}\cos\omega t = \phi_m \cos\omega t$$

$$\text{Total Y-component} = \frac{\phi_m}{2}\sin\omega t - \frac{\phi_m}{2}\sin\omega t = 0$$

$$\text{Resultant flux, } \phi = \sqrt{(\phi_m \cos\omega t)^2 + 0^2} = \phi_m \cos\omega t$$

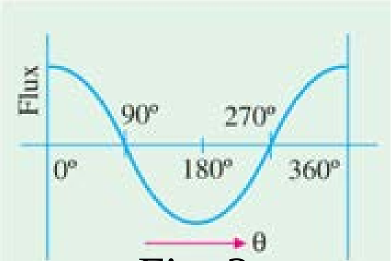


Fig. 2

- When the rotating flux vectors are in phase the resultant vector is $\phi = \phi_m$; when out of phase by 180°, the resultant vector $\phi = 0$ [Fig.-2].
- At standstill, two torques produced by opposite revolving fluxes are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting [Fig.-3].
- If the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor [Fig.-3].

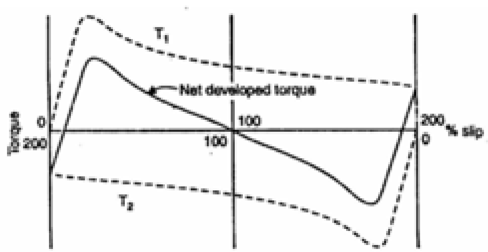


Fig. 3

Cross-Field Theory

- The quadrature pulsating rotor field reacts against the pulsating main field to produce a resultant magnetic field. The resultant magnetic field is a fairly constant rotating magnetic field that rotates in the same direction as the direction of the rotation of the rotor.

- A squirrel- cage induction motor will continue to rotate, producing induction motor torque in a rotating magnetic field, once a rotational emf has been initiated.

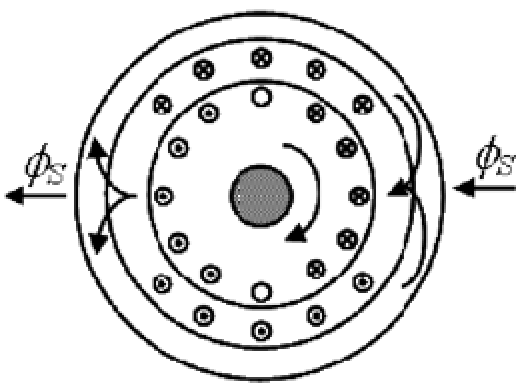


Fig: Stator field Φ_s sets up flux along horizontal axis; rotor rotating in clockwise direction

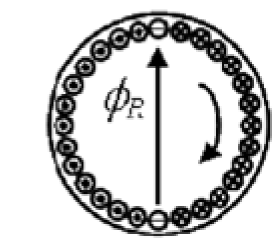


Fig: Cross field when stator field is zero.

Making Single-Phase Induction Motor Self-Starting

- To make a single-phase induction motor self-starting, phase splitting is done by temporarily converting a two-phase motor during the starting period.
- The stator is provided with an extra ***starting (auxiliary)*** winding, in addition to the ***main (running)*** winding.
- The two windings are spaced ***90° electrically apart*** and are connected in parallel across the single-phase supply.
- When the motor attains sufficient speed, the starting (auxiliary) winding may be removed depending upon the type of the motor.
- There are many methods by which the necessary phase-difference between the two currents can be created.
  - **(i) Split-phase motors** - started by two phase motor action through the use of an auxiliary or starting winding.
  - **(ii) Capacitor motors** - started by two-phase motor action through the use of an auxiliary winding and a capacitor.
  - **(iii)Shaded-pole motors** - started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

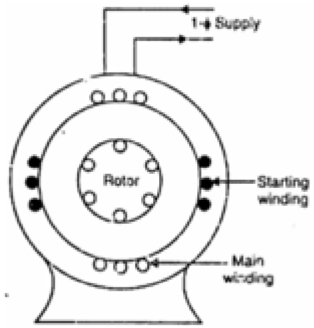
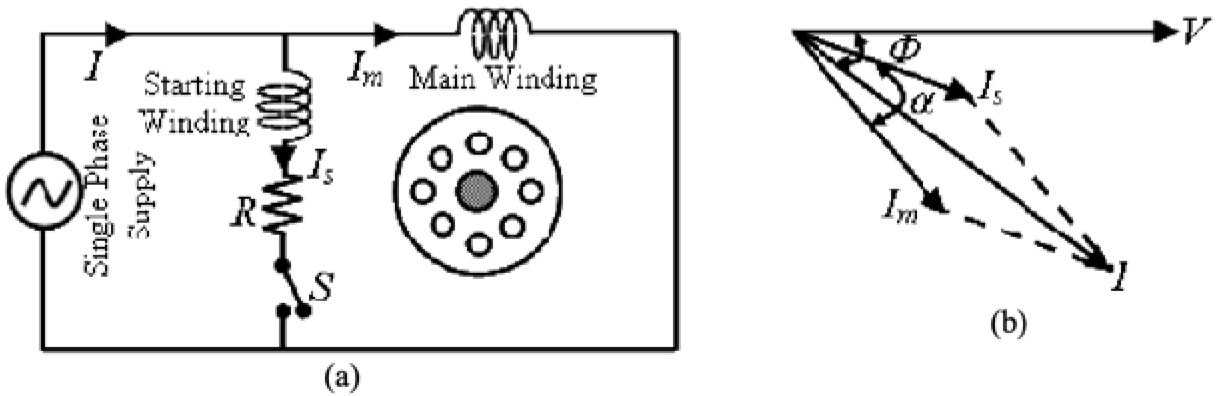


Fig: Auxiliary winding in stator to make 1-Φ motor self starting

Split-Phase Induction Motors

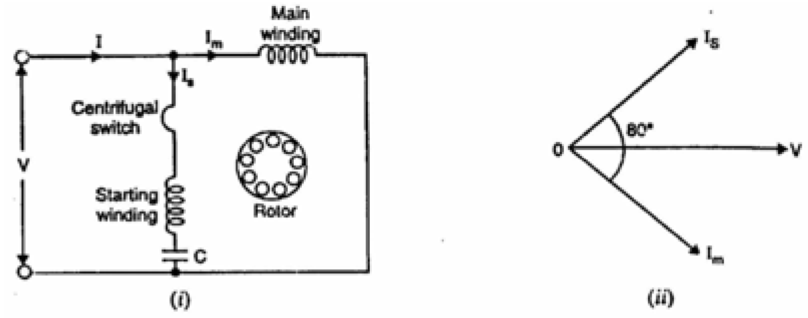
The single-phase induction motor is equipped with an auxiliary winding in addition to the main winding. The starting winding is connected in parallel with the main running winding.



- Due to their low cost, split-phase induction motors are most popular single- phase motors in the market.
- An important characteristic of these motors is that they are essentially constant-speed motors. The speed variation is 2-5% from no-load to full- load.
- These motors are suitable where a ***moderate starting torque*** is required and where starting periods are infrequent e.g., to drive: (a) fans (b) washing machines (c) oil burners (d) small machine tools etc.
- The power rating of such motors generally lies between **60 W and 250 W.**

Capacitor-Start Motor

- The capacitor-start motor is identical to a split-phase motor except that the starting winding has as many turns as the main winding.
- Moreover, a capacitor C is connected in series with the starting winding.



- Although starting characteristics of a capacitor-start motor are better than those of a split-phase motor, both machines possess the same running characteristics because the main windings are identical.
- The phase angle between the two currents is about 80° compared to about 25° in a split-phase motor.
- The starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.
- Capacitor-start motors are used where ***high starting torque*** is required and where the starting period may be long e.g., to drive: (a) compressors (b) large fans (c) pumps (d) high inertia loads
- The power rating of such motors lies between **120 W and 7-5 kW**.

Difference between capacitor start and capacitor run motor

- Capacitor-run motor is identical to a capacitor-start motor except that starting (auxiliary) winding is not opened after starting so that both the windings remain connected to the supply when running as well as at starting.

- In Capacitor-run motor, a single capacitor C is used for both starting and running. This design eliminates the need of a centrifugal switch and at the same time improves the power factor and efficiency of the motor.

- In the Capacitor-start motor, two capacitors C_1 and C_2 are used in the starting winding. The smaller capacitor C_1 required for optimum running conditions is permanently connected in series with the starting winding. The much larger capacitor C_2 is connected in parallel with C_1 for optimum starting and remains in the circuit during starting. The starting capacitor C_2 is disconnected when the motor approaches about 75% of synchronous speed. The motor then runs as a single-phase induction motor.

- Because of the capacitor-run motor produces a constant torque, it is vibration free and can be used in: (a) hospitals (b) studios and (c) other places where silence is important.

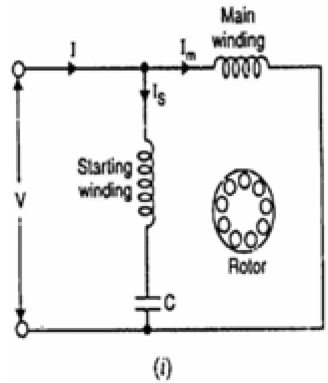


Fig: Capacitor Run Motor

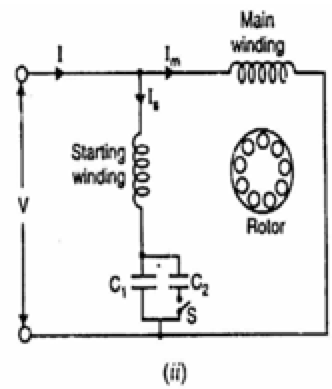
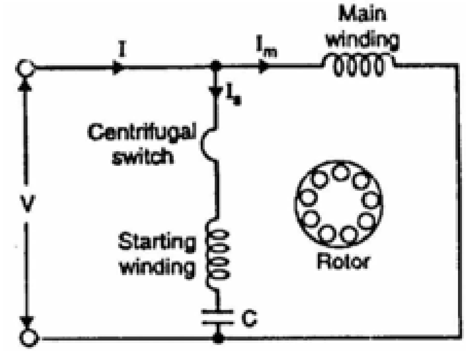


Fig: Capacitor Start Motor

Example Problems

Problem [6.1]: The impedance of the main and auxiliary windings of a 50 Hz single-phase induction motor are $Z_m = (3+j3)\ \Omega$ and $Z_a = (6+j3)\ \Omega$ respectively. What will be the value of the capacitor to be connected in series with auxiliary winding to achieve a phase difference of 90^o between the currents of the two windings?

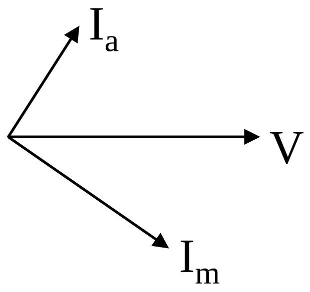


$$I_m = \frac{V\angle 0}{3+j3} = \frac{V\angle 0}{4.24\angle 45°} = \frac{V\angle -45°}{4.24} \qquad I_a = \frac{V\angle 0}{6+j3} = \frac{V\angle 0}{6.7\angle 26.5°} = \frac{V\angle -26.5°}{6.7}$$

The current flowing through the auxiliary winding after connecting a capacitor C in series should make an angle 90^o with I_m or make an angle 90^o-45^o = 45^o with the applied voltage V.
Since the new current of auxiliary winding should be leading the voltage V by an angle of 45^o, the capacitive reactance of the auxiliary circuit is greater than the inductive reactance. Thus

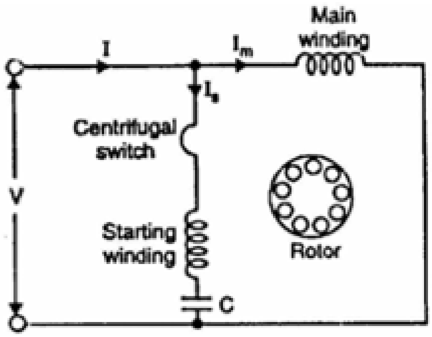
$$\tan 45° = \frac{X_C - X_L}{R} \qquad 1 = \frac{(1/\omega C) - 3}{6} \qquad (1/\omega C) - 3 = 6 \qquad (1/\omega C) = 9$$

$$\omega C = 1/9; \qquad C = 1/9\omega \qquad C = 353.6\ \mu F$$



Example Problems

Problem [6.2]: A 50 Hz Capacitor-Start induction motor has a resistance 5 Ω and an inductive reactance 20 Ω in both main and auxiliary winding. Determine the value of resistance and capacitance to be added in series with auxiliary winding to send the same current in each winding with a phase difference of 90°.



$$Z = 5 + j20 = 20.6\angle 76^\circ$$

$$I_m = I_a = \frac{V\angle 0}{5 + j20} = \frac{V\angle 0}{20.6\angle 76^\circ} = \frac{V\angle -76^\circ}{20.6}$$

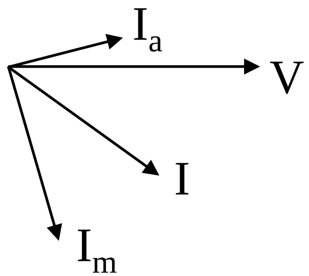
The current flowing through the auxiliary winding after connecting resistor R and a capacitor C in series should make an angle 90° with I_m or make an angle 90°-76° =14° with the applied voltage V. Since the new current of auxiliary winding should be leading the voltage V by an angle of 14°, the capacitive reactance of the auxiliary circuit is greater than the inductive reactance. Thus

$$\cos 14^\circ = \frac{5 + R}{Z}; \qquad 5 + R = Z\cos 14^\circ; \qquad R = 20.6\cos 14^\circ - 5 = 19.99 - 5 = 14.99\ \Omega$$

$$\text{Again } \sin 14^\circ = \frac{X_C - X_L}{Z}; \qquad X_C - X_L = Z\sin 14^\circ; \qquad X_C = 20.6\sin 14^\circ + X_L$$

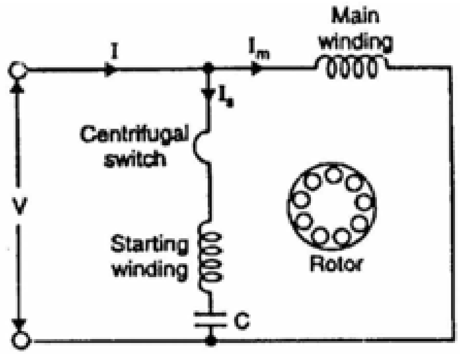
$$X_C = 20.6\sin 14^\circ + 20 = 24.98\ \Omega$$

$$(1/\omega C) = 24.98 \qquad C = 127\ \mu F$$



Example Problems

Example 36.3. A 250 W, 230 V, 50 Hz capacitor-start motor has the following constants for the main and auxiliary winding: Main winding, Z_m=(4.5+j3.7) ohm, Auxiliary winding, Z_a=(9.5+j3.5) ohm. Determine the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.



Solution: Let X_C be the reactance of the capacitor connected in the auxiliary winding.

Then Z_a=9.5+j(3.5-X_C)=(9.5-jX) ohm

where, X is the net reactance

Now, Z_m=(4.5+j3.7)=5.82∠39.4° ohm

Obviously, I_m lags behind V by 39.4°.

Since time phase angle between I_m and I_a has to be 90°,

I_a must lead V by (39.4°- 90°)=-50.6°.

For auxiliary winding,

$\tan\phi_a=(3.5-X_C)/R$

or $\tan(-50.6^\circ)=(3.5-X_C)/9.5=-1.217$

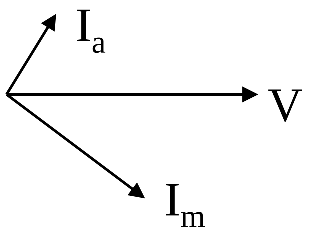
Or $(3.5-X_C)=-9.5\times1.217=-11.56$ ohm

$\therefore X_C=11.56+3.5=15.06$ ohm

Or $1/(2\times\pi\times50\times C)=15.06$

or $C=1/(2\times\pi\times50\times15.06)=211\times10^{-06}$ F

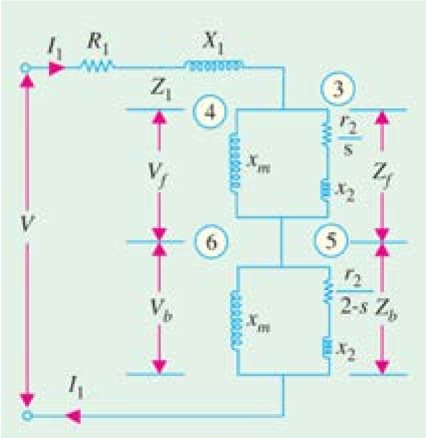
$\therefore C=211$ μF.



Example Problems

Example 36.1. Find the mechanical power output at a slip of 0.05 of the 185-W, 4-pole, 110-V, 60-Hz single-phase induction motor, whose constants are given below:

Resistance of the stator main winding	$R_1 = 1.86$ ohm
Reactance of the stator main winding	$X_1 = 2.56$ ohm
Magnetizing reactance of the stator main winding	$X_m = 53.5$ ohm
Rotor resistance at standstill	$R_2 = 3.56$ ohm
Rotor reactance at standstill	$X_2 = 2.56$ ohm



Solution. Here, $X_m = 53.5\ \Omega$, hence $x_m = 53.5/2 = 26.7\ \Omega$

Similarly, $r_2 = R_2/2 = 3.56/2 = 1.78\ \Omega$ and $x_2 = X_2/2 = 2.56/2 = 1.28\ \Omega$

$$\therefore \quad Z_f = \frac{j x_m \left(\frac{r_2}{s} + j x_2\right)}{\frac{r_2}{s} + j(x_2 + x_m)} = x_m \frac{\frac{r_2}{s} . x_m + j\left[(r_2/s)^2 + x_2 x_0\right]}{(r_2/s)^2 + x_0^2} \quad \text{where } x_0 = (x_m + x_2)$$

$$\therefore \quad Z_f = 26.7 \frac{(1.78/0.05) \times 26.7 + j[(1.78/0.05)^2 + 1.28 \times 27.98]}{(1.78/0.05)^2 + (27.98)^2}$$

$$= 12.4 + j\,17.15 = 21.15 \angle 54.2°$$

$$\text{Similarly,} \quad Z_b = \frac{j x_m \left(\frac{r_2}{2-s} + j x_2\right)}{\frac{r_2}{2-s} + j(x_2 + x_m)} = x_m \frac{\left(\frac{r_2}{2-s}\right) x_m + j\left[\left(\frac{r_2}{2-s}\right)^2 + x_0 x_2\right]}{\left(\frac{r_2}{2-s}\right)^2 + x_0^2}$$

$$= 26.7 \frac{(1.78/1.95) \times 26.7 + j[(1.78/1.95)^2 + 1.28 \times 27.98]}{(1.78/1.95)^2 + (27.98)^2}$$

$$= 0.84 + j\,1.26 = 1.51 \angle 56.3°$$

$$\mathbf{Z_1} = R_1 + j X_1 = 1.86 + j\,2.56 = 3.16 \angle 54°$$

Total circuit impedance is

$$\mathbf{Z_{01}} = \mathbf{Z_1} + \mathbf{Z_f} + \mathbf{Z_b} = (1.86 + j\,2.56) + (12.4 + j\,17.15) + (0.84 + j\,1.26)$$

$$= 15.1 + j\,20.97 = 25.85 \angle 54.3°$$

Motor current $I_1 = 110/25.85 = 4.27$ A

$$V_f = I_1 Z_f = 4.27 \times 21.15 = 90.4 \text{ V}; \; V_b = I_1 Z_b = 4.27 \times 1.51 = 6.44 \text{ V}$$

$$Z_3 = \sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2} = 35.7\ \Omega, \; Z_5 = \sqrt{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} = 1.57\ \Omega$$

$$I_3 = V_f / Z_3 = 90.4/35.7 = 2.53 \text{ A}, \; I_5 = V_b / Z_5 = 6.44/1.57 = 4.1 \text{ A}$$

$$T_f = I_3^2 R_2 / s = 228 \text{ synch. watts}, \; T_5 = I_5^2 r_2/(2-s) = 15.3 \text{ synch. watts.}$$

$$T = T_f - T_b = 228 - 15.3 = 212.7 \text{ synch. watts}$$

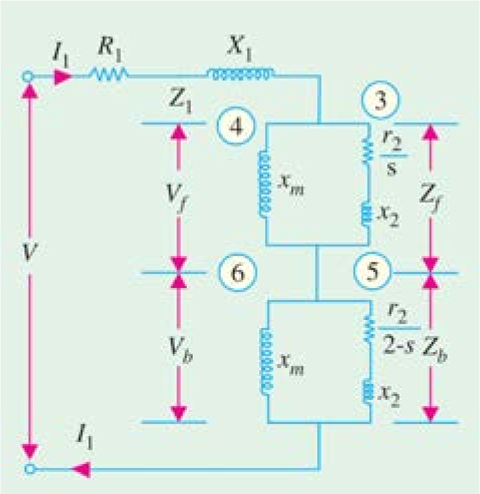
Output $=$ synch. watt $\times (1 - s) = 212.7 \times 0.95 =$ **202 W**

Since friction and windage losses are not given, this also represents the net output.

Example Problems

Example 36.2. Find the mechanical power output of the 185-W, 4-pole, 110-V, 60-Hz single-phase induction motor, whose constants are given below at a slip of 0.05.

R_1 = 1.86 ohm, X_1 = 2.56 ohm, X_m = 53.5 ohm, R_2 = 3.56 ohm, X_2 = 2.56 ohm
Core loss = 3.5 W, Friction and windage loss = 13.5 W.



Solution. It would be seen that major part of this problem has already been solved in Example 36.1. Let us, now, assume that V_f = 82.5% of 110 V = 90.7 V. Then the core-loss current $I_c = 35/90.7 = 0.386$ A ; $r_c = 90.7/0.386 = 235\ \Omega$.

Motor I

$$\text{conductance of core-loss branch} = 1/r_c = 1/235 = 0.00426 \text{ S}$$

$$\text{susceptance of magnetising branch} = -j/x_m = -j/26.7 = -j\,0.0374 \text{ S}$$

$$\text{admittance of branch 3} = \frac{(r_2/s) - j\,x_2}{(r_2/s)^2 + x_2^2} = 0.028 - j\,0.00101 \text{ S}$$

$$\text{admittance of 'motor' } I \text{ is } \mathbf{Y}_f = 0.00426 - j\,0.0374 + 0.028 - j\,0.00101$$
$$= 0.03226 - j\,0.03841 \text{ S}$$
$$\text{impedance } \mathbf{Z}_f = 1/\mathbf{Y}_f = 12.96 + j\,15.2 \text{ or } 19.9\ \Omega$$

Motor II

$$\text{admittance of branch 5} = \frac{\dfrac{r_2}{2-s} - j\,x_2}{\left(\dfrac{r_2}{2-s}\right)^2 + x_2^2} = \frac{0.91 - j\,1.28}{2.469} = 0.369 - j\,0.517$$

$$\text{admittance of 'motor' II, } \mathbf{Y}_b = 0.00426 - j\,0.0374 + 0.369 - j\,0.517$$
$$= 0.3733 - j\,0.555 \text{ S}$$

$$\text{Impedance of 'motor' II, } \mathbf{Z}_b = 1/\mathbf{Y}_b = 0.836 + j\,1.242 \text{ or } 1.5\ \Omega$$

$$\text{Impedance of entire motor (Fig. 36.16) } Z_{01} = \mathbf{Z}_1 + \mathbf{Z}_f + \mathbf{Z}_b = 15.66 + j\,19 \quad \text{or} \quad 24.7\ \Omega$$

$$I_1 = V/Z_{01} = 110/24.7 = 4.46 \text{ A}$$
$$V_f = I_1 Z_f = 4.46 \times 19.9 = 88.8 \text{ V}$$
$$V_b = 4.46 \times 1.5 = 6.69 \text{ V}$$

$$I_3 = 88.8/35.62 = 2.5 \text{ A}$$
$$I_5 = 6.69/1.57 = 4.25 \text{ A}$$
$$T_f = I_3^2(r_2/s) = 222 \text{ synch. watt}$$
$$T_b = I_5^2\left(\frac{r_2}{2-s}\right) = 16.5 \text{ synch. watt}$$
$$T = T_f - T_b = 205.5 \text{ synch. watt}$$

$$\text{Watts converted} = \text{synch. watt } (1 - s)$$
$$= 205.5 \times 0.95 = 195 \text{ W}$$
$$\text{Net output} = 195 - 13.5 = \mathbf{181.5\ W.}$$

Thank You