1. $\dot{x} = y$ $\qquad x(0)=0,\ y(0)=9$

$$\Rightarrow \frac{dx(t)}{dt} = y(t)$$

$$\Rightarrow \mathcal{L}\frac{dx(t)}{dt} = \mathcal{L}\, y(t)$$

$$\Rightarrow sX(s) - x(0) = Y(s)$$

$$\Rightarrow sX(s) - 0 - Y(s) = 0$$

$$\Rightarrow sX(s) - Y(s) = 0$$

$\dot{y} = 16x$

$$\Rightarrow \frac{dy(t)}{dt} = 16\, x(t)$$

$$\Rightarrow \mathcal{L}\frac{dy(t)}{dt} = 16\, \mathcal{L}\, x(t)$$

$$\Rightarrow sY(s) - y(0) = 16\, X(s)$$

$$\Rightarrow sY(s) - 9 + 16X(s) = 0$$

$$\Rightarrow 16X(s) - sY(s) = 9$$

$$X(s) = \frac{\begin{vmatrix} 0 & -1 \\ 4 & -s \end{vmatrix}}{\begin{vmatrix} s & -1 \\ 16 & -s \end{vmatrix}} = \frac{0+4}{-s^2+16} = \frac{4}{16-s^2}$$

$$= -\frac{4}{(s^2-4^2)}$$

$$Y(s) = \frac{\begin{vmatrix} s & 0 \\ 16 & 4 \end{vmatrix}}{\begin{vmatrix} s & -1 \\ 16 & -s \end{vmatrix}} = \frac{4s}{-s^2+16} = -\frac{4s}{(s^2-4^2)}$$

$$\therefore \mathcal{L}^{-1} X(s) = \mathcal{L}^{-1} - \frac{4}{(s^2-4^2)}$$

$$= -\sinh 4t$$

$$\therefore \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} - \frac{4s}{(s^2-4^2)}$$

$$= -4 \cosh 4t$$

$$x(t) = -\sinh 4t$$

$$y(t) = -4 \cosh 4t$$

2. $\dot{x} = -4y$ $\qquad x(0) = 2,\ y(0) = 0$

$$\Rightarrow \frac{dx(t)}{dt} = -4y(t)$$

$$\Rightarrow \mathcal{L}\frac{dx(t)}{dt} = -4\mathcal{L}\,y(t)$$

$$\Rightarrow sX(s) - x(0) = -4Y(s)$$

$$\Rightarrow sX(s) - 2 = -4Y(s)$$

$$\Rightarrow sX(s) + 4Y(s) = 2$$

$\dot{y} = x$

$$\Rightarrow \frac{dy(t)}{dt} = x(t)$$

$$\Rightarrow \mathcal{L}\frac{dy(t)}{dt} = \mathcal{L}\,x(t)$$

$$\Rightarrow sY(s) - y(0) = X(s)$$

$$\Rightarrow sY(s) - 0 = X(s)$$

$$\Rightarrow sY(s) - X(s) = 0$$

$$X(s) = \frac{\begin{vmatrix} 2 & 4 \\ 0 & s \end{vmatrix}}{\begin{vmatrix} s & 4 \\ -1 & s \end{vmatrix}} = \frac{2s}{s^2+4} = \frac{2s}{s^2+2^2}$$

$$Y(s) = \frac{\begin{vmatrix} s & 2 \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} s & 4 \\ -1 & s \end{vmatrix}} = \frac{2}{s^2+4} = \frac{2}{s^2+2^2}$$

$$\mathcal{L}^{-1} X(s) = 2\mathcal{L}^{-1} \frac{s}{s^2+2^2}$$

$$= 2\cos 2t$$

$$\mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \frac{2}{s^2+2^2}$$

$$= \sin 2t$$

3. $\dot{x} = 2x + y$ $\qquad x(0) = 1$

$y(0) = 6$

$\Rightarrow \dfrac{dx(t)}{dt} = 2x(t) + y(t)$

$\Rightarrow \mathcal{L}\dfrac{dx(t)}{dt} = 2\mathcal{L}x(t) + \mathcal{L}y(t)$

$\Rightarrow sX(s) - x(0) = 2X(s) + Y(s)$

$\Rightarrow (s-2)X(s) - Y(s) = 1$

$\dot{y} = 4x + 2y$

$\Rightarrow \dfrac{dy(t)}{dt} = 4x(t) + 2y(t)$

$\Rightarrow \mathcal{L}\dfrac{dy(t)}{dt} = 4\mathcal{L}x(t) + 2\mathcal{L}y(t)$

$\Rightarrow sY(s) - y(0) = 4X(s) + 2Y(s)$

$\Rightarrow -4X(s) + (s-2)Y(s) = 6$

$$X(s) = \frac{\begin{vmatrix} 1 & -1 \\ 6 & s-2 \end{vmatrix}}{\begin{vmatrix} s-2 & -1 \\ -4 & s-2 \end{vmatrix}} = \frac{s-2+6}{s^2-2s-2s+4-4}$$

$$= \frac{s+4}{s^2-4s}$$

$$= \frac{s+4}{s(s-4)} = \frac{-1}{s} + \frac{2}{s-4}$$

$$Y(s) = \frac{\begin{vmatrix} s-2 & 1 \\ -4 & 6 \end{vmatrix}}{\begin{vmatrix} s-2 & -1 \\ -4 & s-2 \end{vmatrix}} = \frac{6s-12+4}{s^2-2s-2s+4-4}$$

$$= \frac{6s-8}{s^2-4s}$$

$$= \frac{6s-8}{s(s-4)}$$

$$= \frac{2}{s} + \frac{4}{s-4}$$

$$\therefore \mathcal{L}^{-1} X(s) = \mathcal{L}^{-1}\left[-\frac{1}{s} + \frac{2}{s-4}\right]$$

$$= -1 + 2e^{4t}$$

$$\mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1}\left[\frac{2}{s} + \frac{4}{s-4}\right]$$

$$= 2 + 4e^{4t}$$

4. $\dot{x} = 3x+y$ $\qquad x(0)=3$

$y(0)=2$

$$\Rightarrow \frac{dx(t)}{dt} = 3x(t)+y(t)$$

$$\Rightarrow \mathcal{L}\frac{dx(t)}{dt} = 3\mathcal{L}x(t)+\mathcal{L}y(t)$$

$$\Rightarrow sX(s) - x(0) = 3X(s) + Y(s)$$

$$\Rightarrow (s-3)X(s) - Y(s) = 3$$

$\dot{y} = 4x+3y$

$$\Rightarrow \frac{dy(t)}{dt} = 4x(t)+3y(t)$$

$$\Rightarrow \mathcal{L}\frac{dy(t)}{dt} = 4\mathcal{L}x(t)+3\mathcal{L}y(t)$$

$$\Rightarrow sY(s) - y(0) = 4X(s)+3Y(s)$$

$$\Rightarrow (s-3)Y(s) - 4X(s) = 2$$

$$\therefore X(s) = \frac{\begin{vmatrix} 3 & -1 \\ 2 & s-3 \end{vmatrix}}{\begin{vmatrix} s-3 & -1 \\ -4 & s-3 \end{vmatrix}} = \frac{3s-9+2}{s^2-3s-3s+9-4}$$

$$= \frac{3s-7}{s^2-6s+5} = \frac{3s-7}{(s-5)(s-1)}$$

$$= \frac{2}{s-5} + \frac{1}{s-1}$$

$$\therefore Y(s) = \frac{\begin{vmatrix} s-3 & 3 \\ -4 & 2 \end{vmatrix}}{\begin{vmatrix} s-3 & -1 \\ -4 & s-3 \end{vmatrix}} = \frac{2s-6+12}{s^2-3s-3s+9-4}$$

$$= \frac{2s+6}{s^2-6s+5} = \frac{2s+6}{(s-5)(s-1)}$$

$$= \frac{4}{s-5} - \frac{2}{s-1}$$

$$\therefore \mathcal{L}^{-1} X(s) = 2\mathcal{L}^{-1}\frac{1}{s-5} + \mathcal{L}^{-1}\frac{1}{s-1}$$

$$= 2e^{5t} + e^{t}$$

$$\therefore \mathcal{L}^{-1} Y(s) = 4\mathcal{L}^{-1}\frac{1}{s-5} - 2\mathcal{L}^{-1}\frac{1}{s-1}$$

$$= 4e^{5t} - 2e^{t}$$