Introduction to Electrical Circuits

Final Term
Lecture - 02

Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11^{th} Edition

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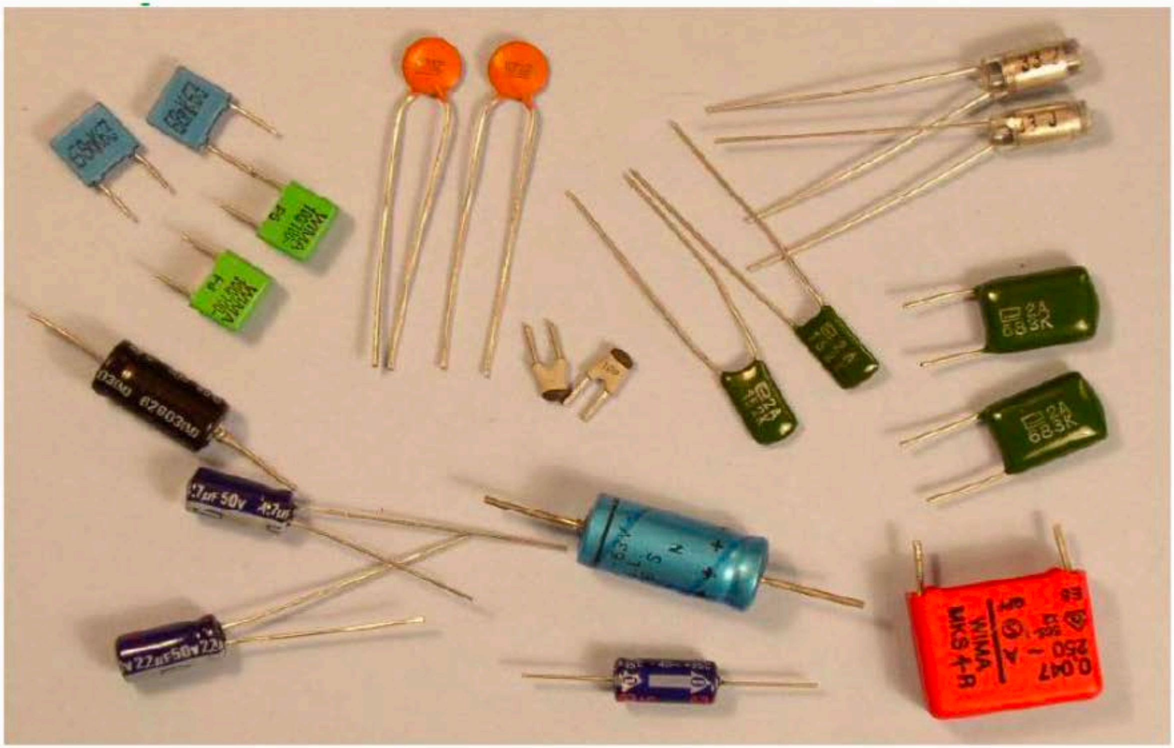
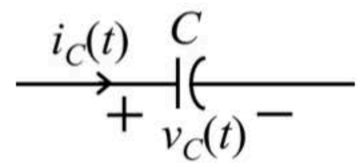
Pure Capacitive Circuit

Response of Basic Capacitor or Condenser Element to a Sinusoidal Voltage or Current





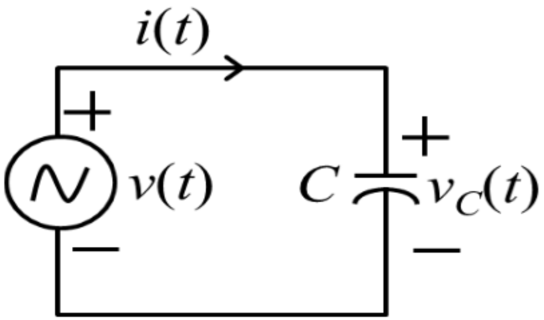
Voltage and current relation in a capacitor:

$$v_C(t) = \frac{1}{C}\int i_C(t)\,dt$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_C(t) = V_m \sin\omega t$

For a capacitance the relation of voltage and current is:



$$i(t) = i_C(t) = C\frac{dv_C(t)}{dt} = CV_m\frac{d(\sin\omega t dt)}{dt} = \omega LV_m \cos\omega t$$

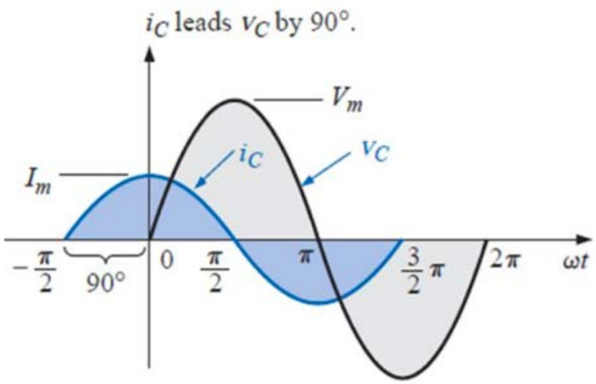
$$i(t) = \omega CV_m \sin(\omega t + 90°) = I_m \sin(\omega t + \theta_i)$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{1}{\omega C} = X_C \quad \Omega$

Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} \quad \Omega$

Angle of current, $\theta_i = 90°$

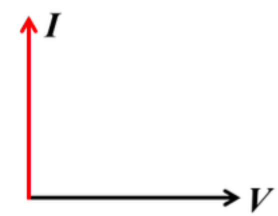
Angle of impedance, $\theta_z = \theta_v - \theta_i = -90°$



Impedance of a Capacitor, $Z = Z_C = X_C\angle{-90°} = -j\,X_C\ \Omega$

The phase difference between voltage across and current through a capacitor is 90°.

For a purely capacitive element, the voltage lags the current through the capacitive element by 90°. Or, the current leads the voltage in a capacitive element by 90°.



Phasor Diagram

EXAMPLE 15.5 Using complex algebra,

a. Find the current i_C for the circuit in Fig. 15.15.
b. Sketch the v_C and i_C curves.

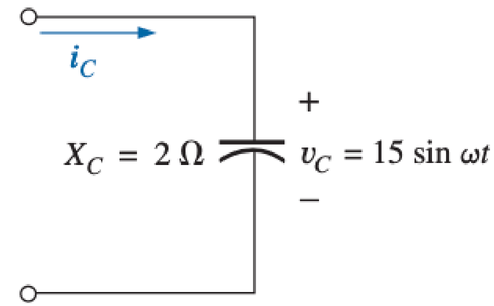


FIG. 15.15
Example 15.5.

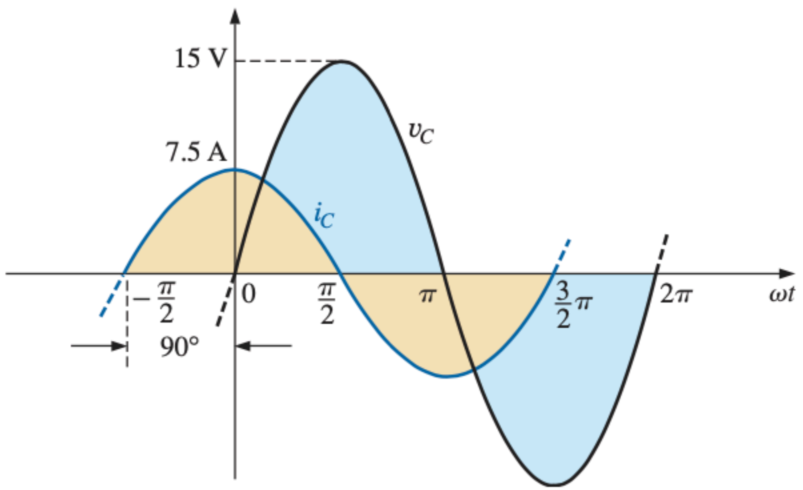
Solution:

a. $$v_C = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90°} = \frac{10.605 \text{ V} \angle 0°}{2\,\Omega \angle -90°} = 5.303 \text{ A} \angle 90°$$

and $$i_C = \sqrt{2}(5.303) \sin(\omega t + 90°) = \mathbf{7.5 \sin (\omega t + 90°)}$$

b. Note Fig. 15.16.



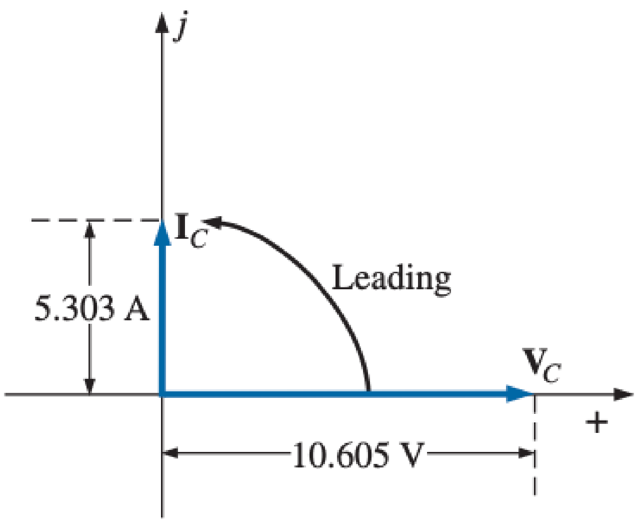


FIG. 15.16
Waveforms for Example 15.5.

EXAMPLE 15.6 Using complex algebra,

a. Find the voltage v_C for the circuit in Fig. 15.17.
b. Sketch the v_C and i_C curves.

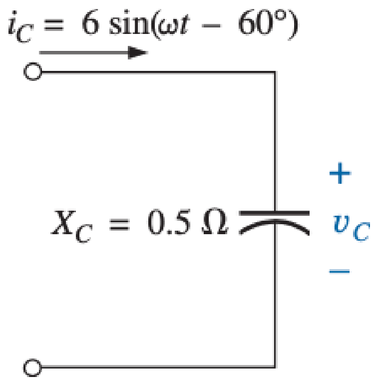


FIG. 15.17
Example 15.6.

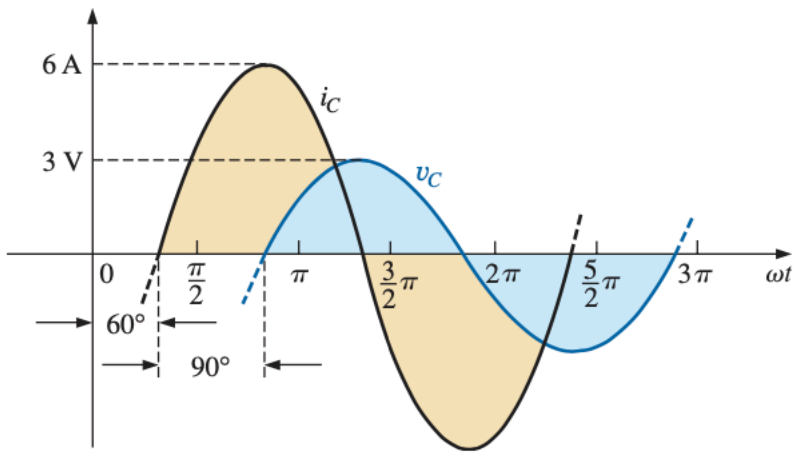
Solution:

a. $i_C = 6\sin(\omega t - 60°) \Rightarrow$ phasor notation $\mathbf{I}_C = 4.242\text{ A} \angle -60°$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90°) = (4.242\text{ A}\angle -60°)(0.5\ \Omega\angle -90°)$$
$$= 2.121\text{ V}\angle -150°$$

and $\quad v_C = \sqrt{2}(2.121)\sin(\omega t - 150°) = \mathbf{3.0\sin(\omega t - 150°)}$

b. Note Fig. 15.18.



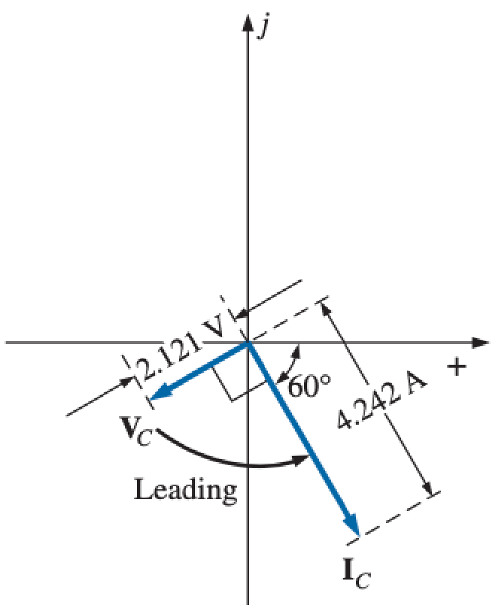


FIG. 15.18
Waveforms for Example 15.6.

Summary For a pure Capacitive Load

Magnitude of impedance, $Z = \frac{V_m}{I_m} = X_C \quad \Omega$ $\qquad \theta_i = \theta_v + 90^\circ \qquad \theta_v = \theta_i - 90^\circ$

Angle of impedance, $\theta_z = \theta_v - \theta_i = -90^\circ$

Impedance of a Capacitor, $\boldsymbol{Z} = \boldsymbol{Z}_C = X_C \angle -90^\circ = -jX_C \ \Omega$

The phase difference between voltage across and current through a capacitor is 90°.

The voltage lags the current in an inductor by 90°.

The current leads the voltage in an inductor by 90°.

The power factor is 0 which is called **zero leading power factor**.

The reactive factor is -1.

The active power is 0 that means **zero**.

The apparent power equals to reactive power.

Capacitor supply the reactive power.

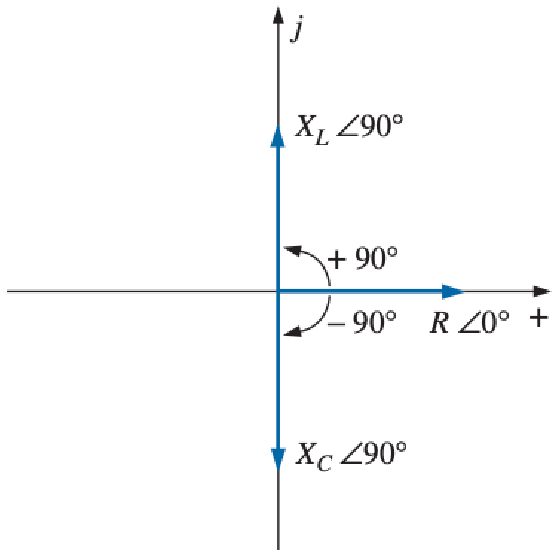
	Resistance	Inductance	Capacitance
Magnitude of impedance(Z) [Ω]	R	X_L	X_C
Angle of impedance($\theta=\theta_z$)	0º	90º	-90º
Impedance ($\boldsymbol{Z}$) [Ω]	$\boldsymbol{Z}_R=R\angle 0^o=R+j0$	$\boldsymbol{Z}_L= X_L\angle 90^o=0+jX_L$	$\boldsymbol{Z}_C= X_C\angle -90^o=0-jX_L$
Phase difference between voltage and current	0º	90º	-90º
Relation between voltage and current	Voltage and current are in phase	Voltage leads current Current **lags** voltage	Voltage lags current Currents **leads** voltage
Power factor (pf=cosθ)	Unity (1)	Zero **lagging** power factor (pf=0)	Zero **leading** power factor (pf=0)
Reactive factor (rf=sinθ)	0	1	-1
Power (P) [W]	$V_{rms}I_{rms}=I_{rms}^2R=V_{rms}^2/R$	0	0
Reactive power ($Q=P_x$) [Var]	0	$V_{rms}I_{rms}=I_{rms}^2X_L$ $=V_{rms}^2/ X_L$	$-V_{rms}I_{rms}=-I_{rms}^2X_C$ $=-V_{rms}^2/ X_C$
Apparent power (S) [VA]	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$

Impedance Diagram

For any network,

- the resistance will *always* appear on the positive real axis,
- the inductive reactance on the positive imaginary axis, and
- the capacitive reactance on the negative imaginary axis.

The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.



Series Configuration

- The overall properties of series ac circuits (Fig. 15.23) are the same as those for dc circuits.
- For instance, the total impedance of a system is the sum of the individual impedances and the current **I** is the same through each impedance.

$$\boxed{\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N}$$

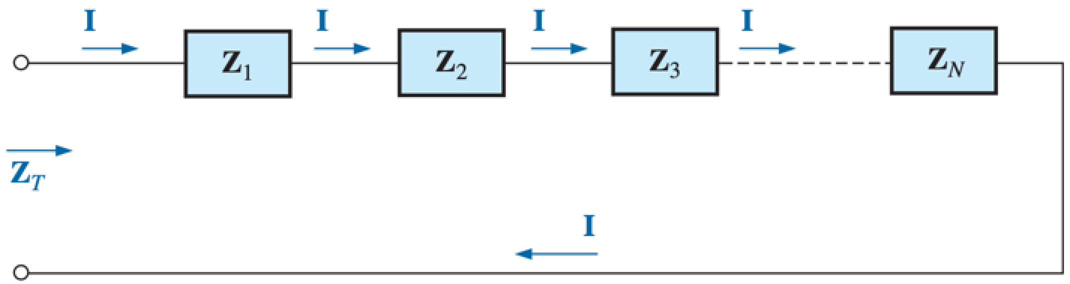


FIG. 15.23
Series impedances.

EXAMPLE 15.9 Draw the impedance diagram for the circuit in Fig. 15.24, and find the total impedance.

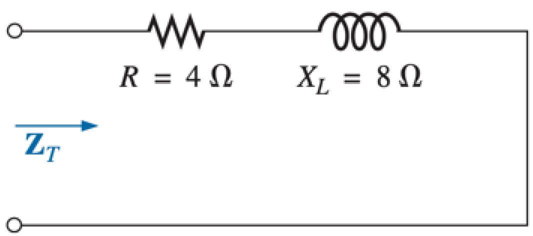


FIG. 15.24
Example 15.9.

Solution: As indicated by Fig. 15.25, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\ &= R \angle 0^\circ + X_L \angle 90^\circ \\ &= R + jX_L = 4\ \Omega + j\,8\ \Omega \\ \mathbf{Z}_T &= \mathbf{8.94\ \Omega \angle 63.43^\circ}\end{aligned}$$

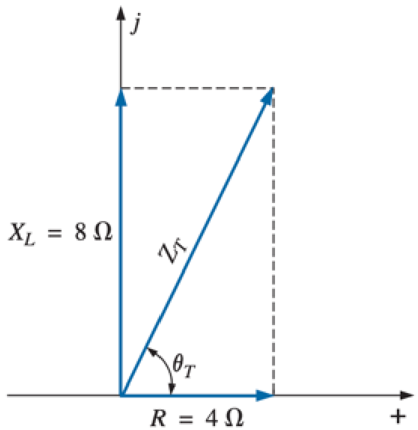


FIG. 15.25
Impedance diagram for Example 15.9.

EXAMPLE 15.10 Determine the input impedance to the series network in Fig. 15.26. Draw the impedance diagram.

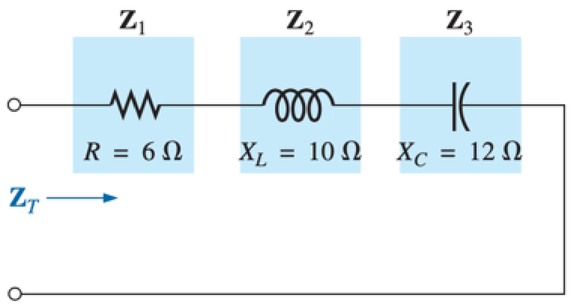


FIG. 15.26
Example 15.10

Solution:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$
$$= R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j\,2\ \Omega$$
$$\mathbf{Z}_T = \mathbf{6.32\ \Omega \angle -18.43^\circ}$$

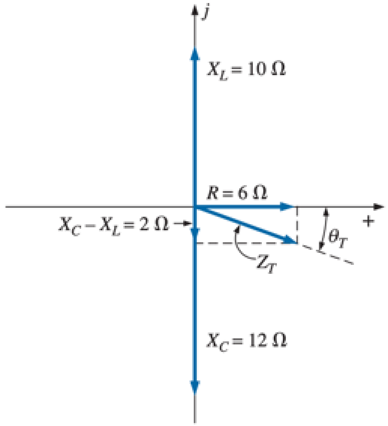


FIG. 15.27
Impedance diagram for Example 15.10.

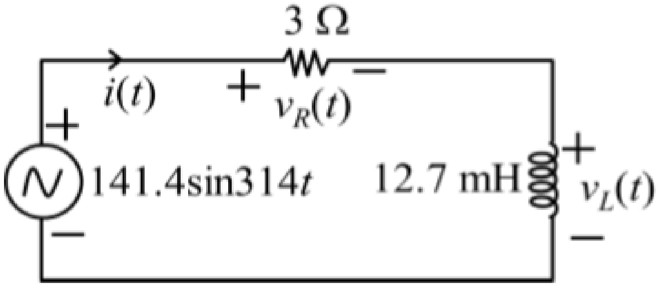
Resistance and Inductance Series Circuit

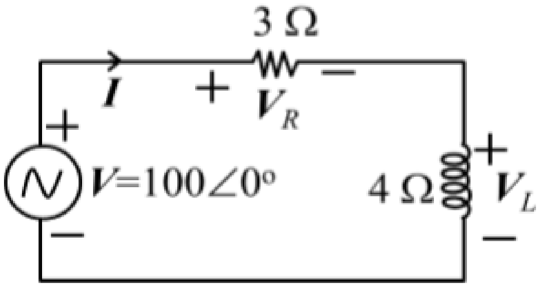
Example

A voltage 141.4sin314t V is applied to a *RL* series circuit which consists *R*=3 ohms, *L* = 12.7 mH. (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the inductive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and inductance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the inductance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: $X_L = \omega L = 314 \times 0.0127 = 4 \ \ \Omega$ $\quad \boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = \frac{141.4}{\sqrt{2}}\angle 0° = 100\angle 0° \ \ \text{V}$

$\boldsymbol{Z}_L = jX_L = j4 = 4\angle 90° \ \ \Omega \qquad \boldsymbol{Z}_R = 3 = 3\angle 0° \ \ \Omega$





$$\boldsymbol{Z} = \boldsymbol{Z}_{RL} = 3 + j4 = 5\angle 53.13^\circ \quad \Omega \qquad V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100 \quad \text{V} \qquad \boldsymbol{V} = 100\angle 0^\circ \text{ V}$$

Current: $$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{100\angle 0^\circ}{5\angle 53.13^\circ} = 20\angle -53.13^\circ \text{ A}$$

Voltage drop across the resistance: $\boldsymbol{V}_R = \boldsymbol{I}\boldsymbol{Z}_R = (20\angle -53.13^\circ)(3\angle 0^\circ) = 60\angle -53.13^\circ \text{ V}$

Voltage drop across the inductance: $\boldsymbol{V}_L = \boldsymbol{I}\boldsymbol{Z}_L = (20\angle -53.13^\circ)(4\angle 90^\circ) = 80\angle 36.87^\circ \text{ V}$

Verification of KVL: $\boldsymbol{V} = \boldsymbol{V}_R + \boldsymbol{V}_L = 60\angle -53.13^\circ + 80\angle 36.87^\circ \text{ V}$

$$\boldsymbol{V} = 36 - j48 + 64 + j48 = 100 \text{ V (equal to supply voltage)}$$

Angle of impedance, $\theta = \theta_z = \theta_v - \theta_i = 53.13^\circ$

Power Factor :$\text{pf} = \cos(\theta_z) = \cos[53.13^\circ] = 0.6$ lagging

Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin[53.13^\circ] = 0.8$

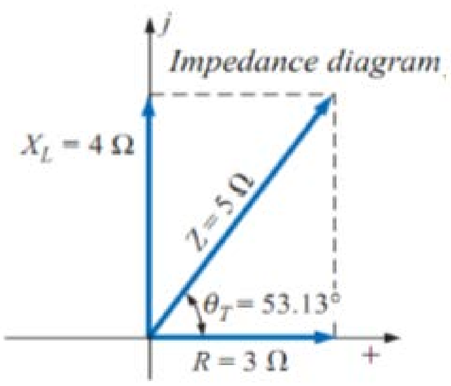
Power or **Active Power** : $P = VI\cos\theta_z = 100 \times 20 \times 0.6 = 1200$ W

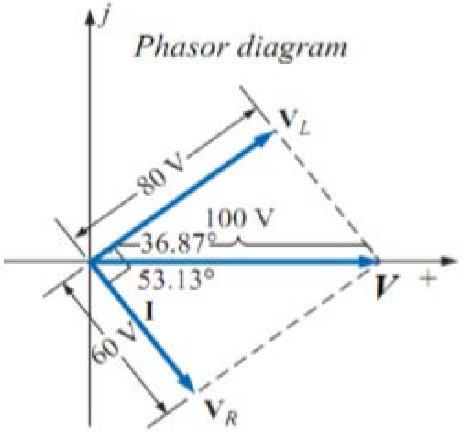
$$P = I^2R = 20^2 \times 3 = 1200\,\text{W} \qquad P = \frac{V_R^2}{R} = \frac{60^2}{3} = 1200\,\text{W}$$

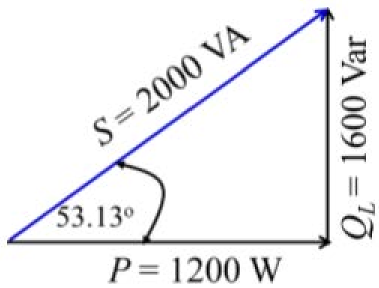
Reactive Power : $Q_L = VI\sin\theta_z = 100 \times 20 \times 0.8 = 1600$ Var

$$Q_L = I^2X_L = 20^2 \times 4 = 1600\ \text{Var} \qquad Q_L = \frac{V_L^2}{X_L} = \frac{80^2}{4} = 1600\ \text{Var}$$

Apparent Power : $S = VI = 100 \times 20 = 2000$ VA







Power Triangle

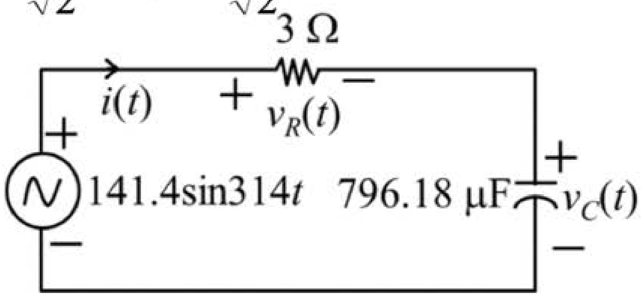
Resistance and Capacitance Series Circuit

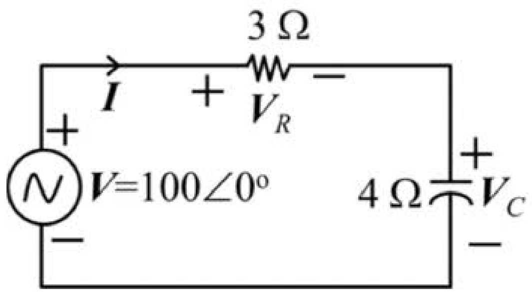
Example

A voltage 141.4sin314t V is applied to a *RC* series circuit which consists *R*=3 ohms, *C*=796.18 μF. (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the capacitive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and capacitance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the capacitance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: **Capacitive reactance**: $X_C = \frac{1}{\omega C} = \frac{1}{314 \times 796.18 \times 10^{-6}} = 4\ \Omega$

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = \frac{141.4}{\sqrt{2}}\angle 0^\circ = 100\angle 0^\circ \text{ V}$$





$$Z_C = -jX_C = -j4 = 4\angle -90° \ \Omega \quad Z_R = 3 = 3\angle 0° \ \Omega$$

$$Z = Z_{RC} = 3 - j4 = 5\angle -53.13° \quad \Omega$$

Current: $$I = \frac{V}{Z} = \frac{100\angle 0°}{5\angle -53.13°} = 20\angle 53.13° \ \text{A}$$

Voltage drop across the resistance: $$V_R = IZ_R = (20\angle 53.13°)(3\angle 0°) = 60\angle 53.13° \ \text{V}$$

Voltage drop across the inductance: $$V_C = IZ_C = (20\angle 53.13°)(4\angle -90°) = 80\angle -36.87° \ \text{V}$$

Verification of KVL: $$V = V_R + V_C = 60\angle 53.13° + 80\angle -36.87° \ \text{V}$$

$$V = 36 + j48 + 64 - j48 = 100 \ \text{V (equal to supply voltage)}$$

Angle of impedance, $\theta = \theta_z = \theta_v - \theta_i = -53.13°$

Power Factor : $\text{pf} = \cos(\theta_z) = \cos[-53.13°] = 0.6 \quad \text{leading}$

Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin[-53.13°] = -0.8$

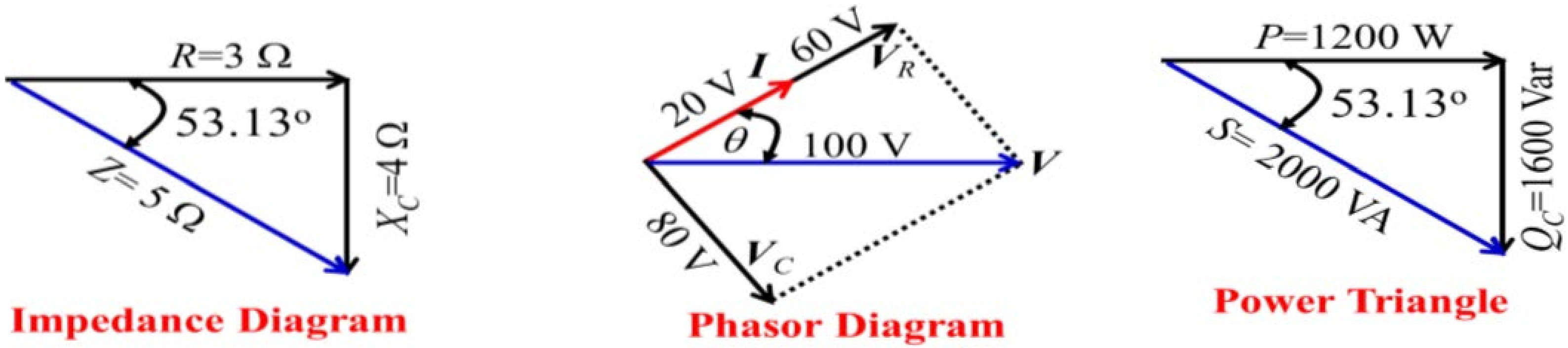
Power or **Active Power** : $P = VI\cos\theta_Z = 100\times 20\times 0.6 = 1200\ \text{W}$

$$P = I^2R = 20^2\times 3 = 1200\,\text{W} \qquad P = \frac{V_R^2}{R} = \frac{60^2}{3} = 1200\,\text{W}$$

Reactive Power : $Q_L = VI\sin\theta_Z = 100\times 20\times -0.8 = -1600\ \text{Var}$

$$Q_L = -I^2X_C = -20^2\times 4 = -1600\ \text{Var} \qquad Q_C = -\frac{V_C^2}{X_C} = -\frac{80^2}{4} = -1600\ \text{Var}$$

Apparent Power : $S = VI = 100\times 20 = 2000\ \text{VA}$



Resistance, Inductance and Capacitance Series Circuit

Example

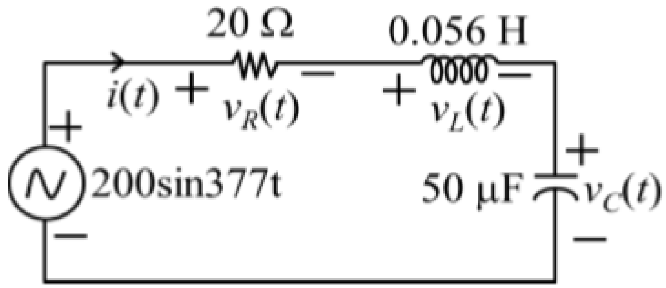
If R=20 ohms, L=0.056 Henry, C = 50 μF and applied voltage 200sin377t, (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the capacitive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and capacitance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the capacitance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: $V_m = 200 \ \text{V} \quad \omega = 377 \ \text{rad/s} \quad R = 20 \ \Omega \quad L = 0.056 \ \text{H} \quad C = 50\times10^{-6} \ \text{F}$

Inductive reactance: $X_L = \omega L = 377\times0.056 = 21.1 \ \Omega$

Capacitive reactance: $X_C = \dfrac{1}{\omega C}$

$$= \frac{1}{377\times50\times10^{-6}} = 53 \ \Omega$$



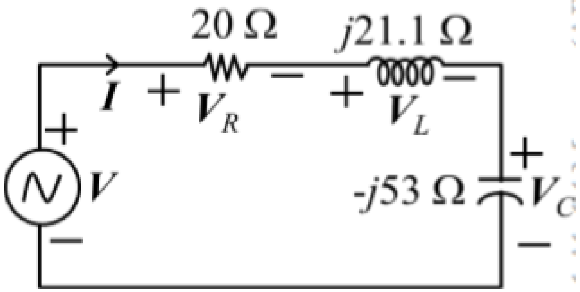
$$Z_R = 20\angle 0^\circ = 20\ \Omega \qquad Z_L = 21.1\angle 90^\circ = j21.1\ \Omega \qquad Z_C = 53\angle -90^\circ = -j53\ \Omega$$

$$Z = Z_R + Z_L + Z_C = 20\angle 0^\circ + 21.1\angle 90^\circ + 53\angle -90^\circ = 20 + j21.1 - j53\ \Omega$$

$$Z = 20 - j31.9 = 37.65\angle -57.9^\circ\ \Omega$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4\ \text{ V}$$

$$V = 141.4\angle 0^\circ\ \text{V}$$



$$I = \frac{V}{Z} = \frac{141.4\angle 0^\circ}{37.56\angle -57.9^\circ} = 3.76\angle 57.9^\circ = 2 + j3.2\ \text{ A}$$

$$V_R = IZ_R = (3.76\angle 57.9^\circ)(20\angle 0^\circ) = 75.1\angle 57.9^\circ = 40 + j63.64\ \text{ V}$$

$$V_L = IZ_L = (3.76\angle 57.9^\circ)(21.1\angle 90^\circ) = 79.24\angle 148^\circ = -67 + j42.1\ \text{ V}$$

$$V_C = IZ_C = (3.76\angle 57.9^\circ)(53\angle -90^\circ) = 199\angle -32.1^\circ = 169 - j106\ \text{ V}$$

Verification of KVL:

$$V = 40 + j63.64 - 67 + j42.1 + 169 - j106 = 141.4\ \text{ V (equal to supply voltage)}$$

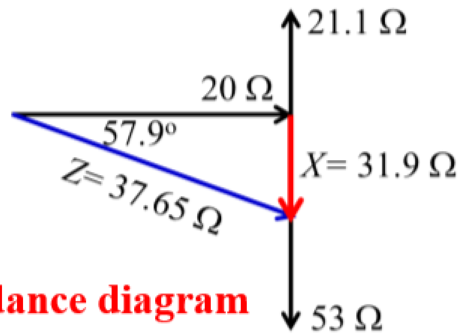
$$\theta = \theta_z = \theta_v - \theta_i = -57.9^\circ\ \Omega \qquad pf = \cos\theta = \cos(-57.9^\circ) = 0.53$$

$$rf = \sin\theta = \sin(-57.9^\circ) = -0.85$$

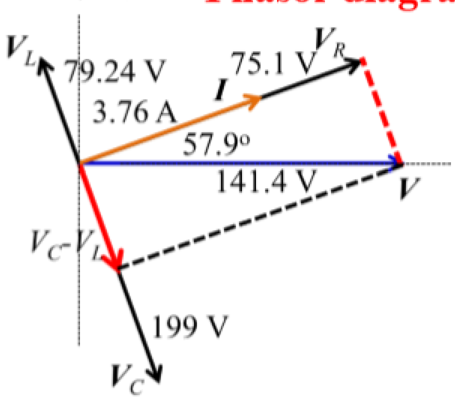
$$S = VI = 141.4 \times 3.76 = 531\ \text{VA} \qquad P = VI\cos\theta_z = 141.4 \times 3.76 \times 0.53 = 282.1\ \ \text{W}$$

$$Q = Q_L - Q_C = VI\sin\theta_z = 141.4 \times 3.76 \times -0.85 = -500\ \ \text{Var}$$

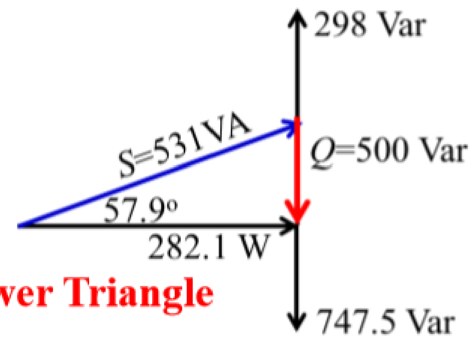
$$Q_L = I^2 X_L = 3.76^2 \times 21.1 = 298\ \text{Var} \qquad Q_C = -I^2 X_C = -3.76^2 \times 53 = -747.5\ \text{Var}$$



Impedance diagram



Phasor diagram



Power Triangle

	RL **series Circuit**	***RC*** **series Circuit**	***RLC*** **series Circuit**
Magnitude of impedance(Z) [Ω]	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Angle of impedance($\theta = \theta_z$)	$\theta = \tan^{-1}\left[\frac{X_L}{R}\right]$	$\theta = -\tan^{-1}\left[\frac{X_C}{R}\right]$	$\theta = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$
Impedance ($\boldsymbol{Z}$) [Ω]	$\boldsymbol{Z} = Z\angle\theta = R + jX_L$	$\boldsymbol{Z} = Z\angle\theta = R - jX_C$	$\boldsymbol{Z} = Z\angle\theta = R + j\left(X_L - X_C\right)$
Phase difference between voltage and current	$\theta = \theta_v - \theta_i > 0$ Between 0° to 90°	$\theta = \theta_v - \theta_i < 0$ Between -90° to 0°	Depends on the value of X_L and X_C
Relation between voltage and current	Voltage leads current i.e. Current lags voltage	Voltage lags current i.e. Current leads voltage	Depends on the value of X_L and X_C

Voltage Divider Rule

The voltage (V_x) across one or more elements in series that have total impedance Z_x, can be given by:

$$V_x = \frac{Z_x}{Z_T} E = \frac{Z_x}{Z_T} V$$

where, E or V is the total voltage appearing across the series circuit, and Z_T is the total impedance of the series circuit.

EXAMPLE 15.11 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.43.

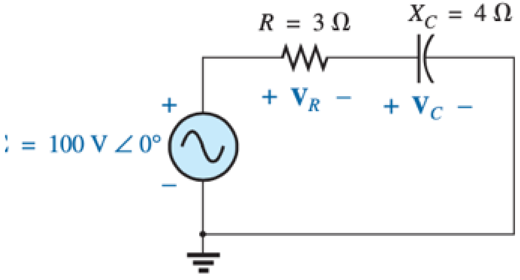


FIG. 15.43
Example 15.11.

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90°)(100\ \text{V} \angle 0°)}{4\ \Omega \angle -90° + 3\ \Omega \angle 0°} = \frac{400\ \text{V} \angle -90°}{3 - j4}$$

$$= \frac{400\ \text{V} \angle -90°}{5 \angle -53.13°} = \mathbf{80\ V \angle -36.87°}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0°)(100\ \text{V} \angle 0°)}{5\ \Omega \angle -53.13°} = \frac{300\ \text{V} \angle 0°}{5 \angle -53.13°}$$

$$= \mathbf{60\ V \angle +53.13°}$$

EXAMPLE 15.13 For the circuit in Fig. 15.46,

a. Calculate **I**, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.
d. Draw the phasor diagram.
e. Obtain the phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and show that it equals the input voltage **E.**
f. Find $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule.

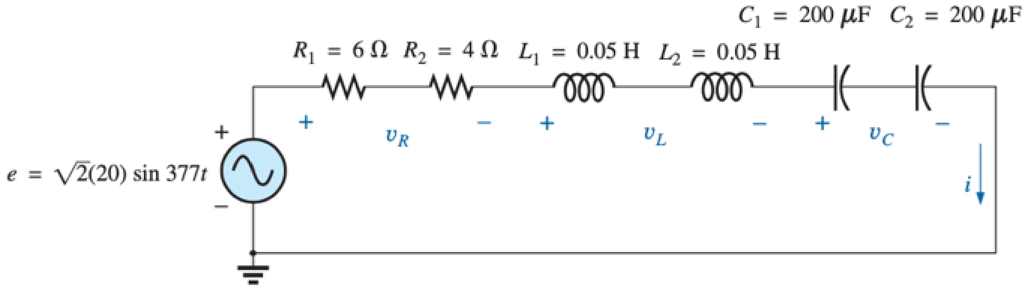


FIG. 15.46
Example 15.13.

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$
$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$
$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$
$$X_L = \omega L = (377\ \text{rad/s})(0.1\ \text{H}) = 37.70\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{rad/s})(100 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$

Redrawing the circuit using phasor notation results in Fig. 15.47.

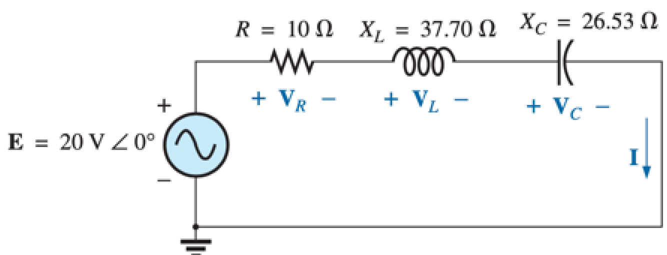


FIG. 15.47

Applying phasor notation to the circuit in Fig. 15.46.

For the circuit in Fig. 15.47,

$$\begin{aligned}\mathbf{Z}_T &= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ &= 10\ \Omega + j\,37.70\ \Omega - j\,26.53\ \Omega \\ &= 10\ \Omega + j\,11.17\ \Omega = \mathbf{15\ \Omega \angle 48.16^\circ}\end{aligned}$$

The current **I** is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\ \text{V} \angle 0^\circ}{15\ \Omega \angle 48.16^\circ} = \mathbf{1.33\ A \angle -48.16^\circ}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\begin{aligned}\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0^\circ) &= (1.33\ \text{A} \angle -48.16^\circ)(10\ \Omega \angle 0^\circ) \\ &= \mathbf{13.30\ V \angle -48.16^\circ} \\ \mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90^\circ) &= (1.33\ \text{A} \angle -48.16^\circ)(37.70\ \Omega \angle 90^\circ) \\ &= \mathbf{50.14\ V \angle 41.84^\circ} \\ \mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90^\circ) &= (1.33\ \text{A} \angle -48.16^\circ)(26.53\ \Omega \angle -90^\circ) \\ &= \mathbf{35.28\ V \angle -138.16^\circ}\end{aligned}$$

b. The total power factor, determined by the angle between the applied voltage **E** and the resulting current **I,** is 48.16°:

$$F_p = \cos\theta = \cos 48.16° = \mathbf{0.667\ lagging}$$

or

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{10\ \Omega}{15\ \Omega} = \mathbf{0.667\ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (20\text{ V})(1.33\text{ A})(0.667) = \mathbf{17.74\ W}$$

e. The phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ is

$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \\ &= 13.30\text{ V}\angle -48.16° + 50.14\text{ V}\angle 41.84° + 35.28\text{ V}\angle -138.16° \\ \mathbf{E} &= 13.30\text{ V}\angle -48.16° + 14.86\text{ V}\angle 41.84°\end{aligned}$$

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\ V}$$

and $\theta_E = \mathbf{0°}$ (from phasor diagram)

and $\mathbf{E} = 20\text{ V}\angle 0°$

d. The phasor diagram appears in Fig. 15.48.

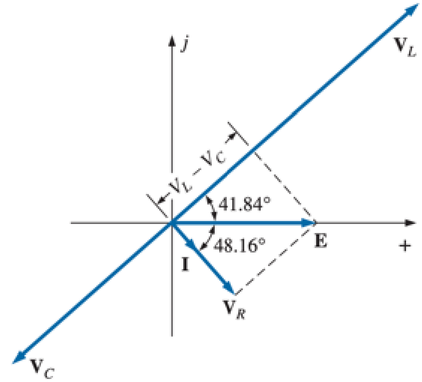


FIG. 15.48
Phasor diagram for the circuit in Fig. 15.46.

f. $$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\angle 0°)(20\text{ V}\angle 0°)}{15\ \Omega\angle 48.16°} = \frac{200\text{ V}\angle 0°}{15\angle 48.16°} = \mathbf{13.3\ V\angle -48.16°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\angle -90°)(20\text{ V}\angle 0°)}{15\ \Omega\angle 48.16°} = \frac{530.6\text{ V}\angle -90°}{15\angle 48.16°} = \mathbf{35.37\ V\angle -138.16°}$$

Thank You