1. $\frac{(1+i)^2}{1-i}$

$$= \frac{(1+i)^2(1+i)}{(1-i)(1+i)}$$

$$= \frac{\cancel{(1+2i+i^2)}\,(1+2i+i^2)(1+i)}{1-i^2}$$

$$= \frac{1+2i+i^2+i+2i^2+i^3}{1-(-1)}$$

$$= \frac{1+3i+3(-1)-i}{2}$$

$$= \frac{-2+2i}{2}$$

$$= -1+i$$

2.(a) $\frac{3-2i}{1-2i} = z$

$$= \frac{(3-2i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{3+6i-2i-4i^2}{1+2i-2i-4i^2}$$

$$= \frac{7+4i}{5}$$

$$= \frac{7}{5} + \frac{4i}{5}$$

$$Re\left\{\frac{3-2i}{1-2i}\right\} = \frac{7}{5}$$

b) $z = \frac{3-2i}{1-2i} = \frac{7}{5} + \frac{4}{5}i$

$$\left|\frac{z}{\bar{z}}\right| = \frac{|z|}{|\bar{z}|}$$

$$\bar{z} = \frac{7}{5} - \frac{4}{5}i$$

$$|z| = \sqrt{\left(\frac{7}{5}\right)^2 + \left(-\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{49}{25} + \frac{16}{25}}$$

$$= \frac{\sqrt{65}}{5}$$

$$|\bar{z}| = \sqrt{\left(\frac{7}{5}\right)^2 + \left(-\frac{4}{5}\right)^2}$$

$$= \frac{\sqrt{65}}{5}$$

$$\therefore \left|\frac{z}{\bar{z}}\right| = \frac{\sqrt{65}}{5} \times \frac{5}{\sqrt{65}}$$

$$= 1$$

c) $z = \frac{7}{5} + \frac{4}{5} i$

$\bar{z} = \frac{7}{5} - \frac{4}{5} i$

$\operatorname{Im}\left\{\frac{z}{\bar{z}}\right\} = \frac{4}{5} \times \left(-\frac{5}{4}\right)$

$= -1$

3. a) $z = -1 + i$

$r = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right)$

$= -\tan^{-1}\left(\tan \frac{\pi}{4}\right) + \pi$

$= -\frac{\pi}{4} + \pi$

$= \cancel{\frac{3\pi}{4}} \quad \frac{3\pi}{4}$

$z = \sqrt{2}e^{i\frac{3\pi}{4}}$

b) $z = -3 - \sqrt{3}i$

$r = \sqrt{(-3)^2 + (-\sqrt{3})^2}$

$= 2\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-3}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi$

$= \frac{\pi}{6} + \pi$

$= \frac{7\pi}{6}$

$z = 2\sqrt{3}e^{i\frac{7\pi}{6}}$

c) $z = \frac{(1-i)^2}{(1+i)}$

$$= \frac{(1-i)^2(1-i)}{(1+i)(1-i)}$$

$$= \frac{1-2i+i^2 - i + 2i^2 - i^3}{1-i^2}$$

$$= \frac{1-3i-1-2+i}{1+1}$$

$$= \frac{-2-2i}{2}$$

$$= -1-i$$

$$r = \sqrt{(-1)^2+(-1)^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-1}{-1}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + 2\pi$$

$= \frac{\pi}{4} + 2\pi$

$= \frac{9\pi}{4}$

$\therefore z = \sqrt{2} e^{i\frac{9\pi}{4}}$

4. a) $z = \sqrt{3} e^{i\frac{\pi}{3}}$

$r = \sqrt{3}$

$a = \sqrt{3} \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$b = \sqrt{3} \sin\frac{\pi}{3} = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$

$\therefore z = \frac{\sqrt{3}}{2} + \frac{3}{2} i$

b) $z = 2e^{i\frac{\pi}{4}}$

$r = 2$

$a = 2\cos\frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

$b = 2\sin\frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

$\therefore z = \sqrt{2} + \sqrt{2}\,i$

$= \sqrt{2}(1+i)$

5. a) $z = (-1-i)^4$

$r = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{-1}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \pi$

$= \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$$\therefore z = \left(\sqrt{2}\, e^{i\frac{5\pi}{4}}\right)^4$$

$$= (\sqrt{2})^4 e^{i5\pi}$$

$$= 4 e^{i5\pi}$$

$\therefore$ Principal argument: $\text{Arg } Z = 5\pi$

$$= 4\pi + \pi$$

$$= \pi$$

b) $z = (-2 + 2\sqrt{3}\, i)^3$

$$r = \sqrt{(-2)^2 + (2\sqrt{3})^2}$$

$$= 4$$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$$

$$= -\tan^{-1}\left(\tan\frac{\pi}{3}\right) + \pi$$

$$= -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

$\therefore z = (4e^{i\frac{2\pi}{3}})^3$

$= 64 e^{i2\pi}$

$\therefore$ Principal argument: $\text{Arg}\, z = 2\pi$

c) $z = \dfrac{(1+i)^3}{(1-i)}$

$= \dfrac{(1+i)^2 (1+i)(1+i)}{(1-i)(1+i)}$

$= \dfrac{(1+2i+i^2)(1+2i+i^2)}{1-i^2}$

$= \dfrac{2i \times 2i}{1+1}$

$= \dfrac{4i^2}{2}$

$= -2$

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$r = \sqrt{(-2)^2}$

$= 2$

$\theta = \tan^{-1}\left(\frac{0}{2}\right)$

$= 0$

$z = 2e^{i \times 0}$

$= 2$

$\therefore$ Principal argument: $\text{Arg } z = 0$