Introduction to Electrical Circuits

Mid Term
Lecture – 8

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Reference Book:

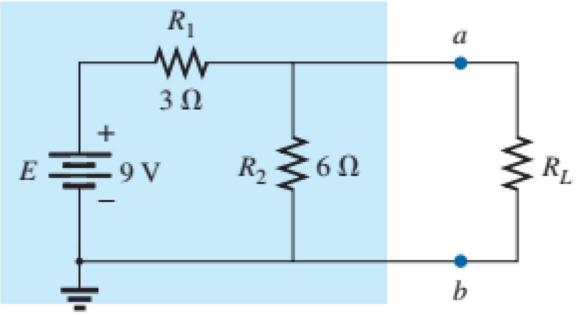
Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W4	MC8	Chapter 9	9.5 MAXIMUM POWER TRANSFER THEOREM	Try Exercise.	18, 19, 20, 21
			9.4 NORTON'S THEOREM	Try Exercise.	
		Chapter 10	10.3 CAPACITANCE (Definition, Symbol, Unit and Equation)	10.2	3, 4
		Chapter 11	11.3 INDUCTANCE (Definition, Symbol, Unit and Equation)	11.2	2,3

9.4 NORTON'S THEOREM

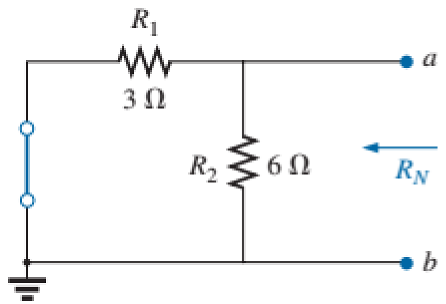
1. ***Remove that portion of the network across which the Norton equivalent circuit is found. Mark the terminals of the remaining two-terminal network.***
2. **Calculate R_N following the same procedure as R_{TH}.**
3. ***Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals.***
4. ***Draw the Norton equivalent circuit.***



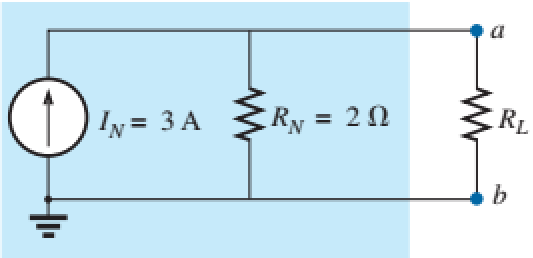
EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area in Fig. 9.61.

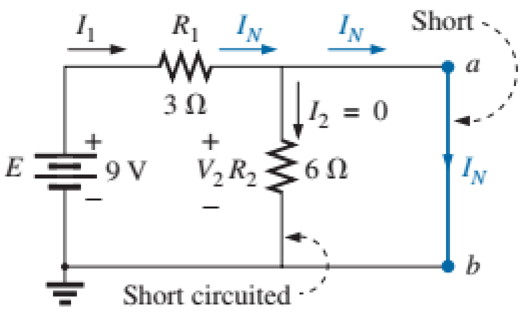
Solution:

$$R_N = R_1 \| R_2 = 3\ \Omega \| 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$



$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$





9.5 MAXIMUM POWER TRANSFER THEOREM

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance or Nortor resistance of the network applied to the load. That is,

$$\boxed{R_L = R_{Th}} \qquad \boxed{R_L = R_N}$$

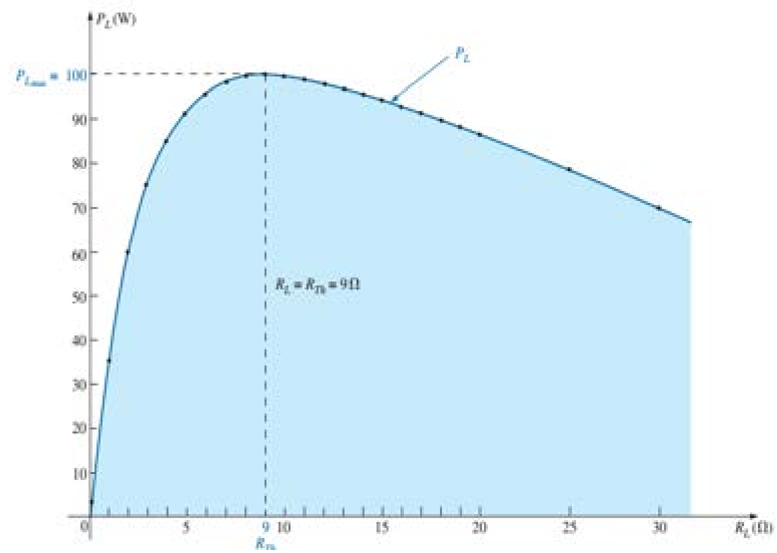
$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Then substitute into the power equation:

$$P_L = I_L^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

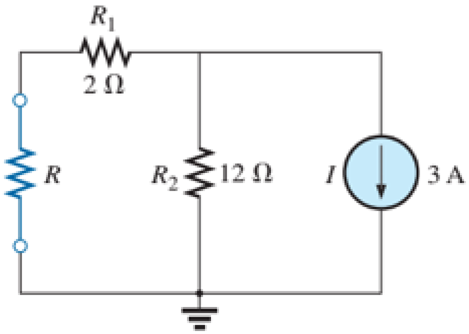
$$\boxed{P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}}$$



Similarly

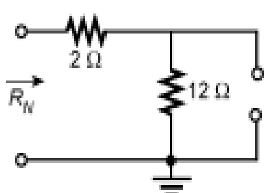
$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \quad \text{(W)}$$

18. Find the Norton equivalent circuit for the network external to the resistor R for the network in Fig. 9.126.



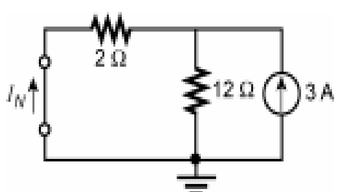
Solution:

R_N:



$$R_N = 2\,\Omega + 12\,\Omega = \mathbf{14\,\Omega}$$

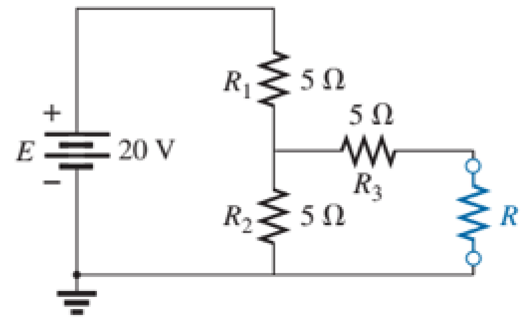
I_N:



$$I_N = \frac{12\,\Omega(3\text{ A})}{12\,\Omega + 2\,\Omega} = 2.57\text{ A}$$

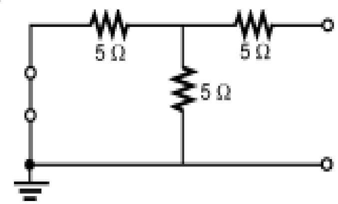
19. **a.** Find the Norton equivalent circuit for the network external to the resistor R for the network in Fig. 9.127.

b. Convert to the Thévenin equivalent circuit, and compare your solution for E_{Th} and R_{Th} to that appearing in the solution for Problem 9.



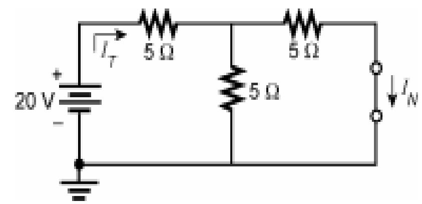
Solution:

a. R_N:



$$\leftarrow R_N = 5\,\Omega + \frac{5\,\Omega}{2} = \mathbf{7.5\,\Omega}$$

I_N:

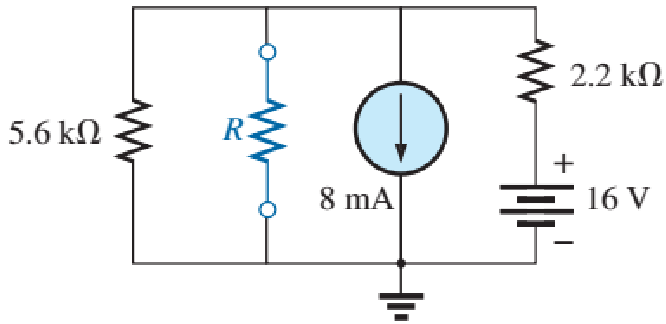


$$I_T = \frac{20\text{ V}}{5\,\Omega + \frac{5\,\Omega}{2}} = 2.67\text{ A}$$

$$I_N = \frac{I_T}{2} = \mathbf{1.34\text{ A}}$$

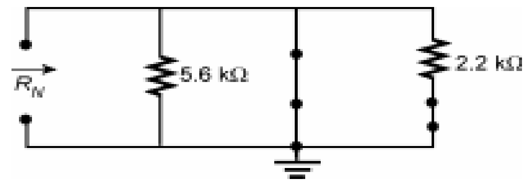
b. $E_{Th} = I_N R_N = (1.34\text{ A})(7.5\,\Omega) = 10.05\text{ V} \cong \mathbf{10\text{ V}}$, $R_{Th} = R_N = \mathbf{7.5\,\Omega}$

20. Find the Norton equivalent circuit for the network external to the resistor R for each network in Fig. 9.129.



Solution:

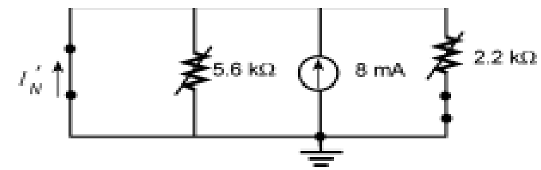
R_N:



$$R_N = 5.6\ \text{k}\Omega \parallel 2.2\ \text{k}\Omega = \mathbf{1.58\ k\Omega}$$

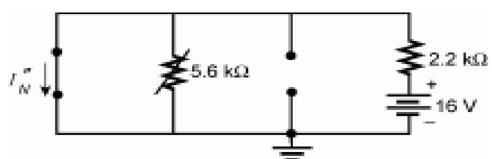
I_N:

I':



$$I'_N = 8\ \text{mA}$$

E:

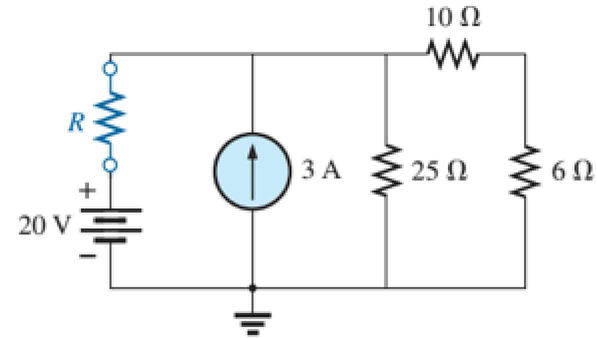


$$I''_N = \frac{16\ \text{V}}{2.2\ \text{k}\Omega} = 7.27\ \text{mA}$$

$$I_N\uparrow = 8\ \text{mA} - 7.27\ \text{mA} = \mathbf{0.73\ mA}$$

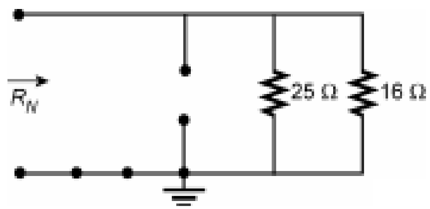
21. **a.** Find the Norton equivalent circuit for the network external to the resistor R for each network in Fig. 9.130.

b. Convert to the Thévenin equivalent circuit, and compare your solution for E_{Th} and R_{Th} to that appearing in the solutions for Problem 12.



Solution:

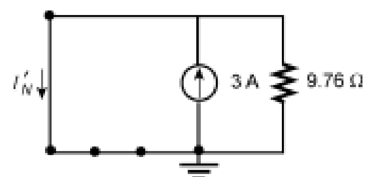
(I): (a)



$R_N = 25\ \Omega \parallel 16\ \Omega = \mathbf{9.76\ \Omega}$

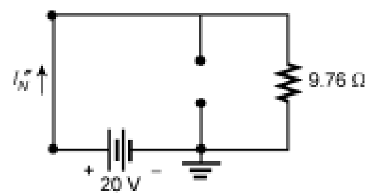
I_N: Superposition:

I:



$I'_N = 3\ \text{A}$

E:



$$I''_N = \frac{20\ \text{V}}{9.76\ \Omega} = 2.05\ \text{A}$$

$I_N = I'_N - I''_N = 3\ \text{A} - 2.05\ \text{A} = \mathbf{0.95\ A}$ (direction of I'_N)

b. $E_{Th} = I_N R_N = (0.95\ \text{A})(9.76\ \Omega) = \mathbf{9.27\ V} \cong 9.28\ \text{V}$, $R_{Th} = R_N = \mathbf{9.76\ \Omega}$

Definition: ***Capacitance is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity. the higher the capacitance of a capacitor, the greater the amount of charge stored on the plates for the same applied voltage.***

Unit: ***The unit of measure applied to capacitors is the farad (F)***

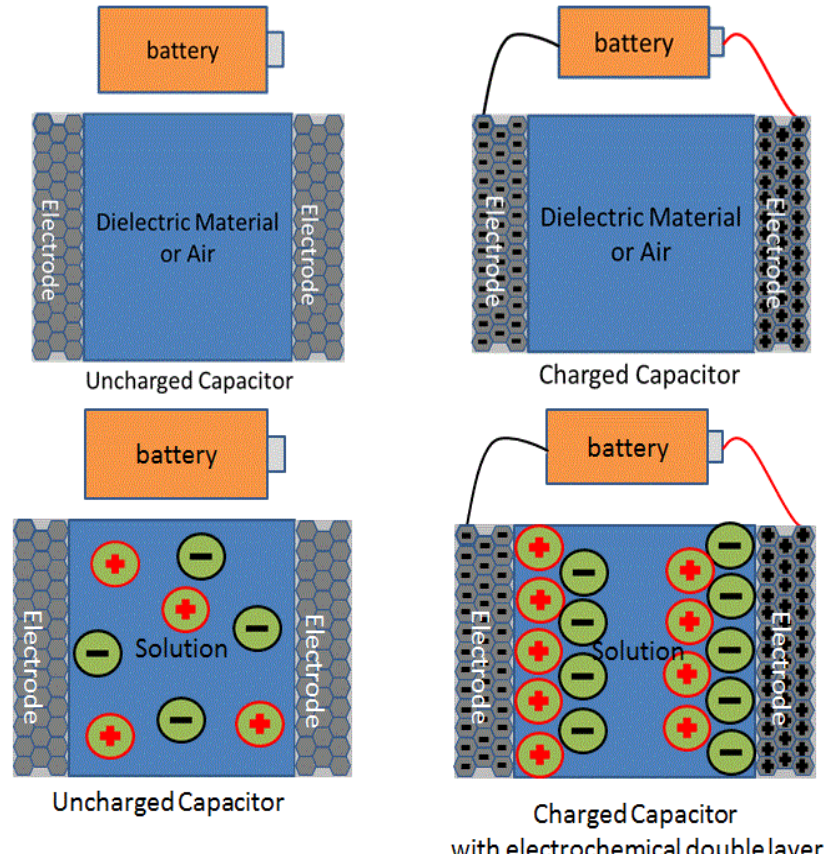
Equation:

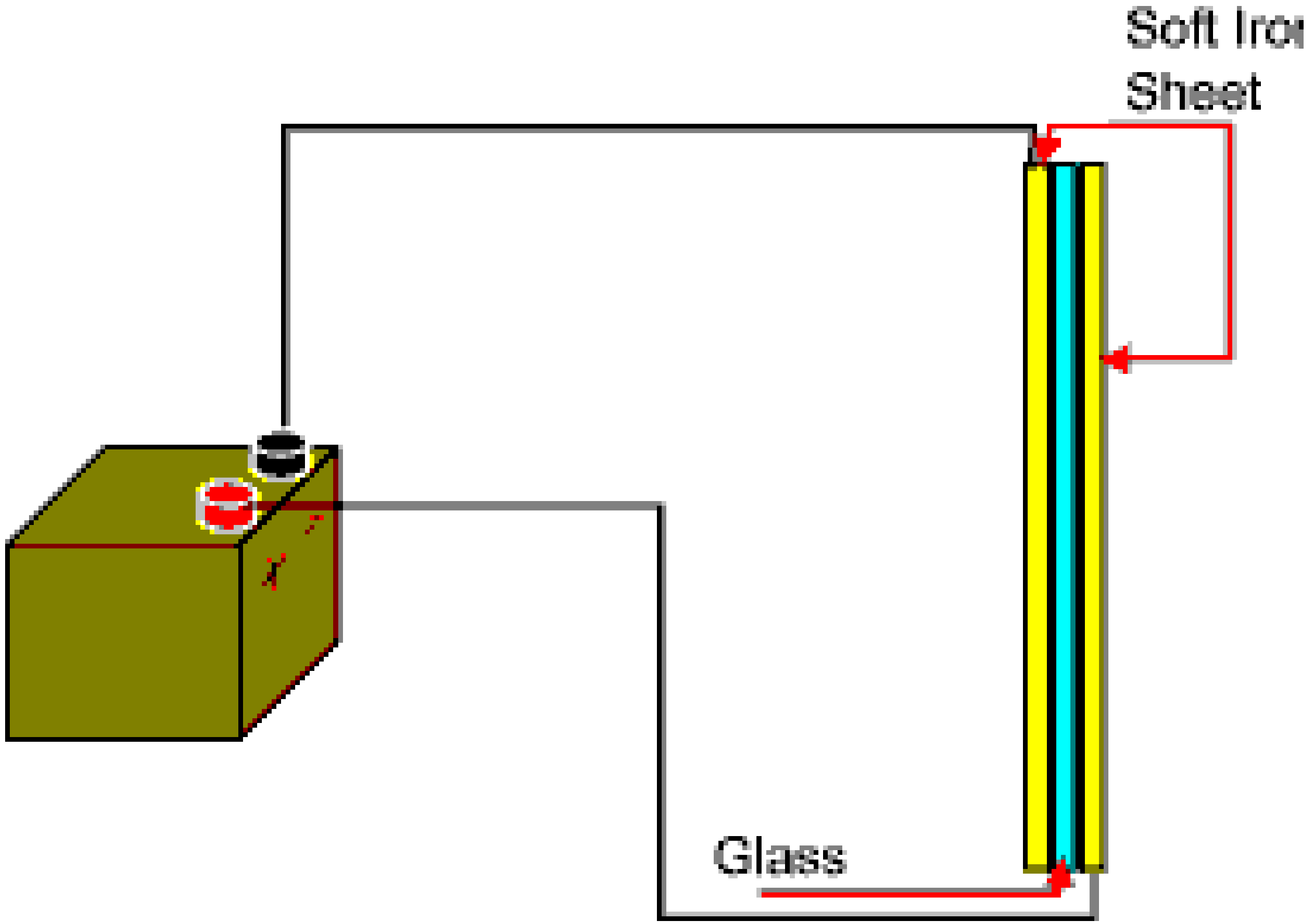
$$\boxed{C = \frac{Q}{V}}$$

C = farads (F)
Q = coulombs (C)
V = volts (V)

Symbol:

FIG. 10.11
Symbols for the capacitor: (a) fixed; (b) variable.




Soft Iron
Sheet
Glass

EXAMPLE 10.2 In Fig. 10.9, if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

Solutions:

a. In Fig. 10.9(a), the area has increased by a factor of three, providing more space for the storage of charge on each plate. Since the area appears in the numerator of the capacitance equation, the capacitance increases by a factor of three. That is,

$$C = 3(C_o) = 3(5\ \mu\text{F}) = \mathbf{15\ \mu F}$$

b. In Fig. 10.9(b), the area stayed the same, but the distance between the plates was increased by a factor of two. Increasing the distance reduces the capacitance level, so the resulting capacitance is one-half of what it was before. That is,

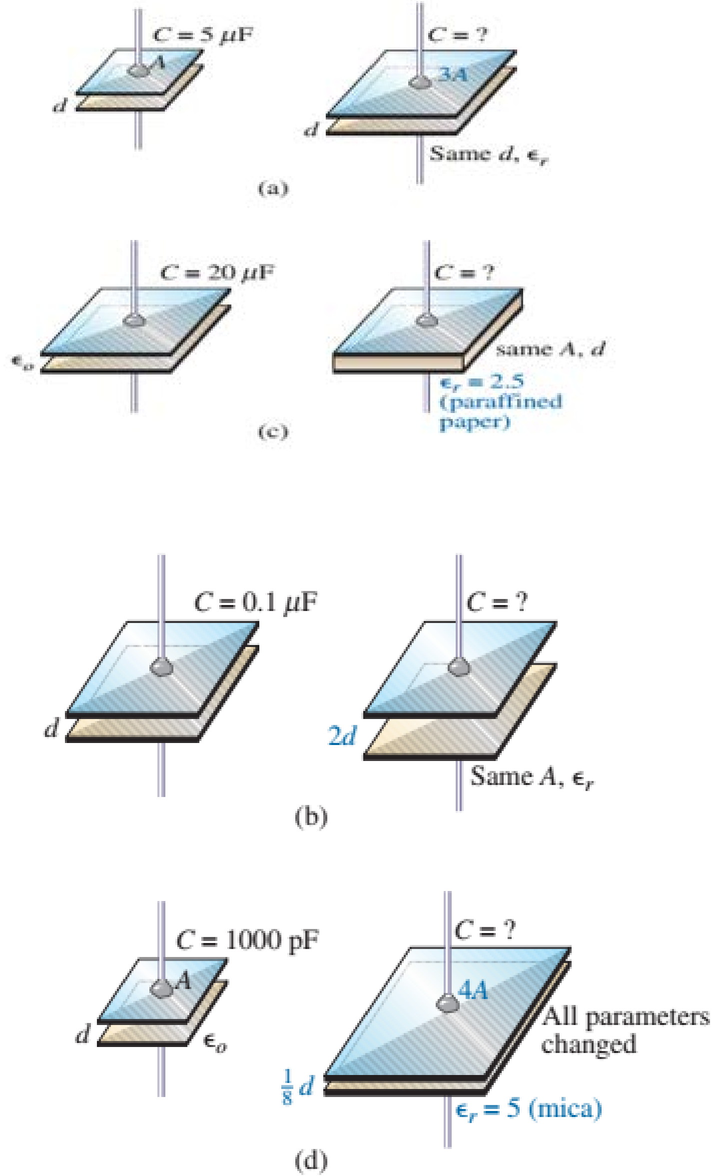
$$C = \frac{1}{2}(0.1\ \mu\text{F}) = \mathbf{0.05\ \mu F}$$

c. In Fig. 10.9(c), the area and the distance between the plates were maintained, but a dielectric of paraffined (waxed) paper was added between the plates. Since the permittivity appears in the numerator of the capacitance equation, the capacitance increases by a factor determined by the relative permittivity. That is,

$$C = \epsilon_r C_o = 2.5(20\ \mu\text{F}) = \mathbf{50\ \mu F}$$

d. In Fig. 10.9(d), a multitude of changes are happening at the same time. However, solving the problem is simply a matter of determining whether the change increases or decreases the capacitance and then placing the multiplying factor in the numerator or denominator of the equation. The increase in area by a factor of four produces a multiplier of four in the numerator, as shown in the equation below. Reducing the distance by a factor of 1/8 will increase the capacitance by its inverse, or a factor of eight. Inserting the mica dielectric increases the capacitance by a factor of five. The result is

$$C = (5)\frac{4}{(1/8)}(C_o) = 160(1000\ \text{pF}) = \mathbf{0.16\ \mu F}$$



Exercise Problems

3. Find the capacitance of a parallel plate capacitor if 1200 μC of charge are deposited on its plates when 10 V are applied across the plates.

Solution:

$$C = \frac{Q}{V} = \frac{1200\ \mu\text{C}}{10\ \text{V}} = \mathbf{120\ \mu F}$$

4. How much charge is deposited on the plates of a 0.15 μF capacitor if 45 V are applied across the capacitor?

Solution:

$$Q = CV = (0.15\ \mu\text{F})(45\ \text{V}) = \mathbf{6.75\ \mu C}$$

Definition: ***Inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.***

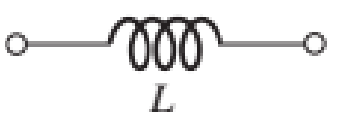
Unit: ***The unit of measure applied to inductors is the henries (H)***

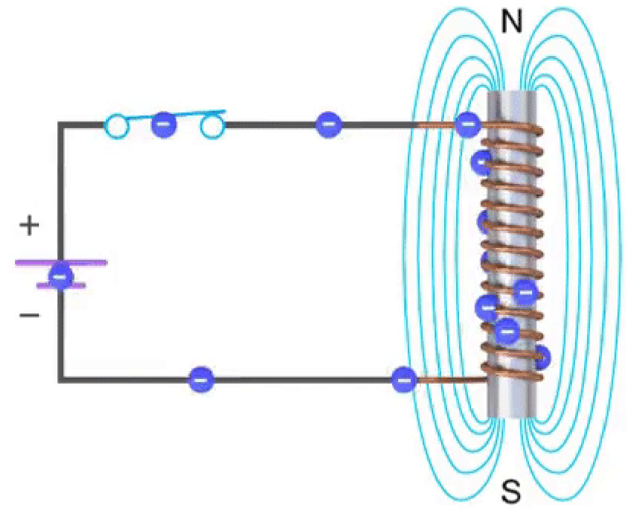
Equation:

$$\boxed{L = \frac{\mu N^2 A}{l}}$$

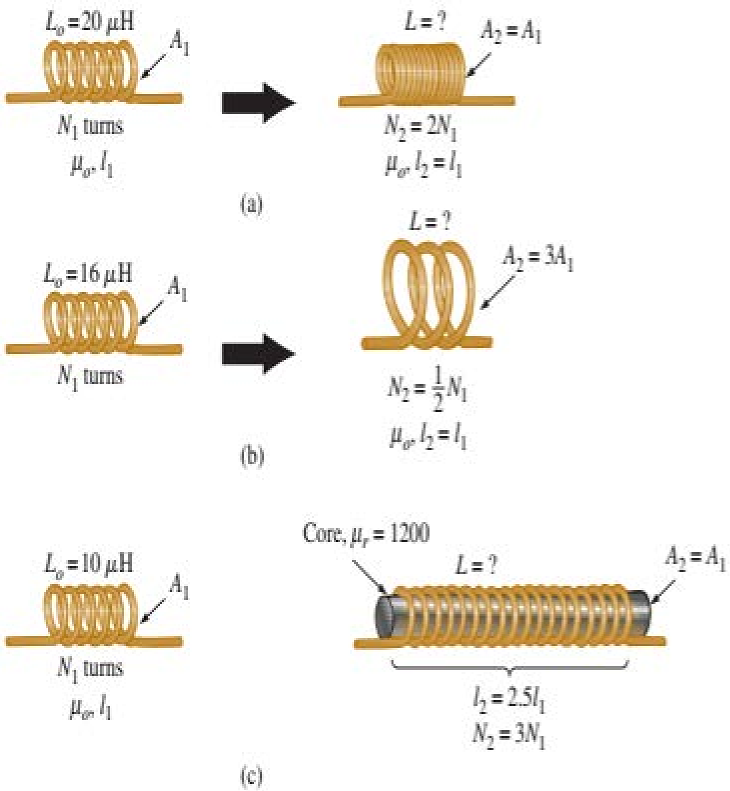
μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

Symbol:





EXAMPLE 11.2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.



Solutions:

a. The only change was the number of turns, but it is a squared factor, resulting in

$$L = (2)^2 L_o = (4)(20\ \mu\text{H}) = \mathbf{80\ \mu H}$$

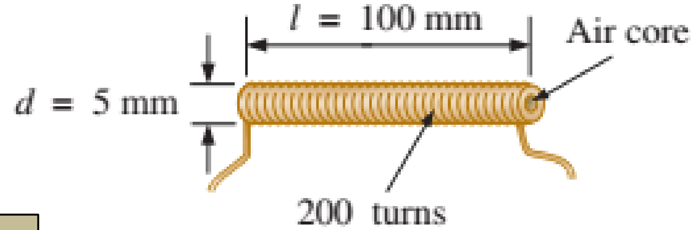
b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L = (3)\left(\frac{1}{4}\right)L_o = \frac{3}{4}(16\ \mu\text{H}) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32 \times 10^3)(10\ \mu\text{H}) = \mathbf{43.2\ mH}$$

2. For the inductor in Fig. 11.82, find the inductance L in henries.



Solution:

$$A = \frac{\pi d^2}{4} = \frac{\pi (5\text{ mm})^2}{4} = 19.63 \times 10^{-6}\text{ m}^2$$

$$L = \frac{N^2 \mu A}{\ell} = \frac{(200\text{ t})^2 (4\pi \times 10^{-7})(19.63 \times 10^{-6}\text{ m}^2)}{100\text{ mm}} = \mathbf{9.87\ \mu H}$$

3. Repeat Problem 2 with $l = 1.6$ in., $d = 0.2$ in., and a ferromagnetic core with $\mu_r = 500$.

Solution:

$$d = 0.2\ \cancel{\text{in.}}\left[\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right] = 5.08\text{ mm}$$

$$A = \frac{\pi d^2}{4} = \frac{(\pi)(5.08\text{ mm})^2}{4} = 20.27 \times 10^{-6}\text{ m}^2$$

$$\ell = 1.6\ \cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 40.64\text{ mm}$$

$$L = \frac{N^2 \mu_r \mu_o A}{\ell} = \frac{(200\text{ t})^2 (500)(4\pi \times 10^{-7})(20.27 \times 10^{-6}\text{ m}^2)}{40.64\text{ mm}} = \mathbf{12.54\ mH}$$


END