Introduction to Electrical Circuits

Mid Term
Lecture - 12

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Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

W6	C11	Chapter 13	13.2 SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS	13.1		Quiz/ Presentation
			13.5 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT	13.8-13.11		
			13.6 PHASE RELATIONS	13.12		
			13.7 AVERAGE VALUE	13.14-13.16		
			13.8 EFFECTIVE (rms) VALUES	13.20-13.24		
	C12	Chapter 14	14.6 COMPLEX NUMBERS	14.16-14.18		

Effective (rms) Values

The **effective** or **root-mean-square** (**rms**) value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

Effective or dc equivalent or RMS value can also be expressed by that steady current (DC) which transfer across any circuit, the **same amount of power dissipated as heat** as is transferred by that alternating current during the same time.

$$\boxed{I_{eq(dc)} = I_{eff} = 0.707 I_m}$$

$$\boxed{I_m = \sqrt{2} I_{eff} = 1.414 I_{eff}}$$

$$\boxed{E_{eff} = 0.707 E_m}$$

$$\boxed{E_m = \sqrt{2} E_{eff} = 1.414 E_{eff}}$$

Effective (rms) Values

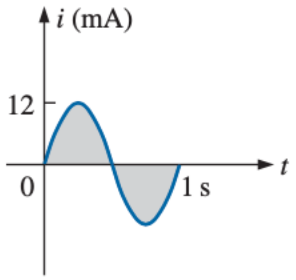
- The formula for power delivered by the ac supply at any time is:

$$P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

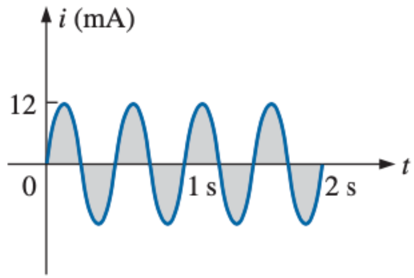
- The average power delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform.

Effective (rms) Values

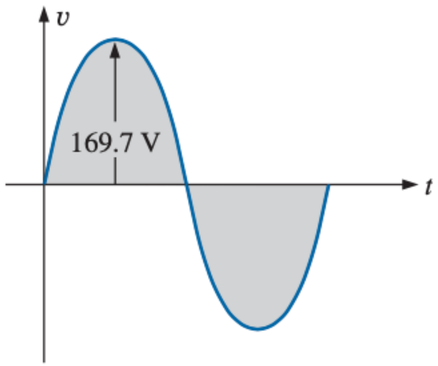
Example 13.20 Find the rms values of the sinusoidal waveform in each part in Fig. 13.60.



(a)



(b)

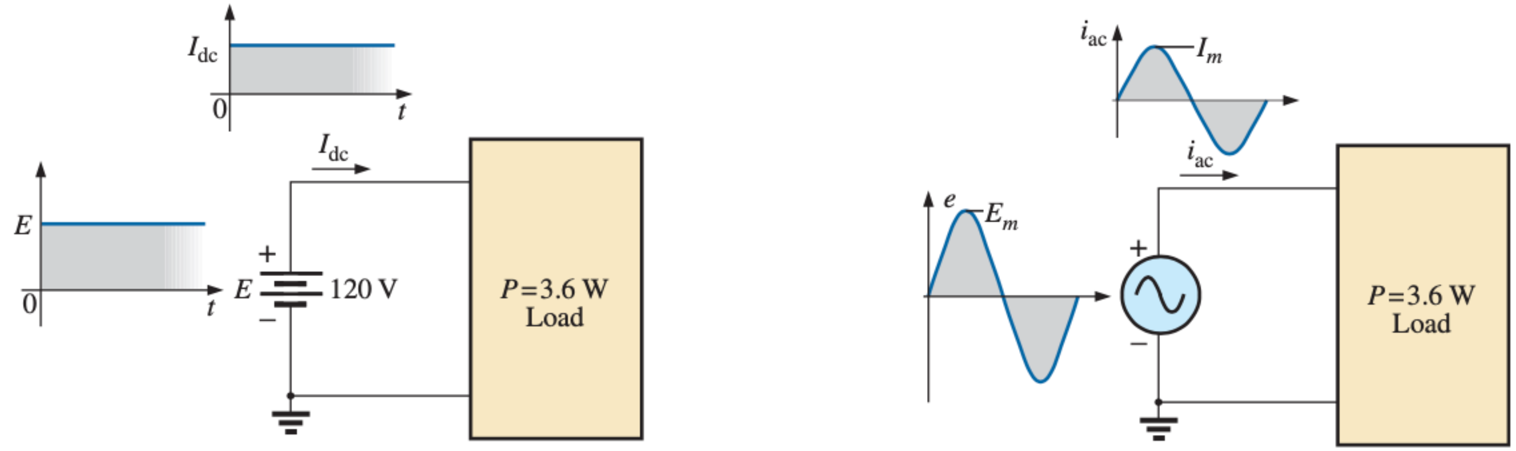


(c)

Solution: For part (a), $I_{\mathrm{rms}} = 0.707(12 \times 10^{-3}\ \mathrm{A}) =$ **8.48 mA.** For part (b), again $I_{\mathrm{rms}} =$ **8.48 mA.** Note that frequency did not change the effective value in (b) compared to (a). For part (c), $V_{\mathrm{rms}} = 0.707(169.73\ \mathrm{V}) \cong$ **120 V,** the same as available from a home outlet.

Effective (rms) Values

Example 13.21 The 120 V dc source in Fig. (a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. (b)] is to deliver the same power to the load.



(a) (b)

Solution:

$$P_{dc} = V_{dc}I_{dc}$$

and $$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2}I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\ mA}$$

$$E_m = \sqrt{2}E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\ V}$$

Effective (rms) Values

Example 13.22 Find the rms value of the waveform in Fig. 13.62.

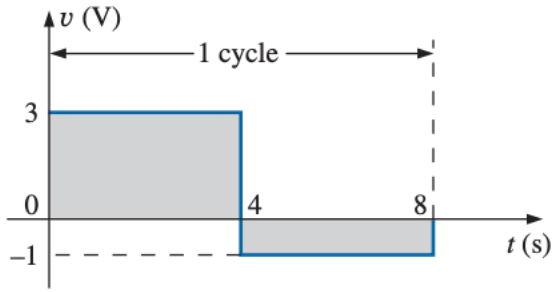


FIG. 13.62
Example 13.22.

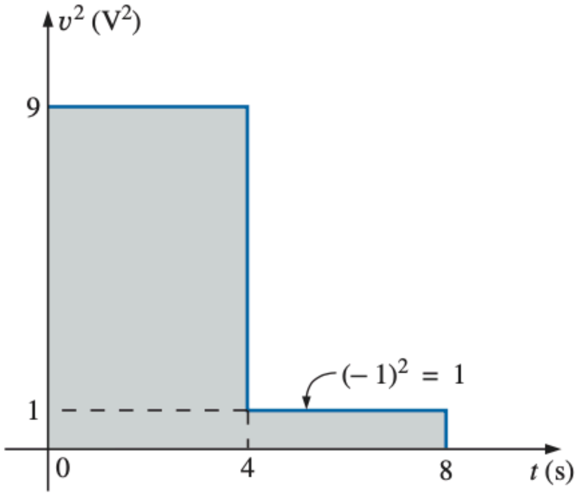


FIG. 13.63
The squared waveform of Fig. 13.62.

Solution: v^2 (Fig. 13.63):

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$

Effective (rms) Values

Example 13.24 Determine the average and rms values of the square wave in Fig. 13.66.

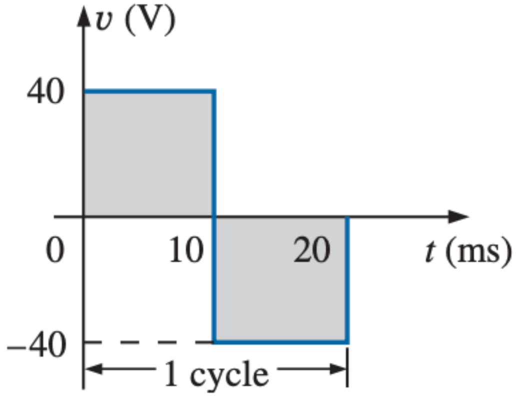


FIG. 13.66
Example 13.24.

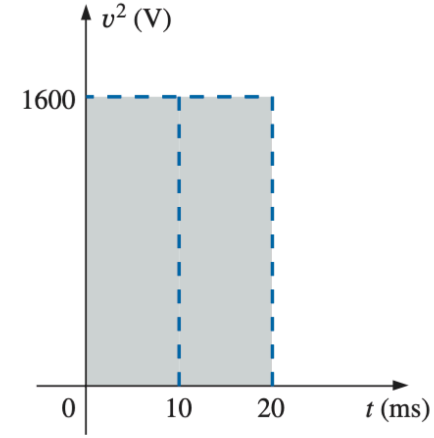


FIG. 13.67
The squared waveform of Fig. 13.66.

Solution: By inspection, the average value is zero.

v^2 (Fig. 13.67):

$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{(32{,}000 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{1600} = \mathbf{40\ V}$$

Complex Numbers

- A **complex number** represents a point in a two-dimensional plane located with reference to two distinct axes.

- This point can also determine a radius vector drawn from the origin to the point.

- The horizontal axis is called the *real* axis, while the vertical axis is called the *imaginary* axis.

Complex Numbers

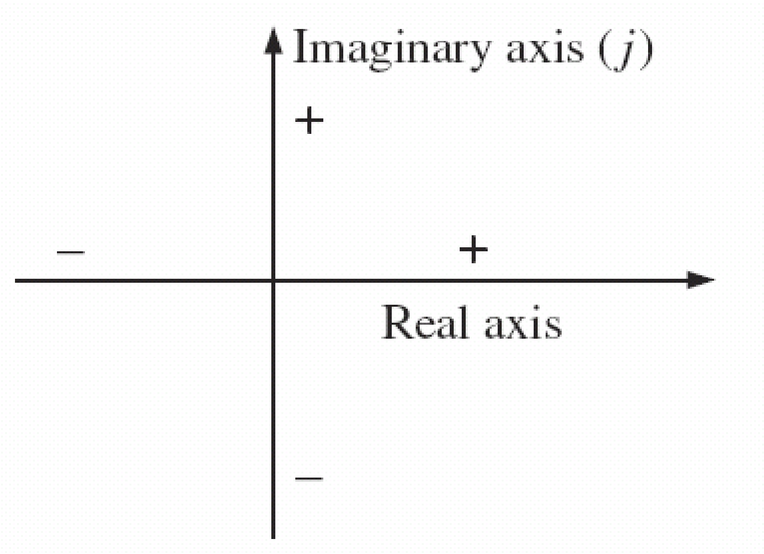


FIG. 14.38 *Defining the real and imaginary axes of a complex plane.*

Rectangular Form

- The format for the **rectangular form** is:

$$\boxed{\mathbf{C} = X + jY}$$

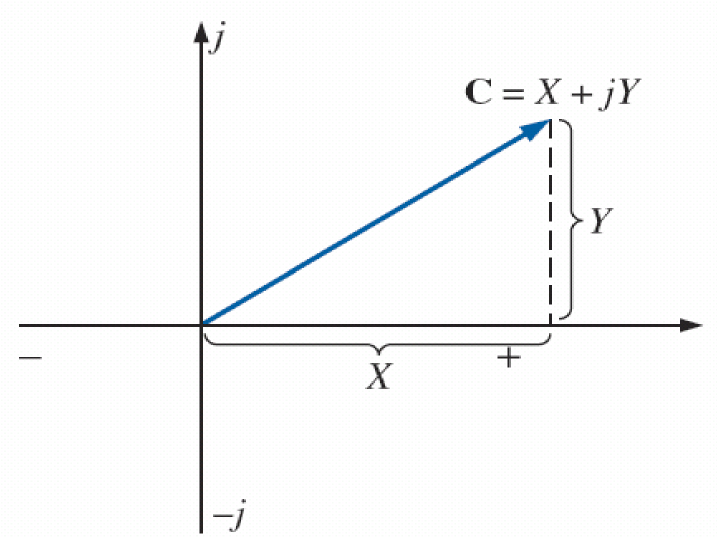


FIG. 14.39 *Defining the rectangular form.*

Polar Form

- The format for the **polar form** is:

$$\boxed{\mathbf{C} = Z \angle \theta}$$

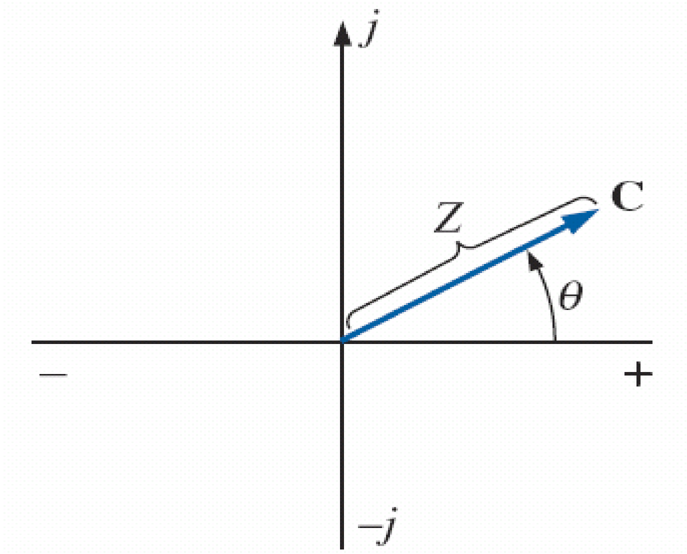


FIG. 14.43 *Defining the polar form.*

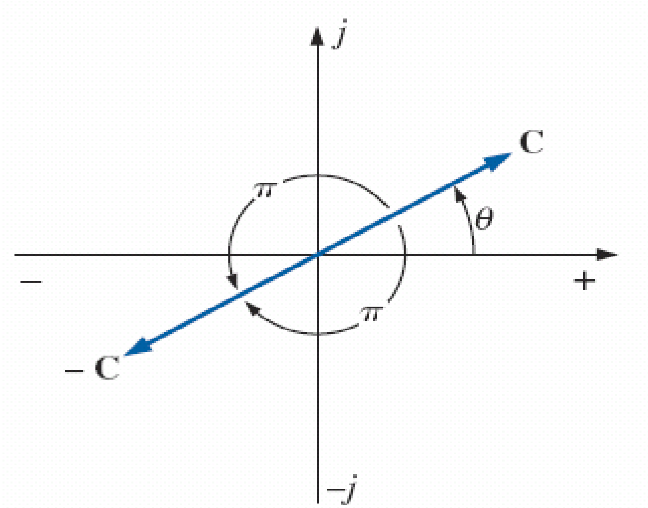


FIG. 14.44 *Demonstrating the effect of a negative sign on the polar form.*

Conversion Between Forms

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}}$$

Polar to Rectangular

$$\boxed{X = Z\cos\theta}$$

$$\boxed{Y = Z\sin\theta}$$

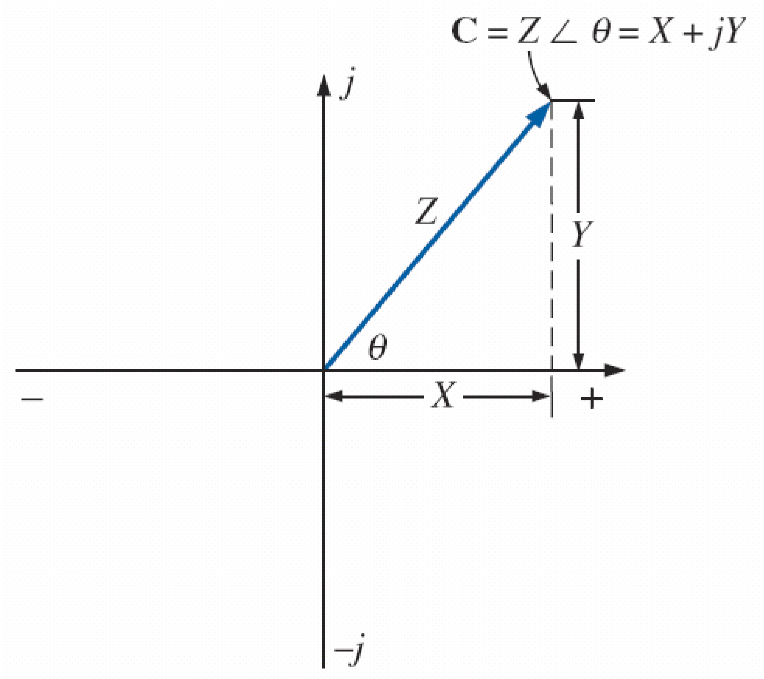


FIG. 14.48 *Conversion between forms.*

Conversion Between Forms

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j\,4 \qquad \text{(Fig. 14.46)}$$

Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

and $\quad \mathbf{C = 5 \angle 53.13^\circ}$

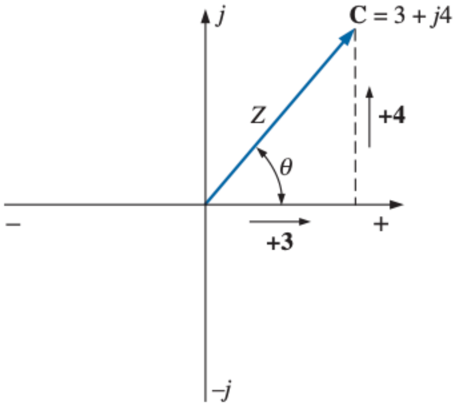


FIG. 14.46
Example 14.15.

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10 \angle 45^\circ \qquad \text{(Fig. 14.47)}$$

Solution:

$$X = 10 \cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10 \sin 45^\circ = (10)(0.707) = 7.07$$

and $\quad \mathbf{C = 7.97 + j\,7.07}$

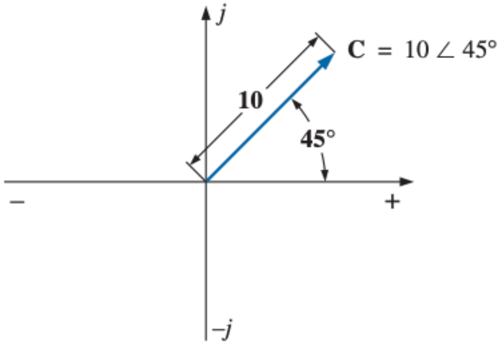


FIG. 14.47
Example 14.16.

Conversion Between Forms

EXAMPLE 14.17 Convert the following from rectangular to polar form:

$$\mathbf{C} = -6 + j\,3 \qquad \text{(Fig. 14.48)}$$

Solution:

$$Z = \sqrt{(6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\beta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57°$$

$$\theta = 180° - 26.57° = 153.43°$$

and $\mathbf{C} = \mathbf{6.71\angle 153.43°}$

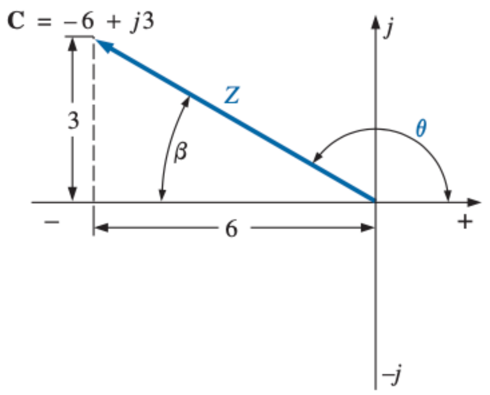


FIG. 14.48
Example 14.17.

EXAMPLE 14.18 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\angle 230° \qquad \text{(Fig. 14.49)}$$

Solution:

$$X = Z\cos\beta = 10\cos(230° - 180°) = 10\cos 50°$$

$$= (10)(0.6428) = 6.428$$

$$Y = Z\sin\beta = 10\sin 50° = (10)(0.7660) = 7.66$$

and $\mathbf{C} = \mathbf{-6.43 - j\,7.66}$

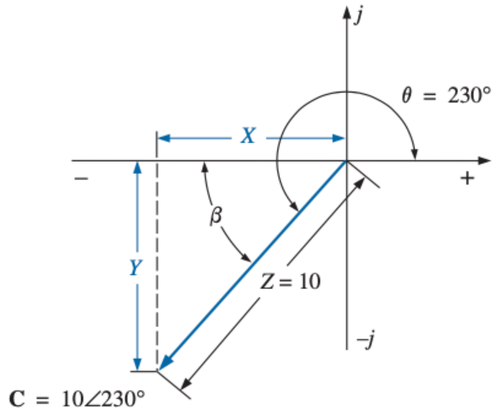


FIG. 14.49
Example 14.18.

Thank You