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Final Assignment: 03

Slide: 08

1. a) $A = \hat{x} zy^3 + \hat{y}\, 2y \sin(xy) + \hat{z}\, 3x^2 \ln z$

$\text{div} A = \nabla \cdot A = \frac{\partial}{\partial x}(zy^3) + \frac{\partial}{\partial y} \sin(xy) + \frac{\partial}{\partial z} 3x^2 \ln z$

$= 0 + 2\left(xy \cos(xy) + \sin xy\right) + 3x^2 \frac{1}{z}$

$= 2xy \cos(xy) + 2\sin(xy) + \frac{3x^2}{z}$

$\text{Curl} A = \nabla \times A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ zy^3 & 2y\sin xy & 3x^2 \ln z \end{vmatrix} = \hat{x}(0-0) - \hat{y}(6x\ln z - y^3) + \hat{z}(2y^2\cos(xy) - 3zy^2)$

$= -\hat{y}(6x\ln z - y^3) + \hat{z}(2y^2\cos(xy) - 3zy^2)$

$\therefore$ A is not solenoidal or conservative.

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b) $A = \hat{r}\frac{\sin\phi}{r} + \hat{\phi}\frac{\cos\phi}{r^2}$

$\operatorname{div} A = \nabla A = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(A_1 h_2 h_3) + \frac{\partial}{\partial u_2}(h_1 A_2 h_3) + \frac{\partial}{\partial u_3}(h_1 h_2 A_3)\right]$

$= \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{\sin\phi}{r}\, r\right) + \frac{\partial}{\partial\phi}\left(\frac{\cos\phi}{r^2}\right) + \frac{\partial}{\partial z}\cdot 0\right]$

$= -\frac{\sin\phi}{r^3}$

$\operatorname{curl} A = \nabla\times A = \begin{vmatrix} \hat{r} & \hat{\phi}\, r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \frac{\sin\phi}{r} & \frac{\cos\phi}{r^2}\, r & 0 \end{vmatrix} = \hat{r}.0 - \hat{\phi}\, r .0 + \hat{z}\left(-\frac{\cos\phi}{r^2} - \frac{\cos\phi}{r}\right)$

$= -\frac{\cos\phi\,(1+r)}{r^2}$

A is not solenoidal or conservative

c) $A = \hat{R}\cos\phi + \hat{\theta}(R - \sin\theta)$

$\operatorname{div} A = \nabla\cdot A = \frac{1}{R^2\sin\theta}\Big[\frac{\partial}{\partial R}(\cos\theta\cdot R^2\sin\theta) + \frac{\partial}{\partial\theta}\big((R-\sin\theta).R\sin\theta\big) +$
$\frac{\partial}{\partial\phi}(0\cdot R)\Big]$

$= \frac{2R\sin\theta\cos\theta}{R^2\sin\theta} + \frac{R^2\cos\theta}{R^2\sin\theta} - \frac{2R\sin\theta\cos\theta}{R^2\sin\theta}$

$= \cot\theta$

Curl A = $\nabla \times A = \begin{vmatrix} \hat{\rho} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \cos\theta & \rho(\rho \sin\theta) & \rho \sin\theta \cdot 0 \end{vmatrix}$

$= \hat{\rho} \cdot 0 - \hat{\theta} \cdot 0 + \hat{\phi}(2\rho \sin\theta + \sin\theta)$

$= 2\rho\hat{\phi}$

A is not solenoidal or conservative

2.a) $A = \hat{x}(\sin y + 1) + \hat{y}(2yz + x\cos y) + \hat{z}(y^2 - 3)$

$\bar{\nabla} \times A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin y + 1 & 2yz + x\cos y & y^2 - 3 \end{vmatrix} = \hat{x}(2y - 2y) - \hat{y}(0 - 0) + \hat{z}(\cos y - \cos y)$

$= 0$

A is conservative force field.

T(x, y, z) is a scalar potential of A

$A = \nabla T$

$T(x, y, z) = \int A \cdot dl$

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$$= \int [\hat{x}(\sin y + 1) + \hat{y}(2yz + x\cos y) + \hat{z}(y^2 - 3)] [\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz]$$

$$= x\sin y + x + y^2 z - 3z + C$$

Work done, ~~Point~~

$$\int_{(1,-1,5)}^{(2,\pi/2,1)} A \cdot dl = (x\sin y + x + y^2 z - 3z)\Big|_{(1,-1,5)}^{(2,\pi/2 \cdot 1)}$$

$$= 2 + 2 + \frac{\pi^2}{4} - 3 + 0.02 - 1 - 5 + 15$$

$$= 10.02 + \frac{\pi^2}{4}$$

b) $A = \hat{r}(2rz - \cos\phi) + \hat{\phi}\sin\phi + \hat{z}(r^2)$

$$\nabla \times A = \begin{vmatrix} \hat{r} & \hat{\phi} r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 2rz - \cos\phi & \sin\phi & r^2 \end{vmatrix} = \hat{r}(0-0) - \hat{\phi} r(2r - 2r) + \hat{z}(\sin\phi - \sin\phi)$$

$$= 0$$

A is conservative force field.

$T(r, \phi, z)$ is a scalar potential of A.

Here, $A = \nabla T$

$$T(r, \phi, z) = \int A \cdot dl$$

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$$= \int [\hat{r}(2rz - \cos\phi) + \hat{\phi}(\sin\phi) + \hat{z}(r^2)] [\hat{r}dr + \hat{\phi}\, r d\phi + \hat{z}dz]$$

$$= r^2 z - r\cos\phi + C$$

Work done:

$$\int_{(1,0,-1)}^{(1,\pi,0)} A \cdot dl$$

$$= \left(r^2 z - r\cos\phi\right)_{1,0,-1}^{1,\pi,0}$$

$$= 0 - \cos\pi + 1 + 1\cos 0$$

$$= 1 + 1 + 1$$

$$= 3$$

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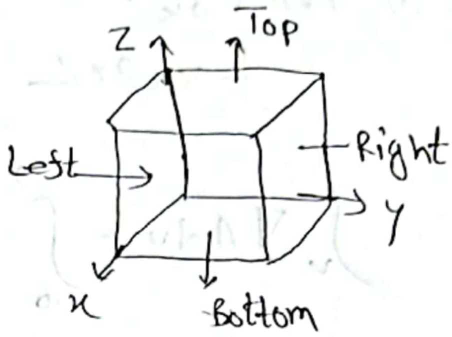
Slide: 09

1. $A = \hat{x}\, xy + \hat{y}\, y^2 z + \hat{z}\, xz$



a). $x = 2$

i) $ds = \hat{x}\, dy\, dz$

$$\int A \cdot ds = \int_0^2 \int_0^2 xy\, dy\, dz$$

$$= \int_0^2 \frac{xy^2}{2} \Big|_0^2 dz$$

$$= 2 \cdot xz \Big|_0^2$$

$$= 4x$$

$$= 4 \times 2 = 8$$

ii)

$x = 0, \quad ds = -\hat{x}\, dy\, dz$

$$\int A \cdot ds = 0$$

iii) $y = 2, \; ds = \hat{y}\, dx\, dz$

$$\int A \cdot ds = \int_0^2 \int_0^2 y^2 z\, dx\, dz$$

$$= 2 \frac{y^2 z^2}{2} \Big|_0^2$$

$$= 16$$

b) $\nabla \cdot A = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(y^2 z) + \frac{\partial}{\partial z}(xy)$

$= y + 2yz$

$$\int_v \nabla \cdot A \, dv = \int_0^2 \int_0^2 \int_0^2 (y + 2yz) \, dx\, dy\, dz$$

$$= \int_0^2 \int_0^2 xy + 2xyz \Big|_0^2 dy\, dz$$

$$= \int_0^2 2 \cdot \frac{y^2}{2} + 4 \cdot \frac{y^2}{2} z \Big|_0^2 dz$$

$$= 4z + \frac{8z^2}{2} \Big|_0^2$$

$$= 8 + 16 = 24$$

$$\oint A \cdot ds = \int_v \nabla \cdot A \, dv$$

So, the divergence theorem is verified.

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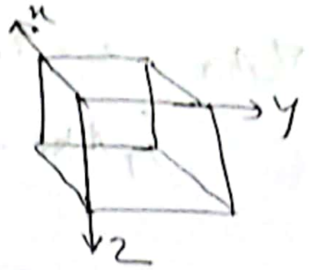
2. $A = \hat{x}x^2 + \hat{y}xy + \hat{z}yz$



$x = 1, \quad ds = \hat{x}\,dy\,dz$

$\int_S A\,dv = \int_0^1 \int_0^1 x^2\,dy\,dz$

$= x^2 z \Big|_0^1$

$= 1$

$x = 0, \quad ds = -\hat{x}\,dy\,dz$

$\int_b A \cdot ds = 0$

$y = 1, \quad ds = \hat{y}\,dx\,dz$

$\int_r A\,ds = \int_0^1 \int_0^1 xy\,dx\,dz$

$= \int_0^1 \frac{x^2 y}{2} \Big|_0^1 dz$

$= \frac{1}{2}$

$y = 0, \quad ds = -\hat{y}\,dx\,dz$

$\int_l A \cdot ds = 0$

For $z = 1$, $ds = \hat{z}\, dx\, dy$

$$\int_T A \cdot ds = \int_0^1 \int_0^1 y z \, dx\, dy$$

$$= y^2/2 \Big|_0^1$$

$$= \frac{1}{2}$$

$z = 0$, $ds = -\hat{z}\, dx\, dy$

$$\int_{b_0} A \cdot ds = 0$$

$$\therefore \oint_S A \cdot ds = 1 + 0 + \frac{1}{2} + 0 + \frac{1}{2} + 0 = 2$$

$$\nabla A = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(yz) = 3x + y$$

$$\int_V \nabla \cdot A\, dv = \int_0^1 \int_0^1 \int_0^1 (3x + y)\, dx\, dy\, dz$$

$$= \int_0^1 \int_0^1 \left(\frac{3x^2}{2} + xy\right)\Big|_0^1 dy\, dz$$

$$= \frac{3z}{2} + \frac{z}{2} \Big|_0^1$$

$$= 2$$

$$\therefore \oint A \cdot dS = \int_V \nabla \cdot A\, dv$$

$\therefore$ The divergence theorem is verified.

(Answer)