Name: Wafinur Leo

Id : 20-42195-1

Math Assignment: 02

Slide: 04

1. $x+2y+4z=-13$

$3x-y+z=5$

$x+3y=3$

Here, $A=\begin{bmatrix}1 & 2 & 4\\ 3 & -1 & 1\\ 1 & 3 & 0\end{bmatrix}$, $X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$, $B=\begin{bmatrix}-13\\ 5\\ 3\end{bmatrix}$

$AX=B$

$X=A^{-1}B$

$$=\begin{bmatrix}-1/13 & 4/13 & 2/13\\ 1/39 & -4/39 & 11/39\\ 10/39 & -1/39 & -7/39\end{bmatrix}\begin{bmatrix}-13\\ 5\\ 3\end{bmatrix}$$

~~Usicang~~

[Using Calculator]

$$=\begin{bmatrix}3\\ 0\\ -4\end{bmatrix}$$

So, $X_1=3$, $X_2=0$, $X_3=-4$

2. $3x+y+2z=14$

$2y+5z=22$

$2x+5y-z=-22$

Here, $AX=B$

$$A=\begin{bmatrix}3 & 1 & 2\\ 0 & 2 & 5\\ 2 & 5 & -1\end{bmatrix},\quad X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix},\quad B=\begin{bmatrix}14\\ 22\\ -22\end{bmatrix}$$

$$X=A^{-1}B$$

$$=\begin{bmatrix}3 & 1 & 2\\ 0 & 2 & 5\\ 2 & 5 & -1\end{bmatrix}^{-1}\begin{bmatrix}14\\ 22\\ -22\end{bmatrix}$$ [Using calculator]

$$=\begin{bmatrix}2\\ -4\\ 6\end{bmatrix}$$

So, $X_1=2$, $X_2=-4$, $X_3=6$

3. $-3x+2y-3z=-8$

$2x-y+z=4$

$x+2y-4z=-2$

Here, $AX=B$

$$A=\begin{bmatrix}-3 & 2 & -3\\ 2 & -1 & 1\\ 1 & 2 & -4\end{bmatrix},\ X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix},\ B=\begin{bmatrix}-8\\ 4\\ -2\end{bmatrix}$$

$$X=A^{-1}B$$

$$=\begin{bmatrix}-3 & 2 & -3\\ 2 & -1 & 1\\ 1 & 2 & -4\end{bmatrix}\begin{bmatrix}-8\\ 4\\ -2\end{bmatrix}$$

$$=\begin{bmatrix}2\\ 2\\ 2\end{bmatrix}$$

So, $X_1=2$, $X_2=2$, $X_3=2$

Slide: 05

1. $x + 2y + 4z = -13$

$3x - y + z = 5$

$x + 3y = 3$

Using Calculator:-

$$D = \begin{vmatrix} 1 & 2 & 4 \\ 3 & -1 & 1 \\ 1 & 3 & 0 \end{vmatrix} = 39.$$

$$D_x = \begin{vmatrix} -13 & 2 & 4 \\ 5 & -1 & 1 \\ 3 & 3 & 0 \end{vmatrix} = 117$$

$$D_y = \begin{vmatrix} 1 & -13 & 4 \\ 3 & 5 & 1 \\ 1 & 3 & 0 \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} 1 & 2 & -13 \\ 3 & -1 & 5 \\ 1 & 3 & 3 \end{vmatrix} = -156$$

$$X = \frac{117}{39} = 3$$

$$Y = \frac{0}{39} = 0$$

$$Z = \frac{-156}{39} = -4$$

Scanned with CamScanner

2. $3x+y+2z=19$

$2y+5z=22$

$2x+5y-z=-22$

Using Calculator: -

$$D=\begin{vmatrix} 3 & 1 & 2 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{vmatrix}=-79 \qquad D_x=\begin{vmatrix} 19 & 1 & 2 \\ 22 & 2 & 5 \\ -22 & 5 & -1 \end{vmatrix}=-158$$

$$D_y=\begin{vmatrix} 3 & 19 & 2 \\ 0 & 22 & 5 \\ 2 & -22 & -1 \end{vmatrix}=448 \qquad D_z=\begin{vmatrix} 3 & 1 & 19 \\ 0 & 2 & 22 \\ 2 & 5 & -22 \end{vmatrix}=-514$$

$$x=\frac{-158}{-79}=2$$

$$y=\frac{448}{-79}=-4$$

$$z=\frac{-514}{-79}=6$$

Scanned with CamScanner

3. $-3x+2y-3z=-8$

$2x-y+z=4$

$x+2y-4z=-2$

Using calculator:

$$D=\begin{vmatrix}-3 & 2 & -3\\ 2 & -1 & 1\\ 1 & 2 & -4\end{vmatrix}=-3 \qquad D_x=\begin{vmatrix}-8 & 2 & -3\\ 4 & -1 & 1\\ -2 & 2 & -4\end{vmatrix}=-6$$

$$D_y=\begin{vmatrix}-3 & -8 & -3\\ 2 & 4 & 1\\ 1 & -2 & -4\end{vmatrix}=-6 \qquad D_z=\begin{vmatrix}-3 & 2 & -8\\ 2 & -1 & 4\\ -1 & 2 & -2\end{vmatrix}=-6$$

$$X=\frac{-6}{-3}=2$$

$$Y=\frac{-6}{-3}=2$$

$$Z=\frac{-6}{-3}=2$$

Scanned with CamScanner

4. $x - 3z = -3$

$2x + \lambda y - z = -2$

$x + 2y + \lambda z = 1$

Here,

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -3 \\ 2 & \lambda & -1 & -2 \\ 1 & 2 & \lambda & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & -3 & -3 \\ 0 & \lambda & 5 & 4 \\ 0 & 2 & \lambda+3 & 4 \end{array}\right] \begin{array}{l} r_2 = r_2 - 2r_1 \\ r_3 = r_3 - r_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -3 & -3 \\ 0 & \lambda & 5 & 4 \\ 0 & 0 & \lambda(\lambda+3)-10 & 4\lambda-8 \end{array}\right] \quad r_3 = \lambda r_3 - 2r_2$$

For, no solution, $\lambda = \frac{10}{\lambda} - 3$ and $\lambda \neq \frac{8}{4}$

For more than one solution, $\lambda = \frac{10}{\lambda} - 3$ and $\lambda = \frac{8}{4}$

For a unique solution $\lambda \neq \frac{10}{\lambda} - 3$

5. $x+y+z=2$

$x+3y+\lambda z=6$

$x+2y+3z=\mu$

Here,
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 3 & \lambda & 6 \\ 1 & 2 & 3 & \mu \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & \lambda-1 & 4 \\ 0 & 1 & 2 & \mu-2 \end{array}\right] \begin{array}{l} r_2 = r_2 - r_1 \\ r_3 = r_3 - r_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & \lambda-1 & 4 \\ 0 & 0 & 3-\lambda & 2\mu-8 \end{array}\right] \quad r_3 = 2r_3 - r_2$$

For no solution $\lambda = 3$ and $\mu \neq 4$

For more than one solution: $\lambda = 3$ and $\mu = 4$

For unique solution $\lambda \neq 3$

6. $x + y + \lambda z = 1$

$x + \lambda y + z = \lambda$

$\lambda x + y + z = \lambda^2$

Here, $\left[\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 1 & \lambda & 1 & \lambda \\ \lambda & 1 & 1 & \lambda^2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & -1+\lambda & 1-\lambda & \lambda-1 \\ \lambda-1 & 1-\lambda & 0 & \lambda-\lambda^2 \end{array}\right]$

$r_2 = r_2 - r_1$

$r_3 = r_3 - r_2$

$= \left[\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & 1 & -1 & 1 \\ 1 & -1 & 0 & -\lambda \end{array}\right]$ $r_2 = \dfrac{r_2}{\lambda-1}$, $r_3 = \dfrac{r_3}{\lambda-1}$

$= \left[\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & 1-\lambda \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & \lambda \end{array}\right]$

For, no solution $\lambda = 0$ and $\lambda \neq 1$

For more than one solution $\lambda = 0$ and $\lambda = 1$

For unique solution $\lambda \neq 0$

Scanned with CamScanner

Slide: 06A

1. $x_4 + x_5 = 500$

$x_1 + x_5 = 1000$

$x_1 + x_2 = 800$

$x_2 + x_4 = 300 + x_3$

$\Rightarrow x_1 + x_4 - x_3 = 300$

So,

$$\begin{aligned} x_4 + x_5 &= 500 \\ x_1 \qquad + x_5 &= 1000 \\ x_1 + x_2 \qquad &= 800 \\ x_2 - x_3 + x_4 \quad &= 300 \end{aligned}$$

Here,

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & 1 & 500 \\ 1 & 0 & 0 & 0 & 1 & 1000 \\ 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \end{array}\right] = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1000 \\ 0 & 1 & 0 & 0 & -1 & -200 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{array}\right]$$

Now, $x_3 = 0$

$x_1 + x_5 = 1000 \Rightarrow x_1 = 1000 - x_5$

$x_2 - x_5 = -200 \Rightarrow x_2 = -200 + x_5$

$x_4 + x_5 = 500 \Rightarrow x_4 = 500 - x_5$

$\therefore 200 \leq x_5 \leq 500$

2. $x_1+x_2=800$

$x_2+x_3=500$

$x_1+400=x_3+700$

$\Rightarrow x_1-x_3=300$

So, $x_1+x_2=800$

$x_2+x_3=500$

$x_1-x_3=300$

Here,

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 800 \\ 1 & 0 & -1 & 300 \\ 0 & 1 & 1 & 500 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 300 \\ 0 & 1 & 1 & 500 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_3=t$

Now, $x_1-t=300 \Rightarrow x_1=300+t$

$x_2+t=500 \Rightarrow x_2=500-t$

$\therefore t \le 500$

Scanned with CamScanner

Slide: 07

3.1

1. a) $f(x) = \sin 5x$

period of $\sin x = 2\pi$

Now, $\quad \sin 2\pi = \sin 5x$

$\Rightarrow 5x = 2\pi$

$\Rightarrow x = \frac{2\pi}{5}$

b) $f(x) = \tan 7x$

period of $\tan x = \pi$

Now, $\quad \tan 7x = \cancel{\pi} \tan \pi$

$\Rightarrow 7x = \pi$

$\Rightarrow x = \frac{\pi}{7}$

c) $f(x) = \cos^2 x$

period of $\cos x = 2\pi$

Now, $\cos^2 x = \cancel{2\pi} (\cos 2\pi)^2$

$\Rightarrow \frac{1}{2}(1 + \cos 2x) = \cos 2\pi$

$\Rightarrow x = \pi$

d) $f(x) = \sin^2 x - \cos^2 x = \cos 2x$

Period of ~~cos 2x~~ $\cos x = 2\pi$

Now, $\cos 2x = \cos 2\pi$

$\Rightarrow 2x = 2\pi$

$\Rightarrow x = \pi$

e) $f(x) = \sin 2x + \cos 3x$

Period of $\sin 2x = \pi$

$\cos 3x = \frac{2\pi}{3}$

~~Now, period =~~ $\cancel{\pi + \frac{2\pi}{3}}$

$\cancel{= \frac{3\pi}{3}}$

Now, period combine $\pi, \frac{2\pi}{3}$

$= 2\pi$

2. a) $f(x) = \sin x$

$f(-x) = -\sin x$

$\therefore f(x) = -f(x)$

$\therefore$ odd function

b) $f(x) = x^3 - x$

$\therefore f(-x) = (-x)^3 - (-x)$

$= -(x^3 - x)$

$\therefore f(x) = -f(-x)$

$\therefore$ odd function

c) $f(x) = \tan x$

$\therefore f(-x) = -\tan x$

$\therefore f(x) = -f(-x)$

$\therefore$ odd function

d) $f(x) = e^{x^2}$

$\therefore f(-x) = (e^{(-x)})^2$

$= e^{x^2}$

$\therefore f(x) = f(-x)$

$\therefore$ even function

e) $f(x) = \cos x$

$\therefore f(-x) = \cos(-x) = \cos x$

$\therefore f(x) = f(-x)$

$\therefore$ even function

f) $f(x) = \frac{e^{3x} + e^{3x}}{2}$

$\therefore f(-x) = \frac{e^{-3x} + e^{-(-3x)}}{2}$

$= \frac{e^{3x} + e^{-3x}}{2}$

$\therefore f(x) = f(-x)$

$\therefore$ even function

g) $f(x) = \frac{e^{x} - e^{-x}}{2}$

$\therefore f(-x) = \frac{e^{-x} - e^{-(-x)}}{2}$

$= -\frac{e^{-x} + e^{x}}{2}$

$\therefore f(x) = f(-x)$

$\therefore$ odd function

h) $f(x) = x^3$

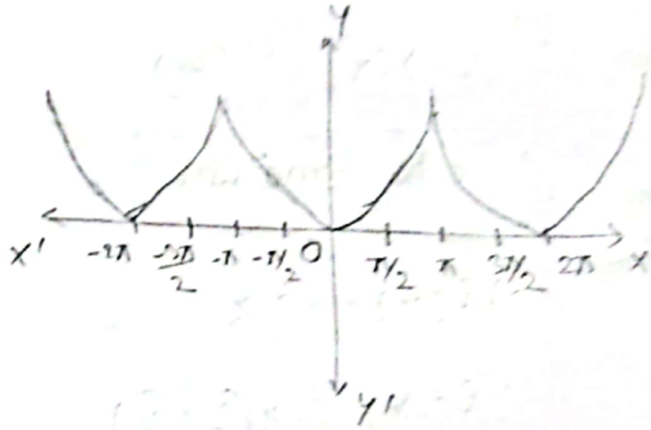
$\therefore f(-x) = (-x)^3 = -x^3$

$\therefore f(x) = -f(-x)$

$\therefore$ odd function

## 3.2

1. $f(x) = x^2 \; ; \quad -\pi \leq x \leq \pi$




Here, interval distance

$$= \pi - (-\pi)$$

$$= \pi + \pi$$

$$= 2\pi$$

Here, $T = 2\pi = 2L$

$$\therefore L = \pi$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\,dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} x^2\,dx$$

$$= \frac{2}{\pi} \left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi} \times \frac{\pi^3}{3}$$

$$= \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x^2 \cos\left(\frac{n\pi x}{\pi}\right) dx$$

$$= \frac{4(-1)^n}{n^2} \quad \text{[using calculator]}$$

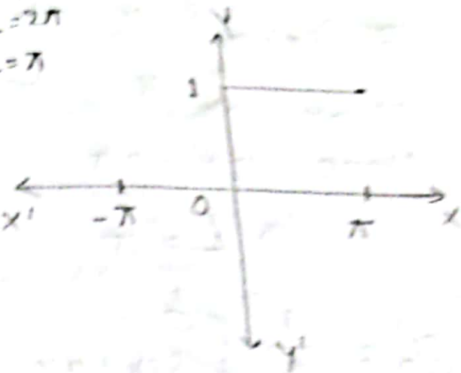
$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin\left(\frac{n\pi x}{\pi}\right) dx = 0$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$

2. $f(x) = \begin{cases} 0 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \end{cases}$ ; Here, $2L = 2\pi$ $\Rightarrow L = \pi$




$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi} \int_{0}^{\pi} 1\,dx$$

$$= 1$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} 0 \cos(nx)\,dx + \int_{0}^{\pi} \cos(nx)\,dx$$

$$= 0$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx$$

$$= \frac{-(-1)^n + 1}{n\pi}$$

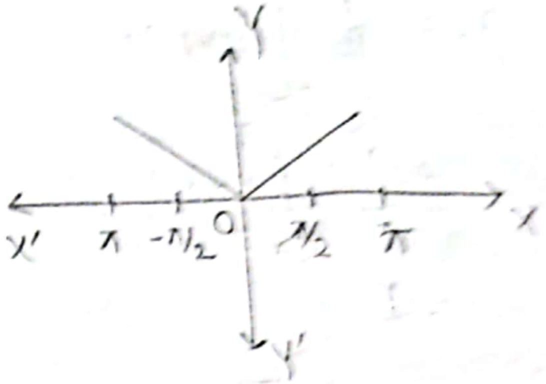
$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} -\frac{1}{n\pi} \left[(-1)^n - 1\right] \sin(nx)$$

3. $f(x) = \begin{cases} -x & \text{when } -\pi < x < 0 \\ x & \text{when } 0 < x < \pi \end{cases}$

Here, $2L = 2\pi$

$\Rightarrow L = \pi$




$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-x)\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$= \pi$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-x)\cos(nx)\,dx + \int_{0}^{\pi} x\cos(nx)\,dx\right]$$

$$= \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-x)\sin(nx)\,dx + \int_{0}^{\pi} x\sin(nx)\,dx\right]$$

$$= 0$$

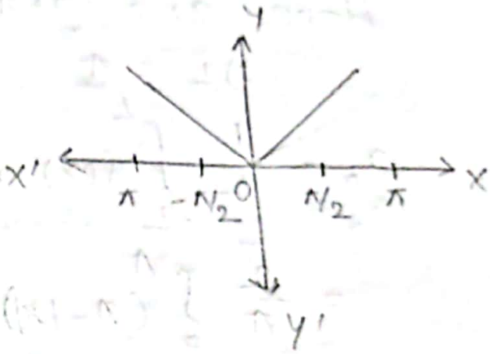
$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left[(-1)^n - 1\right]\cos(nx)$$

9. $f(x) = |x| \; ; \; -\pi \le x \le \pi$

Here, $2L = 2\pi$

$\Rightarrow L = \pi$




$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} |x|\,dx \;\; [\text{even function}]$$

$$= \pi$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{\pi}\int_0^{\pi} |x|\cos(nx)\,dx \;\; [\text{even function}]$$

$$= \frac{2\,[(-1)^n - 1]}{\pi n^2}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{\pi}\int_0^{\pi} |x|\sin(nx)\,dx \;\; [\text{odd function}]$$

$$= 0$$

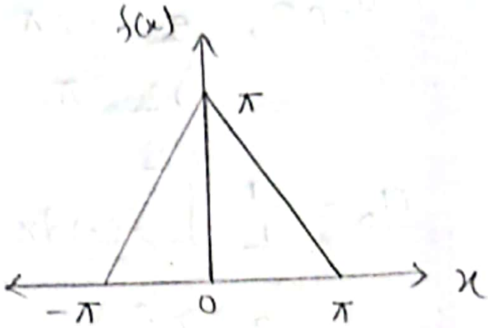
$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2\,[(-1)^n - 1]}{\pi n^2}\cos(nx)$$

Ex: (i) $f(x) = \pi - |x|$

Here, $2L = (\pi + \pi)$

$\Rightarrow L = \pi$




$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi}(\pi - |x|)\,dx$$

$$= \pi$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi}(\pi - |x|)\cos(nx)\,dx \quad [\text{even}]$$

$$= -\frac{2[(-1)^n + 1]}{\pi n^2}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{\pi n x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(\pi - |x|)\sin(nx)\,dx \quad [\text{odd}]$$

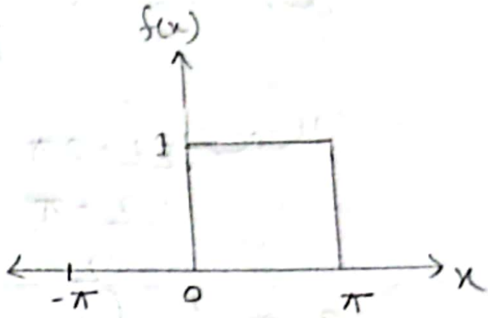
$$= 0$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} -\frac{2}{\pi n^2}[(-1)^n + 1]$$

b) Here, $f(x) = \pi - |x|$

$$2L = (\pi + \pi)$$
$$\Rightarrow L = \pi$$




$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} 1\,dx$$

$$= 1$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(\pi - x)\cos(nx)\,dx$$

$$= 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(\pi - |x|)\sin(nx)\,dx$$

$$= -\frac{1}{n\pi}\left[(-1)^n - 1\right]$$

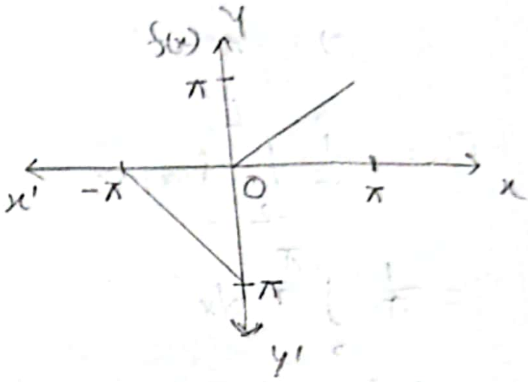
$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} -\frac{1}{nx}\left[(-1)^n - 1\right]\sin(nx)$$

c) ~~Here~~. $f(x) = \begin{cases} x \ ; & 0 < x < \pi \\ -x-\pi \ ; & -\pi < x < 0 \end{cases}$

Here, $2L = 2\pi$

$\therefore L = \pi$




$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} (-x-\pi)\, dx + \int_{0}^{\pi} x\, dx \right]$$

$$= 0$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} (-x-\pi) \cos(nx)\, dx + \int_{0}^{\pi} x \cos(nx)\, dx \right]$$

$$= \frac{2\left[(-1)^n + 1\right]}{\pi n^2}$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} (-x-\pi) \sin(nx)\, dx + \int_{0}^{\pi} x \sin(nx)\, dx \right]$$

$$= -\frac{(-1)^n - \pi}{\pi n}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ \left( a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right) \right]$$

$$= \sum_{n=1}^{\infty} \frac{2\left[(-1)^n + 1\right]}{\pi n^2} - \frac{(-1)^n - \pi}{\pi n}$$