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Final Assignment: 02

Slide: 05

1. a) $P_1 = (-1, \sqrt{3}, -2\sqrt{3})$

Cartesian to cylindrical,

$x = -1,\ y = \sqrt{3},\ z = -2\sqrt{3}$

$r = +\sqrt{(-1)^2 + (\sqrt{3})^2}$

$= +\sqrt{4}$

$= 2$

$\phi = \pi - \tan^{-1}(y/x)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$= \pi + \pi/3 = \frac{4\pi}{3}$

$z = -2\sqrt{3}$

Cartesian to spherical.

$R\ +\sqrt{(-1)^2 + (\sqrt{3})^2 + (-2\sqrt{3})^2}$

$= 4$

$\phi = \pi - \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$= \pi + \pi/3 = \frac{4\pi}{3}$

$\theta = \tan^{-1}\left(\frac{+\sqrt{(-1)^2 + (\sqrt{3})^2}}{-2\sqrt{3}}\right)$

$= 20.1$

b) $P_2 = (4, 0, -4)$

Cartesian to cylindrical,

$x = 4,\ y = 0,\ z = -4$

$r = +\sqrt{4^2 + 0^2}$

$= 4$

$\phi = \tan^{-1}(0/4)$

$= 0$

$z = -4$

Cartesian to spherical,

$R = \sqrt{4^2 + 0^2 + (-4)^2}$

$= 4\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{+\sqrt{4^2 + (0)^2}}{-4}\right)$

$= 75.96$

$\phi = \tan^{-1}(0/4) = 0$

c) $P_3 = (\sqrt{8}, -\sqrt{8}, 4)$

Cartesian to cylindrical,

$x = \sqrt{8},\ y = -\sqrt{8},\ z = 4$

$r = +\sqrt{(\sqrt{8})^2 + (-\sqrt{8})^2}$

$= 4$

$\phi = 2\pi - \tan^{-1}\left(\frac{-\sqrt{8}}{\sqrt{8}}\right)$

$= 2\pi + \frac{\pi}{4}$

$= \frac{9\pi}{4}$

$z = 4$

Cartesian to spherical,

$R = \sqrt{(\sqrt{8})^2 + (-\sqrt{8})^2 + (4)^2}$

$= 4\sqrt{2}$

$\phi = 2\pi - \tan^{-1}\left(\frac{-\sqrt{8}}{\sqrt{8}}\right) = 2\pi + \frac{\pi}{4}$

$= \frac{9\pi}{4}$

$\theta = \tan^{-1}\left(\frac{+\sqrt{(\sqrt{8})^2 + (-\sqrt{8})^2}}{4}\right)$

$= \frac{\pi}{4}$

2. a) $P_1 = (2, \frac{2\pi}{3}, 2\sqrt{3})$

Cylindrical to Cartesian:

$\rho = 2,\ \phi = \frac{2\pi}{3},\ z = 2\sqrt{3}$

$x = 2\cos(\frac{2\pi}{3}) = 1.998$

$y = 2\sin(\frac{2\pi}{3}) = 0.07$

$z = 2\sqrt{3}$

Cylindrical to spherical

$R = +\sqrt{2^2 + (2\sqrt{3})^2} = 4$

$\theta = \tan^{-1}(2/2\sqrt{3}) = \frac{\pi}{6}$

$\phi = \frac{2\pi}{3}$

b) $P_2 = (\sqrt{3}, 0, -1)$

Cylindrical to cartesian:

$r = \sqrt{3},\ \phi = 0,\ z = -1$

$x = \sqrt{3}\cos 0° = \sqrt{3}$

$y = \sqrt{3}\sin 0 = 0$

$z = -1$

Cylindrical to spherical

$R = +\sqrt{(\sqrt{3})^2 + (-1)^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = -\frac{\pi}{3}$

$\phi = 0$

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c) $P_3 = (4\sqrt{3}, \pi, -4)$

Cylindrical to cartesian:

$r = 4\sqrt{3}$, $\phi = \pi$, $z = -4$

$x = 4\sqrt{3}\cos(\pi)$

$= 6.91$

$y = 4\sqrt{3}\sin(\pi) = 0.37$

$z = -4$

Cylindrical to Spherical.

$R = +\sqrt{(4\sqrt{3})^2 + (-4)^2} = 8$

$\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right) = -\frac{\pi}{3}$

$\phi = \pi$

Slide: 07

1. a) $T(x,y,z) = 2x^3yz + y^2x^2 - 5\frac{y}{2}$

$$\nabla T = \hat{x}\frac{\partial}{\partial x}(2x^3yz + y^2x^2 - 5y/2) + \hat{y}\frac{\partial}{\partial y}(2x^3yz + y^2x^2 - 5y/2) + \hat{z}\frac{\partial}{\partial z}(2x^3yz + y^2x^2 - 5y/2)$$

$$= \hat{x}(6x^2yz + 2xy^2) + \hat{y}(2x^3z + 2x^2y - 5/2) + \hat{z}(2x^3y)$$

Point $(0,2,-1)$,

$$= \hat{x}(6\times0\times2\times(-1) + 2\times0\times2^2) + \hat{y}(2\times0^3\times(-1) + 2\times0^2\times2 - 5/2) + \hat{z}(2\times0^3\times(-1))$$

$$= -\hat{y}\frac{5}{2}$$

b) $T(\rho,\phi,z) = \dfrac{z+\sin\phi}{\rho}$

$$\nabla T = \hat{\rho}\frac{\partial}{\partial\rho}\left(\frac{z+\sin\phi}{\rho}\right) + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}\left(\frac{z+\sin\phi}{\rho}\right) + \hat{z}\frac{\partial}{\partial z}\left(\frac{z+\sin\phi}{\rho}\right)$$

$$= \hat{\rho}\frac{(z+\sin\phi)\,2\rho}{\rho^2} + \frac{\hat{\phi}}{\rho}\frac{\cos\phi.\rho}{\rho^2} + \hat{z}\frac{\rho}{\rho^2}$$

$$= \hat{\rho}\frac{2(z+\sin\phi)}{\rho} + \hat{\phi}\frac{\cos\phi}{\rho^2} + \hat{z}\frac{1}{\rho}$$

Point $\left(2, \frac{3\pi}{2}, 1\right)$

$= \hat{r}\,\frac{2(1+\sin\frac{3\pi}{2})}{2} + \hat{\phi}\,\frac{\cos\frac{3\pi}{2}}{4} + \hat{z}\,\frac{1}{2}$

$= 0 + 0 + \hat{z}\,\frac{1}{2}$

$= \frac{1}{2}\hat{z}$

c) $T(R, \theta, \phi) = R^2 \cos\phi \sin\theta$

$\nabla T = \hat{R}\frac{\partial}{\partial R}(R^2\cos\phi\sin\theta) + \frac{\hat{\theta}}{R}\frac{\partial}{\partial\theta}(R^2\cos\phi\sin\theta) + \frac{\hat{\phi}}{R\sin\theta}\frac{\partial}{\partial\phi}(R^2\cos\phi\sin\theta)$

$= \hat{R}\, 2R\cos\phi\sin\theta + \frac{\hat{\theta}}{R} R^2\cos\phi\cos\theta - \frac{\hat{\phi}}{R\sin\theta} R^2\sin\phi\sin\theta$

$= \hat{R}\, 2R\cos\phi\sin\theta + \hat{\theta}\, R\cos\phi\cos\theta - \hat{\phi}\, R\sin\phi$

Point $\left(2, \frac{\pi}{4}, \frac{2\pi}{3}\right)$

$= \hat{R}\, 2\times 2\cos\frac{2\pi}{3}\times\sin\frac{\pi}{4} + \hat{\theta}\, 2\times\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \hat{\phi}\, 2\sin\frac{2\pi}{3}$

$= \hat{R}\, 2\times 2\times -\frac{1}{2}\times\frac{1}{\sqrt{2}} + \hat{\theta}\, 2\times \cancel{\cos} -\frac{1}{2}\times\frac{1}{\sqrt{2}} - \hat{\phi}\, 2\times\frac{\sqrt{3}}{2}$

$= \hat{R}\,\frac{2}{\sqrt{2}} \cancel{+} - \frac{1}{\sqrt{2}}\hat{\theta} - \sqrt{3}\,\hat{\phi}$

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2.a) $T(x,y,z) = x^2y - xz$

$$\nabla T = \hat{x}\frac{\partial}{\partial x}(x^2y - xz) + \hat{y}\frac{\partial}{\partial y}(x^2y - xz) + \hat{z}\frac{\partial}{\partial z}(x^2y - xz)$$

$$= \hat{x}(2xy - z) + \hat{y}.\ x^2 + \hat{z}(-x)$$

Point $(1,0,2)$

$$= \hat{x}(2\times1\times0 - 2) + \hat{y}\times1^2 - \hat{z}\times1$$

$$= -2\hat{x} + \hat{y} - \hat{z}$$

$\hat{x} - 2\hat{y} - 6\hat{z}$, $\quad \hat{a} = \dfrac{\hat{x} - 2\hat{y} - 6\hat{z}}{\sqrt{1^2 + (-2)^2 + (-6)^2}} = \dfrac{1}{\sqrt{41}}\hat{x} - \dfrac{2}{\sqrt{41}}\hat{y} - \dfrac{6}{\sqrt{41}}\hat{z}$

$$\therefore \nabla T.\hat{a} = (-2\hat{x} + \hat{y} - \hat{z})\left(\frac{1}{\sqrt{41}}\hat{x} - \frac{2}{\sqrt{41}}\hat{y} - \frac{6}{\sqrt{41}}\hat{z}\right)$$

$$= -\frac{2}{\sqrt{41}} - \frac{2}{\sqrt{41}} + \frac{6}{\sqrt{41}}$$

$$= \frac{-2-2+6}{\sqrt{41}}$$

$$= \frac{2}{\sqrt{41}}$$

b) $T(r, \phi, z) = r^3 \cos\phi$

$$\nabla T = \hat{r} \frac{\partial}{\partial r}(r^3 \cos\phi) + \frac{\hat{\phi}}{r} \frac{\partial}{\partial \phi}(r^3 \cos\phi) + \hat{z} \frac{\partial}{\partial z}(r^3 \cos\phi)$$

$$= \hat{r}\, 2r^2 \cos\phi + \frac{\hat{\phi}}{r} r^3 (-\sin\phi)$$

$$= \hat{r}\, 2r^2 \cos\phi \cancel{+} - \hat{\phi}\, r^2 \sin\phi$$

Point $(2, \frac{\pi}{4}, 1)$

$$= \hat{r}\, 2 \times 2^2 \times \cos\frac{\pi}{4} - \hat{\phi}\, 2^2 \times \sin\frac{\pi}{4}$$

$$= \hat{r}\, 2 \times 4 \times \frac{1}{\sqrt{2}} - \hat{\phi}\, 4 \times \frac{1}{\sqrt{2}}$$

$$= 8/\sqrt{2}\, \hat{r} - 4/\sqrt{2}\, \hat{\phi}$$

$$\therefore \nabla T . \hat{r} = \left(\frac{8}{\sqrt{2}} \hat{r} - \frac{4}{\sqrt{2}} \hat{\phi}\right) \times \hat{r}$$

$$= \frac{4 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2}$$

c) $T(R, \theta, \phi) = \frac{1}{R}\cos^2\theta$

$$\nabla T = \hat{R}\frac{\partial}{\partial R}\left(\frac{1}{R}\cos^2\theta\right) + \frac{\hat{\theta}}{R}\frac{\partial}{\partial\theta}\left(\frac{1}{R}\cos^2\theta\right) + \frac{\hat{\phi}}{R\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{1}{R}\cos^2\theta\right)$$

$$= -\hat{R}\frac{1}{R^2}\cos^2\theta + \frac{\hat{\theta}}{R} \times \frac{1}{R}(-\sin 2\theta)$$

Point $\left(1, \frac{\pi}{4}, \frac{\pi}{2}\right)$

$$= -\hat{R}\frac{1}{1}\cos^2\frac{\pi}{4} - \hat{\theta} \times \frac{1}{1^2} \times \sin \cancel{2} \times \frac{\pi}{\cancel{4}_2}$$

$$= -\hat{R}\left(\frac{1}{\sqrt{2}}\right)^2 - \hat{\theta}$$

$$= -\frac{1}{2}\hat{R} - \hat{\theta}$$

$$\nabla T \cdot (\hat{R} - \hat{\theta}) = \left(-\frac{1}{2}\hat{R} - \hat{\theta}\right) \cdot (\hat{R} - \hat{\theta})$$

$$= -\frac{1}{2} + 1$$

$$= \frac{-1+2}{2}$$

$$= \frac{1}{2}$$

3. a) $T = 4y^2z^2$

$$\nabla^2 T = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(4y^2z^2)\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(4y^2z^2)\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}(4y^2z^2)\right)$$

$$= \frac{\partial}{\partial y}(8yz^2) + \frac{\partial}{\partial z}(8y^2z)$$

$$= 8z^2 + 8y^2$$

b) $T = xy + zx$

$$\nabla^2 T = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(xy+zx)\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(xy+zx)\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}(xy+zx)\right)$$

$$= \frac{\partial}{\partial x}(y+z) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(x)$$

$$= 0$$

c) $T = 10r^3 \cos 2\phi$

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(10r^3\cos 2\phi)\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\frac{\partial}{\partial \phi}(10r^3\cos$$

$$+ \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}(10r^3\cos 2\phi)\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r \times 20 r^2 \cos 2\theta\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(10 r^3 (-2\sin 2\theta)\right)$$

$$= \frac{1}{r} \times 30 r^2 \cos 2\theta - \frac{1}{r^2} \times 10 r^3 \times 2 \times 2 \cos 2\theta$$

$$= 30 r \cos 2\theta - 40 r \cos 2\theta$$

$$= \cos 2\theta (30 r - 40 r)$$

$$= -10 r \cos 2\theta$$