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Final Assignment : 01

Slide: 01

a) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix}$$

$$|A-\lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 3 = 0$$

$$\Rightarrow (\lambda-3)(\lambda+1) = 0$$

$\therefore \lambda = 3, -1$

Eigenvectors:

$$(A-\lambda I)\,v=0$$

$$\begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=0$$

$\lambda=3$,

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=0$$

$$-2v_1+2v_2=0$$

$$2v_1-2v_2=0$$

$$\Rightarrow 0v_1-0.v_2=0$$

Hence, $v_2=b$

$$-2v_1=-2v_2$$

$$\Rightarrow v_1=b$$

$$\therefore v=\begin{bmatrix} b \\ b \end{bmatrix}$$

$\lambda=1$,

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=0$$

$$2v_1+2v_2=0$$

$$2v_1+2v_2=0$$

$$\Rightarrow 0v_1+0v_2=0$$

$$v_2=b$$

$$2v_1=-2v_2$$

$$\Rightarrow v_1=-b$$

$$v=\begin{bmatrix} -b \\ b \end{bmatrix}$$

b) $A = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1-\lambda & 0 \\ 3 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 0 \\ 3 & 2-\lambda \end{vmatrix} = \lambda^2 - \lambda - 2$$

$\therefore |A - \lambda I| = 0$

$\Rightarrow \lambda^2 - \lambda - 2 = 0$

$\Rightarrow (\lambda - 2)(\lambda + 1) = 0$

$\therefore \lambda = -1, 2,$

Eigenvectors: $\begin{bmatrix} -1-\lambda & 0 \\ 3 & 2-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} =$

$\lambda = -1,$

$$\begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$3v_1 + 3v_2 = 0$

$0v_1 + 0v_2 = 0$

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$v_2 = b$

$\therefore 3v_1 = -3v_2$

$\Rightarrow v_1 = -b$

$\therefore v = \begin{bmatrix} -b \\ b \end{bmatrix}$

$\lambda = 2,$

$$\begin{bmatrix} -3 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$-3v_1 + 0v_2 = 0$

$3v_2 + 0v_2 = 0$

$v_1 = 0, \quad v_2 = b$

$\therefore v = \begin{bmatrix} 0 \\ b \end{bmatrix}$

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c) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$

$\therefore A - \lambda I = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 & 1 \\ 0 & -2-\lambda & 0 \\ 0 & -5 & 2-\lambda \end{bmatrix}$

$\therefore |A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ 0 & -2-\lambda & 0 \\ 0 & -5 & 2-\lambda \end{vmatrix} = (1-\lambda)(\lambda+2)(\lambda-2)$

$\therefore |A - \lambda I| = 0$

$\Rightarrow (1-\lambda)(\lambda+2)(\lambda-2) = 0$

$\therefore \lambda = 1, 2, -2$

Eigenvectors: $(A - \lambda I)\, V = 0$

$$\begin{bmatrix} 1-\lambda & 2 & 1 \\ 0 & -2-\lambda & 0 \\ 0 & -5 & 2-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Sub :

$\lambda = 1$,

$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -5 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$\Rightarrow$ $0v_1 + 2v_2 + v_3 = 0$

$0v_1 - 3v_2 + 0v_3 = 0$

$0v_1 \cancel{+} - 5v_2 + v_3 = 0$

$v_2 = v_3 = 0$, $v_1 = a$. $V = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$

$\lambda = -2$, $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & -5 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$

$3v_1 + 2v_2 + v_3 = 0$

$-5v_2 + 4v_3 = 0$

$0v_1 + 0v_2 + 0v_3 = 0$

$v_3 = C$

$-5v_2 = -4v_3$

$\Rightarrow v_2 = \frac{4C}{5}$

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$$3v_1 + 2\frac{4c}{5} + c = 0$$

$$\Rightarrow v_1 = \frac{-13c}{15}$$

$$V = \begin{bmatrix} -\frac{13c}{15} \\ \frac{4c}{5} \\ c \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & -4 & 0 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$-v_1 + 2v_2 + v_3 = 0$$

$$-4v_2 = 0$$

$$-5v_2 = 0$$

REF

$$-v_1 + 2v_2 + v_3 = 0$$

$$-4v_2 = 0$$

$$0v_1 + 0v_2 + 0v_3 = 0$$

$$v_3 = c$$

$$v_2 = 0$$

$$v_1 = c$$

$$V = \begin{bmatrix} c \\ 0 \\ c \end{bmatrix}$$

d) $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-\lambda & 3 & 3 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{bmatrix}$

$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 3 & 3 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{vmatrix} = (2-\lambda)(\lambda^2 - 1)$

$|A - \lambda I| = 0$

$\Rightarrow (2-\lambda)(\lambda^2 - 1) = 0$

$\lambda = 2, 1, -1$

Eigenvectors: $(A - \lambda I)V = 0$

$$\begin{bmatrix} 2-\lambda & 3 & 3 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

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$\lambda = 2.$

$$\begin{pmatrix} 0 & 3 & 3 \\ 1 & 1 & 2 \\ -1 & -4 & -5 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$0v_1 + 3v_2 + 3v_3 = 0$

$v_1 + v_2 + 2v_3 = 0$

$-v_1 - 4v_2 - 5v_3 = 0$

REF: $-v_1 - 4v_2 - 5v_3 = 0$

$-3v_2 + 3v_3 = 0$

$0v_1 + 0v_2 + 0v_3 = 0$

$v_3 = c$

$-3v_2 = 3v_3$

$\Rightarrow v_2 = -c$

$-v_1 = 5c - 4c$

$\Rightarrow v_1 = -c$

$$V = \begin{bmatrix} -c \\ -c \\ c \end{bmatrix}$$

Sub :

$\lambda = 1,$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$v_1 + 3v_2 + 3v_3 = 0$

$v_1 + 2v_2 + 2v_3 = 0$

$-v_1 - 4v_2 - 4v_3 = 0$

REF

$-v_2 - v_1 = 0$

$0v_1 + 0v_2 + 03 = 0$

$v_3 = c$

$v_1 = \frac{3c}{2}$

$v_2 = -\frac{3c}{2}$

$$v = \begin{bmatrix} \frac{3c}{2} \\ -\frac{3c}{2} \\ c \end{bmatrix}$$

$\lambda = -1,$

$$\begin{bmatrix} 3 & 3 & 3 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$3v_1 + 3v_2 + 3v_3 = 0$

$v_1 + 4v_2 + 2v_3 = 0$

$-v_1 - 4v_2 - 2v_3 = 0$

$v_1 = v_2 = v_3 = 0$

$$v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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Slide: 02

a) $\dot{x}_1(t) = x_1(t) + 2(x_2)(t)$

$\dot{x}_2(t) = 3x_1(t) + 2x_2(t)$

$x_1(0) = 0, \quad x_2(0) = 4$

$$x(t) = \begin{matrix} x_1(t) \\ x_2(t) \end{matrix} \qquad \dot{x}(t) = \begin{matrix} x_1(t) \\ \dot{x}(t) \end{matrix}$$

$$x(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = (\lambda - 4)(\lambda + 1) = 0$$

$$|A - \lambda I| = 0$$

$\Rightarrow (\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4, -1$

Sub

Eigenvectors. $\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$\lambda = 4$.

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$-3v_1 + 2v_2 = 0$

$0v_1 + 0v_2 = 0$

$v_2 = b, \quad v_1 = \frac{2}{3} b$

$$V = \begin{bmatrix} 2/3\, b \\ b \end{bmatrix}$$

$\lambda = -1$, $\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$2v_1 + 2v_2 = 0$

$0v_1 + 0v_2 = 0$

$v_2 = b$

$2v_1 = -2v_2$ $\quad V = \begin{bmatrix} -b \\ b \end{bmatrix}$

$\Rightarrow v_1 = -b$

$$x(t) = c_1 v_1 e^{\lambda_1 t} + c_2 v_2 e^{\lambda_2 t}$$

$$\rightarrow \begin{pmatrix} 0 \\ -4 \end{pmatrix} = c_1 \begin{pmatrix} 2/3\, 5 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$

Here, $\quad \frac{2}{3} 5 c_1 - 5 c_2 = 0$

$\rightarrow \quad 5 c_1 + 5 c_2 = -4$

$$c_1 = -\frac{12}{25}, \quad c_2 = -\frac{8}{25}$$

$$x_1(t) = -\frac{8}{5} e^{4t} + \frac{8}{5} e^{-t}$$

$$x_2(t) = -\frac{12}{5} e^{4t} - \frac{8}{5} e^{-t}$$

b) $\quad \dot{x}_1(t) = -5x_1(t) + x_2(t)$

$$\dot{x}_2(t) = 4x_1(t) - 2x_2(t)$$

$$x_1(0) = 1, \quad x_2(0) = 2$$

$$x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$A = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = (\lambda+6)(\lambda+1)$$

$$|A - \lambda I| = 0$$

$$\Rightarrow (\lambda+6)(\lambda+1) = 0$$

$$\therefore \lambda = -6, -1$$

Eigenvectors:

$$\begin{bmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} =$$

$\lambda = -6$,

$$\begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$v_1 + v_2 = 0$$

$$0 v_1 + 0 v_2 = 0$$

$v_2 = b$,

$v_1 = -b$

$$V = \begin{bmatrix} -b \\ b \end{bmatrix}$$

$\lambda = -1$

$$\begin{bmatrix} -4 & -1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$-4v_1 + v_2 = 0$$

$$0 v_1 + 0 v_2 = 0$$

$v_2 = b$. $v_1 = b/4$

$$V = \begin{bmatrix} b/4 \\ b \end{bmatrix}$$

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$$x(t) = c_1 v_1 e^{\lambda_1 t} + c_2 v_2 e^{\lambda_2 t}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = c_1 \begin{pmatrix} -b \\ b \end{pmatrix} + c_2 \begin{pmatrix} b/4 \\ b \end{pmatrix}$$

$$-bc_1 + b/4\, c_2 = 1$$

$$bc_1 + bc_2 = 2$$

$$c_2 = 12/5b, \quad c_1 = -2/5b$$

$$x_1(t) = \frac{2}{5} e^{-6t} + 3/5\, e^{-t}$$

$$x_2(t) = -2/5\, e^{-6t} + \frac{12}{5} e^{-t}$$

(Ans:)

Slide 03

(a) $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 1 \\ -2 & 3-\lambda & 4 \\ -5 & 5 & 6-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 0 & 1 \\ -2 & 3-\lambda & 4 \\ -5 & 5 & 6-\lambda \end{vmatrix} = \lambda^3 - 11\lambda^2 + 21\lambda - 1$$

$\because |A - \lambda I| = 0$

$$\Rightarrow \lambda^3 - 11\lambda^2 + 21\lambda - 1 = 0$$

$$A^3 - 11A^2 + 21A - I = 0$$

$$\Rightarrow A^2 - 11A + 21I - A^{-1} = 0$$

$$\Rightarrow A^{-1} = A^2 - 11A + 21I$$

$$= \begin{bmatrix} -2 & 5 & -3 \\ -8 & 17 & -10 \\ 5 & -10 & 6 \end{bmatrix}$$

Verification: $A^3 - 11A^2 + 21A - I$

$= 0$

Cayley-Hamilton theorem is verified.

b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 5-\lambda & 3 \\ 1 & 0 & 8-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 5-\lambda & 3 \\ 1 & 0 & 8-\lambda \end{vmatrix} = \lambda^3 - 19\lambda^2 + 46\lambda + 1$$

Sub:

7) $|A-\lambda I|=0$

$\Rightarrow \lambda^3-14\lambda^2+46\lambda+10$

$A^3-14A^2+46A+I=0$

$\Rightarrow A^{-1}=14A-46I-A^2$

$$=\begin{bmatrix} -38 & 20 & 15 \\ 17 & 5 & 3 \\ 7 & -2 & 15 \end{bmatrix}$$

erification $=A^3-14A^2+46A+I$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cayley-Hamilton theorem is verified.

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c) $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & -1 & 1 \\ 2 & -1-\lambda & 0 \\ 1 & -1 & 0-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 & 1 \\ 2 & 1-\lambda & 0 \\ 1 & -1 & 0-\lambda \end{vmatrix} = \lambda^3 + 1$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 + 1 = 0$$

$$A^3 + I = 0$$

$$\Rightarrow A^{-1} = -A^2$$

Slide: 09

1. a) $U (5, 9, 5)$

$U_1 = (1, -1, 3)$

$U_2 = (2, 1, 4)$

$U_3 = (3, 2, 5)$

$\alpha_1 (1, -1, 3) + \lambda_2 (2, 1, 4) + \lambda_3 (3, 2, 5) = 5, 9, 5$

$\alpha_1 + 2\alpha_2 + 3\alpha_3 = 5$

$-\alpha_1 + \alpha_2 + 2\alpha_3 = 9$

$3\alpha_1 + 4\alpha_2 + 5\alpha_3 = 5$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ -1 & 1 & 2 & 9 \\ 3 & 4 & 5 & 5 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & -2 & -2 \end{array}\right]$$

$3\alpha_2 + 5\alpha_3 = 14$

$\Rightarrow \alpha_3 = 1$

$3\alpha_2 = 14 - 5 \qquad \alpha_1 = -4$

$\Rightarrow \alpha_2 = 3.$

$-4u_1 + 3u_2 + u_3 = 0$

b) $\alpha_1(1,2,3) + \alpha_2(1,3,-2) + \alpha_3(1,4,1) = (6,20,2)$

$\alpha_1 + \alpha_2 + \alpha_3 = 6$

$2\alpha_1 + 3\alpha_2 + 4\alpha_3 = 20$

$3\alpha_1 - 2\alpha_2 + \alpha_3 = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 3 & 4 & 20 \\ 3 & -2 & 1 & 2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 3 & 24 \end{array}\right]$$

$8\alpha_3 = 24$

$\Rightarrow \alpha_3 = 3$

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$\alpha_2 = 8-6 = 2$

$\alpha_1 = 6-5 = 1$

$u_1 + 2u_2 + 3u_3 = 4$ (Ans.)

c) $\alpha_1(3,1,0,1) + \alpha_2(1,2,3,1) + \alpha_3(0,3,6,6) = (4,2,1,0)$

$3\alpha_1 + \alpha_2 = 4$

$\alpha_1 + 2\alpha_2 + 3\alpha_3 = 2$

$3\alpha_2 + 6\alpha_3 = 1$

$\alpha_1 + \alpha_2 + 6\alpha_3 = 0$

$$\left[\begin{array}{ccc|c} 3 & 1 & 0 & 4 \\ 1 & 2 & 3 & 2 \\ 0 & 3 & 6 & 1 \\ 1 & 1 & 6 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 3 & 1 & 0 & 4 \\ 0 & 5 & 9 & 2 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$3\alpha_3 = -1$

$\alpha_3 = -1/3$, $\quad \alpha_1 = 3 \quad 3u_1 + u_2 - \frac{1}{3}u_3 = u$

$5\alpha_2 = 2+3$

$\Rightarrow \alpha_2 = 1$

(Ans

2. a) $\alpha_1(2,-1,4)+\alpha_2(3,6,2)+\alpha_3(2,10,-4)=0$

$2\alpha_1+3\alpha_2+2\alpha_3=0$

$-\alpha_1+6\alpha_2+10\alpha_3=0$

$4\alpha_1+2\alpha_2-4\alpha_3=0$

REF. $\begin{bmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 \\ 0 & 15 & 22 \\ 0 & 0 & -32 \end{bmatrix}=0$

$\alpha_3=0$

$15\alpha_2+22\alpha_3=0$

$\Rightarrow \alpha_2=0$

$\alpha_1=0$

∴ Linearly independent (Ans.)

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b) $\alpha_1 (3,0,1,-1) + \alpha_2 (2,-1,0,1) + \alpha_3 (1,1,1,-2) = 0$

$3\alpha_1 + 2\alpha_2 + \alpha_3 = 0$

$-\alpha_2 + \alpha_3 = 0$

$\alpha_1 + \alpha_3 = 0$

$-\alpha_1 + \alpha_2 - 2\alpha_3 = 0$

REF: $\begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

$\alpha_3 = c$, $\alpha_1 = -1$

$\alpha_2 = c$

$\therefore$ linearly dependent. (Ans.)

c) $\alpha_1 (1,-1,2) + \alpha_2 (3,-5,1) + \alpha_3 (2,7,8) + \alpha_4 (-1,1,1) = 0$

REF: $\begin{bmatrix} 1 & 3 & 2 & -1 \\ 1 & -5 & 7 & 1 \\ 2 & 1 & 8 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & -1 \\ 0 & -8 & 5 & 2 \\ 0 & 0 & 7 & 14 \end{bmatrix} = 0$

$\alpha_1 = 0$

$\alpha_1 = \alpha_2 = \alpha_3 = 0$

Linearly independent (Ans:)

d) $\alpha_1(2,1,2) + \alpha_2(0,1,-1) + \alpha_3(4,3,3) = 0$

$2\alpha_1 + 4\alpha_3 = 0$

$\alpha_1 + \alpha_2 + 3\alpha_3 = 0$

$2\alpha_1 - \alpha_2 + 3\alpha_3 = 0$

REF: $\begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & 3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{bmatrix} = 0$

$2\alpha_1 - \alpha_2 + 3\alpha_3 = 0$

$\alpha_2 + 3\alpha_3 = 0$

$-2\alpha_3 = 0$

$\alpha_3 = 0$

$\alpha_1 = \alpha_2 = \alpha_3 = 0$

: Linearly independent (Ans:)