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HW: First Slide

$$A = \begin{bmatrix} -5 & 6 & 1 \\ 2 & -2 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 8 & -4 \\ 1 & 2 & -9 \end{bmatrix}$$

$$7A = 7\begin{bmatrix} -5 & 6 & 1 \\ 2 & -2 & 9 \end{bmatrix} = \begin{bmatrix} -35 & 42 & 7 \\ 14 & -14 & 63 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} -5 & 6 & 1 \\ 2 & -2 & 9 \end{bmatrix} = \begin{bmatrix} -15 & 18 & 3 \\ 6 & -6 & 27 \end{bmatrix}$$

$$2B = 2\begin{bmatrix} 5 & 8 & -4 \\ 1 & 2 & -9 \end{bmatrix} = \begin{bmatrix} 10 & 16 & -8 \\ 2 & 4 & -18 \end{bmatrix}$$

$$3A+2B = \begin{bmatrix} -15 & 18 & 3 \\ 6 & -6 & 27 \end{bmatrix} + \begin{bmatrix} 10 & 16 & -8 \\ 2 & 4 & -18 \end{bmatrix} = \begin{bmatrix} -5 & 34 & -5 \\ 8 & -2 & 9 \end{bmatrix}$$

$$2A = \begin{bmatrix} -10 & 12 & 2 \\ 4 & -4 & 18 \end{bmatrix} \qquad 3B = \begin{bmatrix} 15 & 24 & -12 \\ 3 & 6 & -27 \end{bmatrix}$$

$$2A-3B = \begin{bmatrix} -10 & 12 & 2 \\ 4 & -4 & 18 \end{bmatrix} - \begin{bmatrix} 15 & 24 & -12 \\ 3 & 6 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -12 & 14 \\ 1 & -10 & 45 \end{bmatrix}$$

(Ans:)

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* $A = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 0 \\ -4 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 7 & 0 \\ -4 & 5 \end{bmatrix}$

$= \begin{bmatrix} -7-+16 & 0+20 \\ 0-+12 & 0+15 \end{bmatrix}$

$= \begin{bmatrix} -23 & -20 \\ -12 & 15 \end{bmatrix}$ (Ans:)

$BA = \begin{bmatrix} 7 & 0 \\ -4 & 5 \end{bmatrix} \times \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -7+0 & 28+0 \\ 4+0 & -16+15 \end{bmatrix}$

$= \begin{bmatrix} -7 & 28 \\ 4 & -1 \end{bmatrix}$

(Ans:)

* $A = \begin{bmatrix} 6 & 0 \\ -4 & -2 \\ 9 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 5 \\ -2 & -3 \\ 0 & 3 \end{bmatrix}$

AB is not possible because row number is greater than column number.

BA is not possible because row number is greater than column number.

* $A = \begin{bmatrix} 7 & -3 & 0 \\ 3 & 4 & 3 \\ -5 & 0 & -5 \end{bmatrix}$

$$\det(A) = 7\begin{vmatrix} 4 & 3 \\ 0 & -5 \end{vmatrix} + 3\begin{vmatrix} 3 & 3 \\ -5 & -5 \end{vmatrix} + 0\begin{vmatrix} 3 & 4 \\ -5 & 0 \end{vmatrix}$$

$$= 7(-20-0) + 3(-15+15) + 0$$

$$= -140$$ (Ans:)

* $A = \begin{bmatrix} 10 & -4 & 4\alpha \\ -1 & 2 & 0 \\ 1 & \alpha & -2 \end{bmatrix}$

$$\det(A) = 10\begin{vmatrix} 2 & 0 \\ \alpha & -2 \end{vmatrix} + 4\begin{vmatrix} -1 & 0 \\ 1 & -2 \end{vmatrix} + 4\alpha\begin{vmatrix} -1 & 2 \\ 1 & \alpha \end{vmatrix}$$

$$= 10(-4-0) + 4(2-0) + 4\alpha(-\alpha-2)$$

$$= -40 + 8 - 4\alpha^2 - 8\alpha$$

$$= -4\alpha^2 - 8\alpha - 32$$

For sigular $\det(A) = 0$

$$\Rightarrow -4\alpha^2 - 8\alpha - 32 = 0$$

$$\Rightarrow \alpha^2 + 2\alpha + 8 = 0$$

$$\alpha \Rightarrow \frac{-2 \pm \sqrt{2^2 - 4\times1\times8}}{2\times1} = -1, 7$$

HW: Second slide

$$* \; A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} + 3\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}$$

$$= 1(0-24) - 2(0-20) + 3(0-5)$$

$$= -24 + 40 - 15$$

$$= 1$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix}$$

$$= 0 - 24 = -24$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix}$$

$$= -(0-20) = +20$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}$$

$$= 0 - 5$$

$$= -5$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix}$$

$$= -(0-18)$$

$$= 18$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix}$$

$$= 0 - 15$$

$$= -15$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= -(6-10)$$

$$= 4$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= 8 - 3 = 5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix}$$

$$= -(4-0)$$

$$= -4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= 1-0$$

$$= 1$$

$$\therefore \text{adj } A = \begin{bmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

$\therefore$ Verification: $A A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -24+40-15 & 18-30+12 & 5-8+3 \\ 0+20-20 & 0-15+16 & 0-4+4 \\ -120+120-0 & 90-90+0 & 25-24+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{(Proved)}$$

$$B = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad |B| = 1\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - 5\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix}$$

$$= (-1-0) - 5(0-0) + 2(0-0)$$

$$= -1$$

$$B_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= (-1-0) = -1$$

$$B_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= -(0-0) = 0$$

$$B_{13} = (-1)^{1+3}\begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix}$$

$$= 0$$

$$B_{21} = (-1)^{2+1}\begin{vmatrix} 5 & 2 \\ 0 & 1 \end{vmatrix} = -5$$

$$B_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

$$B_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 5 \\ 0 & 0 \end{vmatrix} = 0$$

$$B_{31} = (-1)^{3+1}\begin{vmatrix} 5 & 2 \\ -1 & 2 \end{vmatrix} = (10+2) = 12$$

$$B_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = -2$$

$$B_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1$$

$$\therefore adj\,B = cof\,B^T = \begin{bmatrix} -1 & 0 & 0 \\ -5 & 1 & 0 \\ 12 & -2 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & -5 & 12 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{adj\,B}{|B|} = \frac{1}{-1}\begin{bmatrix} -1 & -5 & 12 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & -12 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Verification:

$\cancel{BP}$ $\cancel{BB^{-1}}$

$$BB^{-1} = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & -12 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 5-5+0 & -12+0+2 \\ 0+0+0 & 0+1+0 & 0-2+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{(Proved)}$$

$$*A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ 5 & 2 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ 5 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0-15 & -1+4-6 & 1+2+9 \\ 4+0+0 & 2+2+0 & -2+1+0 \\ 8+0+25 & 4-4+10 & -4+2+15 \end{bmatrix}$$

$$= \begin{bmatrix} -17 & -3 & -6 \\ 4 & 4 & -1 \\ 33 & 10 & 19 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & -1 \\ 5 & 2 & -3 \end{bmatrix} \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2-4 & 4+1+2 & -6+0-5 \\ 0+4+4 & 0+2+2 & 0+0+5 \\ -5+4+12 & 10+2+6 & -15+0-15 \end{bmatrix} = \begin{bmatrix} -4 & 7 & -11 \\ 8 & 0 & -5 \\ -13 & 18 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix}$$

$$= -5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix}$$

$$= -(10-0) = -10$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix}$$

$$= -4 - 4 = -8$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ -2 & 5 \end{vmatrix}$$

$$= -(10-6) = -4$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -3 \\ 4 & 5 \end{vmatrix}$$

$$= -5 + 12 = 7$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 2 \\ 4 & -2 \end{vmatrix}$$

$$= -(2-8)$$

$$= 6$$

$$|A| = -1 \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} - 2 \begin{vmatrix} 2 & 0 \\ 4 & 5 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix}$$

$$= -1(5-0) - 2(10-0) - 3(-4-4)$$

$$= -5 - 20 + 24 = -1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix}$$

$$= 0 + 3 = 3$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} -1 & -3 \\ 2 & 0 \end{vmatrix}$$

$$= -(0+6) = -6$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= -1 - 4$$

$$= -5$$

$$\operatorname{adj} A = \cos A^T = \begin{bmatrix} 5 & -10 & -8 \\ -4 & 7 & 6 \\ 3 & -6 & -5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{-1} \begin{bmatrix} 5 & -4 & 3 \\ -10 & 7 & -6 \\ -8 & 6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -10+0-15 & -5+8-6 & 5+4+9 \\ 20+0+30 & 10-14+12 & -10-7-18 \\ 16+0+25 & 8-12+10 & -8-6-15 \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -3 & 18 \\ 50 & 8 & -35 \\ 41 & 6 & -29 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

$$|B| = 2\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 5 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & 2 \\ 5 & 2 \end{vmatrix}$$

$$= 2(-6-2) - 1(0-5) - 1(0-10)$$

$$= -16 + 5 + 10 = -1$$

$$B_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix}$$

$$= -6 - 2 = -8$$

$$B_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 1 \\ 5 & -3 \end{vmatrix}$$

$$= -(0-5)$$

$$= 5$$

$$B_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 5 & 2 \end{vmatrix}$$

$$= (0-10) = -10$$

$$B_{21} = (-1)^{2+1}\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}$$

$$= -(-3+2)$$

$$= 1$$

$$B_{22} = (-1)^{2+2}\begin{vmatrix} 2 & -1 \\ 5 & -3 \end{vmatrix}$$

$$= -6 + 5$$

$$= -1$$

$$B_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 1 \\ 5 & 2 \end{vmatrix}$$

$$= -(4-5) = 1$$

$$B_{31} = (-1)^{3+1}\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1 + 2 = 3$$

$$B_{32} = (-1)^{3+2}\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= -(2-0)$$

$$= -2$$

$$B_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 4 - 0 = 4$$

$$B^{-1}\ \text{adj}\,B = \text{cof}\,B^T = \begin{bmatrix} -8 & 5 & -10 \\ 1 & -1 & 1 \\ 3 & -2 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} -8 & 1 & 3 \\ 5 & -1 & -2 \\ -10 & 1 & 4 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj}\,B}{|B|} = \frac{1}{-1}\begin{bmatrix} -8 & 1 & 3 \\ 5 & -1 & -2 \\ -10 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$$

$$B^{-1}A = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -8-2-12 & 16-1+6 & -24+0-15 \\ 5+2+8 & -10+1-4 & 15+0+10 \\ -10-2-16 & 20-1+8 & -30+0-20 \end{bmatrix}$$

$$= \begin{bmatrix} -22 & 21 & -39 \\ 18 & -13 & 25 \\ -28 & 27 & -50 \end{bmatrix}$$

$$AB = \begin{bmatrix} -17 & -3 & -6 \\ 4 & 4 & -1 \\ 33 & 10 & 9 \end{bmatrix}$$

$$|AB| = -17\begin{vmatrix} 4 & -1 \\ 10 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & -1 \\ 33 & 9 \end{vmatrix} - 6\begin{vmatrix} 4 & 4 \\ 33 & 10 \end{vmatrix}$$

$$= -17(36+10) + 3(36+33) - 6(40-132)$$

$$= -782 + 207 + 552 = -23$$

$$AB_{11} = (-1)^{1+1}\begin{vmatrix} 4 & -1 \\ 10 & 9 \end{vmatrix}$$

$$= (+36+30) = +66$$

$$AB_{12} = (-1)^{1+2}\begin{vmatrix} 4 & -1 \\ 33 & 9 \end{vmatrix}$$

$$= -(+36+33) = -69$$

$$AB_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 4 \\ 33 & 10 \end{vmatrix}$$

$$= 40 - 132$$

$$= -92$$

$$AB_{21} = (-1)^{2+1}\begin{vmatrix} -3 & -6 \\ 10 & 9 \end{vmatrix}$$

$$= -(-27+60)$$

$$= -33$$

$$AB_{22} = (-1)^{2+2}\begin{vmatrix} -17 & -6 \\ 33 & 9 \end{vmatrix}$$

$$= -153 + 198 = 45$$

$$\cancel{= 289 - 264 = 25}$$

$$AB_{23} = (-1)^{2+3}\begin{vmatrix} -17 & -3 \\ 33 & 10 \end{vmatrix}$$

$$= -(-170 - 99)$$

$$= 269$$

$$AB_{31} = (-1)^{3+1}\begin{vmatrix} -3 & -6 \\ 4 & -1 \end{vmatrix}$$

$$= 3 + 24 = +27$$

$$AB_{32} = (-1)^{3+2}\begin{vmatrix} -17 & -6 \\ 4 & -1 \end{vmatrix}$$

$$= -(17+24) = -\text{[illegible]}7$$

$A_{33} = (-1)^{3+3} \begin{vmatrix} -17 & -3 \\ 4 & 9 \end{vmatrix}$

$= -68 + 12$

$= -56$

$$\operatorname{adj} AB = \operatorname{cof} AB^T = \begin{bmatrix} +66 & -69 & -92 \\ -33 & 45 & 269 \\ 27 & -7 & -56 \end{bmatrix}^T$$

$$= \begin{bmatrix} \cancel{-38} & \cancel{29} & \cancel{-23} \\ \cancel{37} & \cancel{25} & \cancel{-19} \\ \cancel{-92} & \cancel{269} & \cancel{-56} \end{bmatrix} = \begin{bmatrix} 66 & -33 & 27 \\ -69 & 45 & -7 \\ -92 & 269 & -56 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\operatorname{adj} AB}{|AB|} = \frac{1}{-523} \begin{bmatrix} +66 & -33 & +27 \\ -69 & 45 & -7 \\ -92 & 269 & -56 \end{bmatrix}$$

$*\begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 0 & 4 \end{bmatrix} \quad r_3' = 2r_3 - 5r_2$

$= \begin{bmatrix} 4 & -1 \\ 0 & 1 \\ 0 & 4 \end{bmatrix} \quad r_2' = 2r_2 - r_1$

$= \begin{bmatrix} 4 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad r_3'' = -4r_2 + r_3'$ [This is REF]

$= \begin{bmatrix} 4 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad r_1' = r_2 + r_1$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad r_1'' = \frac{r_1'}{4}$ [This is RREF]

H.W.: Slide Three

$$\& \; A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 5 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & -4 & -15 & -5 & 0 & 1 \end{array}\right] \quad r_3 \to r_3 - 5r_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right] \quad r_3 \to 4r_2 + r_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right] \quad r_2 \to r_2 - 4r_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 16 & -12 & -3 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right] \quad r_1 = r_1 - 3r_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -24 & 18 & 5 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right] \quad r_1 = r_1 - 2r_2$$

$$\therefore A^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

Verification:

$$AA^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{[Using Calculator]}$$

$$= I$$

$$B = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_2 \to r_2 - 2r_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 5 & 0 & 1 & 0 & -2 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1 \to r_1 - 2r_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 5 & -12 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1 \to r_1 + 5r_2$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 5 & -12 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_2 = \frac{r_2}{-1}$$

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$$\therefore B^{-1} = \begin{bmatrix} 1 & 5 & -12 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Verification:

$$B^{-1}B = \begin{bmatrix} 1 & 5 & -12 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ [Using Calculator

$$= I$$

$$* \; m = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{bmatrix} \quad r_2 = r_1 + r_2$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad r_2 = \frac{r_2}{3}, \quad r_3 = \frac{r_3}{3}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad r_3 = r_3 - r_2$$

Rank: 2

$$N = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -1 \\ -2 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} \quad r_3 = r_2 + r_3$$

$$= \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} r_2 = r_2 + 2r_1 \\ r_3 = \dfrac{r_3}{5} \end{array}$$

Rank: 2

* a)

$$3x + y + 2z = 14$$

$$2y + 5z = 22$$

$$2x + 5y - z = -22$$

Argumented matrix

$$= \left[\begin{array}{ccc|c} 3 & 1 & 2 & 19 \\ 0 & 2 & 5 & 22 \\ 2 & 5 & -1 & -22 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 3 & 1 & 2 & 19 \\ 0 & 2 & 5 & 22 \\ 0 & 13 & -7 & -94 \end{array}\right] \quad r_3 \to 3r_3 - 2r_1$$

$$= \left[\begin{array}{ccc|c} 3 & 1 & 2 & 19 \\ 0 & 2 & 5 & 22 \\ 0 & 0 & -79 & -479 \end{array}\right] \quad r_3 \to 2r_3 - 13r_2$$

So,

$$3x + y + 2z = 19$$

$$2y + 5z = 22$$

$$-79z = -479$$

Here, Ran = 3

$(A|B) = 3$

It is cosistent.

Here, $-79z = -474$

$\therefore z = 6$

$2y + 5 \times 6 = 22$

$\Rightarrow y = -4$

$3x + (-4) + 2 \times 6 = 14$

$\therefore x = 2$

$\therefore (x, y, z) = (2, -4, 6)$

(Answer)

* $x+3y-2z=0$

$-3x+2y+5z=-1$

$2x-y+4z=8$

Angumented matrix:

$$=\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ -3 & 2 & 5 & -1 \\ 2 & -1 & 4 & 8 \end{array}\right]$$

$$=\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ -3 & 2 & 5 & -1 \\ 0 & -7 & 8 & 8 \end{array}\right] \quad r_3 = r_3 - 2r_1$$

$$=\left[\begin{array}{ccc|c} 1 & 3 & -2 & 6 \\ 0 & 11 & -1 & -1 \\ 0 & -7 & 8 & 8 \end{array}\right] \quad r_2 = r_2 + 3r_1$$

$$=\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 11 & -1 & -1 \\ 0 & 0 & 81 & 81 \end{array}\right] \quad r_3 = 11r_3 + 7r_2$$

Here, Rank = 3

(A|B) = 3

It is consistent

So,

$x + 3y - 2z = 0$

$11y - z = -1$

$81z = 81$

Now, $81z = 81$

$\Rightarrow z = 1$

$11y - 1 = -1$

$\Rightarrow y = 0$

$x + 3 \times 0 - 2 \times 1 = 0$

$\Rightarrow x = 2$

$\therefore (x, y, z) = (2, 0, 1)$ (Answer)