Name: Wasihun Leo

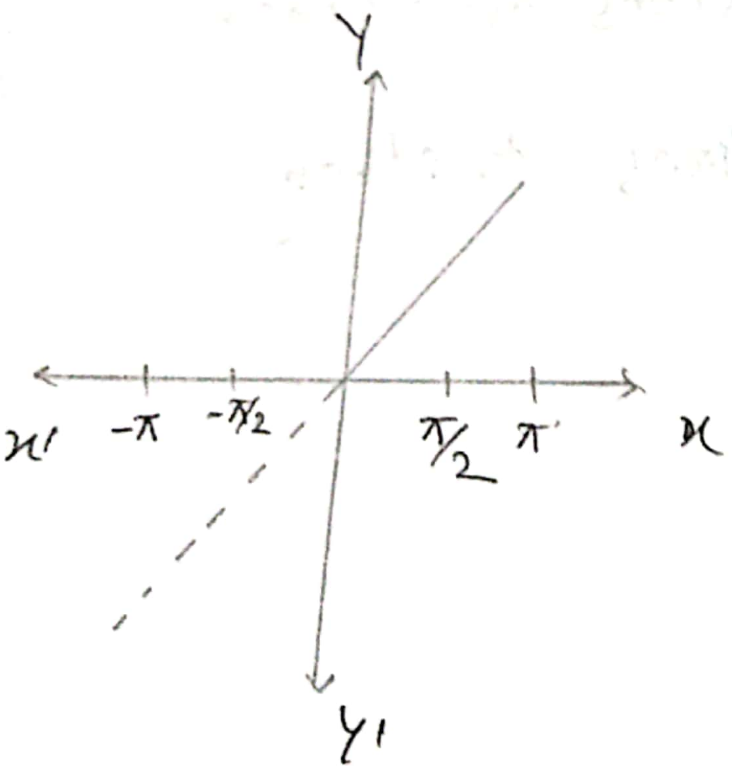
Id : 20-42195-1

Math assignment : 08

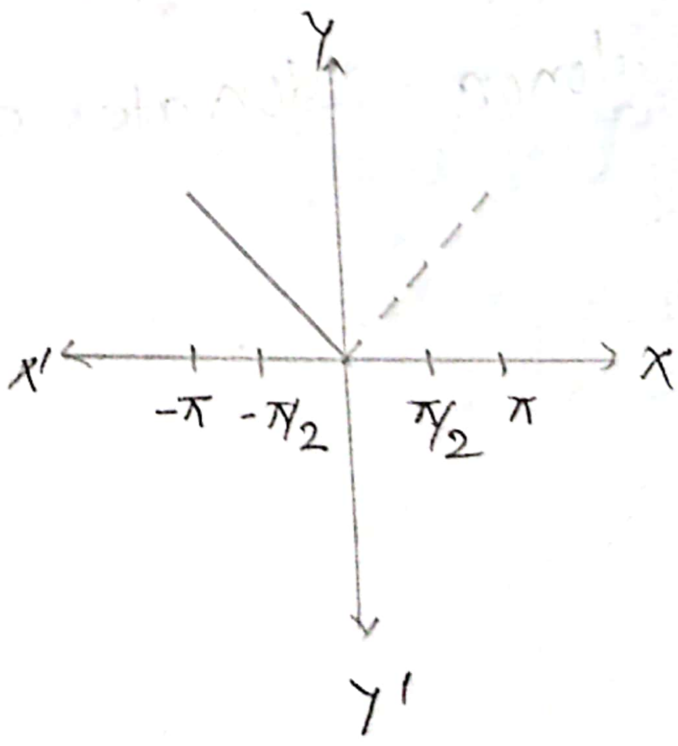
Slide : 08

1. $f(x)=x$; $0 < x < \pi$

Half range sine fourier series:



Half rang cosine fourier series:



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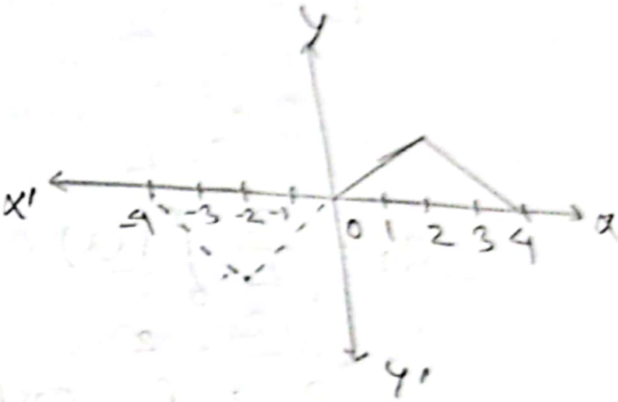
2. $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4-x, & 2 < x < 4 \end{cases}$

Half range sine fourier series:

$2L = 8$

$\Rightarrow L = 4$



$\therefore a_0 = 0$ [odd]

$\therefore a_n = 0$ [odd]

$$\therefore b_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{4}\left[\int_0^2 x \sin\left(\frac{n\pi x}{4}\right) dx + \int_2^4 (4-x) \sin\left(\frac{n\pi x}{4}\right) dx\right]$$

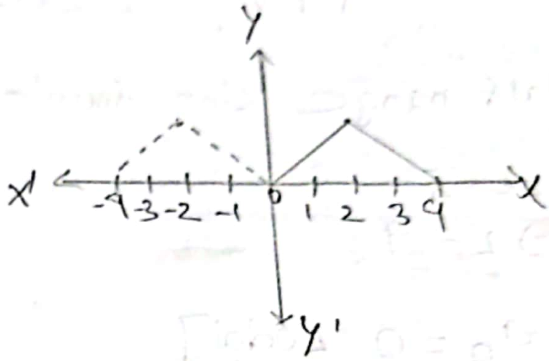
$$= \frac{16}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$$

$$\therefore f(x) = \sum_{n=odd}^{\infty} \frac{16}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi x}{4}\right)$$

Half range cosine Fourier series:

Here, $2L = 8$

$\Rightarrow L = 4$



$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$= 2$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{4}\left[\int_0^2 x\cos\left(\frac{n\pi x}{4}\right)dx + \int_2^4 (4-x)\cos\left(\frac{n\pi x}{4}\right)dx\right]$$

$$= \frac{8}{n^2\pi^2}\left(2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]$$

$b_n = 0$

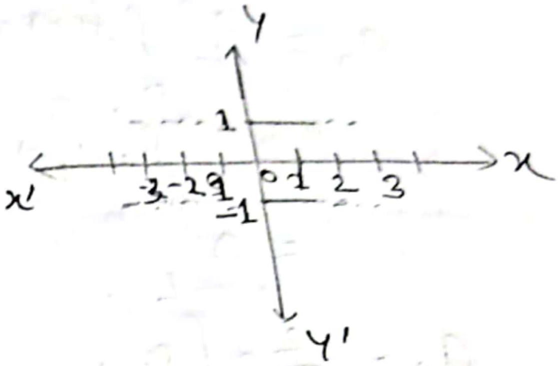
$$\therefore f(x) = 2 + \sum_{n=\text{even}}^{\infty} \frac{8}{n^2\pi^2}\left[2\cos\left(\frac{n\pi}{2}\right) - 1-(-1)^n\right]\cos\left(\frac{n\pi x}{4}\right)$$

3. $f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & 1 < x < 2 \end{cases}$

Half range sine fourier series:

Here, $2L = 4$

$\Rightarrow L = 2$



$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{2}\left[\int_0^1 1\sin\left(\frac{n\pi x}{2}\right)dx + \int_1^2 (-1)\sin\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{2}{n\pi}\left(1 + (-1)^n - \cos\left(\frac{n\pi}{2}\right)\right)$$

$$\therefore f(x) = \sum_{n=odd}^{\infty} \frac{2}{n\pi}\left(1 + (-1)^n - 2\cos\frac{n\pi}{2}\right)\sin\left(\frac{n\pi x}{2}\right)$$

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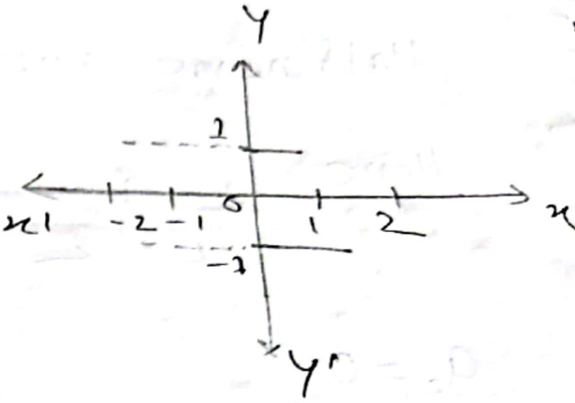
Half range cosine fourier series:

Here, $2L = 4$

$\Rightarrow L = 2$



$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$= \int_0^1 1\,dx + \int_1^2 (-1)\,dx$$

$$= 0$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{2}\left[\int_0^1 1\cos\left(\frac{n\pi x}{2}\right)dx + \int_1^2 \cos\left(\frac{n\pi x}{2}\right)dx\right]$$

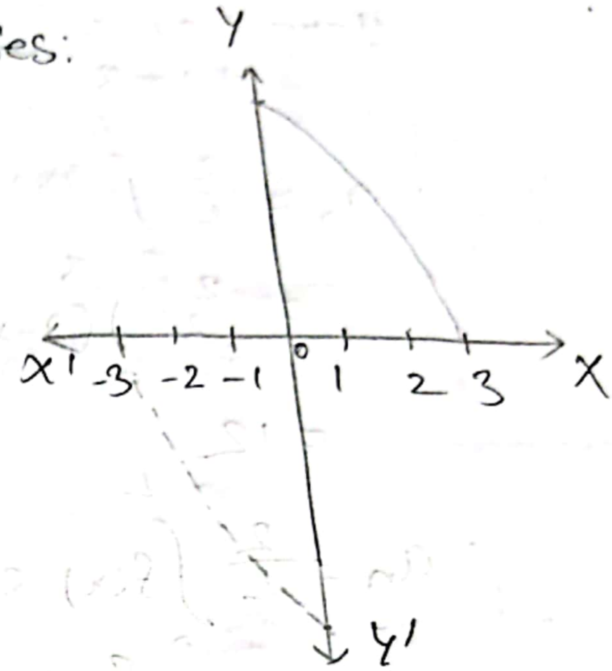
$$= \frac{4}{n\pi}\sin\left(\frac{m\pi}{2}\right)$$

$b_n = 0$

$$\therefore f(x) = \sum_{n = even}^{\infty} \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{2}\right)$$

9. $f(x) = 9 - x^2$; $0 < x < 3$

Half range sine fourier series:



Here, $2L = 6$

$\Rightarrow L = 3$

$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{3} \int_0^3 (9-x)^2 \sin\left(\frac{n\pi x}{3}\right) dx$$

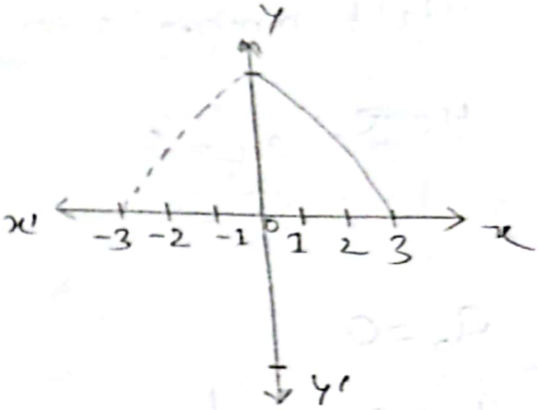
$$= \frac{18}{\pi^3 n^3} \left(\pi^2 n^2 + 2 - 2(-1)^n\right)$$

$$f(x) = \sum_{n=odd}^{\infty} \frac{18}{\pi^3 n^3} \left(\pi^2 n^2 + 2 - 2(-1)^n\right) \sin\left(\frac{n\pi x}{3}\right)$$

Half range cosine fourier series:

Here $2L = 6$

$\Rightarrow L = 3$



$\therefore a_0 = \frac{2}{L} \int_0^L f(x)\, dx$

$= \frac{2}{3} \int_0^3 (9 - x^2)\, dx$

$= 12$

$\therefore a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{2}\right) dx$

$= \frac{2}{3} \left[\int_0^3 (9 - x^2) \cos\left(\frac{n\pi x}{3}\right) dx \right]$

$= \frac{36}{n^2 \pi^2} (-1)^{n+1}$

$\therefore b_n = 0$

$\therefore f(x) = \frac{a_0}{2} + \sum_{n=even}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

$= \frac{12}{2} + \sum_{n=even}^{\infty} \frac{36}{\pi^2 n^2} (-1)^{n+1} (-1)^n \cancel{\cos\left(\frac{n\pi x}{3}\right)}$

$= 6 + \sum_{n=even}^{\infty} \frac{36}{\pi^2 n^2} (-1)^{2n+1}$

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$$1.\ f(x) = \begin{cases} 0 & \text{when } x < 0 \\ 1 & \text{when } x = 0 \\ e^{-x} & \text{when } x > 0 \end{cases}$$

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos(\omega x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\infty}^{0} 0 \cdot \cos(\omega x) dx + \int_{0}^{\infty} e^{-x} \cos(\omega x)\, dx \right]$$

$$= \frac{1}{\pi} \times \frac{1}{\omega^2+1}$$

$$B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin(\omega x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\infty}^{0} 0 \cdot \sin(\omega x) dx + \int_{0}^{\infty} e^{-x} \sin(\omega x)\, dx \right]$$

$$= \frac{1}{\pi} \times \frac{\omega}{w^2+1}$$

$$\therefore f(x) = \int_{0}^{\infty} [A(w) \cos(wx) + B(w) \sin(wx)]\, dw$$

$$= \frac{1}{\pi} \int_{0}^{\infty} \left[\frac{1}{w^2+1} \cos(wx) + \frac{w}{w^2+1} \sin(wx) \right]$$

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2. $f(x) = e^{-kx}$ when $x>0$ and $f(-x) = -f(x)$

$$A(w) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos(wx)\,dx$$

$$= 0 \text{ [odd]}$$

$$B(w) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin(wx)\,dx$$

$$= \frac{2}{\pi} \left[\frac{e^{-kx}(-k\sin(wx) - w\cos(wx))}{w^2+k^2} \right]_0^{\infty}$$

$$= \frac{2}{\pi} \times \frac{w}{w^2+k^2}$$

$$\therefore f(x) = \int_0^{\infty} [A(w)\cos(wx) + B(w)\sin(wx)]\,dw$$

$$\Rightarrow e^{-kx} = \int_0^{\infty} \left[\frac{2}{\pi} \times \frac{w}{w^2+k^2}\right] \sin(wx)\,dw$$

$$\Rightarrow e^{-kx} = \frac{2}{\pi} \int_0^{\infty} \frac{w\sin(wx)}{w^2+k^2}\,dw$$

$$\therefore \int_0^{\infty} \frac{w\sin(wx)}{w^2+k^2}\,dw = \frac{\pi}{2} e^{-kx}, \quad k>0$$

(Proved)

3. $f(x) = e^{-kx}$ when $x>0$ and $f(-x)=f(x)$

$$\therefore A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(wx)\,dx$$

$$= \frac{2}{\pi}\left[\frac{e^{-kx}(-k\cos(wx)+w\sin(wx))}{w^2+k^2}\right]_0^{\infty}$$

$$= \frac{2}{\pi}\times\frac{k}{w^2+k^2}$$

$$B = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx$$

$$= 0 \text{ [odd]}$$

$$\therefore f(x) = \int_0^{\infty}[A(w)\cos(wx)+B(w)\sin(wx)]\,dw$$

$$\Rightarrow e^{-kx} = \frac{2}{\pi}\int_0^{\infty}\frac{k\cos(wx)}{w^2+k^2}\,dw$$

$$\Rightarrow \frac{\pi}{2k}e^{-kx} = \int_0^{\infty}\frac{\cos(wx)}{w^2+k^2}, \quad k>0$$

(Proved)

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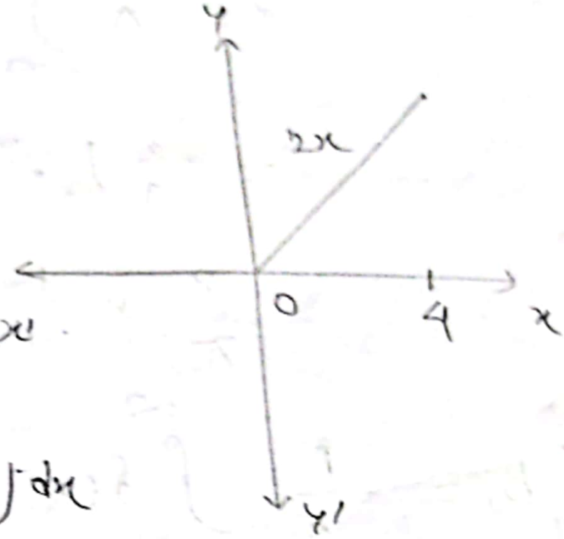
1. $f(x) = 2x, \; 0 < x < 4$

Here, $2L = 4+4$

$\Rightarrow L = 4$



a) finite fourier sine transform:

$$F_s\left(f(x)\right) = \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \int_0^4 2x \sin\left(\frac{n\pi x}{4}\right) dx$$

$$= \frac{32(-1)^{n+1}}{n\pi}$$

b) finite fourier cosine transform:

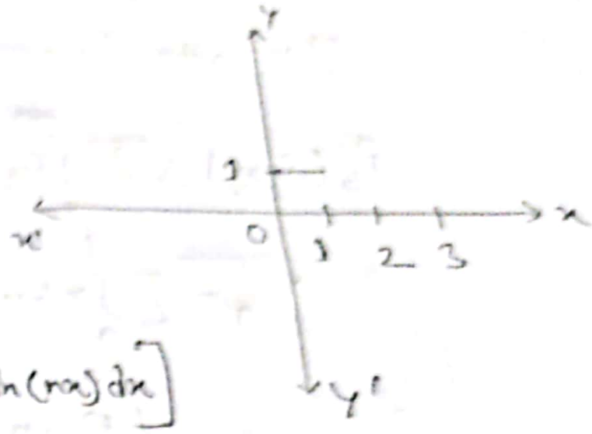
$$F_c(n) = \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \int_0^4 2x \cos\left(\frac{n\pi x}{4}\right) dx$$

$$= \frac{32}{n^2\pi^2}\left((-1)^n - 1\right)$$

2. $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

a) infinite fourier sine transform:



$$\hat{F}_S(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(nx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 1 \sin(nx)\, dx + \int_1^\infty 0 \sin(nx)\, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{1}{n}(-\cos(n) + 1) + 0 \right]$$

$$\hat{F}_S(n) = \sqrt{\frac{2}{\pi}}\ \frac{1}{n}(-\cos(n) + 1)$$

b) infinite fourier cosine transform:

$$\hat{F}_C(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(nx)\, dx$$

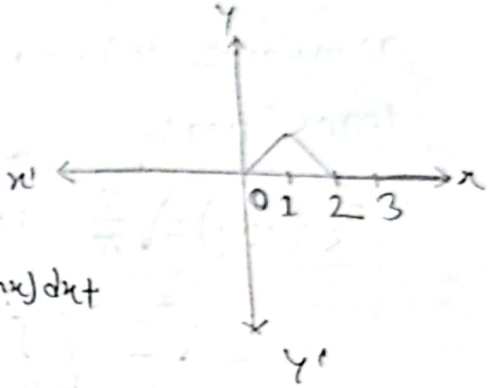
$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 1 \cos(nx)\, dx + \int_1^\infty 0 \cos(nx)\, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{1}{n} \sin(n) + 0 \right]$$

$$\hat{F}_C(n) = \sqrt{\frac{2}{\pi}}\ \frac{1}{n} \sin(n)$$

3. $f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & x > 2 \end{cases}$

a) infinite fourier sine transform:



$$\hat{F}_s(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(nx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 x \sin(nx)\, dx + \int_1^2 (2-x) \sin(nx)\, dx + \int_2^{\infty} 0 \sin(nx)\, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\left(\frac{1}{n^2} \sin(n) - \frac{1}{n} \cos(n)\right) + \frac{1}{n} \cos(n) + \frac{1}{n^2} \sin(n) - \frac{1}{n^2} \sin(2n) + 0 \right]$$

$$\hat{F}_s(n) = \sqrt{\frac{2}{\pi}} \left(\frac{2}{n^2} \sin(n) - \frac{1}{n^2} \sin(2n) \right)$$

b) infinite fourier cosine tranform:

$$\hat{F}_c(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos(nx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 x \cos(nx)\, dx + \int_1^2 (2-x) \cos(nx)\, dx + \int_2^{\infty} 0 \cos(nx)\, dx \right]$$

$$\hat{F}_c(n) = \sqrt{\frac{2}{\pi}} \left[\frac{1}{n} \sin(n) + \frac{1}{n^2} \cos(n) - \frac{1}{n^2} + \frac{n}{2}(\sin(2n) - \sin(n)) - \frac{1}{n^2}\left(2\sin(2n) + \cos(2n) - n\sin n - \cos(n)\right) \right]$$

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1. $h_k = 1.7\,\delta[k]$

$$H_n = \sum_{k=0}^{N-1} h_k e^{i\frac{2\pi}{N} \times nk}$$

$$= \sum_{k=0}^{N-1} 1.7\,\delta[k]\, e^{i\frac{2\pi}{N}\times nk}$$

$$= 1.7$$

2. $h_k = 5.5\,\delta[k-3]$

$$H_n = \sum_{k=0}^{N-1} 5.5\,\delta[k-3]\, e^{i\frac{2\pi}{N}\times nk}$$

$$= 5.5\,\delta[3-3]\, e^{i\frac{6\pi}{N} n}$$

$$= 5.5\left[\cos\left(\frac{6\pi n}{N}\right) + i\sin\left(\frac{6\pi n}{N}\right)\right] \quad [e^{i\theta} = \cos\theta + i\sin\theta]$$

where $n = 0, 1, 2, 3, \cdots N-1$

3. $h_k = 3\delta[k] + 8\,\delta[k-1]$

$$H_n = \sum_{k=0}^{N-1} \{ 3\delta[k] + 8\,\delta[k-1] \}\, e^{i\frac{2\pi}{N}nk}$$

$$= 3\,\delta[0]\, e^0 + 8\,\delta[1-1]\, e^{i\frac{2\pi n}{N}}$$

$$= 3 + 8\, e^{i\frac{2\pi n}{N}}$$

$$= 3 + 8\left[\cos\frac{2\pi n}{N} + i\sin\frac{2\pi n}{N}\right]$$

4. $h_k = 0.5\,\delta[k-2] - 2\delta[k-4]$

$$\therefore H_n = \sum_{k=0}^{N-1} \{0.5\,\delta[k-2] - 2\delta[k-4]\}\, e^{i\frac{2\pi}{N}kn}$$

$$= 0.5\,\delta[2-2]\, e^{i\frac{4\pi n}{N}} - 2\,\delta[4-4]\, e^{i\frac{8\pi n}{N}}$$

$$= 0.5\left(\cos\frac{4\pi n}{N} + i\sin\frac{4\pi n}{N}\right) - 2\left(\cos\frac{8\pi n}{N} + i\sin\frac{8\pi n}{N}\right)$$