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Answer to the Question No. (1)

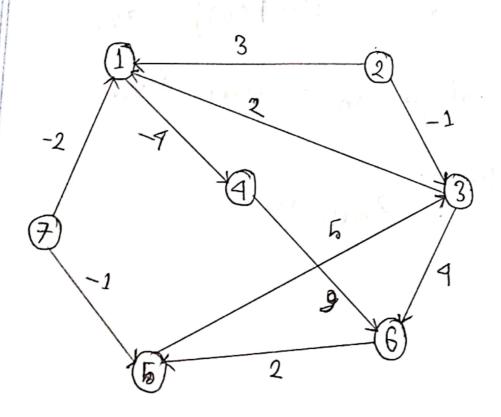Here,

$$a = 2, \quad c = 9, \quad d = 2, \quad e = 1, \quad f = 9, \quad g = 5, \quad h = 1$$

$$b = a + b + h$$
$$= 2 + 0 + 1$$
$$= 3$$

Initial adjacency matrix:

$$D^{(0)} = \begin{array}{c c} & \begin{array}{c c c c c c c} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \left[\begin{array}{c c c c c c c} 0 & \infty & \infty & -4 & \infty & \infty & \infty \\ 3 & 0 & -1 & \infty & \infty & \infty & \infty \\ 2 & \infty & 0 & \infty & \infty & 4 & \infty \\ \infty & \infty & \infty & 0 & \infty & 9 & \infty \\ \infty & \infty & 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 & 0 & \infty \\ -2 & \infty & \infty & \infty & -1 & \infty & 0 \end{array} \right] \end{array}$$

Initial path matrix:

$$\Pi^{(0)} = \begin{array}{c c} & \begin{array}{c c c c c c c} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \left[\begin{array}{c c c c c c c} \phi & \phi & \phi & 1 & \phi & \phi & \phi \\ 2 & \phi & 2 & \phi & \phi & \phi & \phi \\ 3 & \phi & \phi & \phi & \phi & 3 & \phi \\ \phi & \phi & \phi & \phi & \phi & 4 & \phi \\ \phi & \phi & 5 & \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi & 6 & \phi & \phi \\ 7 & \phi & \phi & \phi & 7 & \phi & \phi \end{array} \right] \end{array}$$

VIA NODE = 1

$$D^{(1)} =
\begin{array}{c c}
 & \begin{array}{c c c c c c c} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\left[\begin{array}{c c c c c c c}
0 & \infty & \infty & -4 & \infty & \infty & \infty \\
3 & 0 & -1 & -1 & \infty & \infty & \infty \\
2 & \infty & 0 & -2 & \infty & 4 & \infty \\
\infty & \infty & \infty & 0 & \infty & 9 & \infty \\
\infty & \infty & 5 & \infty & 0 & \infty & \infty \\
\infty & \infty & \infty & \infty & 2 & 0 & \infty \\
-2 & \infty & \infty & -6 & -1 & \infty & 0
\end{array}\right]
\end{array}$$

$$\Pi^{(1)} =
\begin{array}{c c}
 & \begin{array}{c c c c c c c} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\left[\begin{array}{c c c c c c c}
\phi & \phi & \phi & 1 & \phi & \phi & \phi \\
2 & \phi & 2 & 1 & \phi & \phi & \phi \\
3 & \phi & \phi & 1 & \phi & 3 & \phi \\
\phi & \phi & \phi & \phi & \phi & 4 & \phi \\
\phi & \phi & 5 & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & 6 & \phi & \phi \\
7 & \phi & \phi & 1 & 7 & \phi & \phi
\end{array}\right]
\end{array}$$

VIA NODE=2

$$D^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}$$

	1	2	3	4	5	6	7
1	0	∞	∞	-1	∞	∞	∞
2	3	0	-1	-1	∞	∞	∞
3	2	∞	0	-2	∞	4	∞
4	∞	∞	∞	0	∞	9	∞
5	∞	∞	5	∞	0	∞	∞
6	∞	∞	∞	∞	2	0	∞
7	-2	∞	∞	-6	-1	∞	0

$\Pi^{(2)} =$

	1	2	3	4	5	6	7
1	φ	φ	φ	1	φ	φ	φ
2	2	φ	2	1	φ	φ	φ
3	3	φ	φ	1	φ	2	φ
4	φ	φ	φ	φ	φ	4	φ
5	φ	φ	5	φ	φ	φ	φ
6	φ	φ	φ	φ	6	φ	φ
7	7	φ	φ	1	7	φ	φ

VIA NODE = 3

$$
D^{(3)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
0 & \infty & \infty & -4 & \infty & \infty & \infty \\
1 & 0 & -1 & -1 & \infty & 3 & \infty \\
2 & \infty & 0 & -2 & \infty & 4 & \infty \\
\infty & \infty & \infty & 0 & \infty & 9 & \infty \\
7 & \infty & 5 & 3 & 0 & 9 & \infty \\
\infty & \infty & \infty & \infty & 2 & 0 & \infty \\
-2 & \infty & \infty & -6 & -1 & \infty & 0
\end{array}
$$

$$
\Pi^{(3)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\phi & \phi & \phi & 1 & \phi & \phi & \phi \\
3 & \phi & 2 & 1 & \phi & 3 & \phi \\
3 & \phi & \phi & 1 & \phi & 2 & \phi \\
\phi & \phi & \phi & \phi & \phi & 4 & \phi \\
3 & \phi & 5 & 3 & \phi & 3 & \phi \\
\phi & \phi & \phi & \phi & 6 & \phi & \phi \\
7 & \phi & \phi & 1 & 7 & \phi & \phi
\end{array}
$$

VIA NODE = 4

$$D^{(4)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\left[\begin{array}{ccccccc}
0 & \infty & \infty & -4 & \infty & 5 & \infty \\
3 & 0 & -1 & -1 & \infty & 3 & \infty \\
2 & \infty & 0 & -2 & \infty & 4 & \infty \\
\infty & \infty & \infty & 0 & \infty & 9 & \infty \\
7 & \infty & 5 & 3 & 0 & 13 & \infty \\
\infty & \infty & \infty & \infty & 2 & 0 & \infty \\
-2 & \infty & \infty & -6 & -1 & 3 & 0
\end{array} \right] \end{array}$$

$$\Pi^{(4)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\left[\begin{array}{ccccccc}
\emptyset & \emptyset & \emptyset & 1 & \emptyset & 4 & \emptyset \\
3 & \emptyset & 2 & 1 & \emptyset & 3 & \emptyset \\
3 & \emptyset & \emptyset & 1 & \emptyset & 2 & \emptyset \\
\emptyset & \emptyset & \emptyset & \emptyset & \emptyset & 4 & \emptyset \\
3 & \emptyset & 5 & 3 & \emptyset & 3 & \emptyset \\
\emptyset & \emptyset & \emptyset & \emptyset & 6 & \emptyset & \emptyset \\
7 & \emptyset & \emptyset & 1 & 7 & 4 & \emptyset
\end{array} \right] \end{array}$$

VIA NODE = 5

$$D^{(5)} = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & \infty & \infty & -4 & \infty & 5 & \infty \\ 2 & 1 & 0 & -1 & -1 & \infty & 3 & \infty \\ 3 & 2 & \infty & 0 & -2 & \infty & 1 & \infty \\ 4 & \infty & \infty & \infty & 0 & \infty & 9 & \infty \\ 5 & 7 & \infty & 5 & 3 & 0 & 13 & \infty \\ 6 & 9 & \infty & 7 & 5 & 2 & 0 & \infty \\ 7 & -2 & \infty & 1 & -6 & -1 & 8 & 0 \end{array}$$

$$\Pi^{(5)} = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & \phi & \phi & \phi & 1 & \phi & 4 & \phi \\ 2 & 3 & \phi & 2 & 1 & \phi & 3 & \phi \\ 3 & 3 & \phi & \phi & 1 & \phi & 2 & \phi \\ 4 & \phi & \phi & \phi & \phi & \phi & 4 & \phi \\ 5 & 3 & \phi & 5 & 3 & \phi & 3 & \phi \\ 6 & 5 & \phi & 5 & 5 & 6 & \phi & \phi \\ 7 & 7 & \phi & 5 & 1 & 7 & 4 & \phi \end{array}$$

VIA NODE = 6

$$D^{(6)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \left[\begin{array}{ccccccc} 0 & \infty & 12 & -4 & 7 & 5 & \infty \\ 3 & 0 & -1 & -1 & 5 & 3 & \infty \\ 2 & \infty & 0 & -2 & 6 & 4 & \infty \\ 18 & \infty & 16 & 0 & 11 & 9 & \infty \\ 7 & \infty & 5 & 3 & 0 & 13 & \infty \\ 9 & \infty & 7 & 5 & 2 & 0 & \infty \\ -2 & \infty & 4 & -6 & -1 & 3 & 0 \end{array}\right] \end{array}$$

$$\pi^{(6)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \left[\begin{array}{ccccccc} \varnothing & \varnothing & 6 & 1 & 6 & 4 & \varnothing \\ 3 & \varnothing & 2 & 1 & 6 & 3 & \varnothing \\ 3 & \varnothing & \varnothing & 1 & 6 & 2 & \varnothing \\ 6 & \varnothing & 6 & \varnothing & 6 & 4 & \varnothing \\ 3 & \varnothing & 5 & 3 & \varnothing & 3 & \varnothing \\ 5 & \varnothing & 5 & 5 & 6 & \varnothing & \varnothing \\ 7 & \varnothing & 5 & 1 & 7 & 4 & \varnothing \end{array}\right] \end{array}$$

VIA NODE = 7

$$D^{(7)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\left[\begin{array}{ccccccc}
0 & \infty & 12 & -4 & 7 & 5 & \infty \\
1 & 0 & -1 & -1 & 5 & 3 & \infty \\
2 & \infty & 0 & -2 & 6 & 4 & \infty \\
18 & \infty & 16 & 0 & 11 & 9 & \infty \\
7 & \infty & 5 & 3 & 0 & 13 & \infty \\
9 & \infty & 7 & 5 & 2 & 0 & \infty \\
-2 & \infty & 4 & -6 & -1 & 3 & 0
\end{array}\right]
\end{array}$$

$$\Pi^{(7)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\left[\begin{array}{ccccccc}
\emptyset & \emptyset & 6 & 1 & 6 & 4 & \emptyset \\
3 & \emptyset & 2 & 1 & 6 & 3 & \emptyset \\
3 & \emptyset & \emptyset & 1 & 6 & 2 & \emptyset \\
6 & \emptyset & 6 & \emptyset & 6 & 4 & \emptyset \\
3 & \emptyset & 5 & 3 & \emptyset & 3 & \emptyset \\
5 & \emptyset & 5 & 5 & 6 & \emptyset & \emptyset \\
7 & \emptyset & 5 & 1 & 7 & 4 & \emptyset
\end{array}\right]
\end{array}$$

Final Graph (All pair shortest path):-