# Simplification Using Boolean Algebra
# &
# Universal Gates

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# LOGIC SIMPLIFICATION

$$\overline{(AB+C)(A+BC)} = (\overline{AB+C}) + (\overline{A+BC})$$

$$= (\overline{AB})\,\overline{C} + \overline{A}\,(\overline{BC})$$

$$= (\overline{A}+\overline{B})\,\overline{C} + \overline{A}\,(\overline{B}+\overline{C})$$

# LOGIC SIMPLIFICATION

$$\overline{(A+B+C)D} = \overline{A+B+C} + \overline{D}$$

$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})\,(\overline{\overline{C}D})\,(\overline{EF})$$

$$= (\overline{A} + \overline{\overline{B}})\,(\overline{\overline{C}} + \overline{D})\,(\overline{E} + \overline{F})$$

$$= (\overline{A} + B)\,(C + \overline{D})\,(\overline{E} + \overline{F})$$

# LOGIC SIMPLIFICATION

$$\overline{\overline{(A+B)} + \overline{C}} = \left(\overline{\overline{A+B}}\right)\left(\overline{\overline{C}}\right) = (A+B)C$$

$$\overline{(A+B)\overline{C}\,\overline{D} + E + \overline{F}} = \left(\overline{(A+B)\overline{C}\,\overline{D}}\right)\overline{E}\,\overline{\overline{F}}$$

$$= \left(\left(\overline{A+B}\right) + \overline{\overline{C}} + \overline{\overline{D}}\right)\overline{E}F$$

$$= \left(\overline{A}\,\overline{B} + C + D\right)\overline{E}F$$

# LOGIC SIMPLIFICATION

$$\overline{\overline{A+B\bar{C}} + D\overline{(E+\bar{F})}} = \left(\overline{\overline{A+B\bar{C}}}\right)\left(\overline{D\overline{(E+\bar{F})}}\right)$$

$$= (A+B\bar{C})\left(\bar{D} + \overline{\overline{E+\bar{F}}}\right)$$

$$= (A+B\bar{C})(\bar{D} + E + \bar{F})$$

# LOGIC SIMPLIFICATION

a) $AB + A(B+C) + B(B+C) = AB + AB + AC + BB + BC$

$= AB + AC + B + BC$

$= AB + AC + B(1+C)$

$= AB + AC + B$

$= B(A+1) + AC$

$= B + AC$

# LOGIC SIMPLIFICATION

b) $\overline{AB} + \overline{AC} + \bar{A}\bar{B}\bar{C} = \bar{A} + \bar{B} + \bar{A} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$$= \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C} \quad [\bar{A} + \bar{A} = \bar{A}]$$

$$= \bar{A} + \bar{B} + \bar{C}(1 + \bar{A}\bar{B})$$

$$= \bar{A} + \bar{B} + \bar{C}$$

$$= \overline{ABC}$$

# LOGIC SIMPLIFICATION

e) $\left[A\bar{B}(C+BD)+\bar{A}\bar{B}\right]C = (A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B})C$

$$= (A\bar{B}C + A.0.D + \bar{A}\bar{B})C \quad [B\bar{B}=0]$$

$$= (A\bar{B}C + 0 + \bar{A}\bar{B})C \quad [A.0=0]$$

$$= (A\bar{B}C + \bar{A}\bar{B})C \quad [A+0=0]$$

$$= A\bar{B}CC + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C \quad [C\cdot C = C]$$

$$= \bar{B}C(A+\bar{A}) \quad [A+\bar{A}=1]$$

$$= \bar{B}C\cdot 1$$

$$= \bar{B}C \quad [A\cdot 1 = A]$$

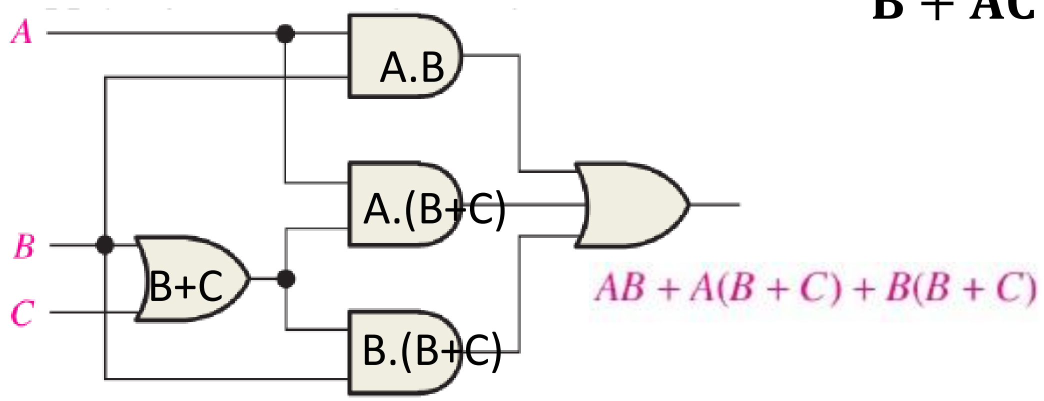
# LOGIC SIMPLIFICATION

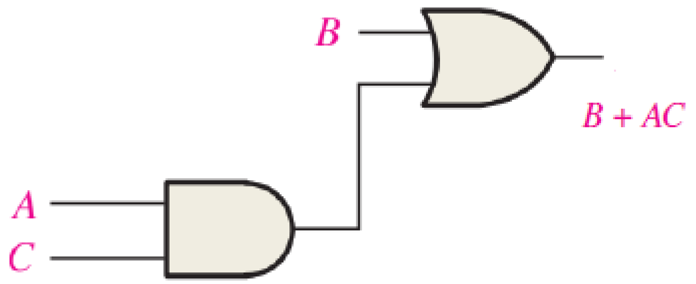
- A large Boolean Logic Expression can often be simplified to a simpler and shorter Boolean Logic Expression. This is done by applying the laws and rules of Boolean Algebra.
- Using Boolean algebra techniques, simplify this expression:

$$\mathbf{AB + A(B + C) + B(B + C)}$$

This can be simplified to:

$$\mathbf{B + AC}$$



# LOGIC SIMPLIFICATION & IMPLEMENTATION

Simplify the Boolean expression $A\overline{B} + A(\overline{B + C}) + B(\overline{B + C})$.

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Simplify the Boolean expression $\overline{AB} + \overline{AC} + \overline{A}\,\overline{B}\,\overline{C}$.

1. Simplify the following Boolean expressions:

   (a) $A + AB + A\overline{B}C$ (b) $(\overline{A} + B)C + ABC$ (c) $A\overline{B}C(BD + CDE) + A\overline{C}$

2. Implement each expression in Question 1 as originally stated with the appropriate logic gates. Then implement the simplified expression, and compare the number of gates.

$$\bar{A}BC + (A + \bar{A})\bar{B}\bar{C} + AC(B + \bar{B})$$

$$\bar{A}BC + \bar{B}\bar{C} + AC$$

$$AC + \bar{A}BC + \bar{B}\bar{C}$$

$$C(A + \bar{A}B) + \bar{B}\bar{C}$$

$$C(A + B) + \bar{B}\bar{C}$$

As $(A + \bar{A}B)$=A+B

$$AC + BC + \bar{B}\bar{C}$$

# Universal Gates

- Although we can implement **any circuit** with **AND/OR/NOT**, we can also implement **any circuit** with **only NAND** or **NOR** gates.
- We might want to do this because of technology considerations, that is, these gates might be cheaper to implement in silicon or they might be the only type of gates we have available.
- Since we can always use only NAND or NOR gates, these gates are sometimes called **universal gates**.
- The "trick" (if you want to call it that) is to see that we can implement the three basic gates (AND, OR, NOT) in terms of NAND or NOR gates.

# Universal Property of NAND and NOR gate

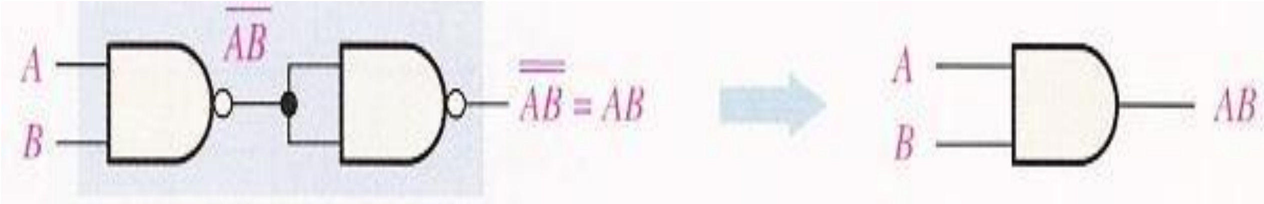
**The NAND gate as a Universal logic gate:**

- 1) <u>One NAND gate used as an inverter (**NOT gate**)</u>:

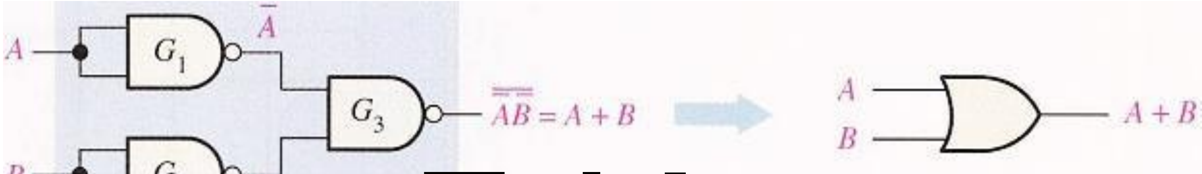

$\overline{A.A} = \bar{A}$

$f = \overline{x \bullet x} = \overline{x}$

- 2) <u>Two NAND gates used as an **AND gate**</u>




- ) <u>Three NAND gates used as an **OR gate**</u>

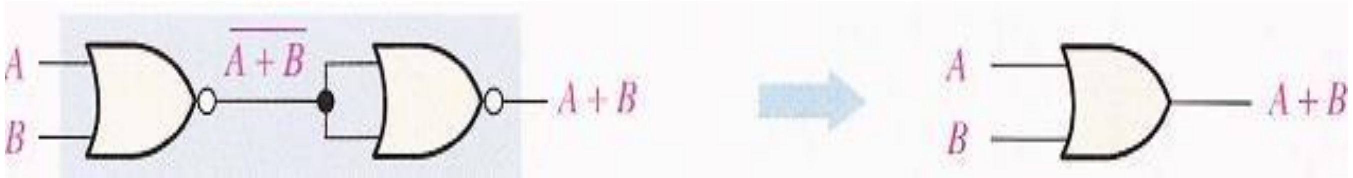



$\overline{\bar{A}.\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B$

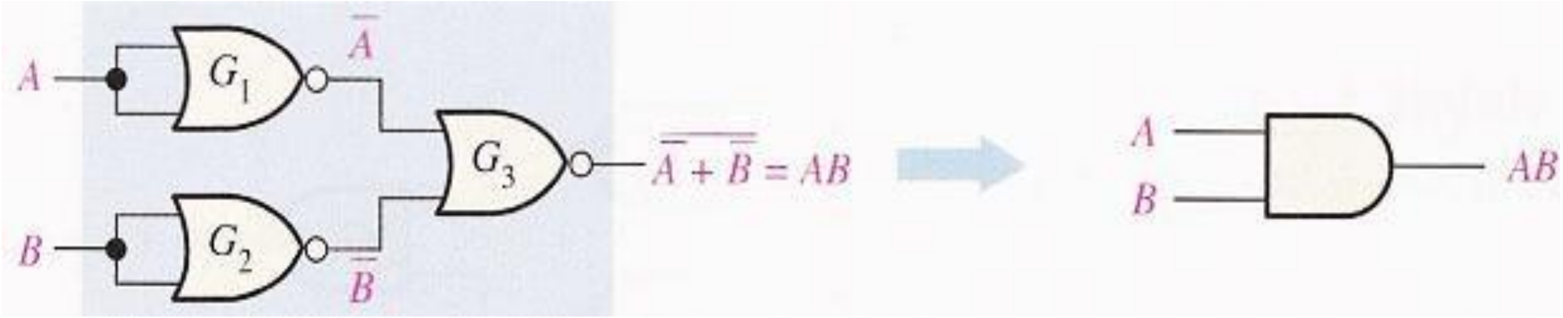
# The NOR gate as a Universal logic gate:

- 1) <u>One NOR gate used as an inverter (**NOT gate**)</u>:


- 2) <u>Two NOR gates used as an **OR gate**</u>:




- 3) <u>Three NOR gates used as an **AND gate**</u>:



# Steps for Construction of Logic circuits Using Universal Gates

1)First construct the logic expression using basic gates ( AND/OR/NOT),
2)Replace each basic gate with its equivalent universal gate implementation,
3)Cancel two consecutive NOT gates (according to Boolean algebra),
4)Redraw the final circuit.

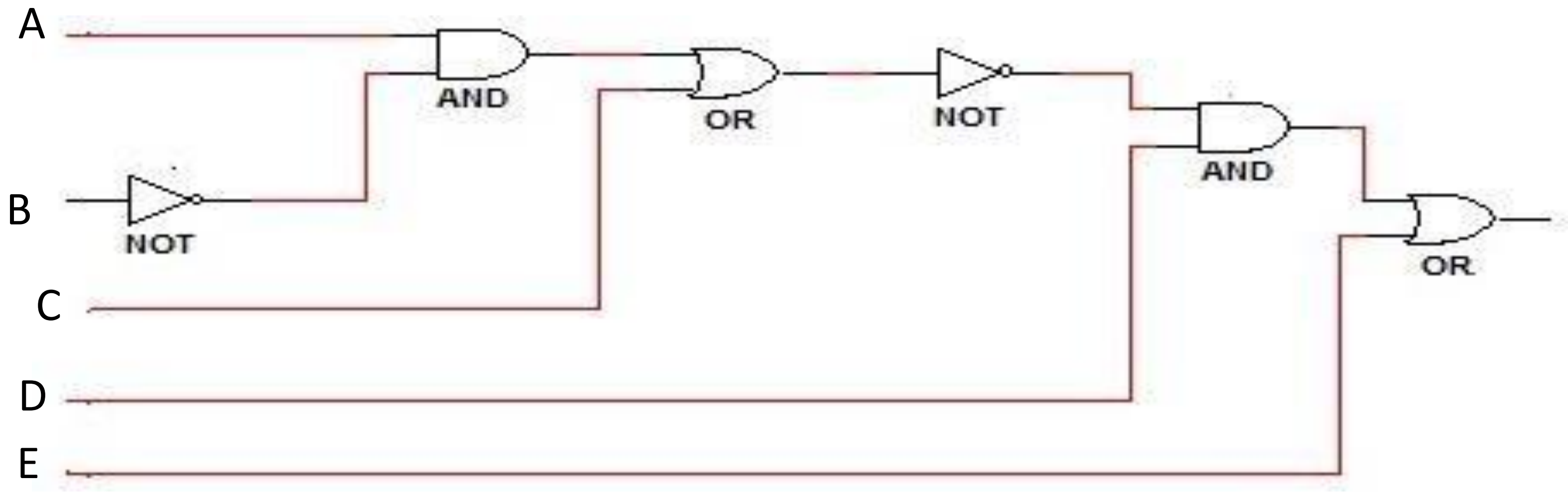
**Example:** Implement the following expression with (a) NAND gates ONLY, (b) NOR gates ONLY.

**Y=(AB'+C)'D+E=** $\overline{[A.\bar{B}+C]}.D+E$

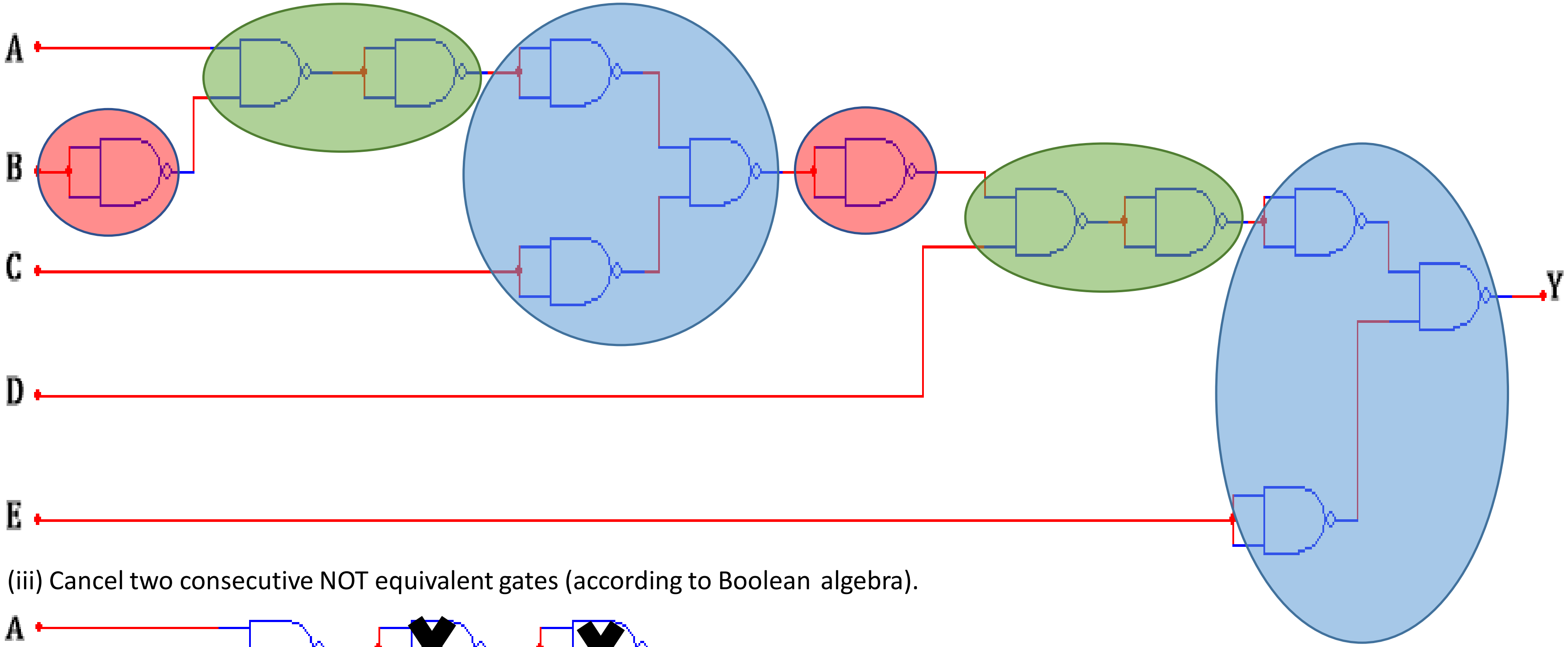
**Solution:**

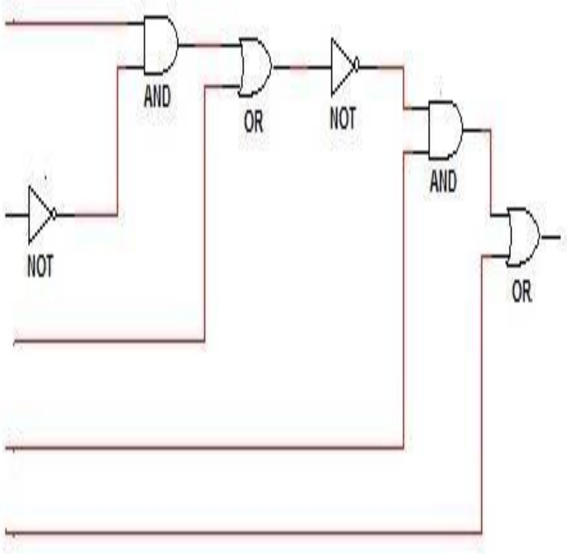
**Implementation using NAND gates ONLY:**

i) Implementation with Basic gates:



(ii) Replace each basic gate with its equivalent NAND gate implementation from Table 1.



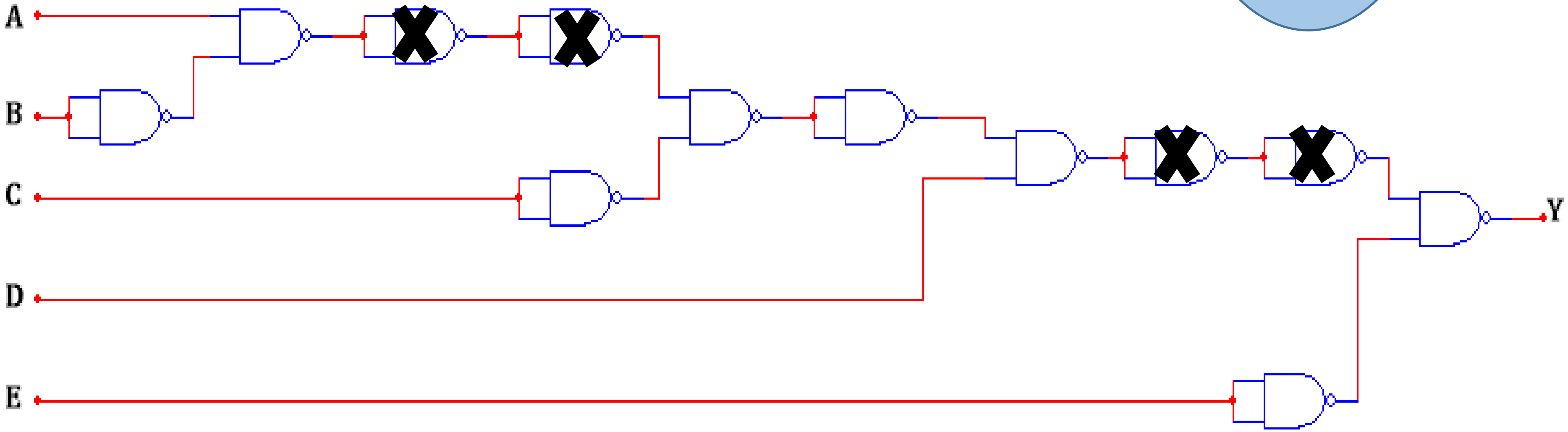




A''=A

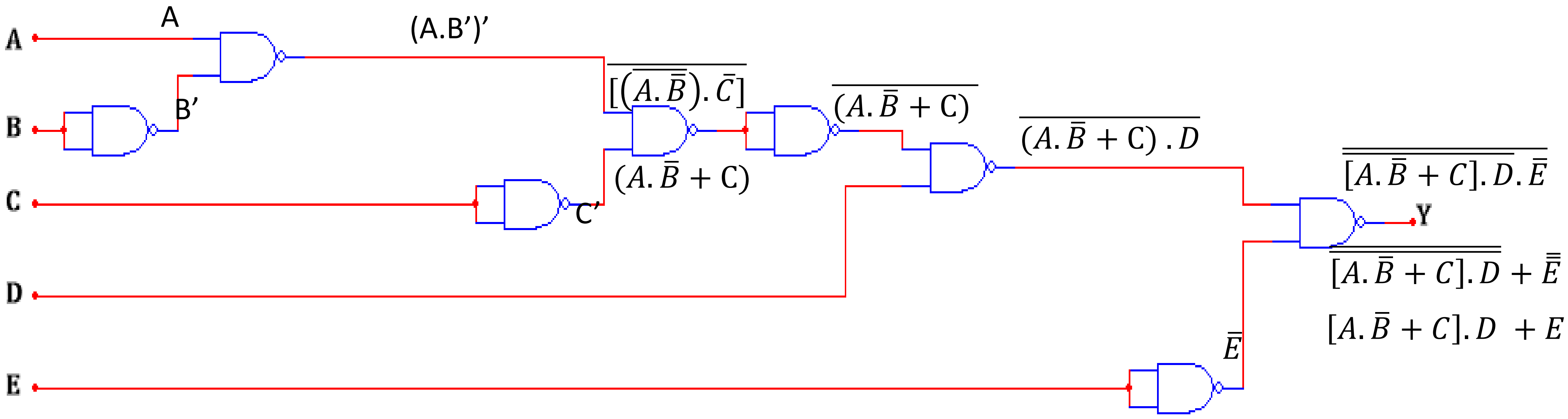
A'

A

(iii) Cancel two consecutive NOT equivalent gates (according to Boolean algebra).

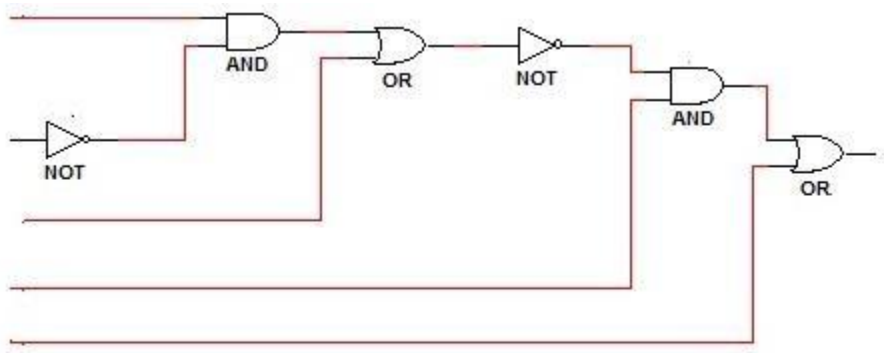


(iv)Redraw the final circuit




**(b) Implementation using NOR gates ONLY:**

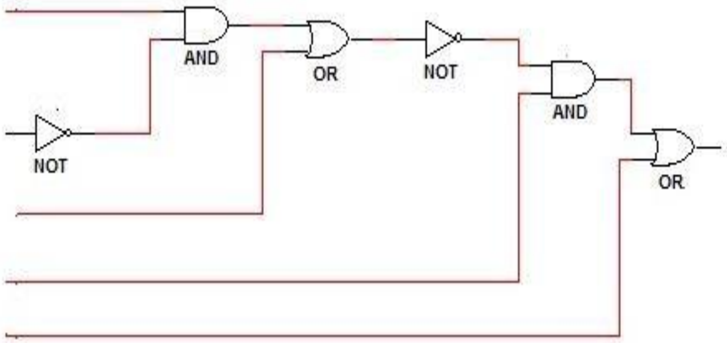
(i) Implementation with Basic gates:



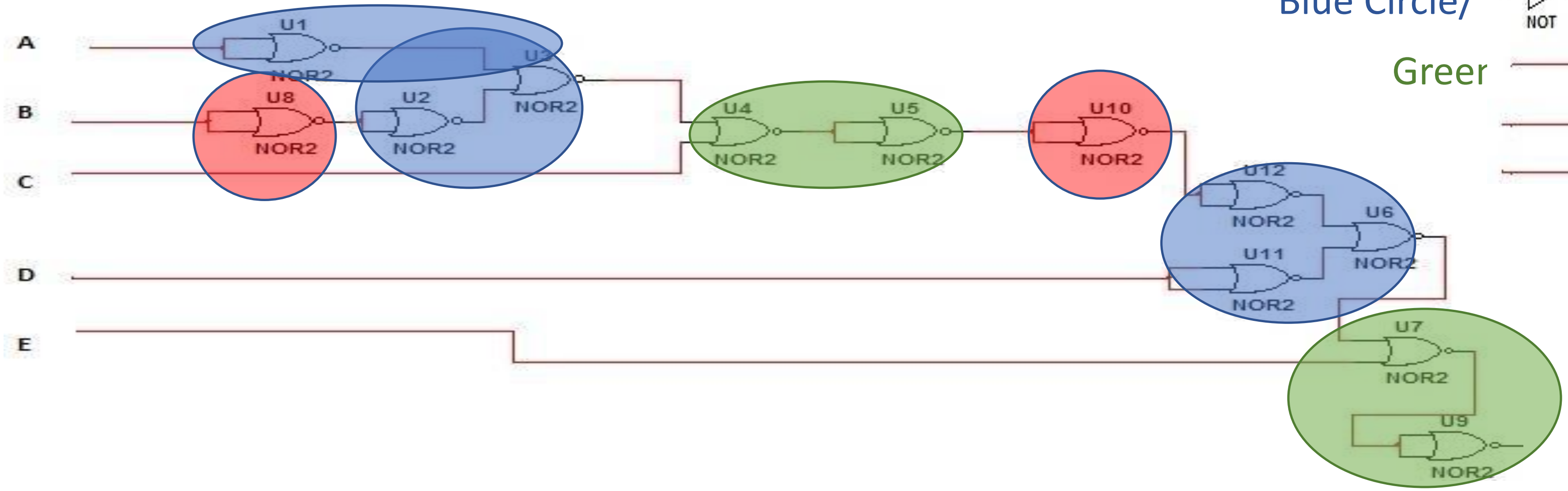
Red Circle = NOT

Blue Circle/

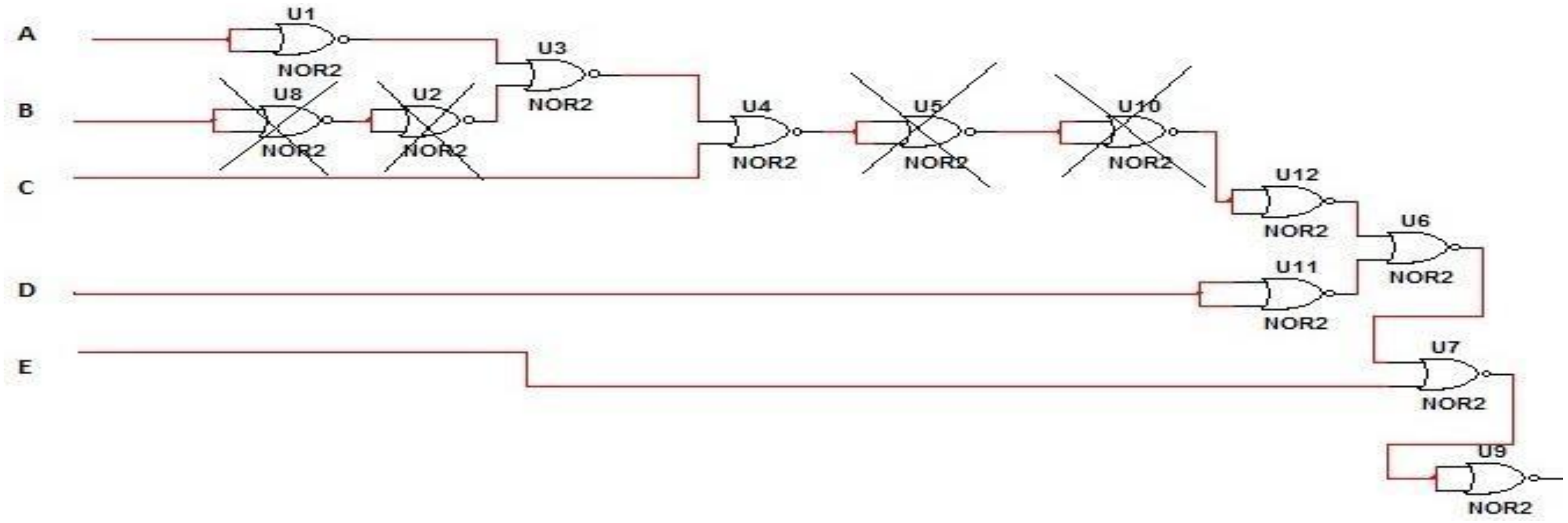
Greer



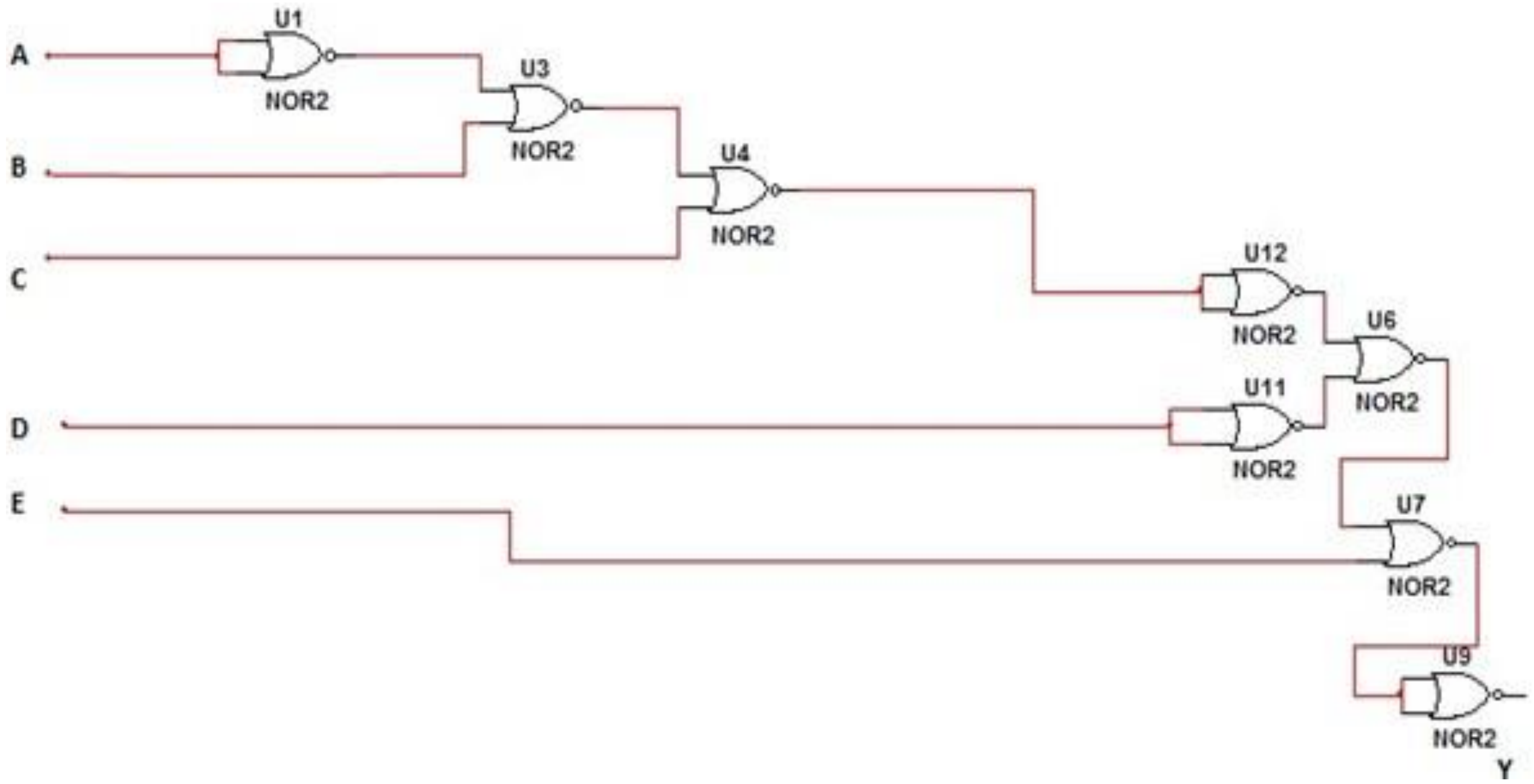

(ii) Replace each basic gate with its equivalent NOR gate implementation from Table 1.




(iii) Cancel two consecutive NOT equivalent gates (according to Boolean algebra).



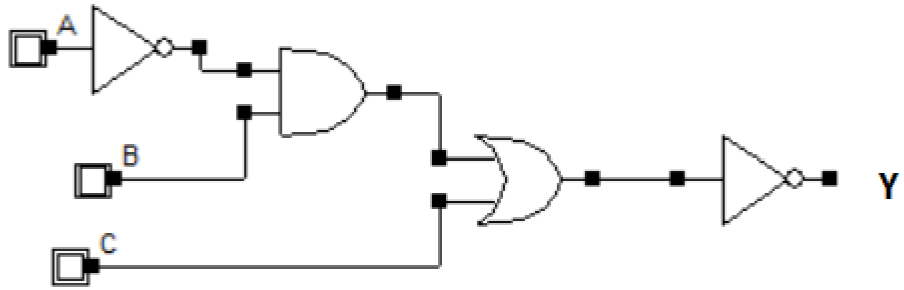
(iv)Redraw the final circuit.



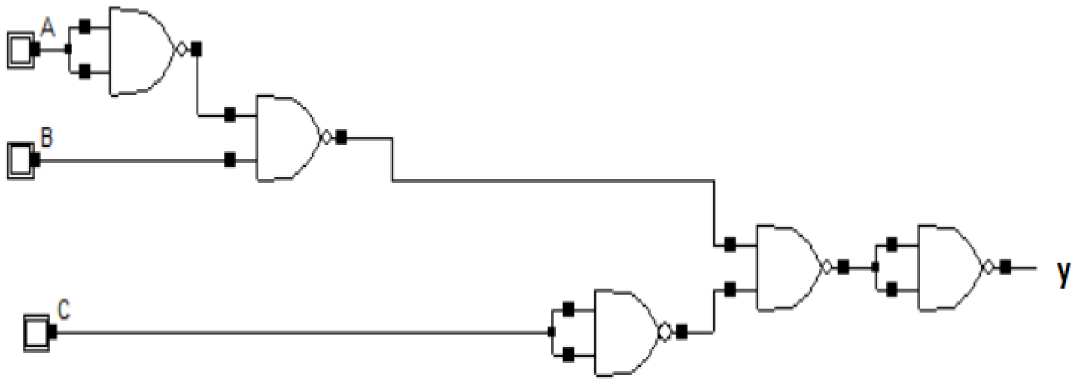
# In Class Activity

- Design the following equation using i) NAND gates only ii) NOR gates only
- Count the number of gates for each design and comment on which one is better.
  - $Y=(A'.B+C)'$

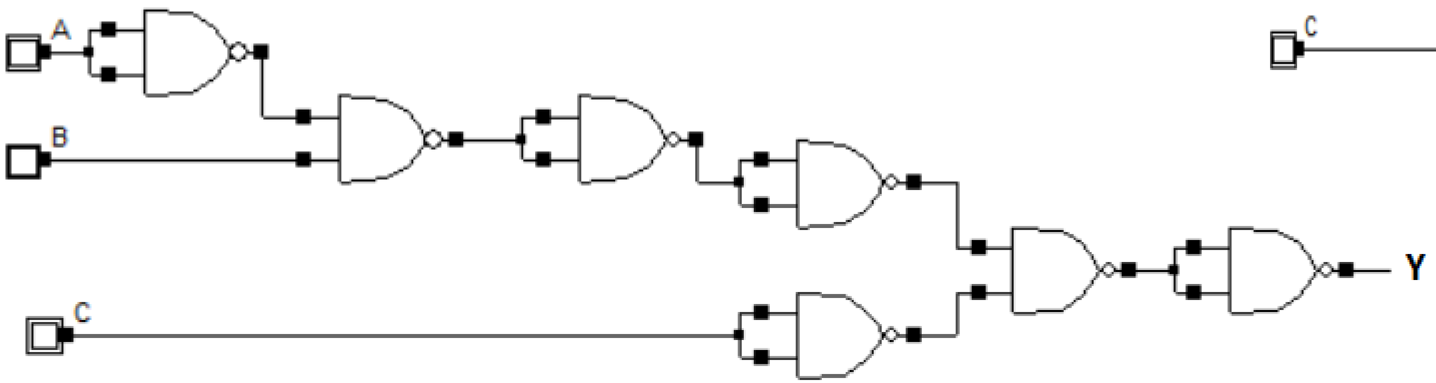
i) Basic Gate Design



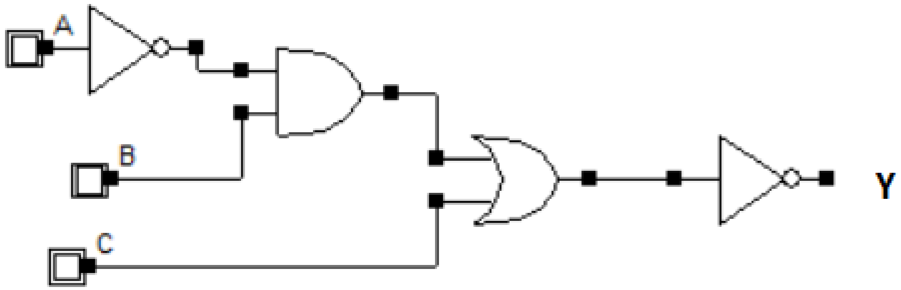

iii) Final Circuit after reduction



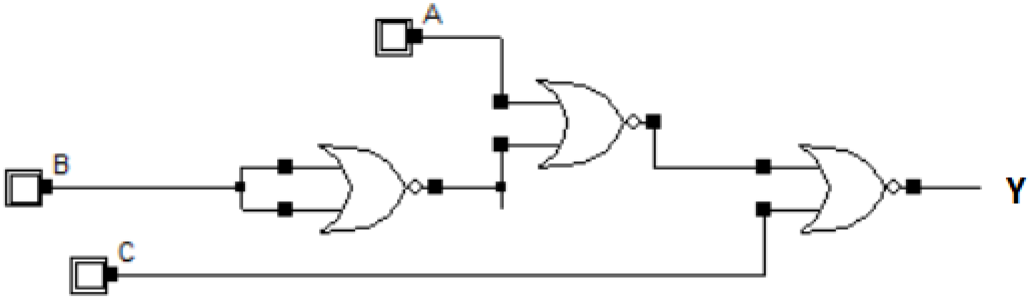

ii) Replacement using NAND only



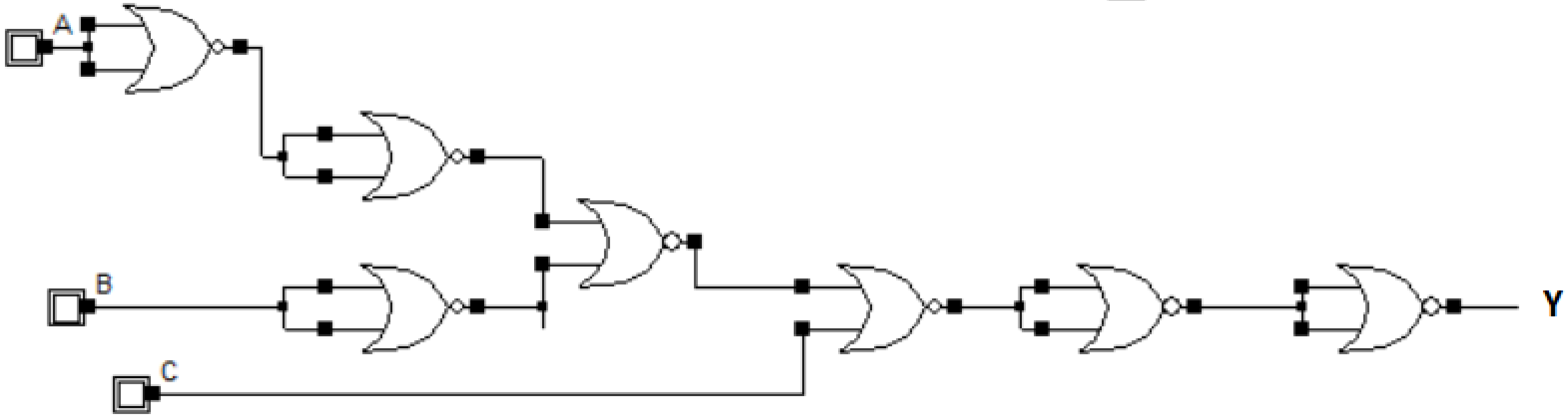
i) Basic Gate Design




iii) Final Circuit after reduction




ii) Replacement using NOR only



# Comment

- Since after design both the circuits using NAND and NOR gates it is found that NAND gate design requires 5 gates in the final design while, the NOR gate design requires only 3 gates. So in my opinion, NOR gate design is more preferred as it has lesser number of gates and cost effective.