Experiment No. 1:
Experiment Name: To determine the acceleration due to gravity by means of a compound pendulum.

Objectives:
The main objective of this lab is to determine the acceleration due to gravity in the lab with a compound pendulum.

Prelab:
Student should read the lab manual and have clear idea about the objective, time frame and outcomes of the lab.

Outcomes:
After completing this experiment student should be able to answer the following questions:
- What is acceleration due to gravity?
- How a compound pendulum can be constructed and what are its criteria?
- What is the difference between a simple pendulum and a compound pendulum?
- How effective length (equivalence to simple pendulum length) can be measured from graph?
- How acceleration due to gravity can be calculated from the effective length of a bar pendulum?

Timing and Length of Investigation (Total 3 Hours):

- **Lab Preparation (15 minutes):**
  - Students will sit for the lab class with preparations and class attendance will be taken.
- **Lecture on Theory (30 minutes):**
  - Teacher will clarify the objective and theory of the experiment.
- **Lecture on Procedure (15 minutes):**
  - Students will learn about the procedure of the experiment through a video lecture.
- **Experimental Work (90 to 100 minutes):**
  - A sample data table will be provided to students and teacher will clarify every part of it.
  - Students will do all the calculations, draw graphs in excel and complete the result part.
- **Post Lab Discussion (15 to 20 minutes):**
  - Teacher will summarize the total lab work and have a discussion with the students related with the questions given in the outcomes part.
- **Report Submission:**
  - After completing the lab reports students will upload their lab reports as groups in teams in the same day.

Theory:

Bar Pendulum:

The bar pendulum consists of a metallic bar of about one meter long. A series of circular holes each of approximately 5 mm in diameter are made along the length of the bar. The bar is suspended from a horizontal knife-edge passing through any of the holes (Fig.). The knife edge, in turn, is fixed in a platform provided with the screws. By adjusting the rear screw, the platform can be made horizontal.
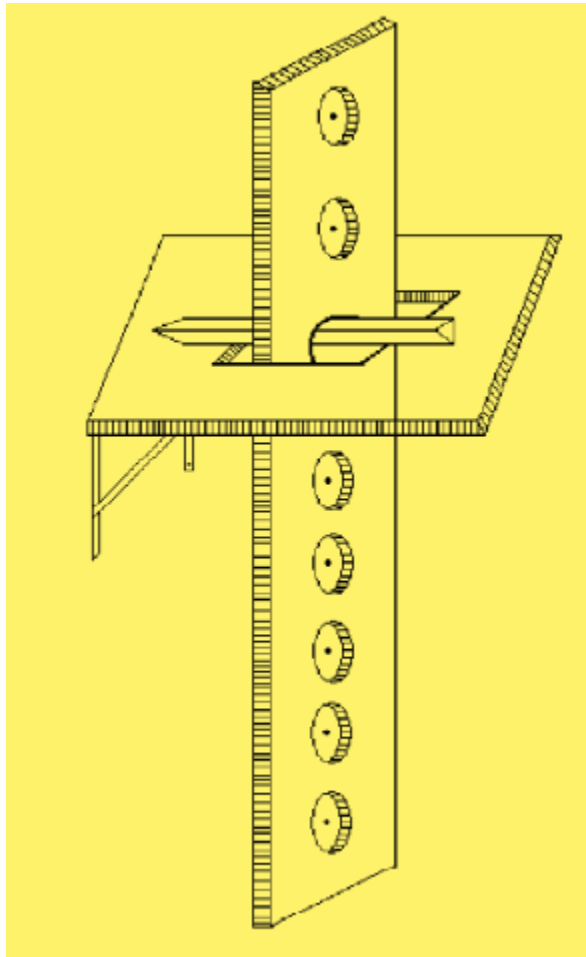

Fig: Bar Pendulum

A simple pendulum consists of a small body called a "bob" (usually a sphere) attached to the end of a string the length of which is great compared with the dimensions of the bob and the mass of which is negligible in comparison with that of the bob. Under these conditions the mass

of the bob may be regarded as concentrated at its center of gravity, and the length of the pendulum is the distance of this point from the axis of suspension. When the dimensions of the suspended body are not negligible in comparison with the distance from the axis of suspension to the center of gravity, the pendulum is called a compound, or physical, pendulum. A rigid body mounted upon a horizontal axis so as to vibrate under the force of gravity is a compound pendulum.

In Fig. a body of irregular shape is pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle θ. In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is l and the mass of the body is m. The restoring torque for an angular displacement θ is

$$\tau = -mg\, l\, \sin\theta \qquad \ldots(1)$$

For small amplitudes ($\theta \approx 0$),

$$I\frac{d^2\theta}{dt^2} = -mgl\theta, \qquad \ldots(2)$$

where I is the moment of inertia of the body through the axis P. Eq. (2) represents a simple harmonic motion and hence the time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad \ldots(3)$$

Now $I = I_G + ml^2$, where I_G is the moment of inertia of the body about an axis parallel with axis of oscillation and passing through the center of gravity G.

$$I_G = mK^2 \qquad \ldots(4)$$

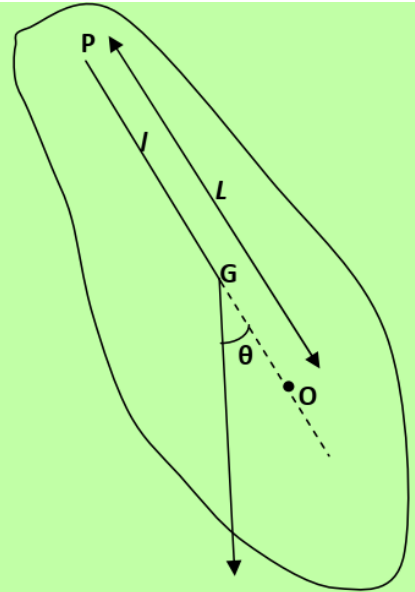where K is the radius of gyration about the axis passing through

Fig. 1

$$T = 2\pi\sqrt{\frac{mK^2+ml^2}{mgl}} = 2\pi\sqrt{\frac{\frac{K^2}{l}+l}{g}} \qquad \ldots(5)$$

The time period of a simple pendulum of length L, is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \ldots(6)$$

Comparing with Eq. (5) we get

$$L = l + \frac{K^2}{l} \qquad \ldots(7)$$

This is the length of "equivalent simple pendulum". If all the mass of the body were concentrated at a point O (See Fig.1) such that $OP = \frac{k^2}{l} + l$, we would have a simple pendulum with the same time period. The point O is called the 'Centre of Oscillation'. Now from Eq. (7)

$$l^2 - lL + K^2 = 0 \qquad \ldots(8)$$

i.e. a quadratic equation in l. Equation 6 has two roots l_1 and l_2 such that

$$l_1 + l_2 = L$$

and $\qquad\qquad\qquad l_1 l_2 = K^2 \qquad\qquad\qquad$ …(9)

Thus both l_1 and l_2 are positive. This means that on one side of C.G there are two positions of the centre of suspension about which the time periods are the same. Similarly, there will be a pair of positions of the centre of suspension on the other side of the C.G about which the time periods will be the same. Thus there are four positions of the centers of suspension, two on either side of the C.G, about which the time periods of the pendulum would be the same. The distance between two such positions of the centers of suspension, asymmetrically located on either side of C.G, is the length L of the simple equivalent pendulum. Thus, if the body was supported on a parallel axis through the point O (see Fig. 1), it would oscillate with the same time period T as when supported at P. Now it is evident that on either side of G, there are infinite numbers of such pair of points satisfying Eq. (9). If the body is supported by an axis through G, the time period of oscillation would be infinite. From any other axis in the body the time period is given by Eq. (5). From Eq.(6) and (9), the value of g and K are given by

$$g = 4\pi^2 \frac{L}{T^2} \qquad\qquad\qquad …(10)$$

$$K = \sqrt{l_1 l_2} \qquad\qquad\qquad …(11)$$

By determining L, l_1 and l_2 graphically for a particular value of T, the acceleration due to gravity g at that place and the radius of gyration K of the compound pendulum can be determined.

Apparatus and Accessories:

(i) A bar pendulum, (ii) a knife–edge with a platform, (iv) a precision stopwatch, (v) a meter scale and

Procedure:

(i) Suspend the bar using the knife edge of the hook through a hole nearest to one end of the bar.
(ii) Allow the bar to oscillate in a vertical plane with small amplitude (within 4^0 of arc).
(iii) Note the time for 20 oscillations by a precision stopwatch by observing the transits of the vertical line on the bar through the telescope. Make this observation three times and find the mean time t for 20 oscillations. Determine the time period T.
(iv) Measure the distance d of the axis of the suspension, i.e. the hole from one of the edges of the bar by a meter scale.
(v) Repeat operation (i) to (iv) for the other holes till center of gravity (C.G) of the bar is approached where the time period becomes very large.
(vi) Invert the bar and repeat operations (i) to (v) for each hole starting from the extreme top.
(vii) Draw a graph with the distance d of the holes as abscissa and the time period T as ordinate. The nature of graph will be as shown in Fig.

Data Tables:

Table 1: Observation for the time period and the distance of the point of suspension from CG for End- A.

| Hole no.	Distance From CG L (cm)	Time for 20 Oscillations (s)	Mean time t (s)	Period T = t / 20 (s)
1	45	(1)		
		(2)		
2	40	(1)		
		(2)		
3	35	(1)		
		(2)		
4	30	(1)		
		(2)		
5	25	(1)		
		(2)		
6	20	(1)		
		(2)		
7	15	(1)		
		(2)		

Table 2: Observation for the time period and the distance of the point of suspension from CG for End- B.

Hole no.	Distance From CG L (cm)	Time for 20 Oscillations (s)		Mean time t (s)	Period T = t / 20 (s)
1	45	(1)			
		(2)			
2	40	(1)			
		(2)			
3	35	(1)			
		(2)			
4	30	(1)			
		(2)			
5	25	(1)			
		(2)			
6	20	(1)			
		(2)			
7	15	(1)			
		(2)			

Draw the horizontal line ABCDE parallel to the X-axis. Here A, B, D and E represent the point of intersections of the line with the curves. Note that the curves are symmetrical about a vertical line which meets the X-axis at the point G, which gives the position of the C.G of the bar. This vertical line intersects with the line ABCDE at C. Determine the length AD and BE and find the length L of the equivalent simple pendulum from $L = \frac{AD+BE}{2} = \frac{L_x}{2}$.

Find also the time period T corresponding to the line ABCDE and then compute the value of g. Draw several horizontal lines parallel to X-axis and adopting the above procedure find the value of g for each horizontal line. Calculate the mean value of g. Alternatively, for each horizontal line obtain the values of L and T and draw a graph with T^2 as abscissa and L as ordinate. The graph would be a straight line. By taking a convenient point on the graph, g may be calculated.

Similarly, to calculate the value of K, determine the length AC, BC or CD, CE of the line ABCDE and compute $\sqrt{AC \times BC}$ or $\sqrt{CD \times CE}$. Repeat the procedure for each horizontal line. Find the mean of all K.

L vs T
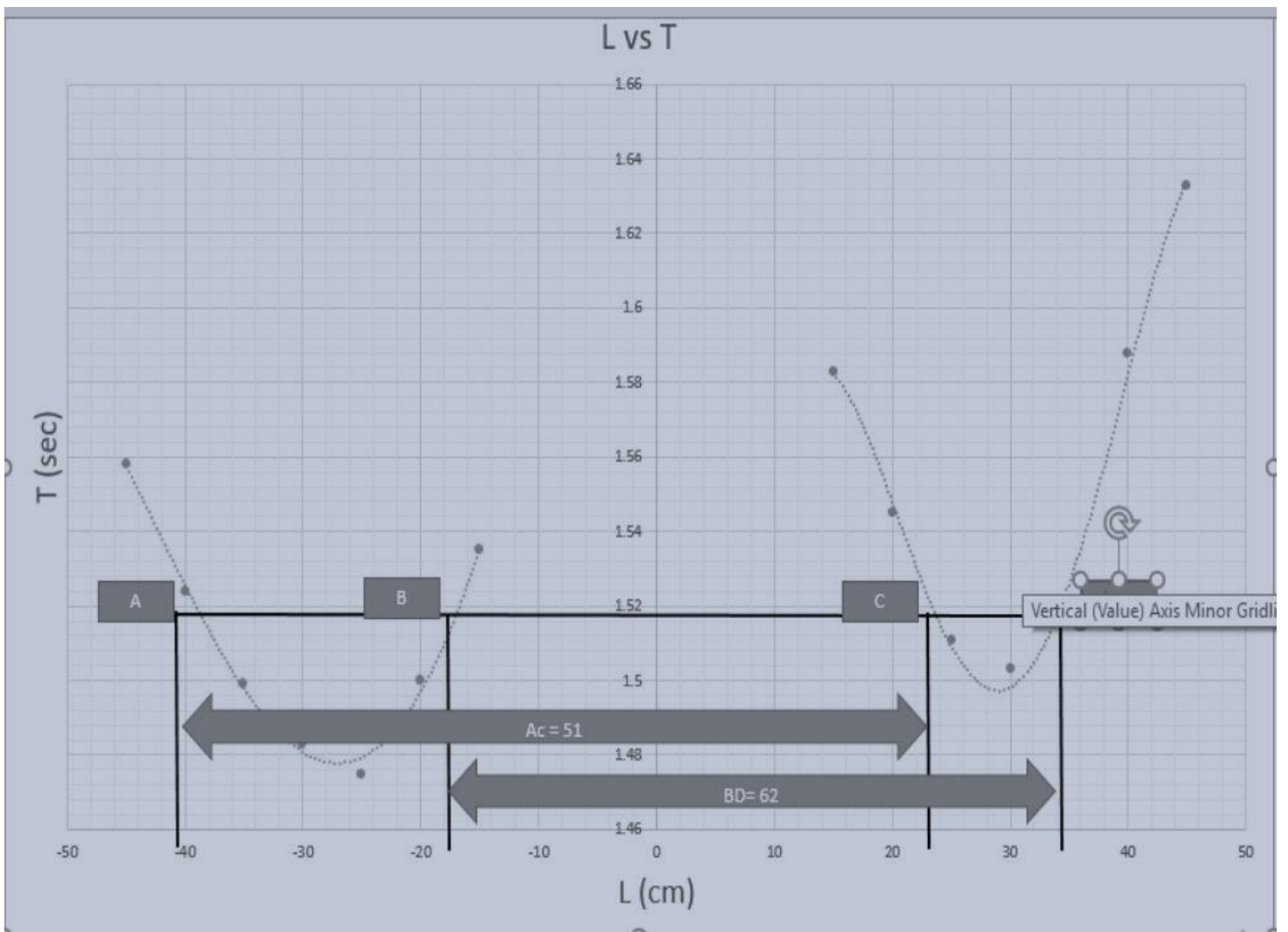
From the T vs L graph:

Length AC = --------- cm,   Length BD = ---------- cm.

Mean length, L = $\dfrac{AC+BD}{2}$ = ----------- cm.

Equivalent length of compound pendulum, L = ---------------cm.

Equivalent time period of compound pendulum, T = ------------ sec.

Calculation:

$$g = \frac{4\pi^2 L}{T^2} =$$

$$= \text{---------------} \, cm/s^2$$

Result:

The acceleration due to gravity is ----------------- cm/s^2

References:

1. Fundamentals of Physics: Resnick & Halliday
2. Practical physics: R.K. Shukla, Anchal Srivatsava, New Age International (P) Ltd, New Delhi
3. Eric J. Irons, American Journal of Physics, Vol. 15, Issue 5, pp.426 (1947)

Oral Questions and their Answers.

1. **What is a compound pendulum?**
   See theory of Expt.

2. **Which is superior—compound pendulum or a simple pendulum?**
   The ideal conditions of a simple pendulum cannot be attained in practice. In a compound pendulum the length of an equivalent simple pendulum can be determined and hence the value of 'g' can be accurately found out. The compound pendulum oscillates as a whole and due to its heavy mass, goes on oscillating for a long time. Hence compound pendulum is superior to simple pendulum.

3. **What do you mean by centre of supension and centre of oscillation?**
   It is possible to find out two points on the opposite side of the centre of gravity of the pendulum such that the periods of oscillation of the pendulum about these points are equal. One point is called the centre of suspension and the other point is called the centre of oscillation.