Introduction to Logic, Logic Gates and Boolean Algebra

Topics to be covered

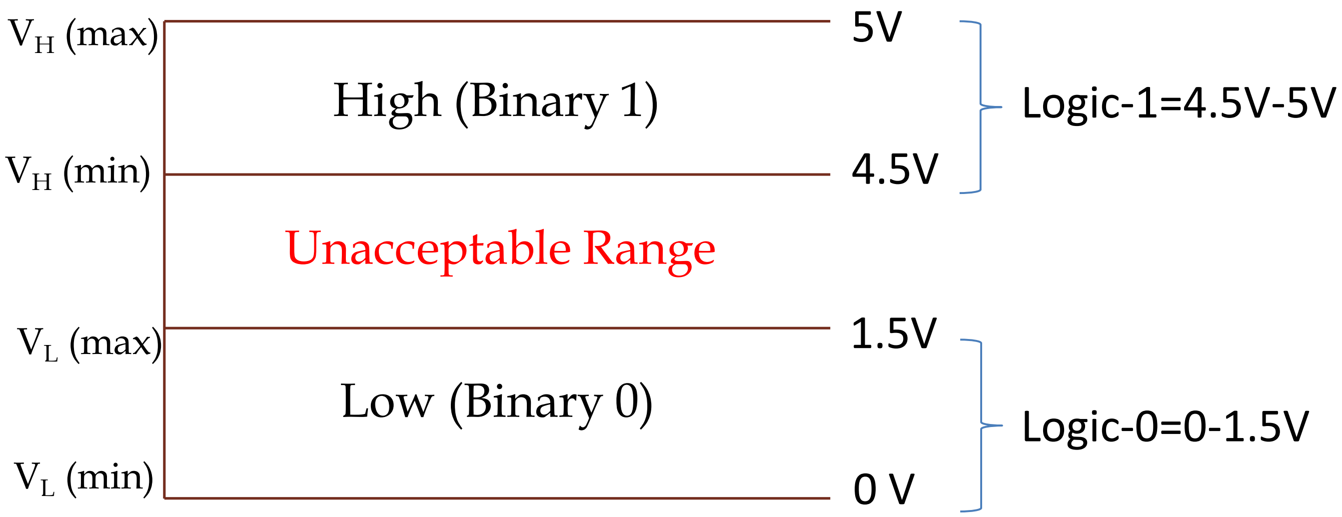
- Logic Gates (NOT, AND, OR, NAND, NOR, XOR, XNOR)
- Rules of Boolean Algebra
- DeMorgan's Theorem

Logic Gates

- Logic Gates are the basic building blocks of any digital system. A logic gate can have one or more than one input but only one output. The relationship between the input/s and the output is based on a **certain logic**. The gates are named based on the logic.
- The names of the logic gates are:

- Basic Gates:
  - NOT Gate or Inverter
  - AND Gate
  - OR Gate
- Universal Gates:
  - NAND Gate
  - NOR Gate
- Exclusive Gates:
  - Exclusive-OR Gate
  - Exclusive-NOR

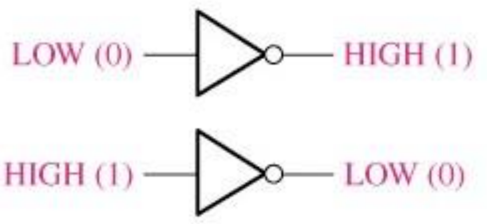
- **Bit:** in binary system we know that there are two digits 0 (low voltage) and 1(high voltage). The voltages used to represent a '1' or '0' are called logic levels



- Pulse is when clock frequency is applied to a circuit.
- Rising edge and falling edge.
- Periodic and non-periodic waveforms.
- Timing diagram

Inverter(NOT gate)

Logic:-Output of an inverter is opposite/complement of its input.



Truth table

A	X
0	1
1	0

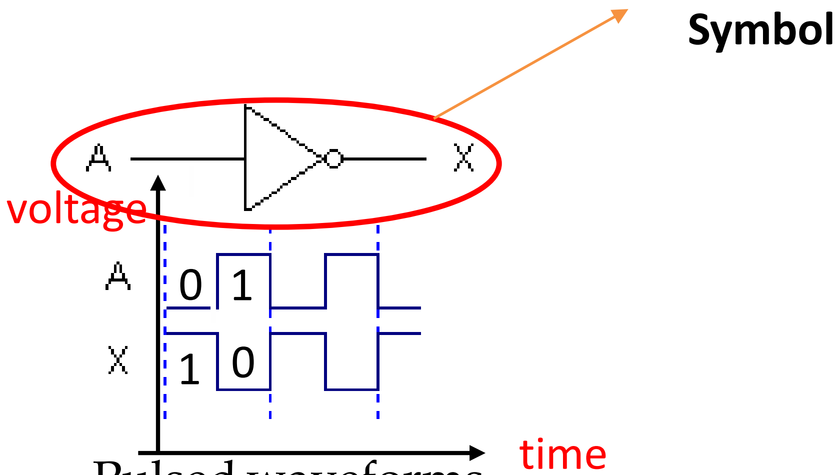
0 = LOW
1 = HIGH

Boolean output expression

$X = \overline{A}$ $X=A'$

When the input is LOW, the output is HIGH

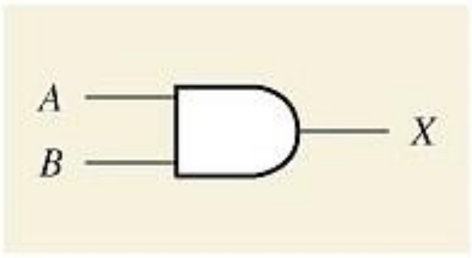
When the input is HIGH, the output is LOW



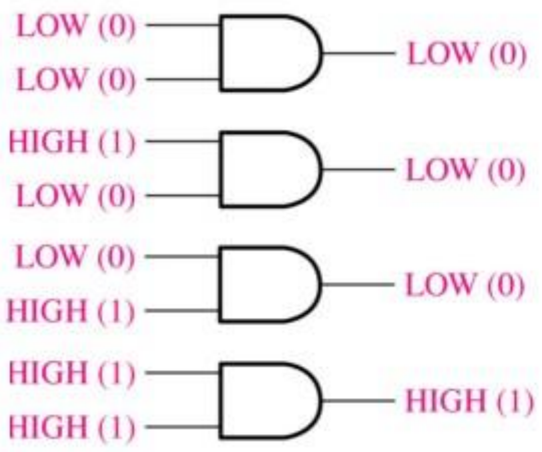
Pulsed waveforms

AND Gate

Symbol



Distinctive shape symbol



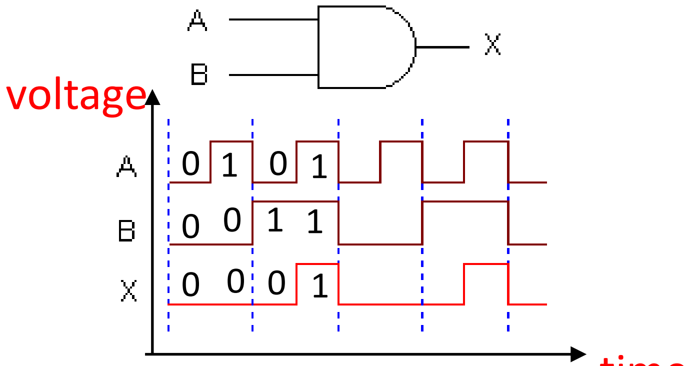
Truth table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

0 = LOW
1 = HIGH

N =2^n

Logic:-The output of an AND gate is HIGH only when all inputs are HIGH. For all other cases of input, output is low.



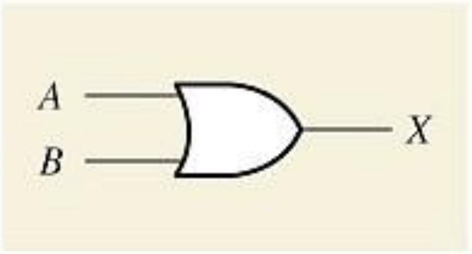
Pulsed waveforms

Boolean output expression

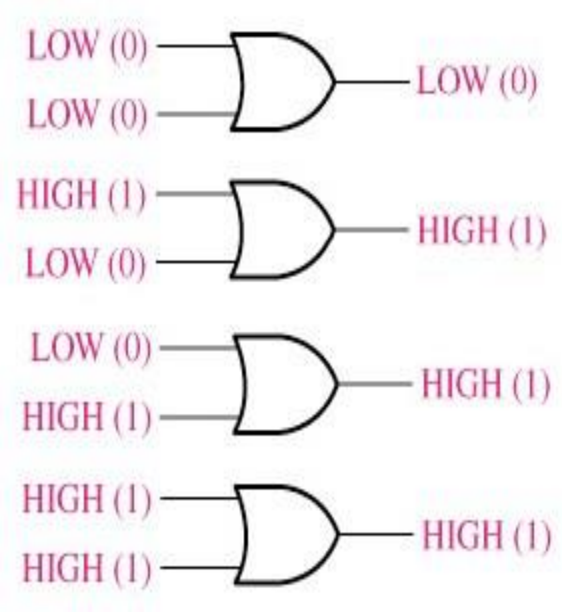
$$X = A \, . \, B$$

The OR Gate

Symbol



Distinctive shape symbol



0 = LOW

1 = HIGH

- **Logic:-The output of an OR gate is HIGH whenever one or more inputs are HIGH. For all other cases of input states , the output is low.**

Truth table

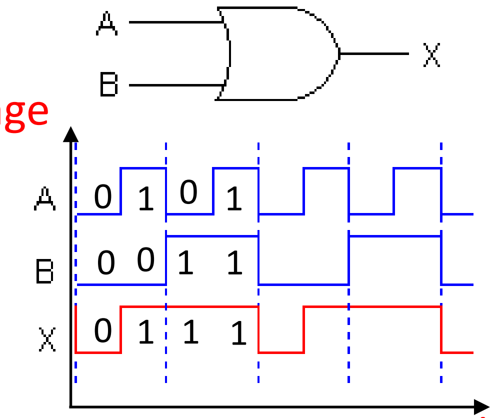
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Boolean output expression

$X = A + B$

Logical OR operator

voltage



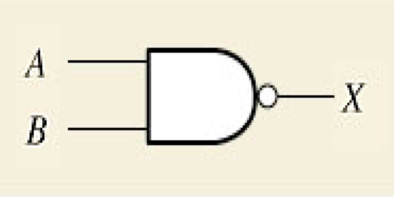
time

Pulsed waveforms

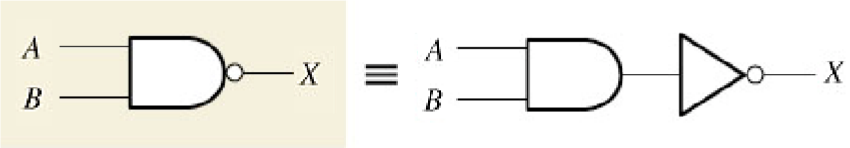
UNIVERSAL GATE

The NAND Gate

Symbol



Distinctive shape symbol



<u>Logic</u>:- The output of an NAND gate is LOW only when all inputs are HIGH. For all other cases of input, output is high.

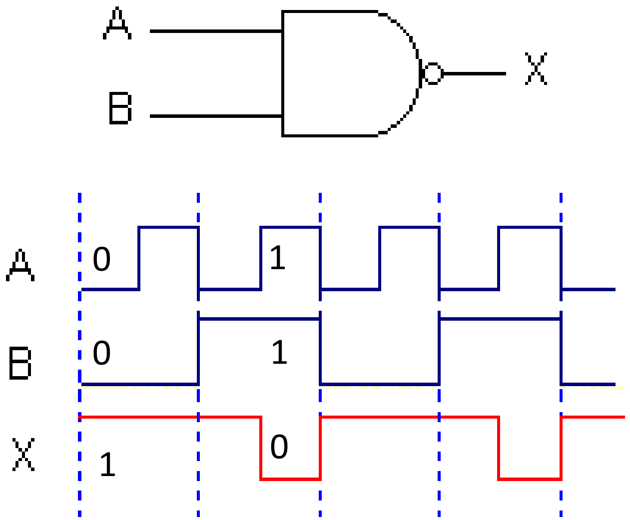
Truth-table

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Boolean output expression

$$X = \overline{A\,.\,B}$$

0 = LOW
1 = HIGH

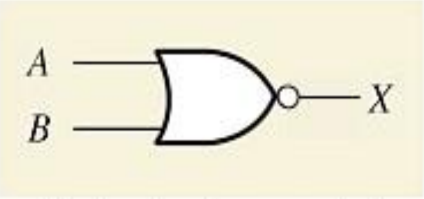


Pulsed waveforms

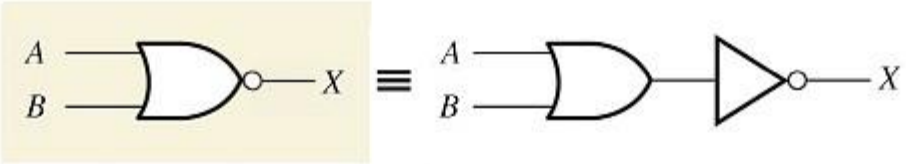
UNIVERSAL GATE

The NOR Gate

Symbol



Distinctive shape symbol



Logic:-The output of an NOR gate is LOW whenever one or more inputs are HIGH. For all other cases of input states , the output is HIGH.

Truth table

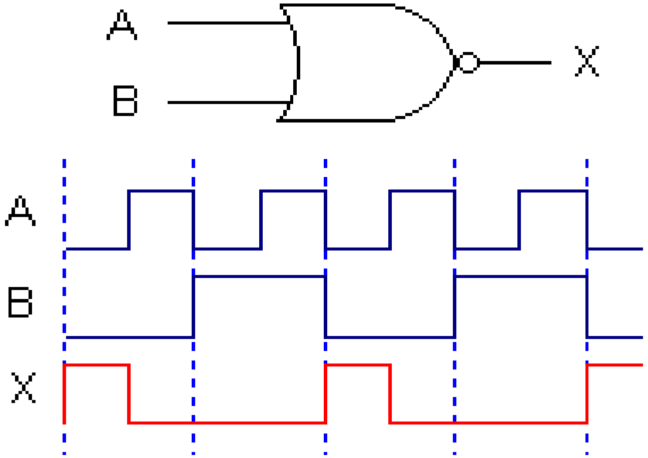
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

0 = LOW

1 = HIGH

Boolean output expression

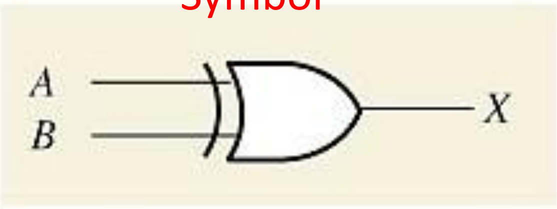
$$X = \overline{A + B}$$



Pulsed waveforms

Exclusive-OR (XOR)

Symbol



Distinctive shape symbol

Logic:- **The output for XOR gate is HIGH when odd number of inputs are HIGH. For all other cases of input, output is LOW.**

Truth table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

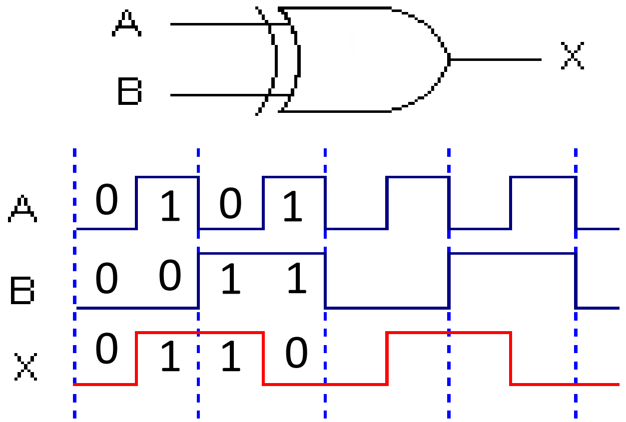
0 = LOW

1 = HIGH

Boolean output expression

$$X = A \oplus B$$

X=A’.B+A.B’

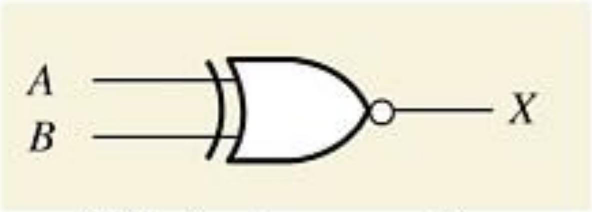


Pulsed waveforms

The output of an XOR gate is HIGH when there are ODD number of 1’s on the inputs to the gate

Exclusive-NOR Gate (XNOR)

Symbol



Distinctive shape symbol

Logic:- **The output for XNOR gate is LOW when odd number of inputs are HIGH. For all other cases of input, output is HIGH.**

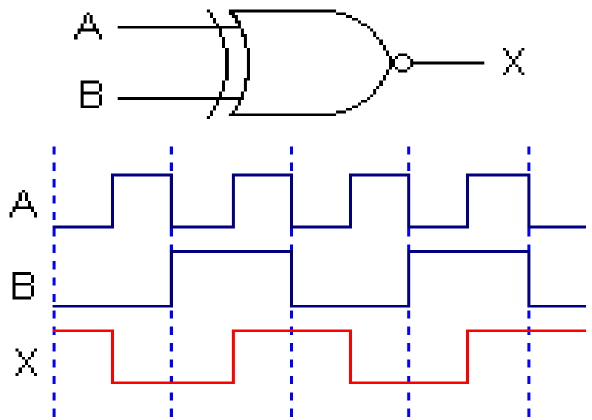
Truth table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

0 = LOW
1 = HIGH

Boolean output expression

$$X = \overline{A \oplus B}$$

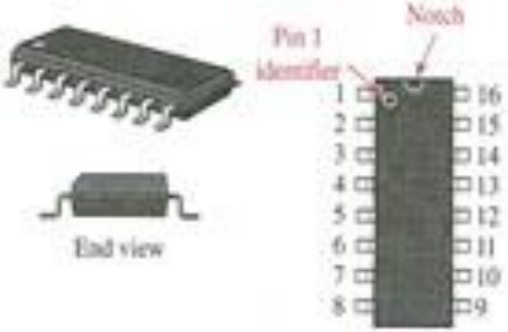


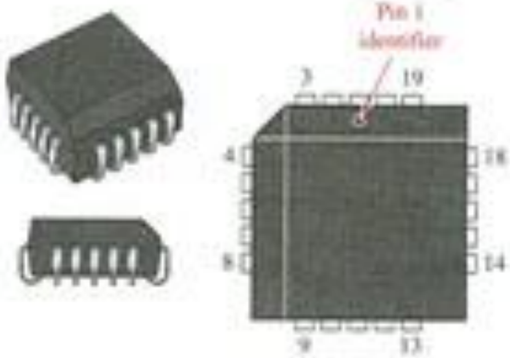
Pulsed waveforms

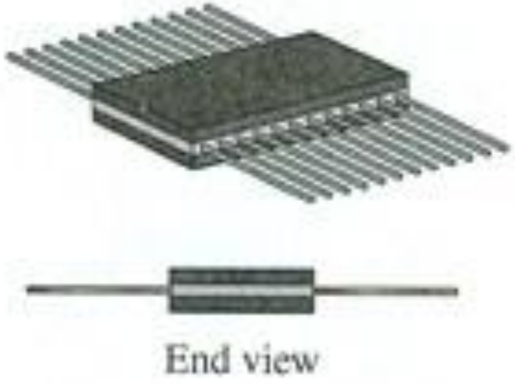
Fixed-Function Integrated Circuits

IC package styles

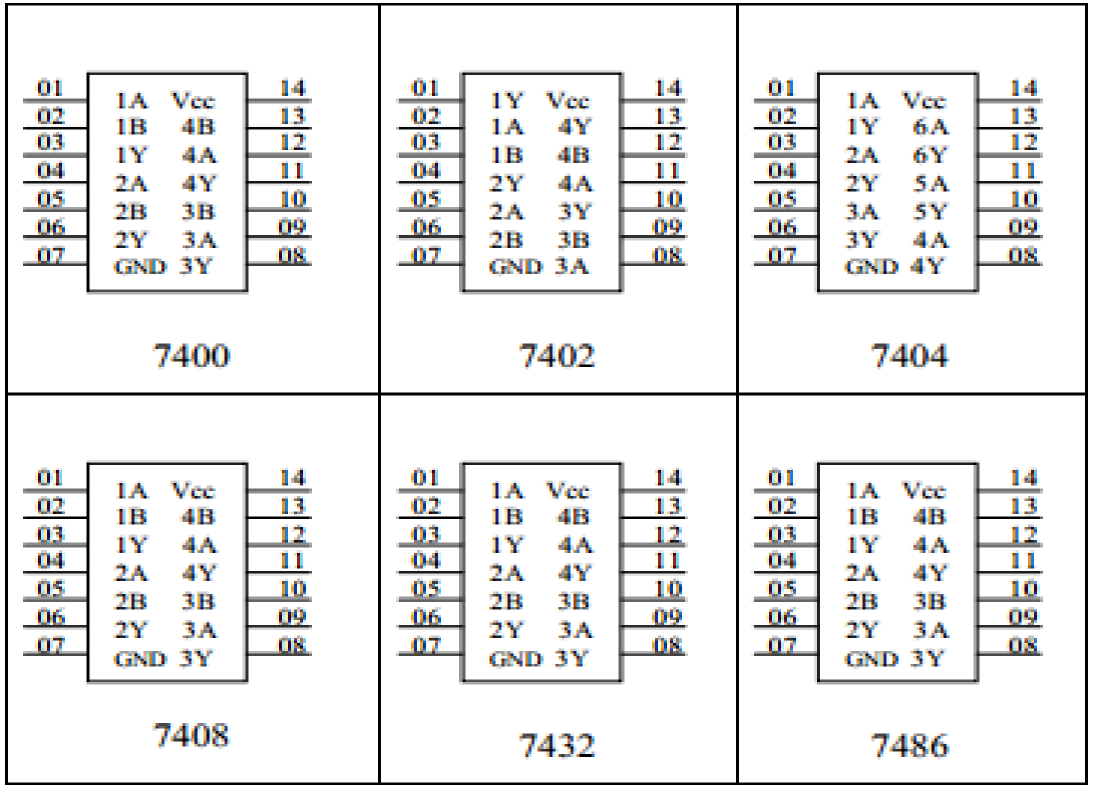
- Dual in-line package (DIP)
- Small-outline IC (SOIC)
- Flat pack (FP)
- Plastic-leaded chip carrier (PLCC)
- Leadless-ceramic chip carrier (LCCC)







IC configurations:



Integrated Circuits (ICs):

7400 :- Quad 2 I/p NAND.

7402 :- Quad 2 I/p NOR.

7404 :- Hex Inverter.

7408 :- Quad 2 I/p AND.

7432 :- Quad 2 I/p OR.

7486 :- Quad 2 I/p X-OR.

RULES OF BOOLEAN ALGEBRA

Basic rules of Boolean algebra.

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $\overset{0}{}\ A \cdot \overset{1}{\overline{A}} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $\underset{0}{A} + \underset{1}{\overline{A}} = 1$	12. $(A + B)(A + C) = A + BC$

Rule 1. $A + 0 = A$

Rule 2. $A + 1 = 1$

Rule 3. $A \cdot 0 = 0$

Rule 4. $A \cdot 1 = A$

Rule 5. $A + A = A$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rule 6. $A + \bar{A} = 1$

Rule 7. $A \cdot A = A$

Rule 8. $A \cdot \bar{A} = 0$

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$
10. $A + AB = A$
11. $A + \bar{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

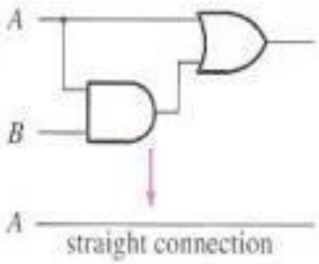
Rule 9. $\bar{\bar{A}} = A$

Rule 10. $A + AB = A$

$$
\begin{aligned}
A + AB &= A(1 + B) && \text{Factoring (distributive law)} \\
&= A \cdot 1 && \text{Rule 2: } (1 + B) = 1 \\
&= A && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



$$A + B(A + \bar{A})$$

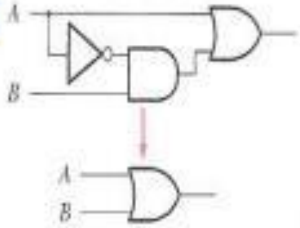
$$A + B.1 = A + B$$

Rule 11. $A + \bar{A}B = A + B$

$A + \bar{A}B = (A + AB) + \bar{A}B$	Rule 10: $A = A + AB$
	Rule 7: $A = AA$
	Rule 8: adding $A\bar{A} = 0$
	Factoring
	Rule 6: $A + \bar{A} = 1$
	Rule 4: drop the 1

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rule 12. $(A + B)(A + C) = A + BC$

$(A + B)(A + C) = AA + AC + AB + BC$	Distributive law
$= A + AC + AB + BC$	Rule 7: $AA = A$
	Factoring (distributive law)
	Rule 2: $1 + C = 1$
	Factoring (distributive law)
	Rule 2: $1 + B = 1$
	Rule 4: $A \cdot 1 = A$

$$= A(1 + C + B) + BC$$

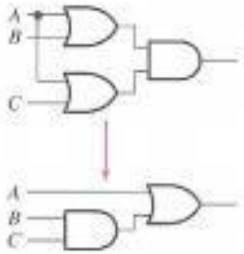
$$= A.1 + BC$$

$$= A + BC$$

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \bar{A} = 0$
3. $A \cdot 0 = 0$	9. $\bar{\bar{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \bar{A}B = A + B$
6. $A + \bar{A} = 1$	12. $(A + B)(A + C) = A + BC$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Simplification using boolean algebra

Obama + Obama + Obama = Obama

Simplification means fewer gates for the same function

EXAMPLE

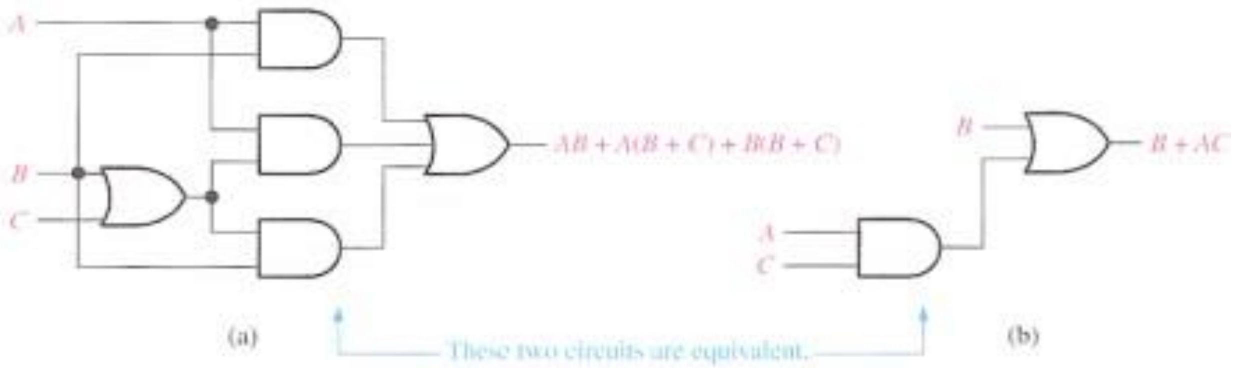
$AB + A(B + C) + B(B + C)$ ▪ $B + AC$

$AB + A(B + C) + B(B + C)$

$AB + AB + AC + BB + BC$

$AB + AB + AC + B + BC$

B(1+C+A+A)+AC
B.1+AC
B+AC



EXAMPLE

$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$ $\overline{B}C$

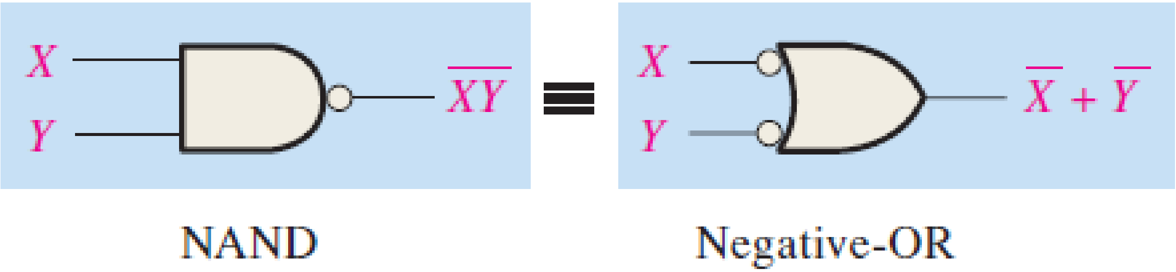
$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$ $BC + A\overline{B} + \overline{B}\,\overline{C}$

▪

DEMORGAN'S THEOREMS

- The first theorem is stated as follows:
  The complement of a product of variables is equal to the sum of the complements of the variable.
  The formula of this theorem for two variables is written as

$$\overline{XY} = \overline{X} + \overline{Y}$$

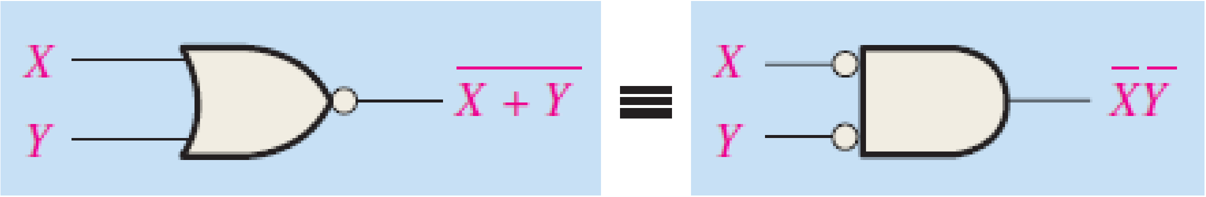


Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

DEMORGAN'S THEOREMS

- The second theorem is stated as follows:
  The complement of a sum of variables is equal to the product of the complements of the variables.
  The formula of this theorem for two variables is written as

$$\overline{\mathbf{X} + \mathbf{Y}} = \overline{X} \,.\, \overline{Y}$$



NOR Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

APPLICATION OF DEMORGAN'S THEOREM

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to each expression:

(a) $\overline{(\overline{A + B}) + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{(\overline{A + B}) + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = \overline{((A + B)\overline{C}\,\overline{D})}(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

APPLICATION OF DEMORGAN'S THEOREM

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}}$.

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

Starting with the expression for a 4-input NAND gate, use DeMorgan's theorems to develop an expression for a 4-input negative-OR gate.

Apply DeMorgan's theorems to the following expressions:

(a) $\overline{ABC} + (\overline{\overline{D} + E})$ **(b)** $\overline{(A + B)C}$ **(c)** $\overline{A + B + C} + \overline{\overline{D}E}$

Textbooks:

- [1] Thomas L. Floyd, "Digital Fundamentals" 11th edition, Prentice Hall.
- [2] M. Morris Mano, "Digital Logic & Computer Design" Prentice Hall.