Digital Logic Design

Representation of Boolean Expressions and Creating Truthtables

Boolean Expressions can be written in two forms

- Sum of Products (SOP)
  - Non Standard SOP
  - Standard SOP
- Product of Sum (POS)
  - Non Standard POS
  - Standard POS

Sum of Product

- This form of Boolean expression representation uses multiplication of inputs which are then summed up together.
  - $\bar{A}BC + A\overline{D} + \overline{BC}D$
  - $\bar{A}BC + AB\bar{C}+ \overline{AB}C$

Non Standard SOP Expression

In this form of representation, all the product terms will not have all the inputs associated with the system.

$$\bar{A}BC + A\overline{D} + \overline{BC}D$$

In the above expression, the total number of inputs for the system is 4 (A, B, C, D). But for the first product term, we see there is input A,B,C is present, D is missing, for the second term, we see A and D is present, but B and C is missing and for the last product term, B,C and D is present but A is missing.

Sum of Product

Standard SOP Expression

In this form of representation, all the product terms should have all the inputs associated with the system.

$$\bar{A}BC + AB\bar{C} + \overline{AB}C$$

In the above expression, the total number of inputs for the system is 3 (A, B, C). We can see that the first product term has all 3 inputs, the second product term has all 3 inputs and the third product term also has 3 inputs.

Product of Sum

- This form of Boolean expression representation uses addition of inputs which are then multiplied together.
  - $(A + \bar{B} + C)(A + B + \bar{C})(\bar{A} + \bar{B} + C)$
  - $(A + \bar{B})(B + \bar{C})(\bar{A} + C)$

Non Standard POS Expression

In this form of representation, all the sum terms will not have all the inputs associated with the system.

$$(A + \bar{B})(B + \bar{C})(\bar{A} + C)$$

In the above expression, the total number of inputs for the system is 3 (A, B, & C). But for the first sum term, we see there is input A & B is present, C is missing, for the second term, we see B and C is present, but A is missing and for the last sum term, A & C is present but B is missing.

Product of Sum

Standard POS Expression

In this form of representation, all the sum terms should have all the inputs associated with the system.

$$(A + \bar{B} + C)(A + B + \bar{C})(\bar{A} + \bar{B} + C)$$

In the above expression, the total number of inputs for the system is 3 (A, B, C). We can see that the first sum term has all 3 inputs, the second sum term has all 3 inputs and the third sum term also has 3 inputs.

Converting Non-Standard SOP to Standard SOP

- Y=A.B+B’.C+A’.B’.C
- We see from the above expression that the first product term is missing input C and the second product term is missing input A. So the above expression is a NON STANDARD SOP.

Converting Non-Standard SOP to Standard SOP

- Y=A.B+B’.C+A’.B’.C
- Bringing input C in the first product term
- A.B = A.B(C+C’)=A.B.C + A.B.C’
- Bringing input A in the second product term.
- B’.C = B’.C.(A’+A) = A’.B’.C + A.B’.C

- After putting the standardized form
- Y = A.B.C + A.B.C’ + A’.B’.C + A.B’.C + A’.B’.C
- Y = A.B.C + A.B.C’ + A’.B’.C + A.B’.C

Converting Non-Standard SOP to Standard SOP

- Y=A.B.D + B.C’ + A’.C
- We see from the above expression that the first product term is missing input C and the second product term is missing inputs A & D. The third product term is missing inputs B and D. So the above expression is a NON STANDARD SOP.
- B.C’=B.C’ (A+A’) = A.B.C’ + A’.B.C’
- =A.B.C’ + A’.B.C’ = (A.B.C’ + A’.B.C’)(D+D’)
- = A.B.C’.D + A.B.C’.D’ + A’.B.C’.D + A’.B.C’.D’
- B.C’ = A.B.C’.D + A.B.C’.D’ + A’.B.C’.D + A’.B.C’.D’
- A’.C = A’.B’.C.D’+ A’.B’.C.D+A’.B.C.D’ + A’.B.C.D
- A.B.D.= A.B.C’.D+A.B.C.D
- Y= A.B.C.D+A.B.C’.D + A.B.C’.D’ + A’.B.C’.D + A’.B.C’.D’+A’.B’.C.D’+ A’.B’.C.D + A’.B.C.D’ + A’.B.C.D

Converting Non-Standard POS to Standard POS

- Y = (A'+B).(B'+C).(A'+B'+C)
- We see from the above expression that the first sum term is missing input C and the second sum term is missing input A. So the above expression is a NON STANDARD POS.
- A'+B = (A' + B) + C'.C
  X Y Z

r.12. X+Y.Z=(X+Y)(X+Z)

- A'+B = (A' +B + C').(A' +B + C)
  X Y X Z

Converting Non-Standard POS to Standard POS

- Y = (A’+B).(B’+C).(A’+B’+C)
- B’+ C = (B’ + C) + A’.A r.12. X+Y.Z=(X+Y)(X+Z)

- B’+ C = (B’ +C + A’).(B’ +C + A) = (A’ + B’ + C). (A + B’ + C)
- Y = (A’ +B + C’).(A’ +B + C). (A’ + B’ + C). (A + B’ + C). (A’+B’+C)

- Y = (A’ +B + C’).(A’ +B + C). (A’ + B’ + C). (A + B’ + C)

Converting Non-Standard POS to Standard POS

- Y = (A’+D).(B’+C).(A’+B’+C)
- (A’+D) = (A’+D) +B.B’= (A’+D+B). (A’+D+B’)
- (A’+D+B) = (A’+D+B) +C.C’ = (A’+D+B +C). (A’+D+B+C’)
- (A’+D+B’) = (A’+D+B’) + C.C’ = (A’+D+B’+C). (A’+D+B’+C’)
- The standard version of (A’+D):
- (A’+D) = (A’+B+C +D). (A’+B+C’+D). (A’ +B’+C+D). (A’+B’+C’+D)

- COMPLETE THE REST BY YOURSELF

- Y = (A’+D).(B’+C).(A’+B’+C)
- (A’+D) = (A’+B+C +D). (A’+B+C’+D). (A’ +B’+C+D). (A’+B’+C’+D)
- (B’+C)=(A+B’+C+D).(A+B’+C+D’).(A’+B’+C+D’).(A’+B’+C+D)
- (A’+B’+C)=(A’+B’+C+D’). (A’+B’+C+D)

- Y =(A’+B+C +D). (A’+B+C’+D). (A’ +B’+C+D). (A’+B’+C’+D). (A+B’+C+D).(A+B’+C+D’).(A’+B’+C+D’)

Active High Low I/O System and Truth-tables

Active High- Low I/O System

- Active High I/O System
- A=1, B=1, C=1 Y=1
- A'=0, B'=0, C'=0 Y'=0
- SOP expressions are represented with active high I/O system

Example

Y = A.B.C + A.B.C' + A'.B'.C + A.B'.C

1 = 1 .1 .1+ 1.1.0 + 0 .0 .1 + 1.0.1

- Active Low I/O System
- A'=1, B'=1, C'=1 Y'=1
- A=0, B=0, C=0 Y=0
- POS expressions are represented with active low I/O system

Example

Y = (A' +B + C').(A' +B + C). (A' + B' + C). (A + B' + C)

0 = (1 + 0 + 1). (1 + 0 + 0). (1 + 1 + 0). (0 + 1 + 0)

Developing truth tables

- A truthtable is a chart that represents a relationship of the output with all the possible combination of inputs. This is like finding the behavior of the circuit.
- Points to remember for developing truthtables.
  - **Make the output expression standard**.
  - Identify the type, whether it is SOP/POS expression.
  - Apply the concept of active high/low I/O system depending on the type of expression.
  - Populate the table based upon the above findings.

Convert a Standard SOP expression to a truth table

Example

Y = A.B.C + A.B.C’ + A’.B’.C + A.B’.C

1 = 1 .1 .1+ 1.1.0 + 0 .0 .1 + 1.0.1

A	B	C	Y	
0	0	0	0	→ POS Position -> (A+B+C)
0	0	1	1	→ SOP Position
0	1	0	0	POS Position (A+B’+C)
0	1	1	0	(A+B’+C’)
1	0	0	0	(A’+B+C)
1	0	1	1	SOP Position
1	1	0	1	
1	1	1	1	

Standard POS expression from the above truthtable is:

Y = (A+B+C). (A+B’+C).(A+B’+C’).(A’+B+C)

Converted a Standard POS to a truth table

Example

Y = (A’ +B + C’).(A’ +B + C). (A’ + B’ + C). (A + B’ + C)

0 = (1 + 0 + 1). (1 + 0 + 0). (1 + 1 + 0). (0 + 1 + 0)

A	B	C	Y	
0	0	0	1	→ SOP Position -> A’.B’.C’
0	0	1	1	→ SOP Position -> A’.B’.C
0	1	0	0	→ POS Position
0	1	1	1	→ SOP Position -> A’.B.C
1	0	0	0	} POS Position
1	0	1	0	}
1	1	0	0	}
1	1	1	1	→ SOP Position -> A.B.C

Standard SOP expression from the above truthtable is:

Y= A’.B’.C’+ A’.B’.C + A’.B.C + A.B.C

Classroom Task

- Y=A+B’C

1. Standardize the above expression
2. From the standard SOP expression, develop a truthtable
3. Find the standard POS expression from the truthtable found in 2.

Y=A+B’C

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

STANDARD SOP EXPRESSION

Y=A.B’C’+AB’C+ABC’+ABC+A’B’C

1=1.0.0 + 101 + 110 + 111+ 001

STANDARD POS EXPRESSION

Y=(A+B+C).(A+B’+C).(A+B’+C’)

Home Task

- Y=(A+B’).C

1. Standardize the above expression
2. Develop a truthtable
3. Find the standard SOP expression, from the truthtable, found in 2.