



COMBINATIONAL LOGIC CIRCUITS

Digital Logic & Circuit Design

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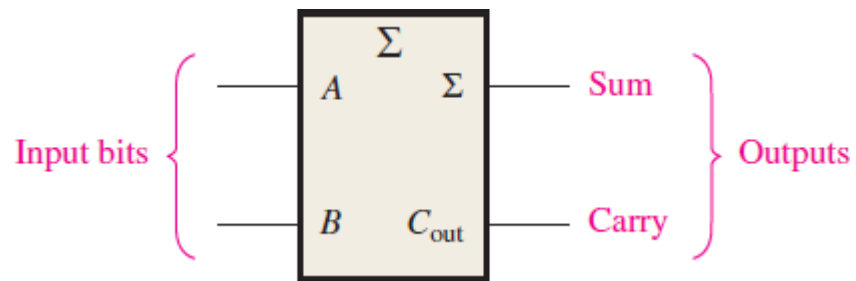


Steps for Digital System Designing

1. You need to draw a block diagram to identify how many inputs and outputs are there
2. Identify and write down the operation/behavior of the system.
3. From the operation/behavior develop a truthtable to relate the effects of input on the output
4. From the truthtable, write down the standard output expressions. (in class I discuss standard SOP expressions only.)
5. Minimize the standard output expression and find the simplified output expression using either rules of Boolean algebra or using k-map.
6. Draw the logic gate circuit diagram using the simplified output expressions.

HALF ADDER

1. Block Diagram



Operation & Basic Rules for Binary Addition

2. Operation = A + B

0 + 0 =	0
0 + 1 =	1
1 + 0 =	1
1 + 1 =	10

3. Half-adder truth table.

A	B	C _{out}	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

4. STANDARD Output SOP Expression

$$\text{Sum} = A'B + AB'$$

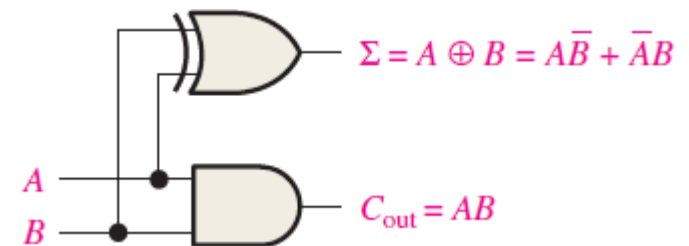
$$C_{\text{out}} = A.B$$

5. Simplified Output SOP Expression

$$\Sigma = A \oplus B$$

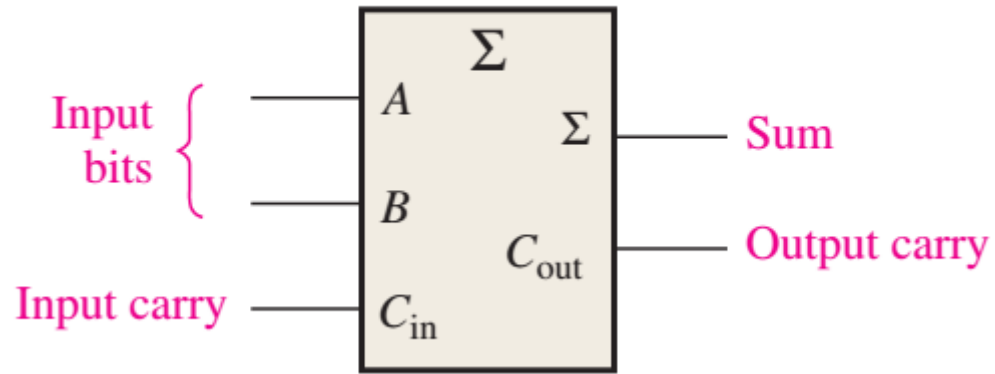
$$C_{\text{out}} = AB$$

6. Logic Gate Diagram

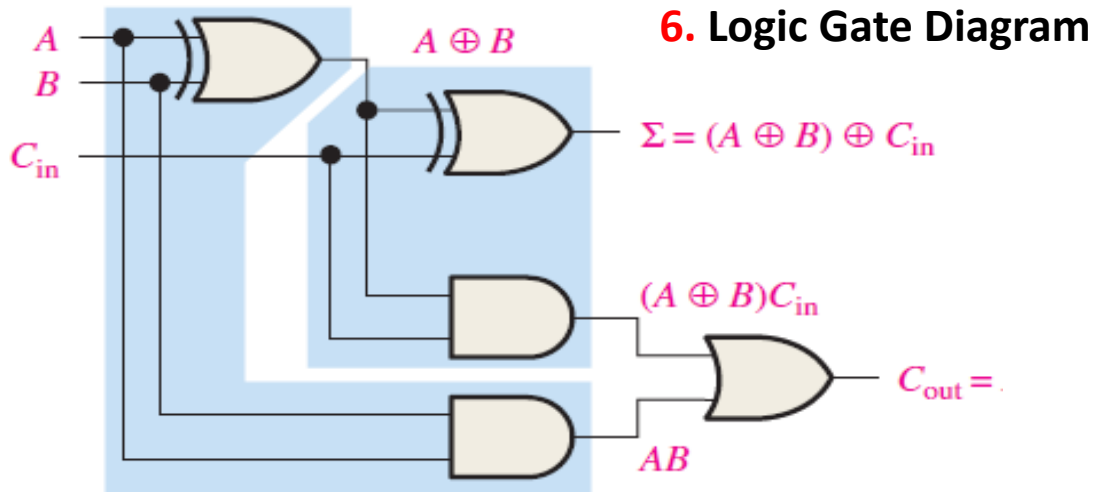


FULL ADDER

1. Block Diagram



2. Operation = $A + B + C_{in}$



6. Logic Gate Diagram

3. TABLE 6-2

Full-adder truth table.

A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$\begin{aligned}(A \oplus B) &= Z \\ (A \oplus B)' &= Z' \\ C_{in}Z' + C_{in}'Z &= Z \oplus C_{in}\end{aligned}$$

C_{in} = input carry, sometimes designated as CI

C_{out} = output carry, sometimes designated as CO

Σ = sum

A and B = input variables (operands)

4. STANDARD Output SOP Expression

$$\text{Sum} = A'B'C_{in} + A'BC_{in}' + A.B'.C_{in}' + A.B.C_{in}$$

$$= C_{in} (A'B' + AB) + C_{in}' (A'B + AB')$$

$$= C_{in} \cdot (A \oplus B) + C_{in}' (A \oplus B)'$$

$$= C_{in} \cdot (A \oplus B)' + C_{in}' (A \oplus B)$$

$$= [(A \oplus B) \oplus C_{in}]$$

$$C_{out} = A'BC_{in} + AB'C_{in}' + A.B.C_{in}' + A.B.C_{in}$$

$$= C_{in} (A'B + AB') + AB(C_{in}' + C_{in})$$

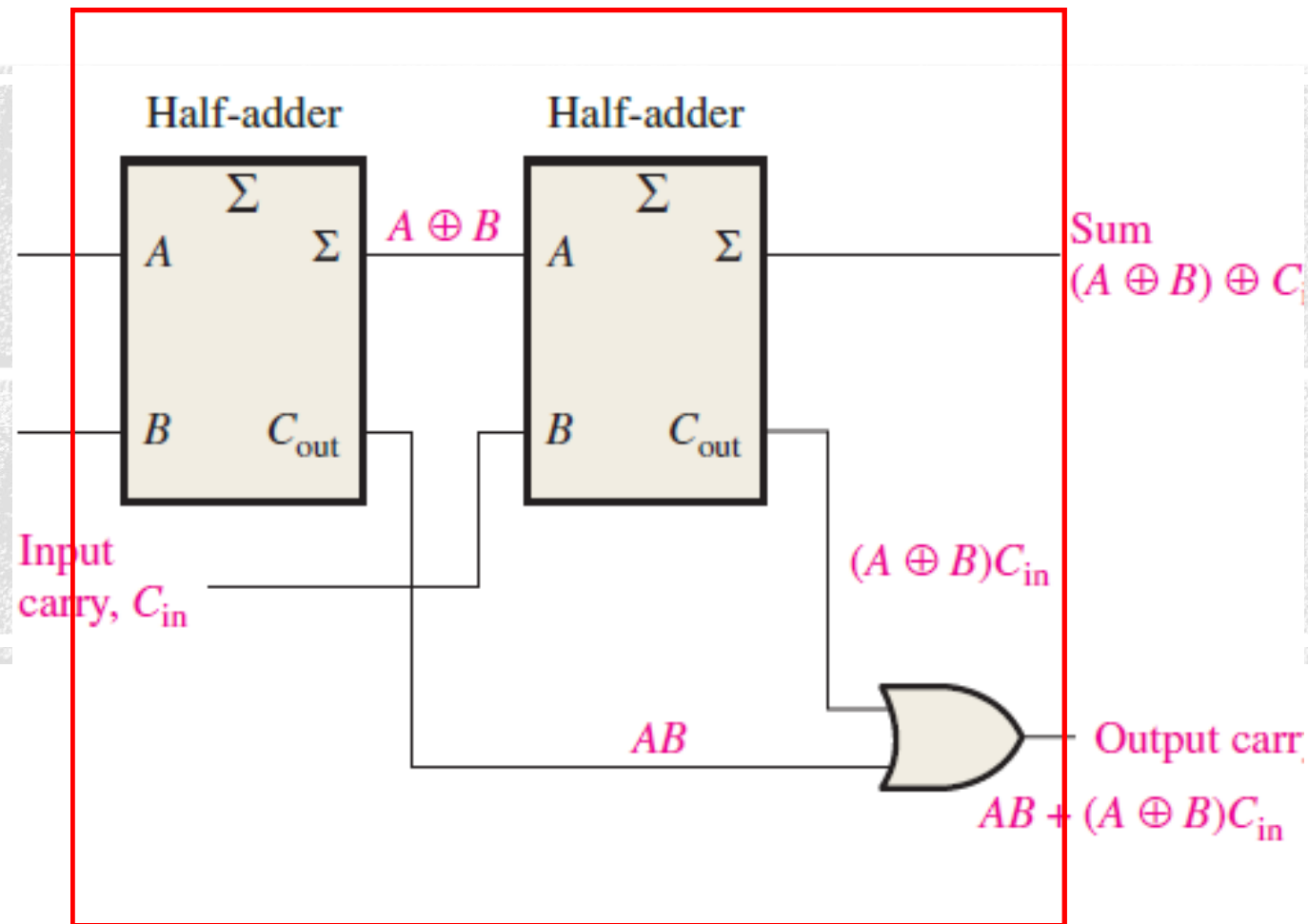
$$= C_{in} \cdot (A \oplus B) + AB$$

5. Simplified Output SOP Expression

$$\Sigma = (A \oplus B) \oplus C_{in}$$

$$C_{out} = AB + (A \oplus B)C_{in}$$

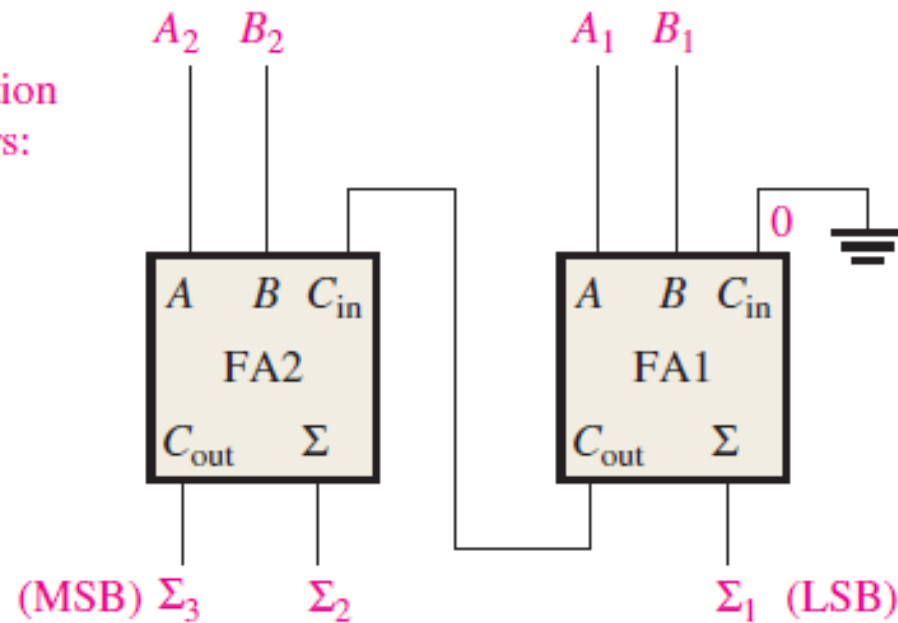




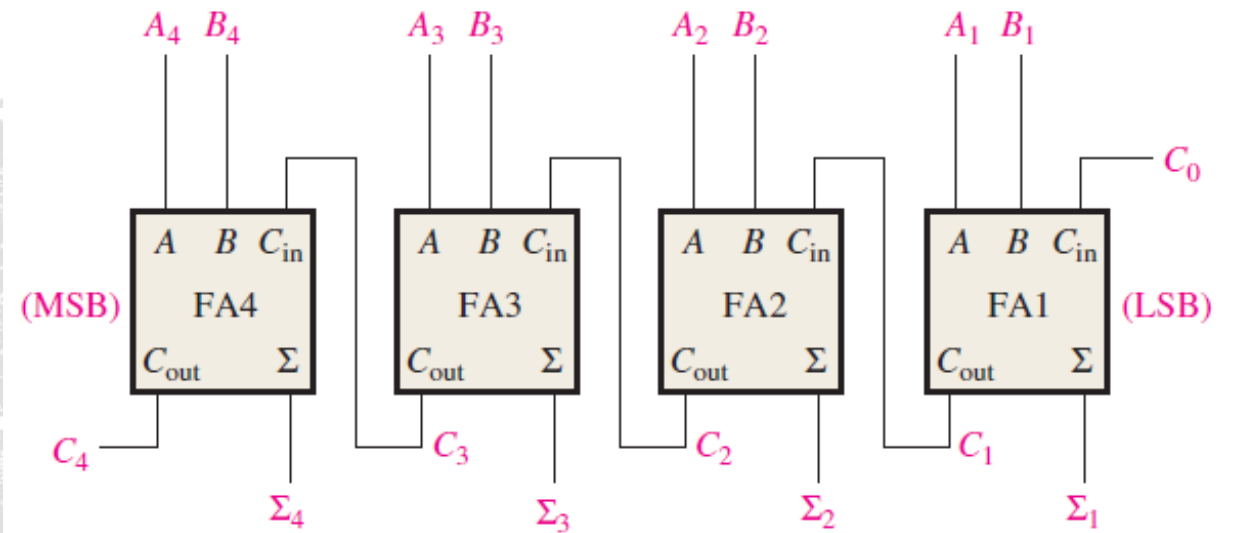
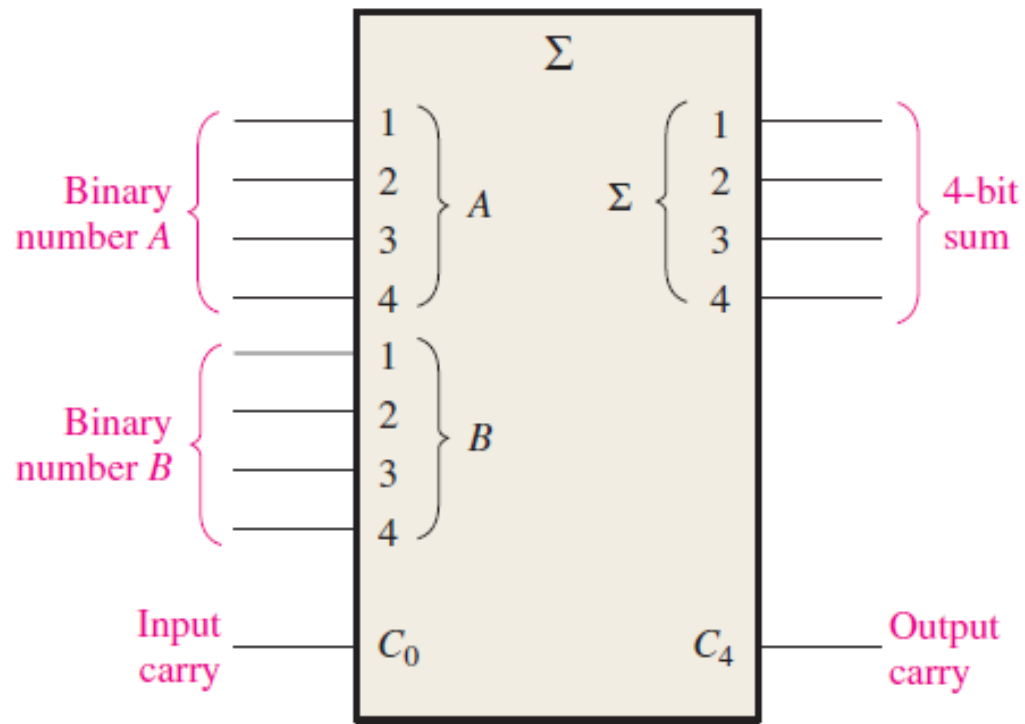
FULL ADDER WITH HALF ADDERS

General format, addition
of two 2-bit numbers:

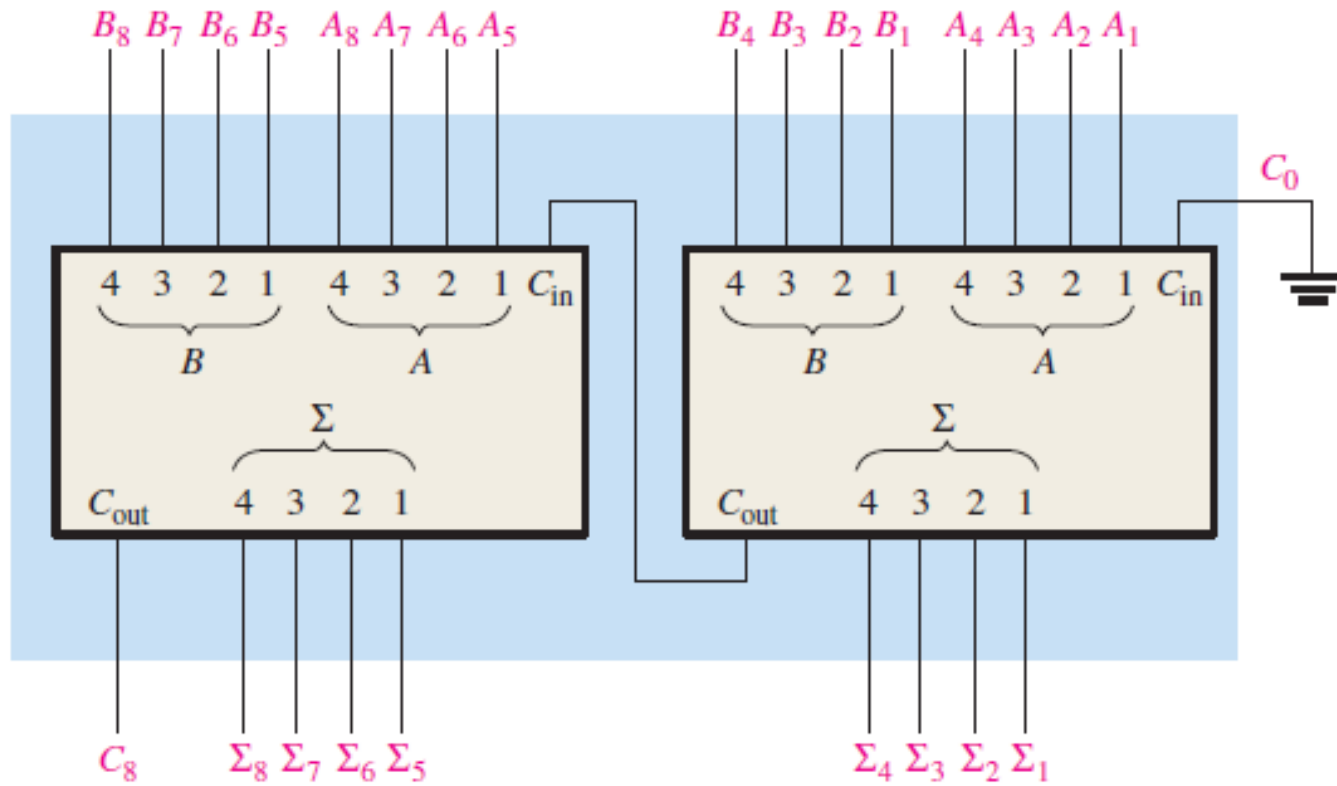
$$\begin{array}{r} A_2A_1 \\ + B_2B_1 \\ \hline \Sigma_3\Sigma_2\Sigma_1 \end{array}$$



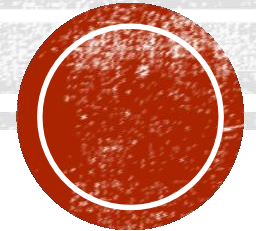
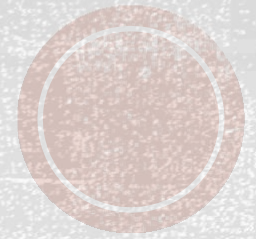
2-BIT PARALLEL ADDER

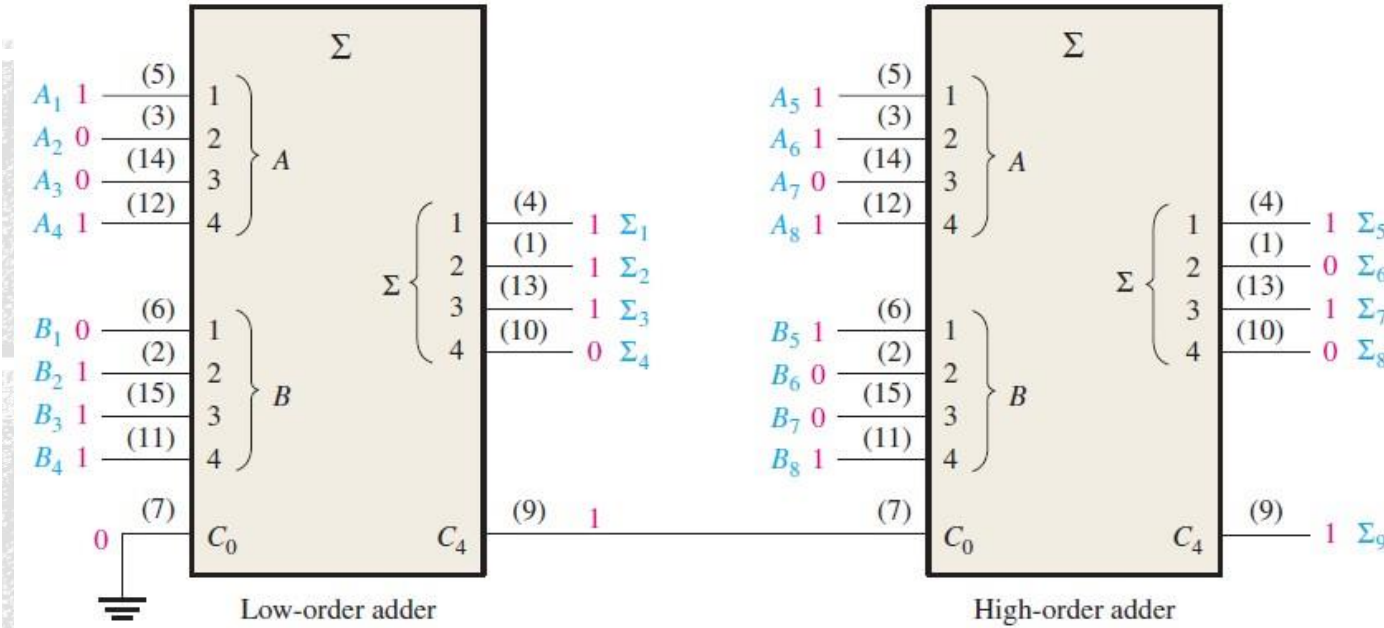


4-BIT PARALLEL ADDER



ADDER EXPANSION



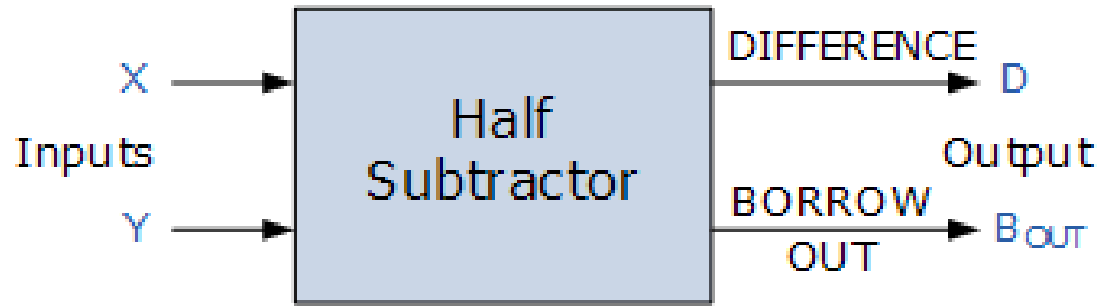


ADDER EXPANSION EXAMPLE

10111001
+10011110

HALF SUBTRACTOR

Block Diagram



Operation = $x - y$

Truthtable

x	y	d	bout
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

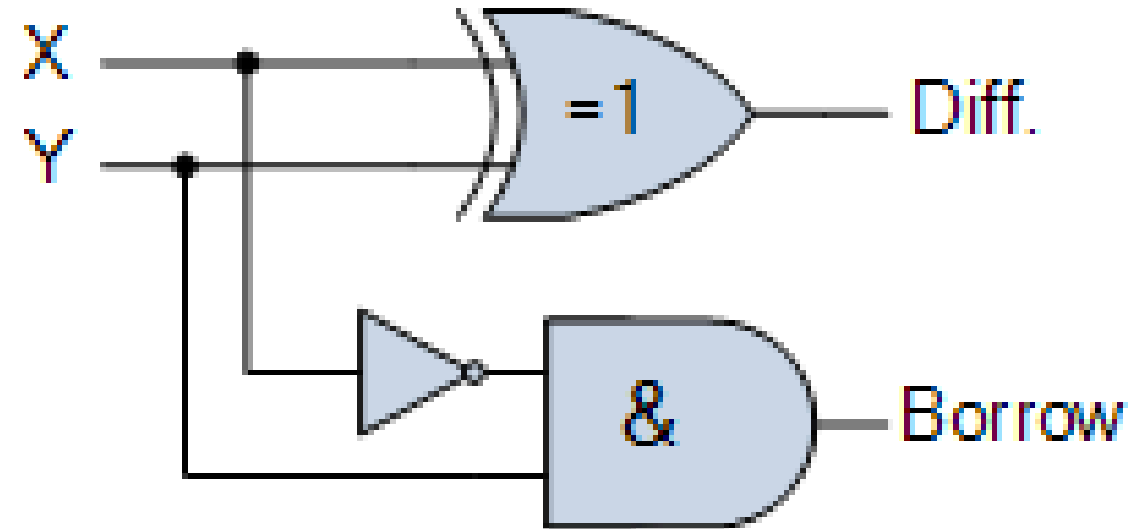
Output Expressions

$$D = x'y + xy'$$

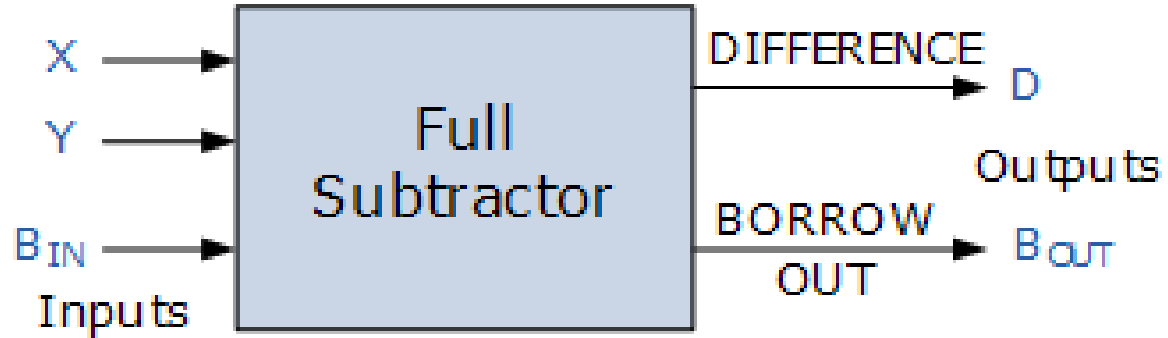
$$D = x \oplus y$$

$$B_{out} = x'y$$

Logic Gate Diagram



Block Diagram



$$\text{Operation} = X - Y - B_{IN}$$

$$= X - (Y + B_{IN})$$

X	Y	B _{IN}	D	B _{OUT}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = X'Y'B_{in} + X'YB_{in}' + X.Y'.B_{in}' + X.Y.B_{in}$$

$$= B_{in} (X'Y' + XY) + B_{in}' (X'Y + XY')$$

$$= B_{in} \cdot (X \odot Y) + B_{in}' (X \oplus Y)$$

$$= B_{in} \cdot (X \odot Y)' + B_{in}' (X \odot Y)$$

$$= [(X \odot Y) \oplus B_{in}]$$

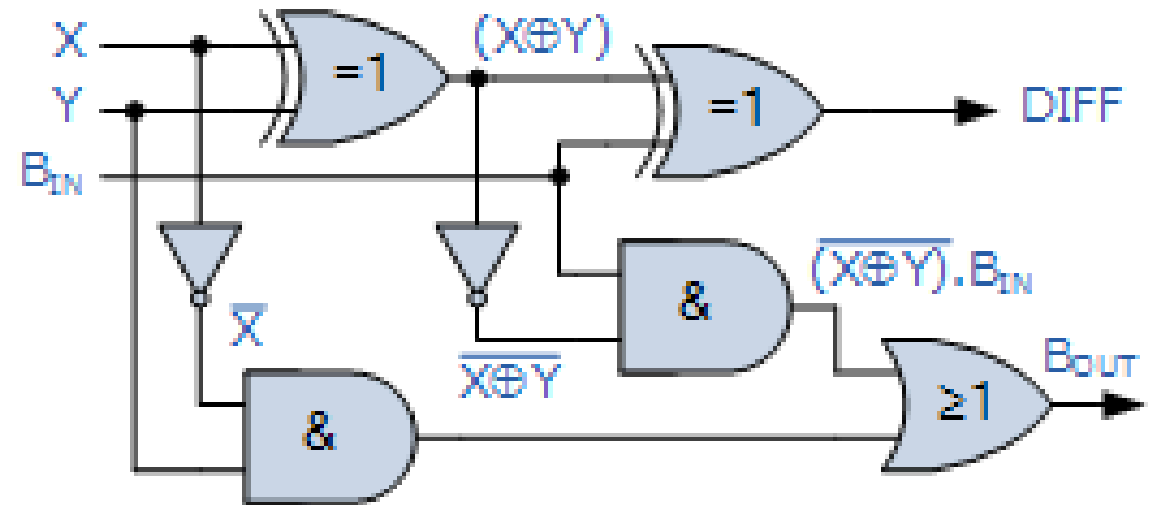
$$B_{out} = X'Y'B_{in} + X'YB_{in}' + X'.Y.B_{in} + X.Y.B_{in}$$

$$= B_{in} (X'Y' + XY) + X'Y(B_{in}' + B_{in})$$

$$= B_{in} \cdot (X \odot Y) + X'Y$$

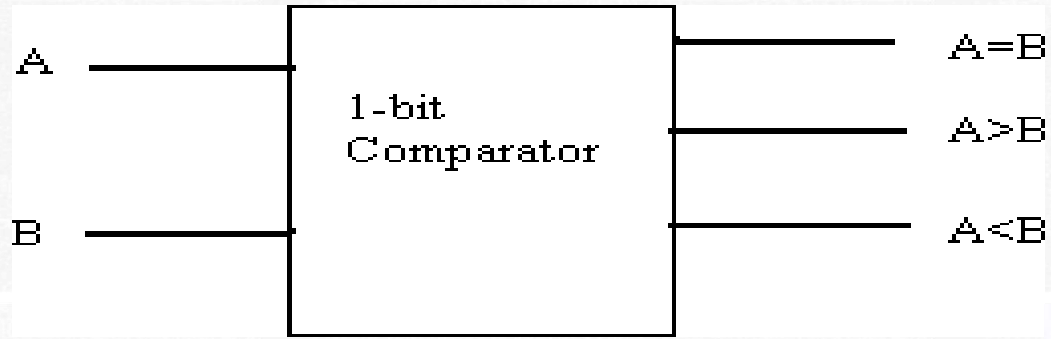
$$B_{in} \cdot (X \odot Y)' + X'Y$$

FULL SUBTRACTOR



COMPARATORS

1-bit Comparator



A	B	A=B	A>B	A<B
0	0	1	0	0
0	1	0	0	1
1	0	0	1	0
1	1	1	0	0

Operation:

A Comparator Compares the two inputs given and tells whether they are equal, one is greater than the other or one is less than the other. Only one output is high for a given input condition of A & B.

OUTPUT EXPRESSION

$$\begin{aligned}(A=B) &= A'B' + AB \\ &= A \odot B = (A \oplus B)' \\ &= (A'B + AB')'\end{aligned}$$

$$(A < B) = A'B$$

$$(A > B) = AB'$$

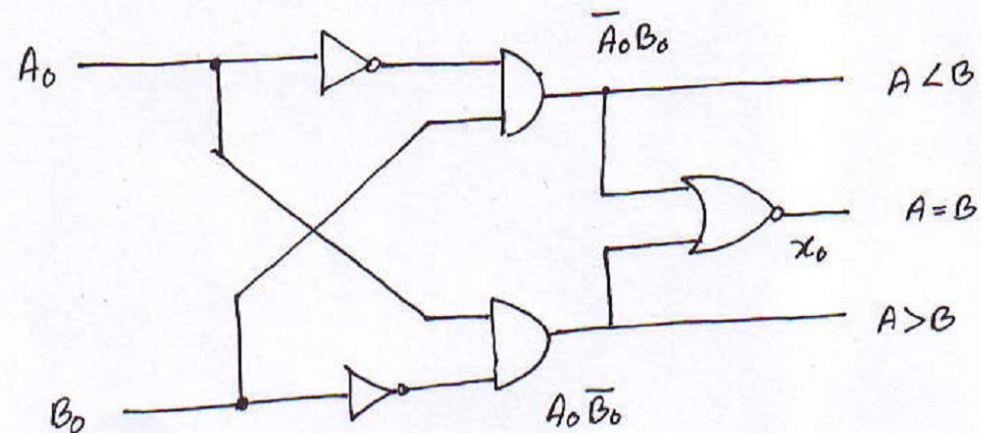
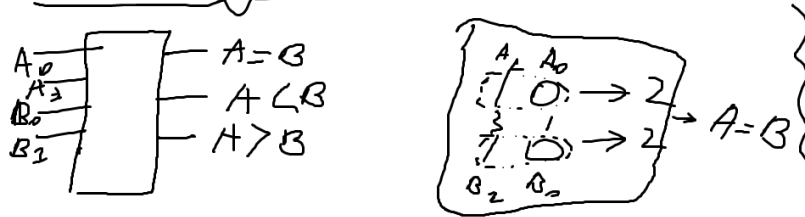


Fig : 1-bit magnitude comparator

2-bit Comparator

2 bit Comparator / $A_1 A_0 \rightarrow A$ $B_1 B_0 \rightarrow B$ \rightarrow Words

BLK Diagram:



Logics:

$\frac{A=B}{\neg f(A=B) \wedge (A=B), \text{ then } A=B}$
 $A \leq B$

If $(A_1 \leq B_1) \vee R$, then $A \leq B$

If $(A_1 = B_1) \text{ \& } (A_0 \subset B_0)$, then $A \subset B$

$$\underline{A > B}$$

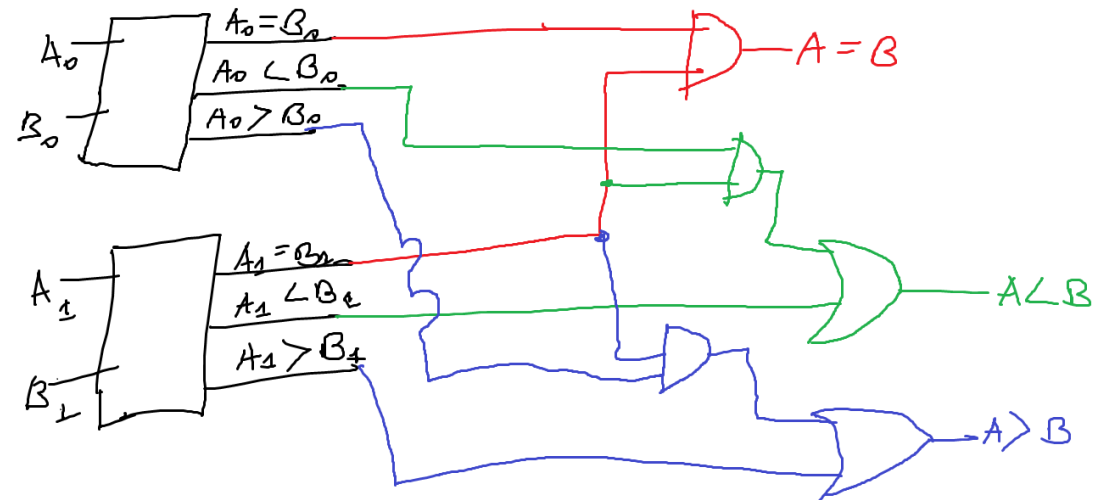
If $(A_1 > B_1)$, then $(A > B)$

If $(A_1 = B_1)^R$ & $(A_0 > B_0)$, then $(A > B)$

$$\begin{array}{|l} 0 \ 1 \rightarrow 1 \\ 1 \ 0 \rightarrow 2 \end{array} \rightarrow A < B$$

$$\begin{array}{|l} 1 \ 0 \rightarrow 2 \\ 1 \ 1 \rightarrow 3 \end{array} \rightarrow A < B$$

Logic diagram:



**THANK YOU FOR
YOUR
ATTENTION**

