

DAT600: Algorithm Theory

Assignment – 1: Compulsory

Sorting Algorithms and Running Times

Code of Conduct

The principle is trust, participation and collaboration for better learning.

So please, do your best to learn and don't try to cheat. It is allowed and encouraged to collaborate with others but for the purpose of learning, not just copy-pasting. You are allowed to consult and get inspired by external resources, but you should mention them in your report.

NB!

You may be asked to explain your work to the student assistants and/or the course responsible. Typically via a random selection, or some other relevant reasons such as the permission to use a good quality work as a good example, or of course in case of suspicion of plagiarism.

Delivery

Write a short and concise report in which you solve the following tasks. Add code sections when ever necessary, and a link to your GitHub repository if you find more convenient.

Task 1 counting the steps

Implement the Four algorithms listed in Table 1 in python using Jupyter notebook or design your own python library. Then vary the size of the input and record the number of steps. Plot the number of steps as a function the input size (n) to confirm that the plotted functions match the asymptotic running time shown on the Table 1.

Hint: see the demo from the lecture

Task 2 Compare true execution time

Choose one or several of the listed algorithm and implement them in two different programming languages. For example python and C or Go or C# or whatever language that you like. Run a test by varying the input size and measuring the execution time on your computer. Comment the differences.

| Algorithm | Worst-case running time | Average-case/expected running time |
|----------------|-------------------------|------------------------------------|
| Insertion sort | $\Theta(n^2)$ | $\Theta(n^2)$ |
| Merge sort | $\Theta(n \lg n)$ | $\Theta(n \lg n)$ |
| Heapsort | $O(n \lg n)$ | — |
| Quicksort | $\Theta(n^2)$ | $\Theta(n \lg n)$ (expected) |

Table 1: Comparison of 4 sorting algorithms

Task 3 Basic proofs

- Show that for any real constants a and b , where $b > 0$, $(n + a)^b = \Theta(n^b)$
- Show that $n^2 / \lg(n) = o(n^2)$
- Show that $n^2 \neq o(n^2)$

Hint: use the the definitions

Task 4 Divide and Conquer Analysis

Using the Master theorem and the recursion tree method, find the running time (Big-O). Time taken for the Karatsuba's multiplication algorithm is given by the following recurrence equation:

$$T(n) = 3T(n/2) + \Theta(n).$$