Black hole Information Paradox

by Dr. Suvrat Raju

mundane physics $\xrightarrow[\text{relative speeds}]{\text{high}}$ new notions of spacetime (SR) → GR

mundane physics → QM

QM: new notions of reality

(SR) → QFT

new notions of reality → QFT

new notions of reality → Quantum gravity?

GR → Quantum gravity?

Why study black-hole information?

quantum gravity features ← teaches us

1) BH & intro to paradox
2) PURE & mixed states
3) holography of information
4) refinement of the paradox

resource!
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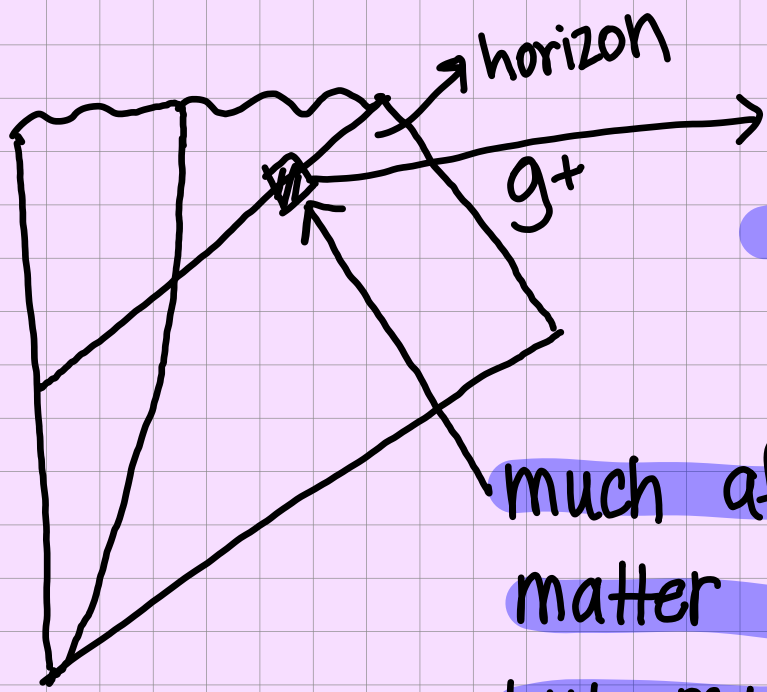
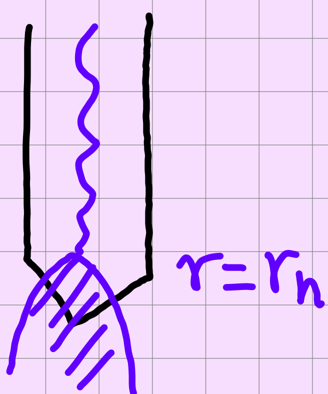


time translational invariance!

much after the infalling matter has fallen in but much before the bh evaporates

Oppenheimer, Sneider
B. datt → described the geometry



r_h = radius of horizon

for the marked part

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{\mu}{r^{d-2}}$$

in $(d+1)$ spacetime dimensions

$$\mu = 8\pi^{(2-d)/2} \, \Gamma\left(\frac{d}{2}\right) \frac{GM}{d-1}$$

$\mu \propto M$

$t \to t + \delta t$ (our ds^2 remains invariant!)

tortoise coordinates

$$dr_* = \frac{dr}{F(r)}$$

as $r \to \infty$, $f(r) \to 1$

$r_* \to \infty$

$f(r) \to 0$ at $r = r_h$,

as $r \to r_h$

$f(r) \to 2K(r - r_h)$

$$K = \frac{f'(r_h)}{2}$$

as $r \to r_h$

$$dr_* \to \frac{dr}{2k(r-r_h)}$$

$$r_* \to \frac{1}{2k} \log\left[(r-r_h)2k\right]$$

?

make a coordinate change

$$U = -\frac{1}{k} e^{k(r_*-t)}$$

(we don't like coordinate singularity)

$$V = \frac{1}{k} e^{k(r_*+t)}$$

$$\therefore dU = -(dr_* - dt)\, e^{k(r_*-t)}$$

$$dV = (dt + dr_*)\, e^{k(r_*+t)}$$

$$-dUdV = (dr_*^2 - dt^2)\, e^{2kr_*}$$

$$e^{2kr_*} = 2k(r-r_h) + O(r-r_h)^2$$

now, as $f(r) \to 0$,

$$ds^2 = -dUdV + r^2 d\Omega^2$$

1) At late times
$t \to t + \delta t$ isometry

2) At late times
the horizon is just like
empty space for a large black hole

3) near the horizon, wave eqn
simplifies and we can identify ingoing
and outgoing modes.

4) 2 point function of near horizon
modes yields thermal outgoing spectrum

5) thermal rad. causes BH to evaporate
to a final state seemingly indepen-
dent of initial conditions.

consider we are in r_* coordinates

$$(\Box - m^2)\phi = \underset{\sim}{0} \to V(\phi)$$

Box

$$\Box = \frac{1}{\sqrt{-g}} \partial_\mu g^{\mu\nu} \sqrt{-g}\, \partial_\nu$$

$$\sqrt{-g} = f(r)\, r^{d-1} \sqrt{g_r}$$

$$g^{**} = g^{tt} = \frac{1}{f(r)}$$

in r_*, t coordinates
$ds^2 = f(r)(-dt^2 + dr_*^2) + r^2 d\Omega^2$

$$\frac{1}{f(r)}\, r^{d-1}\, \partial_* r^{d-1} \partial_* \phi - \frac{1}{f(r)} \partial_t^{\,2} \phi + \frac{1}{r^2} B_\Omega \phi - m^2 \phi = 0$$

$\partial_* =$ derivative w.r.t. r_*

near the horizon, as $r \to r_h$

$\partial_* r = f(r)$

the equation becomes,

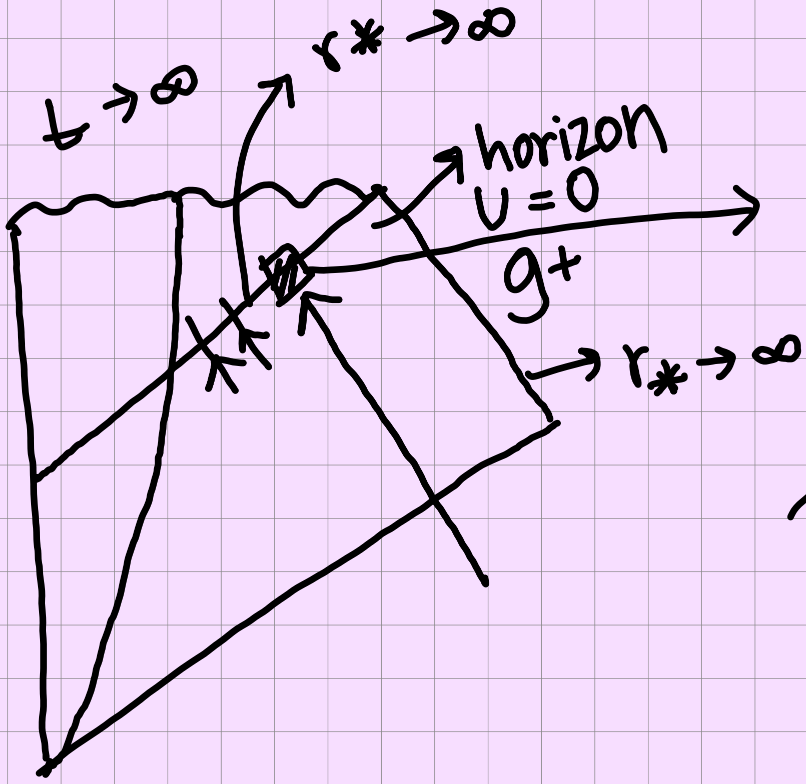
$$\boxed{\partial_*^2 \phi - \partial_t^2 \phi = 0}$$

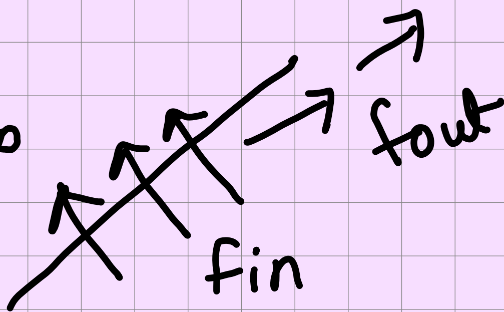
$$\phi = e^{-i\omega t} \cdot y_\ell(\Omega) \begin{cases} e^{-i\omega r_*} \\ e^{i\omega r_*} \end{cases}$$

two possible solutions

$$\phi = e^{-i\omega t}\, y_\ell(\Omega)\, e^{-i\omega r_*}$$

$$\phi = e^{-i\omega t}\, y_\ell(\Omega)\, e^{i\omega r_*}$$



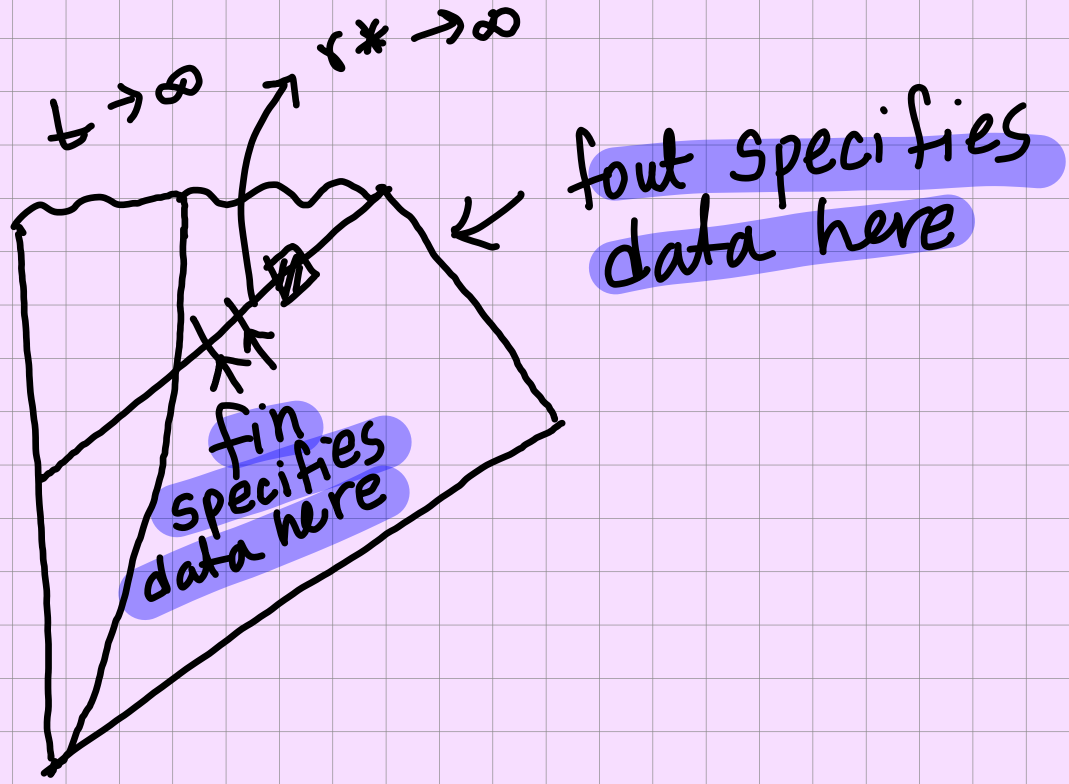


$$\phi = \sum_{\omega,l} e^{-i\omega t} A_{\omega,l} Y_l(\Omega) f_{in}(r_*) + e^{-i\omega t} B_{\omega,l} Y_l(\Omega) f_{out}(r_*) + h.c.$$

$$f_{in}(r_*) \xrightarrow{r_* \to -\infty} h_{\omega,l} e^{-i\omega r_*}$$

$$f_{out}(r_*) \xrightarrow{r_* \to -\infty} g_{\omega,l} e^{-i\omega r_*} + e^{i\omega r_*}$$

$$f_{out}(r_*) \xrightarrow{r_* \to \infty} e^{i\omega r} r^{(1-d)/2}$$



$$\phi \sim \sum_{\omega,\ell} B_{\omega,\ell}\left[(-U)^{i\omega/\kappa} + g_{\omega,\ell} V^{i\omega/\kappa} + A_{\omega,\ell} V^{-i\omega/\kappa} h_{\omega,\ell}\right] Y_\ell(\Omega) + \text{h.c.}$$

$$[A_{\omega,\ell}, A^\dagger_{\omega',\ell'}] \sim \delta(\omega-\omega')\delta_{\ell\ell'}$$

$$[B_{\omega,\ell}, B^\dagger_{\omega',\ell'}] \sim \delta(\omega-\omega')\delta_{\ell\ell'}$$

$$[A,B]=0$$

$$[\phi(t, r_*, \Omega), \dot{\phi}(t, r_*', \Omega')] \sim$$

$$i\delta(r_* - r_*')\,\delta(\Omega, \Omega')/\sqrt{g}$$

reference

CLICK!

$$\langle B^{\dagger}_{\omega,\ell} B_{\omega,\ell} \rangle \sim \int \langle \partial_U \phi \, \partial_{U'} \phi \rangle$$

$$(-U)^{i\omega/\kappa} (-U')^{-i\omega'/\kappa} dU dU'$$

$$\sim \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega') \delta_{\ell\ell}$$

when $U - U'$ and $V - V'$ are small,

Ω is close to Ω'.

$$\phi(U, V, \Omega), \; \phi(U', V', \Omega)$$

$$(-U)^{i\omega/\kappa} = e^{i\omega/\kappa \log(-U)}$$

$$\langle \phi(U, V, \Omega)\, \phi(U', V', \Omega) \rangle \sim S^{(1-d)/2}$$

$$\beta = \frac{2\pi}{\kappa}$$

$\kappa \rightarrow$ surface gravity

$$\langle \phi(\vec{x},t)\,\phi(0,0)\rangle \sim \frac{1}{t^2-\vec{x}^2}$$

$$\langle B_{\omega,l}\, B_{\omega,l}^{\dagger} \rangle \sim \frac{1}{1-e^{-\beta\omega}}\,\delta(\omega-\omega')\,\delta_{ll'}$$

$\beta \to$ temperature

thermal occupancy

the blackhole is radiating

Hawking radiation

the outgoing modes are thermally occupied

$$H = \omega a^{\dagger} a$$

$$\frac{1}{Z(\beta)}\mathrm{Tr}\left(e^{-\beta H}a^{\dagger}a\right),\ \frac{1}{Z(\beta)}\mathrm{Tr}\left(e^{-\beta H}a a^{\dagger}\right)$$

$$Z(\beta) = \mathrm{Tr}\left(e^{-\beta H}\right)$$

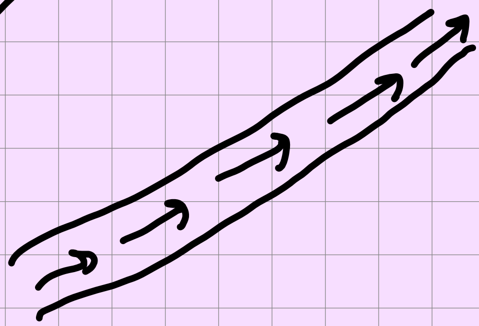
$$\beta \sim \frac{1}{T}$$

$$\langle A_{\omega l}^{\dagger} A_{\omega' l'} \rangle = 0$$

$U=0$

$U=\varepsilon$

Unruh vaccum



$$T = \frac{\kappa}{2\pi} \qquad \kappa = \frac{f'(r_h)}{2}$$

$$dM = \frac{\kappa\, dA}{8\pi} + \text{work terms}$$

$$dU = T\,dS + \text{work terms}$$

$$S = \frac{A}{4}$$ blackhole entropy

$S \sim 10^{76} \leftarrow$ for Solar mass blackhole

$T \sim 10^{-8}\,K$

hard to detect hawking radiation due to the CMBR

What is the Paradox ?!

if the BH is radiating it must be losing some energy.

$$\frac{dM}{dt} = -cAT^{d+1}$$

in 4D,

$$\frac{dM}{dt} = -cAT^{4}$$

d is the spacial dimension

$$f(r) = 1 - \frac{2m}{r}$$

$$f'(r_h) \propto \frac{1}{M}$$

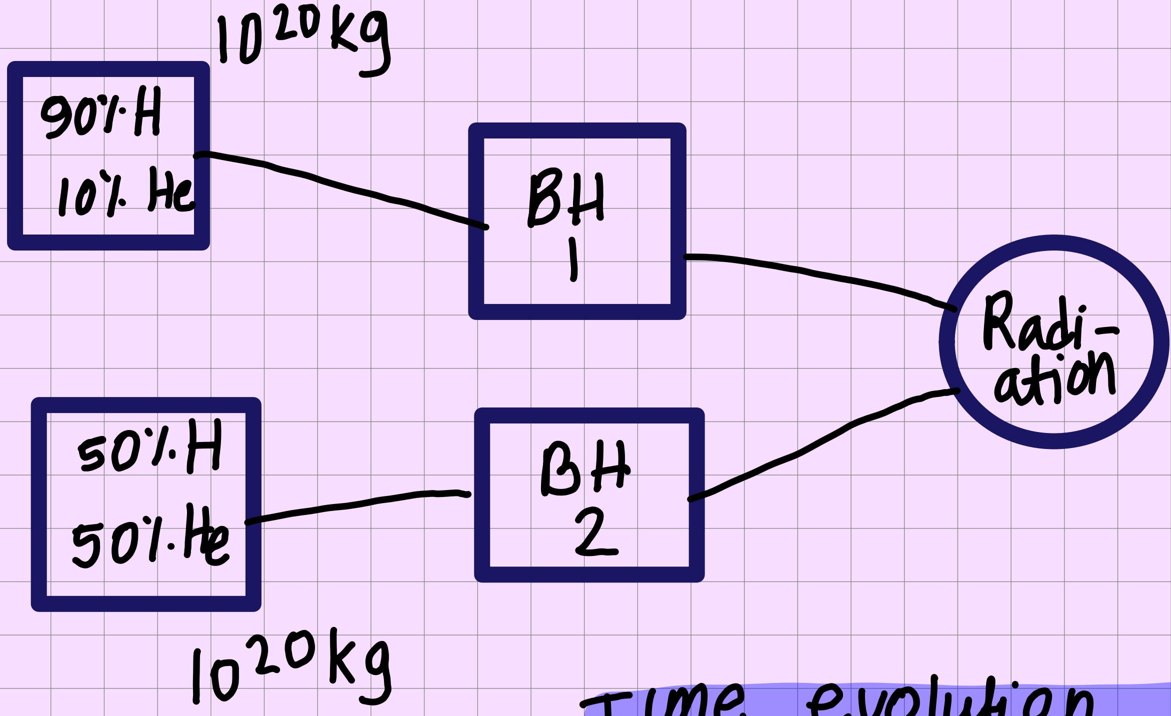
$$T \propto \frac{1}{M} \; ; \; A \propto M^2$$

$$\frac{dM}{dt} \propto \frac{-1}{M^2}$$

in $t \propto M^3$, the BH evaporates completely.

it looks like,

after evaporation, we are left with 'B' excitations which are thermal. $\rightarrow$ paradox



Time evolution is unitary!

but the radiation is same for any two black holes with the same mass.
(independent of initial conditions)

$$\rho = \frac{e^{-\beta H}}{Z(\beta)}$$

hamiltonian for a SHO

$$Z(\beta) = tr(e^{-\beta H})$$

$tr(PO) = \langle O \rangle_\beta$

$$\boxed{\begin{aligned} &H = \omega a^\dagger a \\ &tr(\rho a^\dagger a), \\ &tr(\rho a a^\dagger) \end{aligned}}$$

assigment: find the traces

how close are pure & mixed states?

$\frac{1}{\sqrt{2}}(|$  $\rangle + |$  $\rangle)$

$$\frac{1}{\sqrt{2}}(|1\rangle + |0\rangle) = |\psi_{pure}\rangle$$

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \rho_{mixed}$$

$$\langle \sigma_3 \rangle_{pure} = 0 \qquad \sigma_{(x)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\langle \sigma_3 \rangle_{mixed} = 0$$

$$\langle \sigma_x \rangle_{pure} = \Big(\langle 1|\sigma_x|1\rangle + \langle 0|\sigma_x|0\rangle + \langle 1|\sigma_x|0\rangle + \langle 0|\sigma_x|1\rangle \Big) \frac{1}{2}$$

$$\langle O \rangle_{pure} = \langle \psi_{pure} | O | \psi_{pure} \rangle$$

$$\langle O \rangle_{mixed} = tr(\rho_{mixed}\, O)$$

$$\langle \sigma_x \rangle_{mixed} = 0$$

consider a system with many energy eigenstates in a land of energies ΔE

$(E, E+\Delta E) \longrightarrow e^S$ states

$$P_{micro} = \frac{1}{e^S} \sum_i |E_i\rangle\langle E_i|$$

$$|\psi_{pure}\rangle = \sum a_i |E_i\rangle, \qquad \sum_i |a_i|^2 = 1$$

typical pure state $\boxed{\langle\langle P\rangle\rangle - tr(P\rho_{micro})}$

$$\int \underbrace{\langle \psi_{pure}| P |\psi_{pure}\rangle}_{\text{expectation value of a projector } P} dH = \langle\langle P\rangle\rangle$$

$$dH = \left(\prod_i d^2 a_i\right) \delta\left(\sum_i |a_i|^2 - 1\right) \frac{1}{N}$$

$$N = \frac{2\pi e^S}{\Gamma(e^S)} \qquad \int dH = 1$$

$$\langle \psi_{pure}| = \sum a_j^* \langle E_j|$$

$$\int dH \langle \psi_{pure}| P |\psi_{pure}\rangle = \int \sum_{i,j} a_i a_j^* \langle E_j|P|E_i\rangle dH$$

$$= \int \langle E_i|P|E_i\rangle |a_i|^2 dH$$

$$= \sum_i \langle E_i| P |E_i\rangle \underbrace{\int |a_i|^2 dH}_{\to \frac{1}{e^S}}$$

$$\langle\langle P \rangle\rangle = \sum_{e^s} \langle E_i | P | E_i \rangle$$

$$\mathrm{tr}(P \rho_{micro}) = \sum_i \frac{1}{e^s} \langle E_i | P | E_i \rangle$$

$$\int \left(\langle \psi_{pure} | P | \psi_{pure} \rangle - \mathrm{tr}(\rho_{micro} P) \right)^2 dH$$

$$\int dH \sum_{\substack{i,j \\ k,l}} \left(\langle E_j | P | E_i \rangle \left(a_i a_j^* - \frac{\delta_{ij}}{e^s} \right) \right)$$

$$\left(\langle E_l | P | E_k \rangle \left(a_k a_l^* - \frac{\delta_{kl}}{e^s} \right) \right)$$

$i=j$, $k=l$ or

$i=l$, $k=j$

$$\int a_i a_j^* a_k a_l^* \, dH$$

$$= \left(\delta_{ij} \delta_{kl} + \delta_{il} \delta_{jk} \right) \frac{1}{e^s(e^s+1)}$$

$$\left\langle \sum_{\substack{i,j,\\k,l}} \frac{\delta_{il}\,\delta_{jk}}{e^s(e^s+1)} \right. \langle E_j | P | E_i \rangle \langle E_l | P | E_k \rangle$$

$$= \frac{1}{e^s(e^s+1)} \sum \langle E_j | P | \underbrace{E_i \rangle \langle E_i | P}_{P^2} | E_j \rangle$$

$$< \frac{1}{e^s+1}$$

$$\int dH \sum_{\substack{i,j\\k,l}} \left(\langle E_j | P | E_i \rangle \left(a_i a_j^* - \frac{\delta_{ij}}{e^s} \right) \right)$$

$$\left(\langle E_l | P | E_k \rangle \left(a_k a_l^* - \frac{\delta_{kl}}{e^s} \right) \right)$$

$$< \frac{1}{e^s+1}$$

$$\int \left(\langle \psi_{pure} | P | \psi_{pure} \rangle - tr(\rho_{micro} P) \right)^2 < \frac{1}{e^S + 1}$$

$$\boxed{\langle \psi_{pure} | P | \psi_{pure} \rangle = tr(\rho_{micro} P) + O(e^{-S/2})}$$

1) At late times
$t \to t + \delta t$ isometry emerges
2) The horizon is just like empty Space for a large black hole
3) $\langle B^{\dagger}_{\omega, l} B_{\omega' l'} \rangle$ $\qquad \beta = 1/T, \; T = \frac{K}{2\pi}$

$$= \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega') \delta_{l l'}$$

4) Leads to seeming paradox

5) typical pure & microcanonical mixed states are the same for any observation up to $O(e^{-S/2})$

kinematic result

- von neumann entropy can differentiate mixed & pure states

not directly an observable

$$\mathrm{tr}(\rho \log \rho) \neq \mathrm{tr}(\rho A)$$

6) this calculation is not precise enough to lead to a paradox

Hawking

small correction theorem

$$\frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right) \otimes \left(|0\rangle|1\rangle + |1\rangle|0\rangle\right) \ldots\ldots$$

N times

$$\downarrow + \Sigma_1 |0\rangle|0\rangle + \Sigma_2 |0\rangle|1\rangle + \Sigma_3 |1\rangle|0\rangle + \Sigma_4 |1\rangle|1\rangle$$

$$\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

... N times

small corrections $\neq$

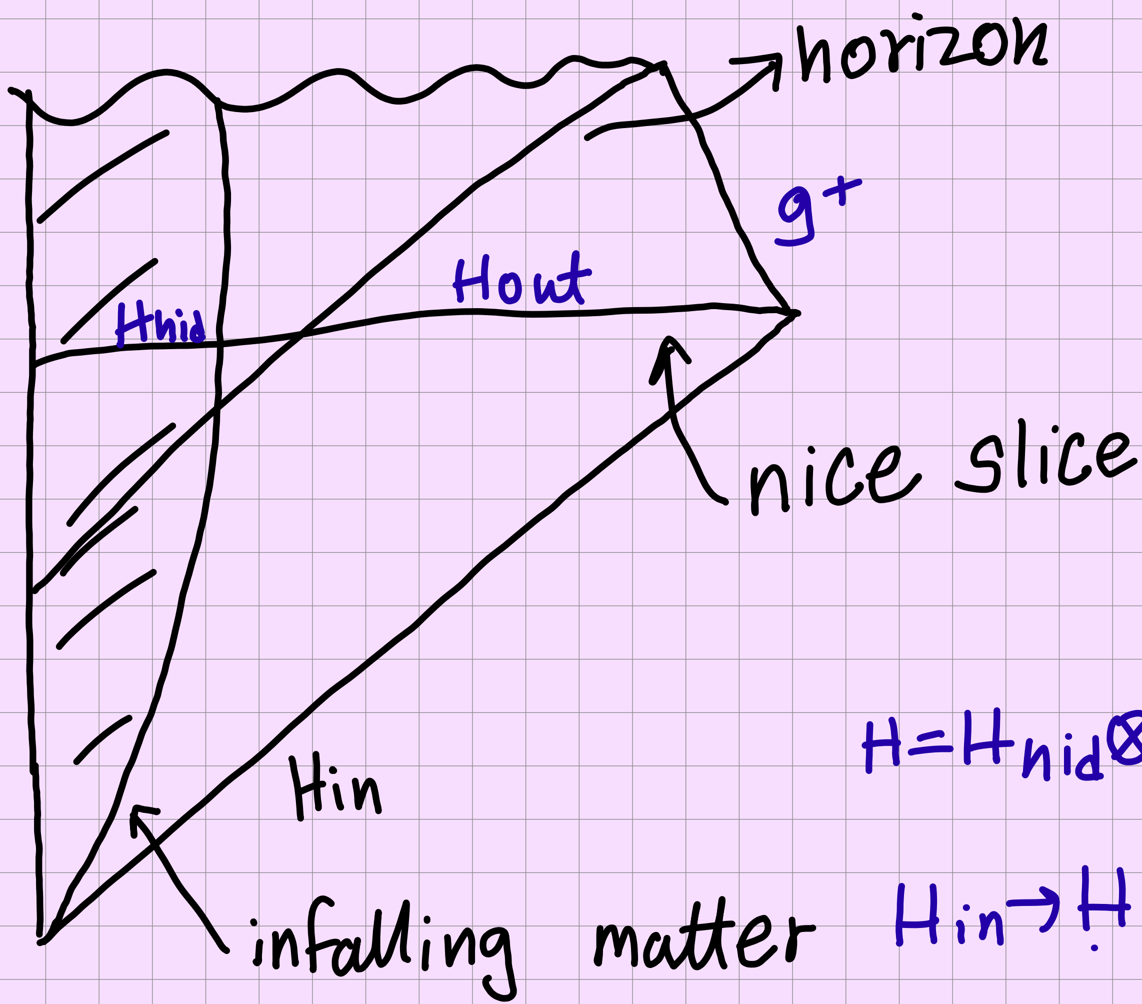
$|\psi_{pure}\rangle$ is close to ρ_{micro} as a state

$$\mathrm{tr}\left(\left(\rho_{micro} - |\psi_{pure}\rangle\langle\psi_{pure}|\right)^2\right) = O(1)$$

7) Typical observations are "close" but the states are not

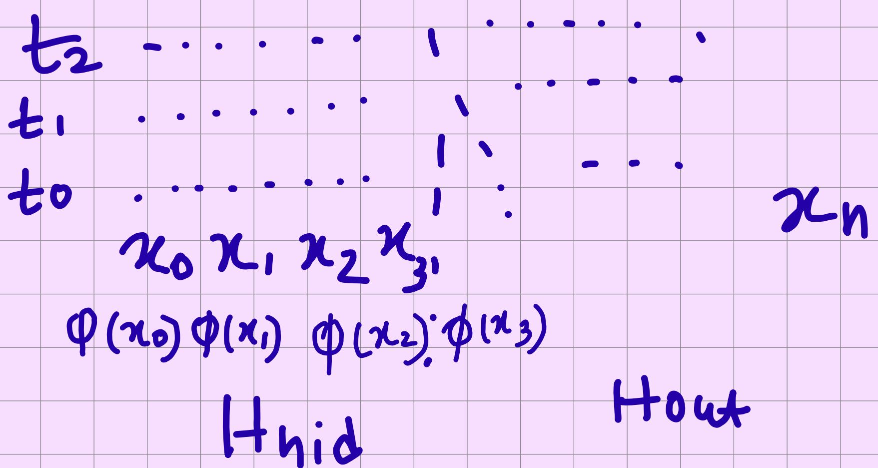
hawking's sophisticated argument

Hawking's paper



$$H = H_{hid} \otimes H_{out}$$

$$H_{in} \to H_{hid} \otimes H_{out}$$



$\rho_{out} = tr(|\psi\rangle\langle\psi|)$

will be mixed $H_{hid}(M, Q, L)$

"principle of ignorance"

observer outside only knows
M, Q, L inside.

[many configurations inside which give the same observables outside]

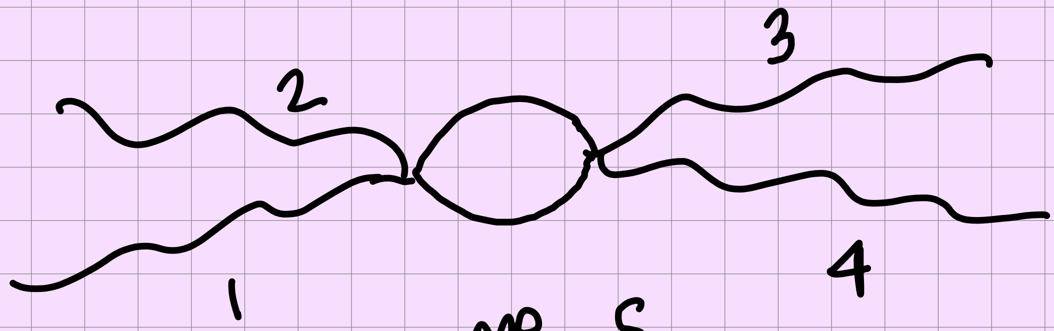
$H = H_{hid} \otimes H_{out}$

$H_{in} \rightarrow H_{hid} \otimes H_{out}$

$H_{in} \rightarrow H_{out}$ not unitary

pure state $\quad$ mixed state

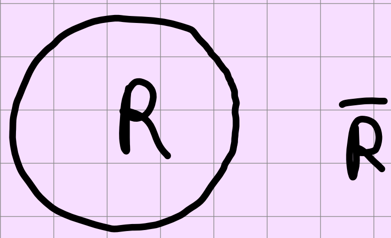
8) principle of ignorance, assumption of factorization suggests state outside is mixed



$$e^{-MR} \quad e^{S}$$

In gravity, principle of holography of information

$$\langle\psi| A_{\bar{R}} |\psi\rangle = \langle\psi| U_R^{\dagger} A_{\bar{R}} U_R |\psi\rangle$$



observables in the complement of a bounded region are sufficient to specify the state completely.

$$\dot{x}_0 \ \dot{x}_1 \,\Big|\, \dot{x}_2 \ \dot{x}_3 \ \dot{x}_4 \,\Big|\, \cdots \cdots \ \dot{x}_n$$

ϕ_1 $\quad \phi(x_2)\,\phi(x_3)$

$\phi(x_4)$

$$\int \vec{g}\cdot\hat{n}\,dA = M$$

Physical intuition

1) Gauss's Law

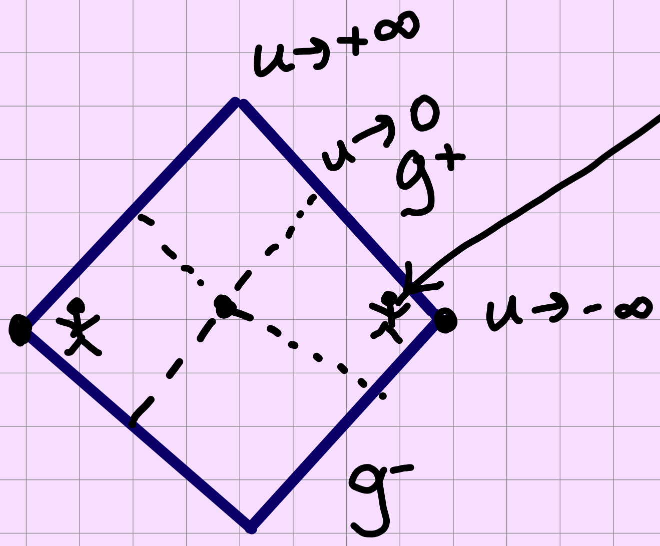
2) Uncertainty principle

$\downarrow$

Operators that keep M fixed fail to commute with some other operators at ∞.

Birkhoff's theorem

suggest that fixing energy + other observables at ∞ fixes the state.



Observers need to determine the profile of the excitation

Let $\phi(u, r, \Omega)$ be a massless field.

$$u = t - r$$

take $r \to \infty$ while keeping u fixed

$$\phi(u, r, \Omega) \xrightarrow[r \to \infty]{} \frac{1}{r} \underbrace{O(u, \Omega)}_{\substack{\text{intrinstic} \\ \text{to } \mathcal{I}^+ \\ \text{scri}}}$$

aymptotic observables

important result:

$$\langle \partial_{u'} O(u',\Omega') \, O(u,\Omega) \rangle = \frac{-1}{4\pi} \frac{1}{u'-u-i\varepsilon} \, \delta^2(\Omega,\Omega')$$

$$|\psi\rangle = e^{i\lambda \int f(u,\Omega) O(u,\Omega)} |0\rangle$$

where $f(u,\Omega)$ has support for $u \in (0,1)$

working to $O(\lambda)$ using correlators in $u' \in (-\infty, -\frac{1}{\varepsilon})$ determine $f(u,\Omega)$

$\langle \psi | H O(u',\Omega') | \psi \rangle$ to $O(\lambda)$

$$\langle 0| e^{-i\lambda \int f(u'',\Omega'')\, O(u'',\Omega'')\, du''\, d\Omega''} |0\rangle$$

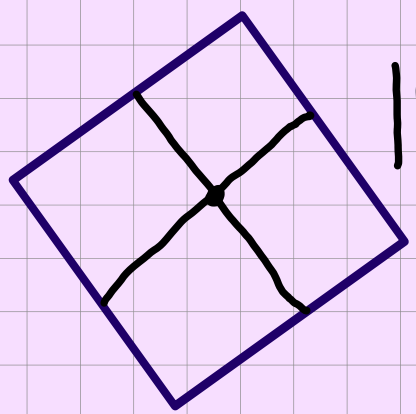
$$\boxed{O(u'',\Omega'')H = H\,O(u'',\Omega'') + \frac{i\partial}{\partial u''} O(u'',\Omega'')}$$

$$= \frac{\lambda}{4\pi}\int \frac{f(u'',\Omega')}{u''-u'+i\varepsilon}\, du'' \quad + O(\lambda^2)$$

$$= -\frac{\lambda}{4\pi}\sum_n \int f(u'',\Omega')\frac{(u'')^n}{(u')^{n+1}}\, du''$$

$$= \sum_{n=0}^{\infty} -\frac{\lambda}{4\pi (u')^{n+1}} \int (u'')^n f(u'',\Omega')$$

9) observables in complement of a bounded region determine the state for pure states.



$$|\psi\rangle = e^{i\int f(x,\Omega)O(x,\Omega)\,dx\,d\Omega}|0\rangle$$

$$\langle\psi| HO(u,\Omega)|\psi\rangle \quad , u \in \left(-\infty, -\frac{1}{\varepsilon}\right)$$

$$\sim \int_0^1 \frac{f(x,\Omega)}{x-u}\,dx$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$U_1|\psi\rangle$

$$\langle\psi|U_1^{\dagger} A_2 U_1|\psi\rangle = \langle\psi|A_2|\psi\rangle$$

$\phi, \tilde{\phi}$

related symmetry

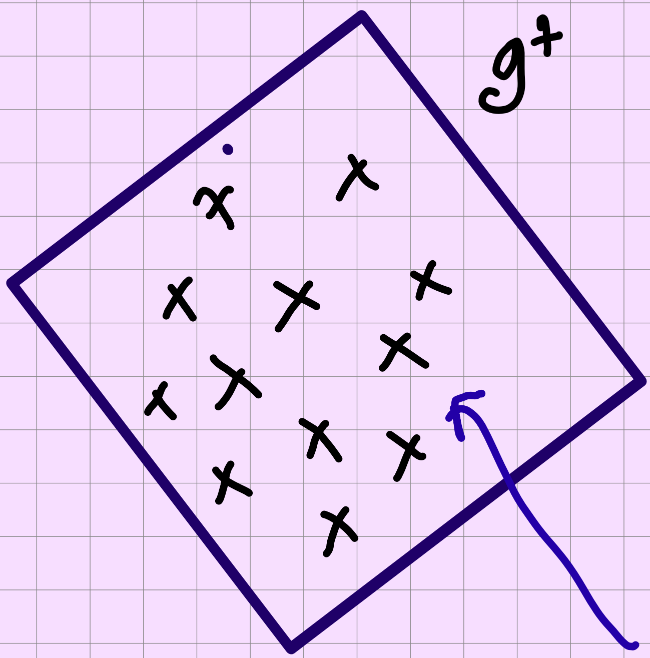
$\phi \xrightarrow[r\to\infty]{} \frac{1}{r} O \, . \; \tilde{\phi} \xrightarrow[r\to\infty]{} \frac{1}{r}\tilde{O}$

$$|\tilde{\psi}\rangle = e^{i\lambda \int f(x,\Omega)\tilde{O}(x,\Omega)\, dx\, d\Omega} |0\rangle$$

$$\langle\psi| H\tilde{O} |\psi\rangle = 0 + O(\lambda^2)$$

$$\langle\tilde{\psi}| H\tilde{O} |\tilde{\psi}\rangle = \langle\psi| HO |\psi\rangle = \int_0^1 \frac{f(x,\Omega)\, dx}{x-u}$$

$$\langle\tilde{\psi}| HO |\tilde{\psi}\rangle = 0 + O(\lambda^2)$$



$A(g^+)$ – algebra of operators of g^+.

bulk is unknown.

asymptotically flat spacetime

$A_{-\infty}$ - algebra of operators for $u \in (-\infty, -\frac{1}{2})$

every element of $A(g^+)$ can be approximated arbitrarily well by an element of $A_{-\infty}$

Assumptions:

1) these algebras continue to make sense.

$$A(g^+) = \text{Span}\{O(u_1, \Omega_1), \ldots\ldots O(u_n, \Omega_n) \ldots\}$$

this also includes metric fluctuations.

$$\phi(r, u, \Omega) \xrightarrow{r \to -\infty} \frac{1}{r} O(u, \Omega)$$

flat space

$$ds^2 \xrightarrow{r \to \infty} -du^2 - 2du\,dr + r^2 d\Omega^A d\Omega^B \gamma_{AB} +$$

$$\underbrace{\frac{2m}{r}}_{\text{mass aspect}} du^2 + r C_{AB} d\Omega^A d\Omega^B + D^B C_{AB} du\, d\Omega^A$$

$$\boxed{\underset{\text{news}}{N_{AB}} = \partial_u \underset{\text{sheer}}{C_{AB}}}$$

$$A_{-\infty} = \text{span}\{ O(u_1) \dots O(u_n)$$
$$C_{AB}(u_1') \dots C_{AB}(u_m')$$
$$m(u_1'') \dots m(u_m'')\}$$

$$u_i \in (-\infty, -\tfrac{1}{\varepsilon})$$

2) hilbert space:

energy is positive

$$H = A(g^{+})|0\rangle$$

$\forall |n\rangle \in H$, $\exists X_n \in A_{-\infty}$

such that $X_n|0\rangle = |n\rangle$

has nothing to do with gravity

Reeh-Schleider theorem

consider

$$\int_{-\infty}^{\infty} O(u) f(u)|0\rangle = |f\rangle$$

we can find that $\int_{-\infty}^{-1/\varepsilon} du\, O(u) g(u)|0\rangle = |f\rangle$

so that

$$\| |f\rangle - |g\rangle \| \approx 0$$

Proof by Contradiction

say $\exists f$ such that $\langle f | O(u) | 0 \rangle = 0$
$\forall u \in (-\infty, -\frac{1}{\varepsilon})$

$$= \sum_E \langle f | E \rangle \langle E | O(u) | 0 \rangle$$

$$= \sum_E \langle f | E \rangle \langle E | O(0) | 0 \rangle e^{iEu}$$

It is analytic when u is extended in the upper half plane

$\Rightarrow \langle f | O(u) | 0 \rangle = 0 \ \forall$ real u.
which is absurd.

$\therefore$ no such $|f\rangle$ exists. $\blacksquare$

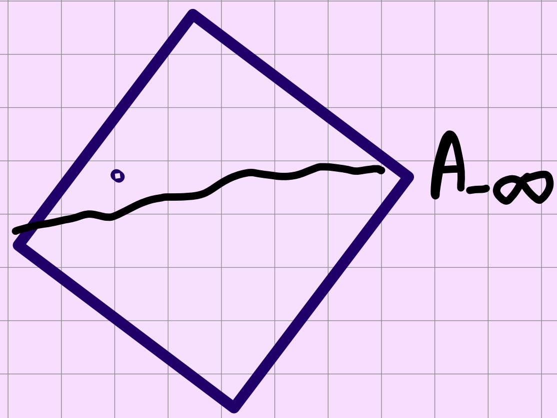
edge - of - wedge theorem

PCT, spin statistics and all that.

further reading

3) $\hat{H} \in A_{-\infty} = \frac{\#}{G_N} \lim_{u \to -\infty} \int m(u, \Omega) d^2\Omega$

$\hat{H} \in A_{-\infty} \leftarrow$ requires gravity



Assumption:

this remains true in the UV-complete theorem.

gauss's law?

$P_0 = |0\rangle\langle 0| \in A_{-\infty}$

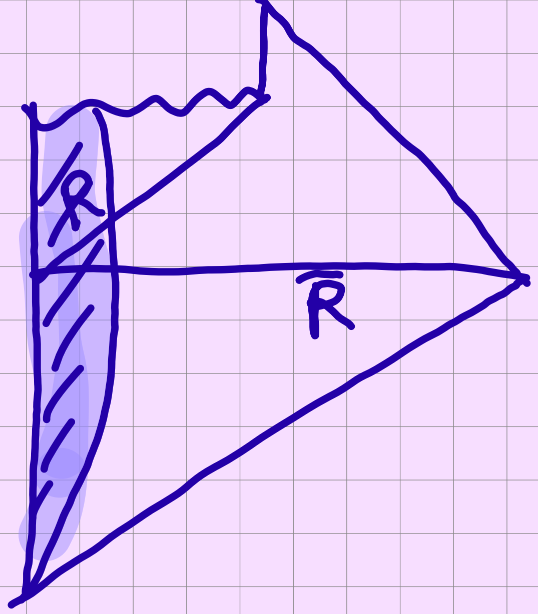
* $\hat{H}$ = hamiltonian
* H = hilbert space

$$T = \sum_{n,m} C_{nm} |n\rangle\langle m|$$

$$= \sum_{n,m} C_{nm} X_n |0\rangle\langle 0| X_m^\dagger$$

$$X_n, X_m \in A_{-\infty} \quad = \underbrace{\sum_{n,m} C_{nm} X_n P_0 X_m^\dagger}_{\text{sum of product of 3 operators from } A_-}$$

10) All elements of $A(g^+)$ can be approximated arbitrarily well in $A_{-\infty}$.



$H = H_R \otimes H_{\bar{R}}$

wrong assumption

1) $H \neq H_R \otimes H_{\bar{R}}$ when R is bounded and $\bar{R}$ is its compliment

2) If we coarse grain observation, hilbert space might factorise effectively.

Page curve

$\dim(H_m) = m$
$\dim(H_n) = n$
assume that
$m < n$

$H = H_m \otimes H_n$

Consider "generic" state

$$|\psi\rangle = \sum_{j=1}^{n} \sum_{i=1}^{m} A_{ij} |i\rangle |j\rangle$$

by a change of basis

$$A = m\left\{ \underbrace{\begin{pmatrix} \tilde{A}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{A}_{22} & & & & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & & & & & & \\ 0 & 0 & & \ddots & & & & & \\ & & & & \ddots & & & & \\ & & & & & & \tilde{A}_{mm} & 0 & 0 \end{pmatrix}}_{n} \right.$$

$$\text{eigen}(P_m) = \left(|\tilde{A}_{11}|^2, |\tilde{A}_{22}|^2 \ldots |\tilde{A}_{mm}|^2 \right)$$

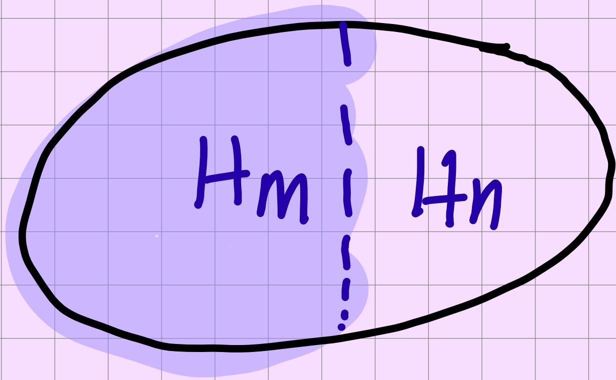
$$S_m = -\mathrm{tr}(P_m \log P_m)$$

$$= -\sum_i |\tilde{A}_{ii}|^2 \log(|\tilde{A}_{ii}|^2)$$

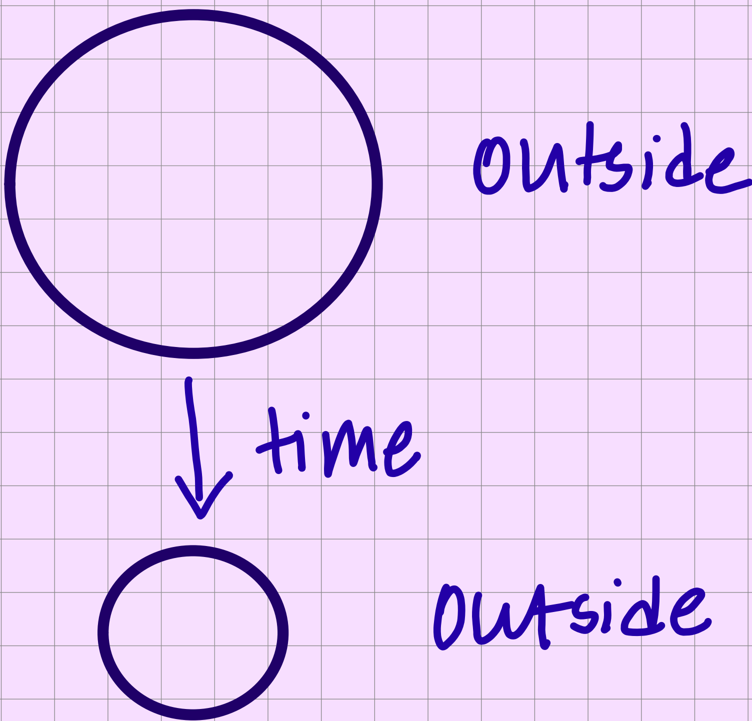
$$= \sum_{i=1}^{m} \frac{1}{m} \log m = \log m$$

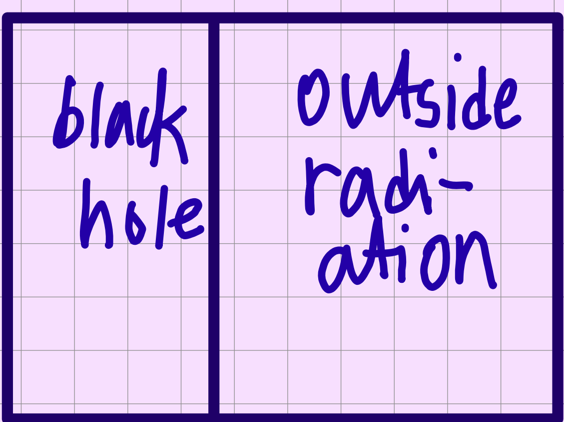
$$\boxed{\sum_{i=1}^{m} |\tilde{A}_{ii}|^2 = 1}$$

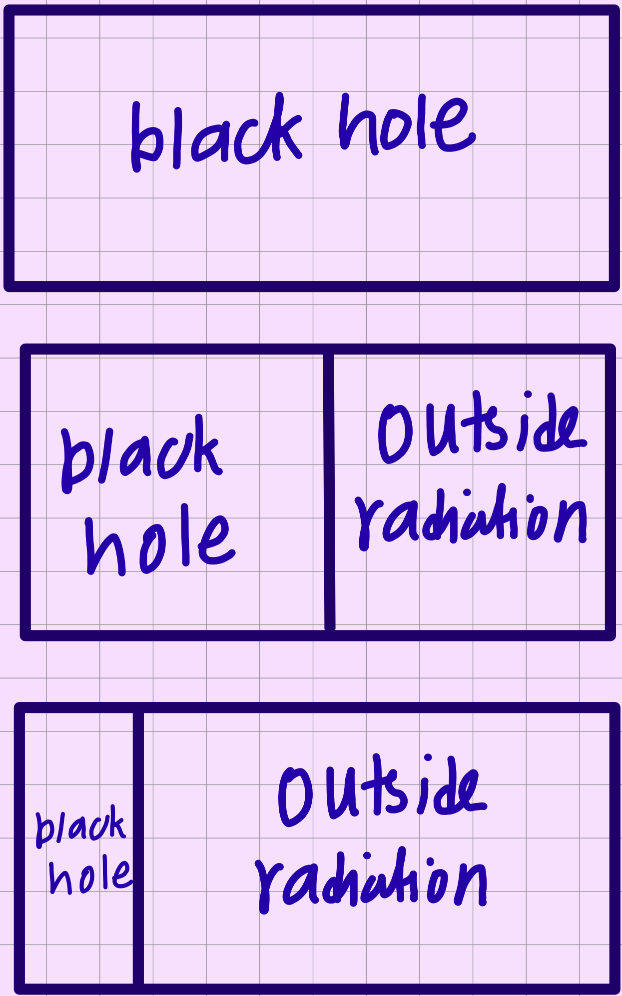
$$\langle S_m \rangle = \log(\min(m,n))$$



Page's argument:



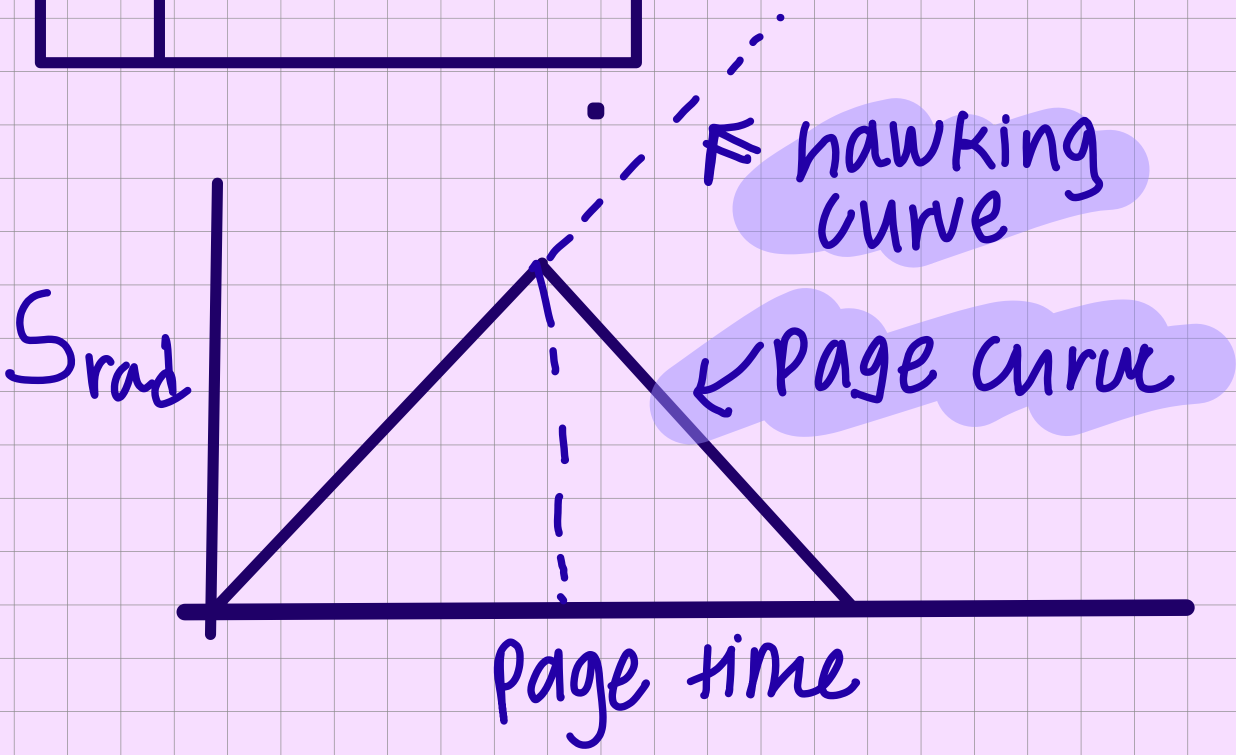




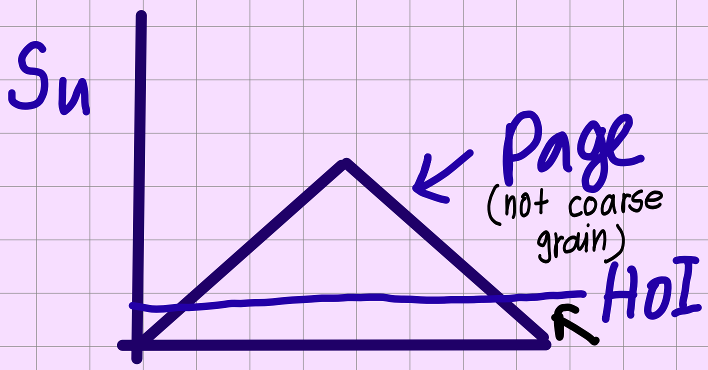
initial state

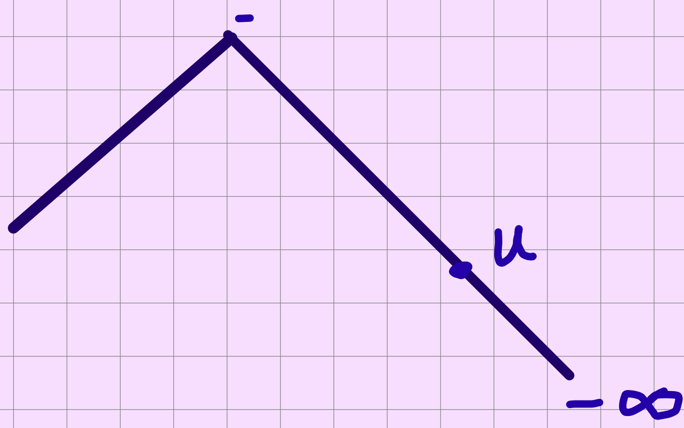
intermediate

final



Page argument also assumes factorization of Hilbert space into inside & outside



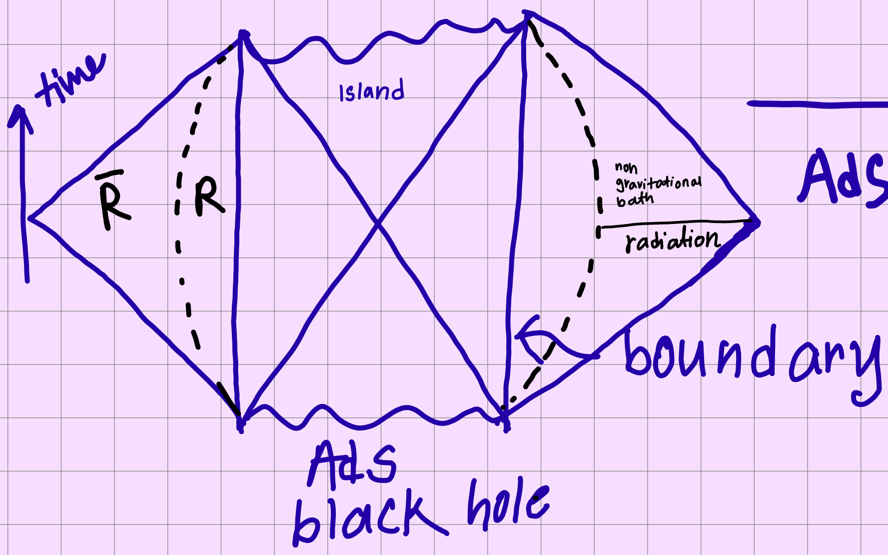


11) $\langle S_m \rangle = \log(\min(m,n))$

12) $H \neq H_R \otimes H_{\bar{R}}$

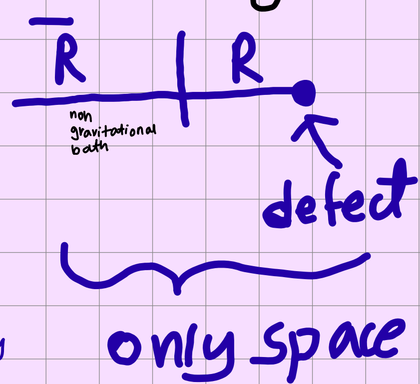
$$S_{fine} = -\mathrm{tr}(\rho \log \rho)$$

fine-grained von-Neumann entropy where you keep track of everything



AdS/CFT

BCFT = CFT with a boundary

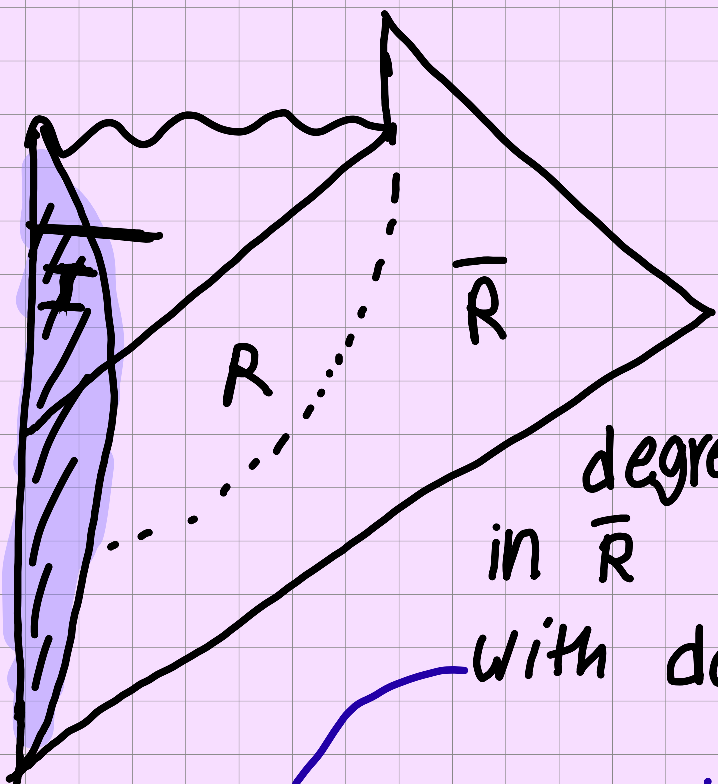


13) AdS black hole coupled to non-gravitational system with a boundary defect.

In this setting, we can compute the page curve of part of a bath.

$$\mathrm{ext}\left(\frac{A I}{4G} + S_{QFT}(\mathrm{rad}^n \cup \mathrm{Island})\right)$$

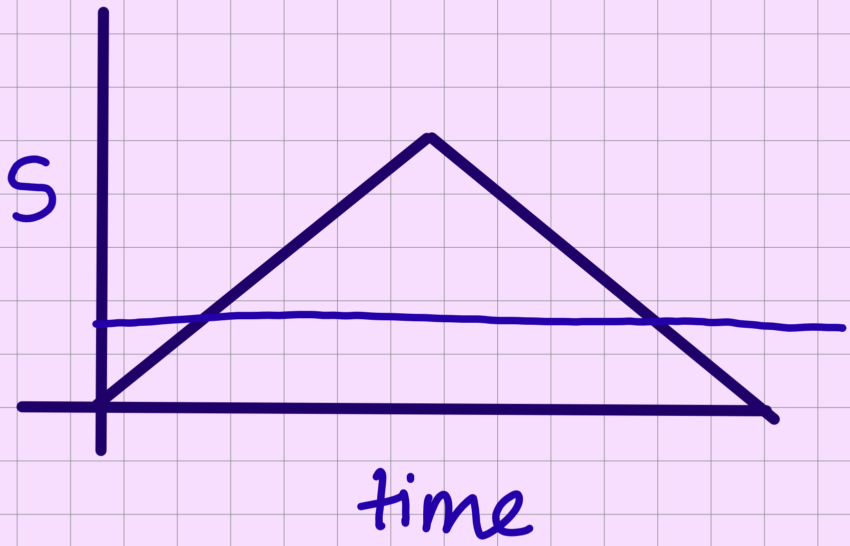
14)



degrees of freedom in $\bar{R}$ are redundant with dof in I.

weak gravity ≠ no gravity

all are redundant with all dof



notes by nazlee
(or nafisa)