Probability Questions

Problem 1: Joint PMF for Rolling a Die Twice

An experiment consists of rolling an unbiased die two times. The random variables $X_i \sim \text{Uniform}\{1, 2, 3, 4, 5, 6\}$ represent the number on the ith roll, where $i = 1, 2$. Calculate:

$$f_{X_1, X_2}(3, 2)$$

Problem 2: Drawing Queens and Kings from a Deck

From a well-shuffled deck of 52 cards, four cards are selected at random. Let the random variable X denote the number of queens drawn, and let the random variable Y denote the number of kings drawn. Find:

$$f_{X,Y}(2, 1)$$

Problem 3: Joint PMF of Two Discrete Random Variables

The joint probability mass function of two discrete random variables X and Y is given by:

$$f_{X,Y}(x, y) = \frac{xy}{9}, \quad x, y \in \{1, 2\}$$

Calculate:

$$f_X(1) + f_X(2)$$

Problem 4: Conditional Probability from a Joint PMF Table

Let X and Y be two random variables with joint PMF $f_{X,Y}(t_1, t_2)$ given by:

$t_2 \backslash t_1$	1	2	3
1	0	0.10	0.08
2	0.20	0.10	0
3	0.02	0.30	0.20

Find:

1. The range of $(Y \mid X = 1)$.

2. $f_{X|Y=2}(1)$.

1) 1/36

$$P(X1 = x1, X2 = x2) = P(X1 = x1) \cdot P(X2 = x2)$$

2) 1056/270725

$$f_{X,Y}(2, 1) = \frac{\binom{4}{2} \times \binom{4}{1} \times \binom{44}{1}}{\binom{52}{4}}$$

3) 1

$$f_X(x) = \sum_y f_{X,Y}(x, y)$$

4. 1) 1:{2,3}
   2) 0.2

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$