A Fuzzy Filter for Images Corrupted by Impulse Noise

Fabrizio Russo, *Member, IEEE*, and Giovanni Ramponi, *Member, IEEE*

***Abstract*— A new operator is presented which adopts a fuzzy logic approach for the enhancement of images corrupted by impulse noise. The proposed operator is based on two-step fuzzy reasoning, and it is able to perform a very strong noise cancellation while preserving image details very well. The new fuzzy filter is favorably compared with other nonlinear operators in the literature.**

I. Introduction

Nonlinear techniques for image enhancement are acquiring growing importance due to their specific ability to perform an effective noise cancellation without degrading the image structure. In this framework, methods based on fuzzy logic are emerging as a valuable resource: indeed, a family of operators based on fuzzy rules has been recently introduced for image filtering and enhancement [1]–[4]. Other techniques still resorting to fuzzy logic have been very recently proposed in the same application area [5]–[8].

In this letter, a new fuzzy operator is presented for the suppression of impulse noise: this operator is based on the approach proposed in [4] and is able to perform a very strong noise cancellation while preserving image details very well. As it will be shown, this behavior is obtained by suitably implementing fuzzy reasoning at two different stages.

II. The Operator Structure

The proposed fuzzy operator is composed of two cascaded subunits adopting fuzzy reasoning. The first subunit (*action detection module*) aims at detecting noise pulses by considering luminance differences among neighboring pixels: as a result, a possible correction term is selected. The second subunit (*action adaptation module*) suitably modifies the value of this correction in order to further improve the detail preservation.

A. Action Detection Module

Let P_0 and P_0', respectively, be a pixel in the input image and the corresponding one in the output image and define the neighborhood

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 8. \end{matrix}$$

The proposed operator is applied recursively to the data. Its input variables are the luminance differences:

$$x_j = \begin{cases} P_j' - P_0, & j = 1, \cdots, 4 \\ P_j - P_0, & j = 5, \cdots, 8. \end{cases} \tag{1}$$

The output variable y is a luminance difference too and represents a candidate correction term which could be added to P_0 in order to cancel the noise. In order to implement fuzzy reasoning, we use triangular shaped fuzzy sets described by a two-parameter membership function $m(u)$

$$m(u) = \begin{cases} 0, & u \le c - w \\ (w - |u - c|)/w, & c - w < u < c + w \\ 0, & u \ge c + w \end{cases} \tag{2}$$

where c and w define the position of the center and the half-width of the isosceles fuzzy set, respectively. We adopt, for the proposed operator, two fuzzy sets labeled *positive* (PO) and *negative* (NE). If the image has L gray levels, we define the membership function parameters as follows: $c_{\mathrm{PO}} = L - 1$; $c_{\mathrm{NE}} = -L + 1$; $w_{\mathrm{PO}} = w_{\mathrm{NE}} = 2(L - 1)$.

Fuzzy reasoning resorts to a set of rules in order to detect noise pulses: as it will be shown, each fuzzy rule deals with a particular pattern of neighboring pixels. As an example, let us consider the luminance differences defined by the set of indexes $I_1 = \{2, 5, 7\}$. A pair of fuzzy rules acting on this pattern can be easily designed as follows:

$$\text{IF } (x_2, \text{PO}) \text{ AND } (x_5, \text{PO}) \text{ AND } (x_7, \text{PO}) \text{ THEN } (y, \text{PO}) \tag{3}$$

$$\text{IF } (x_2, \text{NE}) \text{ AND } (x_5, \text{NE}) \text{ AND } (x_7, \text{NE}) \text{ THEN } (y, \text{NE}). \tag{4}$$

It can be observed that rule (3) exploits the luminance differences x_2, x_5, and x_7 defined by relation (1) in order to detect a *negative* noise pulse in position 0: it is activated if the luminance of P_0 is lower than the one of the neighborhood. Once a negative noise pulse has been detected, this rule aims at reducing its amplitude: in fact, the correction term y specified by (3) is positive. For the same pattern defined by set I_1, rule (4) similarly addresses the case of a *positive* noise pulse.

In order to consider many possible situations, where groups of adjacent noise pixels occur, the overall rulebase of the new operator takes care of 13 different patterns of neighboring pixels defined by the following sets of indexes: $I_1 = \{2, 5, 7\}$, $I_2 = \{5, 7, 4\}$, $I_3 = \{7, 4, 2\}$, $I_4 = \{4, 2, 5\}$, $I_5 = \{1, 3, 8, 6\}$, $I_6 = \{1, 2, 3, 5\}$, $I_7 = \{2, 3, 5, 8\}$, $I_8 = \{3, 5, 8, 7\}$, $I_9 = \{5, 8, 7, 6\}$, $I_{10} = \{8, 7, 6, 4\}$, $I_{11} = \{7, 6, 4, 1\}$, $I_{12} = \{6, 4, 1, 2\}$ and $I_{13} = \{4, 1, 2, 3\}$. Since

Manuscript received July 19, 1995. This work was supported by M.U.R.S.T. and by the European Research Project under ESPRIT LTR-20229-Noblesse. The associate editor coordinating the review of this letter and approving it for publication was Prof. R. Mersereau.

The authors are with the DEEI, University of Trieste, 34127 Trieste, Italy.

Publisher Item Identifier S 1070-9908(96)04930-9.

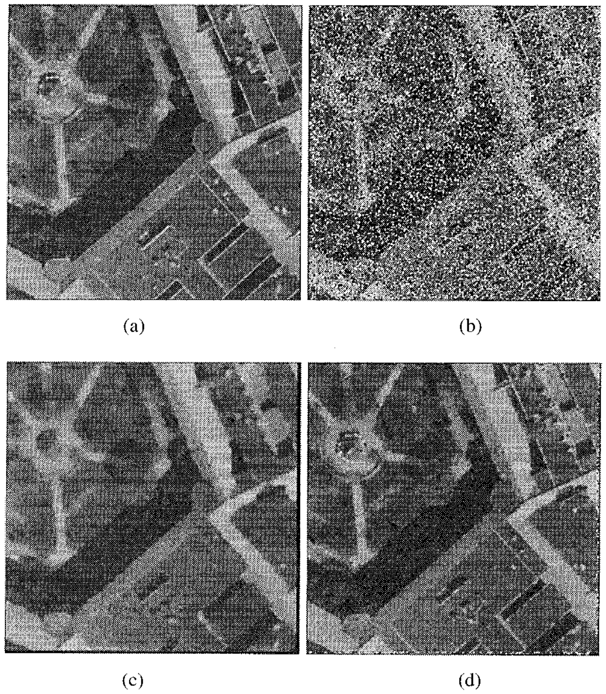

Fig. 1. (a) Original image. (b) Noisy image. (c) Result yielded by a 5×5 median filter. (d) Result yielded by the fuzzy filter.

each pattern involves two fuzzy rules, a total of $2 \times 13 = 26$ fuzzy rules analogous to (3) and (4) is defined.

Once a specific rulebase has been designed, the output y is obtained as the result of the inference process [2]. In our approach, y is evaluated by means of the following group of four relations:

$$\lambda_1 = \mathrm{MAX}\,\{\mathrm{MIN}\,\{m_{\mathrm{PO}}(x_j);\ j \in I_i\};\ i = 1, \cdots, 13\} \quad (5)$$

$$\lambda_2 = \mathrm{MAX}\,\{\mathrm{MIN}\,\{m_{\mathrm{NE}}(x_j);\ j \in I_i\};\ i = 1, \cdots, 13\} \quad (6)$$

$$\lambda_0 = \mathrm{MAX}\,\{0,\ 1 - \lambda_1 - \lambda_2) \quad (7)$$

$$y = (L-1)(\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2 + \lambda_0) \quad (8)$$

where m_{PO} and m_{NE} are the membership functions of fuzzy sets PO and NE, respectively. The above-mentioned procedure is an improved version of the inference mechanism described in [4].

B. Action Adaptation Module

The purpose of this module is to avoid small luminance corrections which are almost useless in reducing the effects of noise and impair the quality of fine details and textures. The operation of this module can be simply described by the following fuzzy reasoning: *if the absolute value of y is small then further reduce it*. More formally, the output y' of this module is yielded by the following relationship:

$$y' = y(1 - m_{\mathrm{SM}}(|y|)) \quad (9)$$

TABLE I
MSE Values for Different Processing Techniques as a Function of Percentage of Noise

NOISE PROB.	FUZZY	3x3 MED	5x5 MED	SD-ROM
0.10	13	72	127	26
0.18	29	111	135	41
0.26	50	214	147	60
0.33	77	431	161	87
0.40	120	785	186	130

where m_{SM} is a two parameter membership function which describes the fuzzy set *small* (SM)

$$m_{\mathrm{SM}}(u) = \begin{cases} 1, & u \le a \\ (a+b-u)/b, & a < u \le a+b \\ 0, & u > a+b. \end{cases} \quad (10)$$

The choice of parameters a and b is not critical: indeed, the same pair of values has been adopted for all the tests described in the next section. Finally, the output pixel takes the value $P_0 + y'$.

III. Experimental Results

In order to assess the performances of the proposed method, some computer simulations have been performed. Five test images have been created, superposing impulse (salt and pepper) noise with probabilities 0.1 to 0.4 to the 512×512 Pentagon image.

First, a quantitative evaluation of the filter can be given by estimating the mean-square error (MSE) of the images processed with the proposed and other techniques. Table I reports the values obtained from the fuzzy method ($a = 40$, $b = 32$) in the first column. The second and third columns, respectively, report the values yielded by a conventional median filter acting on a 3×3 and on a 5×5 support. Observe that the proposed fuzzy technique largely outperforms the median operator: the MSE values of the latter are several times larger. The fuzzy method has also been compared with state-of-the-art filtering techniques. Recently, a signal-dependent, rank ordered mean (SD-ROM) filter has been presented which is able to very effectively suppress impulse noise [9]. It is based on a detection-estimation strategy: if a corrupted sample is detected, it is replaced with an estimation of the true value based on neighborhood information. The MSE values obtained from this algorithm are shown in the last column of Table I. It is seen that still the fuzzy method yields better results, especially for relatively low noise probability but, to a smaller extent, even for highly corrupted data.

It should be mentioned that an even better performance can be obtained, in particular for the case of high probability of corruption, simply by iterated processing of the data. As an example, the MSE value which is yielded by two passes of the fuzzy algorithm in the case of noise probability 0.4 is 96; this means a further reduction by 20% with respect to the single-pass fuzzy filter.

Finally, in order to appraise the power of the proposed method from a subjective point of view, one of the mentioned test images is reproduced. Fig. 1(a) and (b), respectively, show a 256×256 quadrant of the Pentagon image and its corrupted version (noise prob. 0.26). Fig. 1(c) shows the effect of processing the previous picture using a median filter of size 5×5: even if the noise has completely been eliminated, many small details in the scene have disappeared. The same details are much better preserved by our method [Fig. 1(d)].

IV. Conclusions

A new double-action recursive fuzzy filter has been presented. By exploiting fuzzy reasoning at two different stages, the filter is able to perform a very strong noise cancellation without degrading the image structure. The experimental results have shown that the operator is very effective in case of highly corrupted images too.

References

[1] F. Russo and G. Ramponi, "Fuzzy operator for sharpening of noisy images," *IEE Electron. Lett.*, vol. 28, no. 18, pp. 1715–1717, Aug. 1992.

[2] ——, "Nonlinear fuzzy operators for image processing," *Signal Processing*, vol. 38, pp. 429–440, Aug. 1994.

[3] ——, "A fuzzy operator for the enhancement of blurred and noisy images," *IEEE Trans. Image Processing*, to be published.

[4] ——, "Introducing the fuzzy median filter," in *Proc. 7th Europ. Signal Processing Conf. EUSIPCO-94*, Edinburgh, Scotland, Sept. 13–16, 1994, pp. 963–966.

[5] A. Taguchi, H. Takashima, and Y. Murata, "Fuzzy filters for image smoothing," in *Proc. SPIE Conf. Nonlinear Image Processing V*, San Jose, CA, Feb. 7–9, 1994, pp. 332–339.

[6] S. Peng and L. Lucke, "Fuzzy filtering for mixed noise removal during image processing," in *Proc. 3rd IEEE Int. Conf. Fuzzy Systems*, Orlando, FL, June 26–29, 1994, pp. 89–93.

[7] V. Chatzis and I. Pitas, "Mean and median of fuzzy numbers," in *Proc. 1995 IEEE Workshop Nonlinear Signal Image Processing*, Neos Marmaras, Halkidiki, Greece, June 20–22, 1995.

[8] K. N. Plataniotis, D. Androutsos, and A. N. Venetsanopoulos, "Color image processing using fuzzy vector directional filters," in *Proc. 1995 IEEE Workshop Nonlinear Signal Image Processing*, Neos Marmaras, Halkidiki, Greece, June 20–22, 1995.

[9] E. Abreu and S. K. Mitra, "A signal-dependent rank ordered mean (SD-ROM) filter: a new approach for removal of impulses from highly corrupted images," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, ICASSP-95*, Detroit, MI, May 1995.