Binary Trees

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Lecture 08

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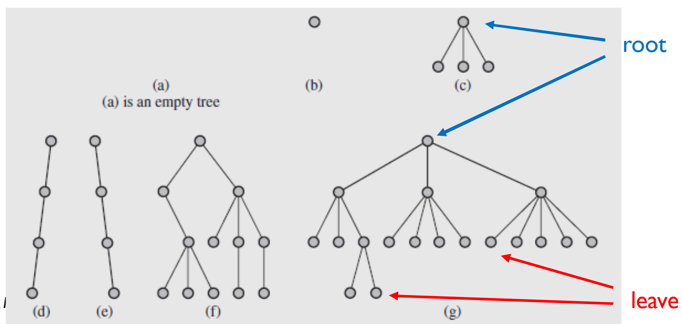
Adapted partially from Data Structures and Algorithms in C++, Adam Drozdek, 4th Edition, Cengage Learning; and Algorithms and Data Structures, Douglas Wilhelm Harder, Mmath

Trees, Binary Trees, and Binary Search Trees

- **Limitations** of linked lists, stacks, and queues,
  - **Linked lists:**
    - linear in form and cannot reflect hierarchically organized data
  - **Stacks and queues**
    - one-dimensional structures and have limited expressiveness

Trees, Binary Trees, and Binary Search Trees (cont.)

- A new data structure, the **tree**,
  - two components, ***nodes*** and ***arcs*** (or ***edges***)
  - the ***root*** at the top, and "grow" down
  - the ***leaves*** of the tree (also called ***terminal nodes***)
    - at the bottom of the tree



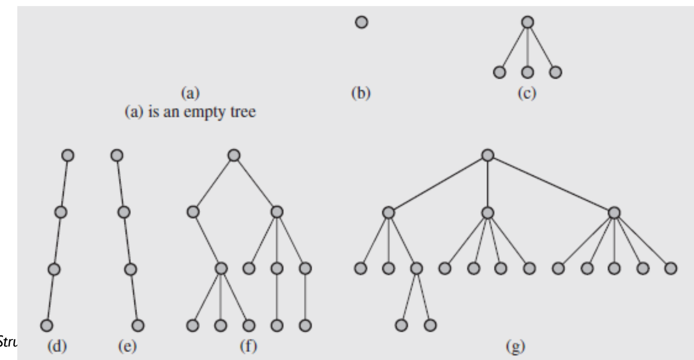
Trees, Binary Trees, and Binary Search Trees (cont.)

- Trees can be defined recursively,
  1. A tree with no nodes or edges (an empty structure) is an **empty tree**
  2. If we have a set $t_1 \cdots t_k$ of disjoint trees, the structure whose root has as its children the roots of $t_1 \cdots t_k$ is also a tree
  3. Only structures generated by rules 1 and 2 are trees

- Every node in the tree must be accessible
  - from the root through a **unique sequence** of edges,
  - a ***path***
- The number of edges in the path
  - path's ***length***
- The length of the path to that node, **plus 1**
  - a node's ***level***

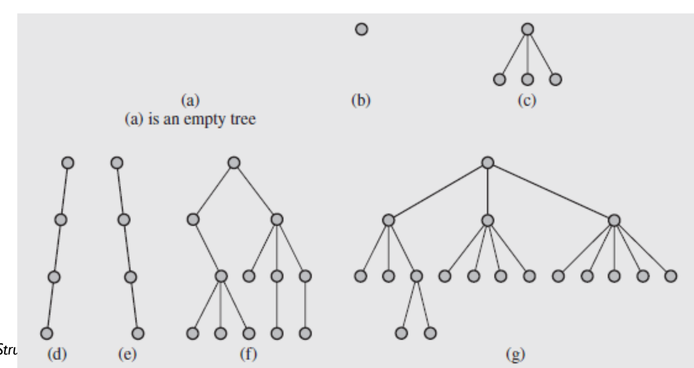
Trees, Binary Trees, and Binary Search Trees (cont.)

- The maximum level of a node in a tree: the tree's ***height***
- An empty tree: **height 0**
- A tree of **height 1**: a single node, which is both the tree's **root** and **leaf**
- The level of a node: must be between 1 and the tree's height



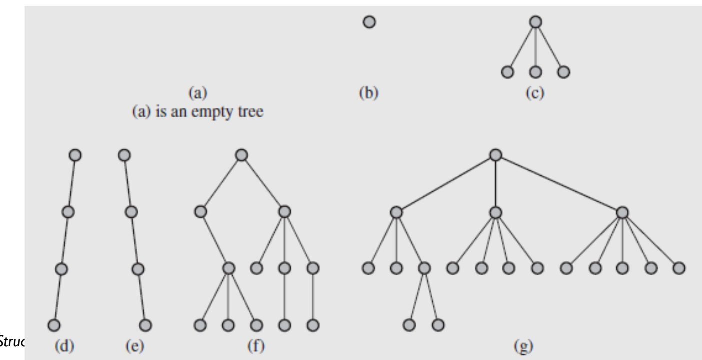
Trees, Binary Trees, and Binary Search Trees (cont.)

- The number of children of a given node?
  - can be **arbitrary**
- Using trees to improve the process of **searching** for elements??



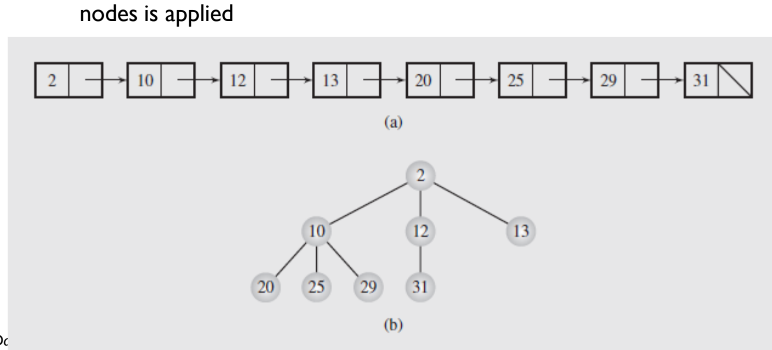
Trees, Binary Trees, and Binary Search Trees (cont.)

- In order to find a particular element in a **list** of ***n*** elements,
  - examine all nodes, if the list is **ordered**
- If the elements of a list are stored in a **tree**??
  - the number of elements that must be looked at can be reduced



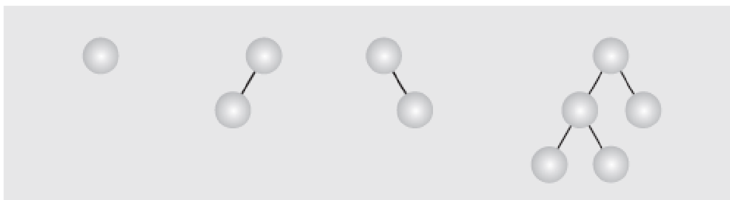
Trees, Binary Trees, and Binary Search Trees (cont.)

- Linked list,
  - no consideration of searching incorporated into design
- Tree,
  - considerable savings in searching if a **consistent ordering** to the nodes is applied



Trees, Binary Trees, and Binary Search Trees (cont.)

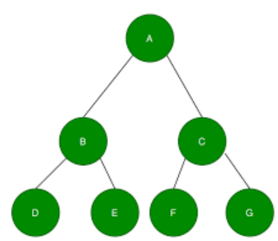
- A binary tree is a tree
  - each node has only **two children**, the ***left child*** and the ***right child***
  - these children can be **empty**

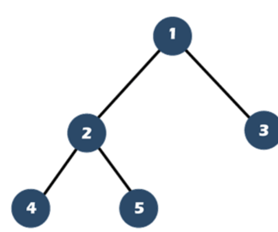


  - check the number of leaves
  - useful in **assessing efficiency** of algorithms

Trees, Binary Trees, and Binary Search Trees (cont.)

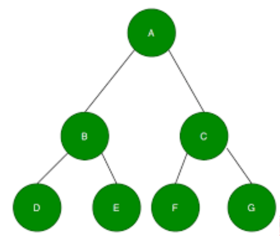
- The **level** of a node,
  - **the number of edges** between it and the root, **plus 1**
  - e.g., the root: level 1, its children: level 2, etc.
- ***Complete Binary Tree:*** if each node at any given level (except the last) had two children,
  - 2^0 nodes at level 1, 2^1 nodes at level 2, etc.
  - in general, **2^i nodes at level $i + 1$**
    - all **nonterminal nodes** have both children
    - all **leaves** are on the same level
- **Decision Tree**: a binary tree, all nodes have **either zero or two** nonempty children
  - the number of leaves (m) = number of nonterminal nodes (k) + 1



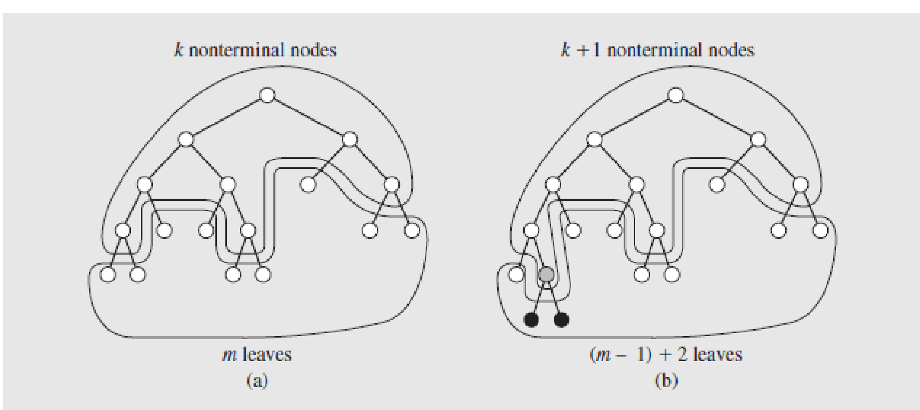


Trees, Binary Trees, and Binary Search Trees (cont.)

- For any given tree for which the condition holds,
  - **attaching two leaves to an existing leaf** will make it nonterminal
  - decreases the leaf nodes by 1 but increase the number of nonterminals by 1
- The two new leaves increase the number of leaves by 2
  - the relation becomes **$(m - 1) + 2$** = **$(k + 1)$** + 1
  - → $m = k + 1$
- An i + 1 level **complete decision tree** has,
  - 2^i leaves and $2^i - 1$ nonterminal nodes
  - totaling $2^{i+1} - 1$ nodes

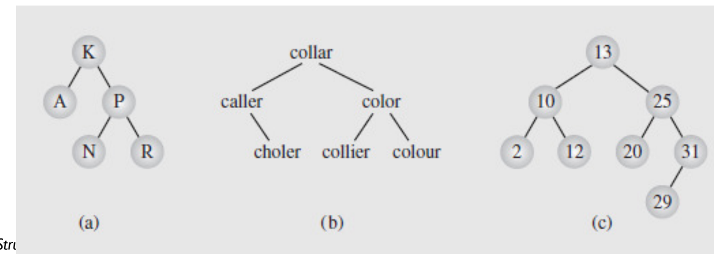


Trees, Binary Trees, and Binary Search Trees (cont.)

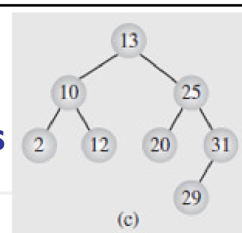


Trees, Binary Trees, and Binary Search Trees (cont.)

- In a ***binary search tree*** (or ***ordered binary tree***),
  - values stored in the **left subtree** of a given node are less than the value stored in that node
  - values stored in the **right subtree** of a given node are greater than the value stored in that node
  - the values stored are considered **unique**;
  - attempts to store **duplicate values** can be treated as an **error**



Implementing Binary Trees



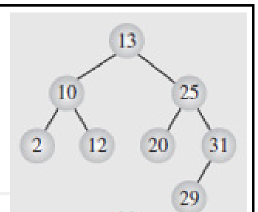
- Use arrays or linked structures to implement binary trees
- If using an array,
  - an information field
  - two "**pointer**" fields containing the indexes of the array locations of the **left** and **right** children
  - -1, an empty child
- The root of the tree
  - always in the first cell of the array

Index	Info	Left	Right
0	13	4	2
1	31	6	–1
2	25	7	1
3	12	–1	–1
4	10	5	3
5	2	–1	–1
6	29	–1	–1
7	20	–1	–1

Implementing Binary Trees (cont.)

- Drawbacks, **binary tree arrays**
  - need to keep track of the locations of each node,
  - have to be located sequentially
  - **deletion operation**??
    - requiring tag to mark empty cells,
    - moving elements around, or
    - requiring updating values
- Use a **linked implementation**
  - an information data member
  - **two pointer** data members

Searching a Binary Search Tree



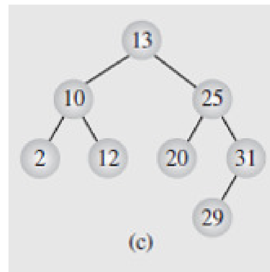
(c)

- Locating a specific value in a binary tree:
  - compare the value to the target value; if match, the search is done
  - If the target is **smaller**, branch to the **left subtree**
  - If the target is **larger**, branch to the **right subtree**
  - If at any point we cannot proceed further,
    - search has failed and the target isn't in the tree

```
template<class T>
T* BST<T>::search(BSTNode<T>* p, const T& el) const {
    while (p != 0)
        if (el == p->el)
             return &p->el;
        else if (el < p->el)
             p = p->left;
        else p = p->right;
    return 0;
}
```

Searching a Binary Search Tree (cont.)

- Find the value 31??
  - only three comparisons
- Finding (or not finding) the values 26 – 30
  - the maximum of four comparisons;
- Allowing **duplicates** requires additional searches:
  - If there is a duplicate,
    - either locate the first occurrence and ignore the others, or
  - locate each duplicate,
    - search until no path contains another instance of the value
- This search will always terminate at a **leaf node**



Searching a Binary Search Tree (cont.)

- The number of comparisons performed during the search
  - determine the **complexity** of the search
  - depend on the number of nodes encountered on the path from the root to the target node
- The complexity??
  - the length of the path plus 1
  - influenced by the shape of the tree and location of the target
- Searching in a binary tree
  - quite efficient, even if it isn't balanced