

Sorting (cont.)

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Lecture 18

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Adapted partially from Data Structures and Algorithms in C++, Adam Drozdek, 4th Edition, Cengage Learning; and Algorithms and Data Structures, Douglas Wilhelm Harder, Mmath

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1

Efficient Sorting Algorithms: Shell Sort

- In insertion sort,
 - work well when the input element is “almost sorted”
 - inefficient to move the elements
- Shell sort
 - improve over insertion sort
 - number of data movements

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2



Efficient Sorting Algorithms: Shell Sort (cont.)

- The pseudo code of shell sort:

```
divide data into h subarrays;
for i = 1 to h
    sort subarray datai;
sort array data;
```
- if h is **too small**
 - the subarrays could be too large and the resulting sort would be inefficient
- if h is **too big**,
 - too many subarrays
- use several different subarrays,
 - apply the same process separately to each subarray

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3



Efficient Sorting Algorithms: Shell Sort (cont.)

- The pseudo code of shell sort (cont.):

```
determine numbers  $h_t \dots h_1$  of ways of dividing array
data into subarrays;
for (h= $h_t$ ;  $t > 1$ ;  $t--$ ,  $h=h_t$ )
    divide data into h subarrays;
for i = 1 to h
    sort subarray datai;
sort array data;
```
- called,
 - diminishing increment sort, shell sort, or shell's method

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4

Efficient Sorting Algorithms: Shell Sort (cont.)

- Perform the shell sort,
 - arrange the elements in the form of a table
 - sort the columns
 - repeat with smaller number of long columns

63, 19, 7, 90, 81, 36, 54, 45, 72, 27, 22, 9, 41, 59, 33

Result:

63	19	7	90	81	36	54	45	63	19	7	9	41	36	33	45
72	27	22	9	41	59	33		72	27	22	90	81	59	54	

63, 19, 7, 9, 41, 36, 33, 45, 72, 27, 22, 90, 81, 59, 54

Result:

63	19	7	9	41				22	19	7	9	27			
36	33	45	72	27				36	33	45	59	41			
22	90	81	59	54				63	90	81	72	54			

CS2413: Data 22, 19, 7, 9, 27, 36, 33, 45, 59, 41, 63, 90, 81, 72, 54



5

Efficient Sorting Algorithms: Shell Sort (cont.)

- Perform the shell sort, (cont.)
 - arrange the elements in the form of a table
 - sort the columns
 - repeat with smaller number of long columns

22, 19, 7, 9, 27, 36, 33, 45, 59, 41, 63, 90, 81, 72, 54

Result:

22	19	7						9	19	7					
9	27	36						22	27	36					
33	45	59						33	45	54					
41	63	90						41	63	59					
81	72	54						81	72	90					

9, 19, 7, 22, 27, 36, 33, 45, 54, 41, 63, 59, 81, 72, 90

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6

Efficient Sorting Algorithms: Shell Sort (cont.)

- Perform the shell sort, (cont.)
 - arrange the elements in the form of a table
 - sort the columns
 - repeat with smaller number of long columns

	Result:
9	7
19	9
7	19
22	22
27	27
36	33
33	36
45	41
54	45
41	54
63	59
59	63
81	72
72	81
90	90

7, 9, 19, 22, 27, 33, 36, 41, 45, 54, 59, 63, 72, 81, 90

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7

Efficient Sorting Algorithms: Heap Sort

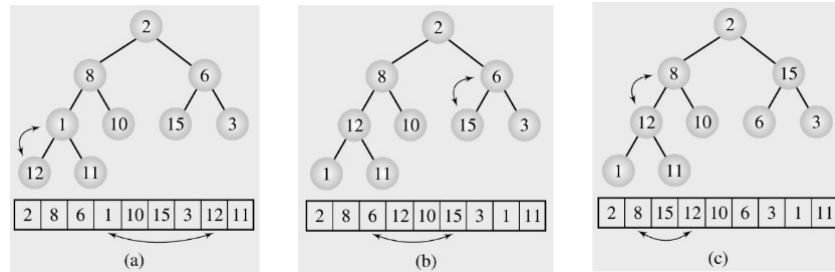
- Motivation
 - selection sort, fairly inefficient ($O(n^2)$),
 - relatively few moves of the data
 - recall, selection sort finds the smallest element in the list and places it first, then the next smallest, etc.
 - if the comparison portion of the sort can be improved?
 - performance can likewise improve
- Two phrases:
 - for **ascending order**,
 - place the **largest element last** in the array, then put the next largest in front of that, etc.
 - 1st: build a heap out of the data
 - 2nd: remove the largest item from the heap and insert that item into the sorted array

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8

Efficient Sorting Algorithms: Heap Sort (cont.)



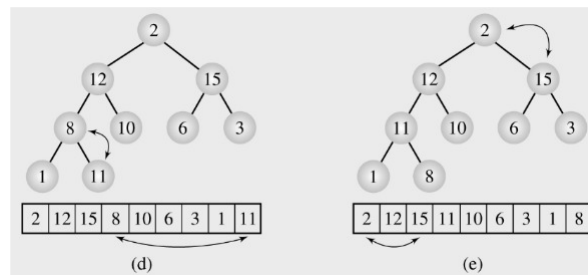
swap

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9

Efficient Sorting Algorithms: Heap Sort (cont.)

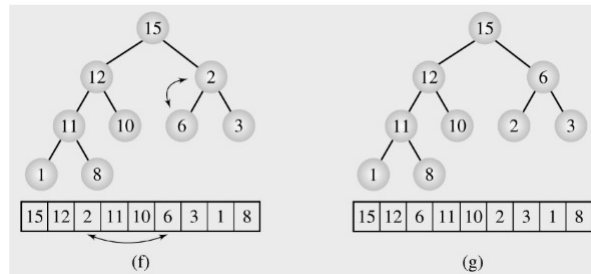


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10

Efficient Sorting Algorithms: Heap Sort (cont.)

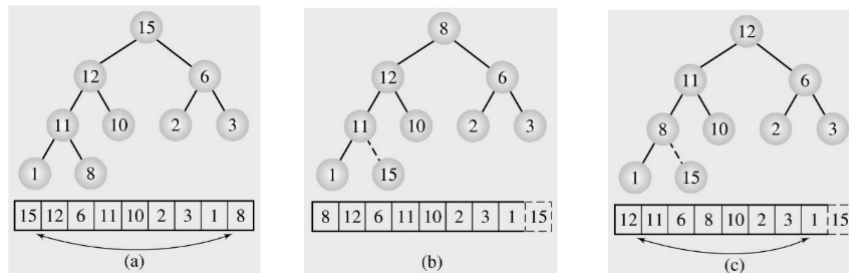


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11

Efficient Sorting Algorithms: Heap Sort (cont.)

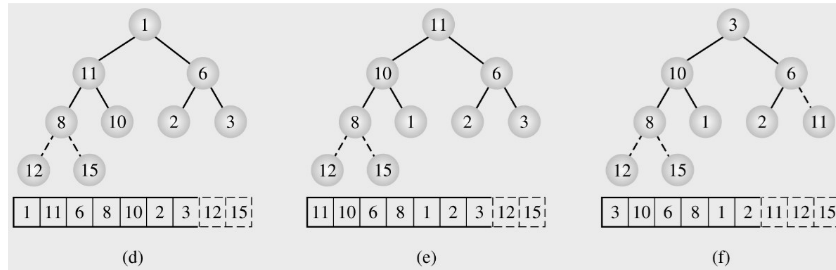


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12

Efficient Sorting Algorithms: Heap Sort (cont.)

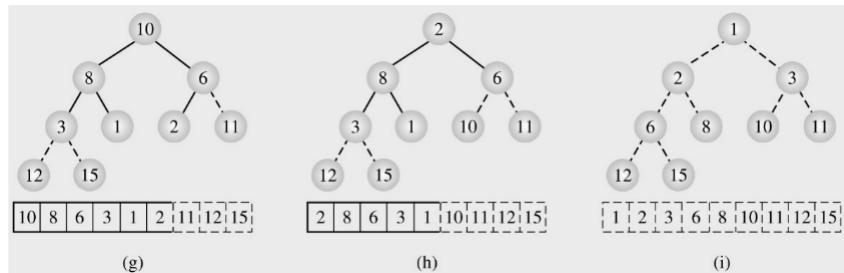


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13

Efficient Sorting Algorithms: Heap Sort (cont.)



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14



Efficient Sorting Algorithms: Heap Sort (cont.)

```
template<class T>
void heapsort(T data[], int n) {
    for (int i = n/2 - 1; i >= 0; --i) // create a heap;
        moveDown (data,i,n-1);
    for (int i = n-1; i >= 1; --i) {
        swap(data[0],data[i]); // move the largest item to data[i];
        moveDown(data,0,i-1); // restore the heap property;
    }
}
```

- In the 1st step, create the heap, $O(n)$
- In the 2nd step,
 - exchange $n - 1$ times the root with the element in position i
 - restore the heap $n - 1$ times
 - in worst case, cause `moveDown()` to iterate $\lg i$ times to bring the root down to the level of the leaves, $\sum_{i=1}^{n-1} (\lg i)$, $O(n \log n)$

CS2413: ■ In total, $O(n) + O(n \log n) + O(n - 1) = O(n \log n)$



15



Efficient Sorting Algorithms: Quick Sort

- Motivation
 - shell sort
 - divide the original array into subarrays, sort those
 - divide the partially sorted array into new subarrays to be sorted
 - until the entire array is in order
 - divide-and-conquer

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16

Efficient Sorting Algorithms: Quick Sort (cont.)

```
quicksort(array[])
  if length(array) > 1
    choose bound; // partition array into subarray1 and subarray2
    while there are elements left in array
      include element either in subarray1 = {el: el ≤ bound}
      or in subarray2 = {el: el ≥ bound};
    quicksort(subarray1);
    quicksort(subarray2);
```

- Two operations for partitioning the array
 - choose a bound (or **pivot**)
 - move the elements to the proper subarrays
- Choosing the bound is non-trivial
 - the goal is to have the two subarrays to be nearly equal in length

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17

Efficient Sorting Algorithms: Quick Sort (cont.)

- e.g., sort the elements using quick sort algorithm

27	10	36	18	25	45
----	----	----	----	----	----

We choose the first element as the pivot.
Set $loc = 0$, $left = 0$, and $right = 5$.

27	10	36	18	25	45
loc					right
left					

Scan from right to left. Since $a[loc] < a[right]$, decrease the value of right.

27	10	36	18	25	45
loc					right
left					

Start scanning from left to right. Since $a[loc] > a[left]$, increment the value of left.

25	10	36	18	27	45
					right
				loc	

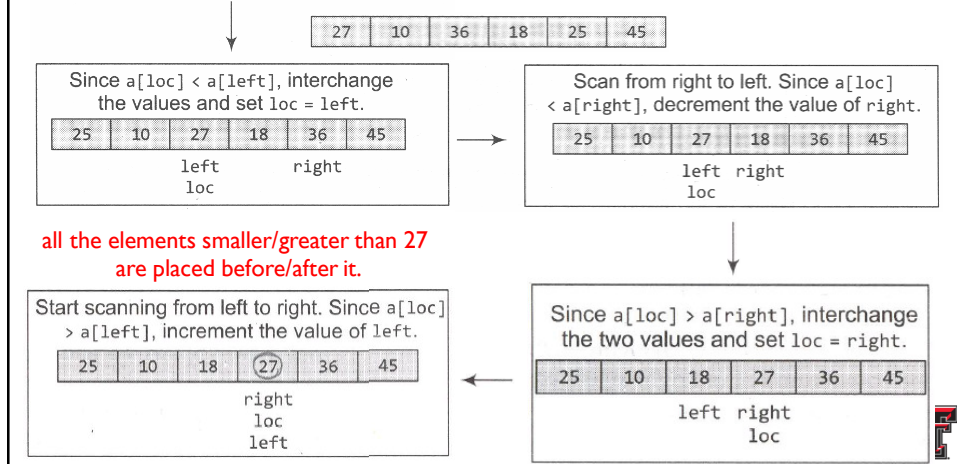
Since $a[loc] > a[right]$, interchange the two values and set $loc = right$.

25	10	36	18	27	45
					right
				loc	

18

Efficient Sorting Algorithms: Quick Sort (cont.)

- e.g., sort the elements using quick sort algorithm



19

Efficient Sorting Algorithms: Quick Sort (cont.)

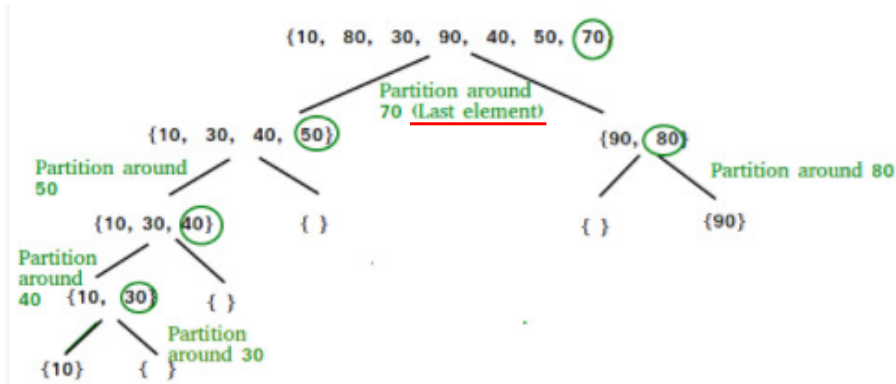
- Choosing the pivot element
 - e.g., randomly choose an element as pivot, pick first element as pivot, pick last element as pivot, or pick median as pivot
 - $O(n \log n)$
 - depending on the element chosen as the pivot
- Faster than
 - insertion, selection, and bubble sorts
- Complex and massively recursive...



20

Efficient Sorting Algorithms: Quick Sort (cont.)

- e.g., pick the last element as a pivot



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21

Efficient Sorting Algorithms: Merge Sort

- Motivation
 - quick sort, difficulty of the partitioning process
 - numerous techniques of choosing a bound (or pivot)
 - no assurance that any approach will result in arrays that are equal in size
- Three major operations:
 - **divide**
 - partition the n -element array to be sorted into two sub-array of $n/2$ elements
 - **conquer**
 - sort the two sub-arrays recursively
 - **combine**
 - merge the two sorted sub-arrays of size $n/2$ to produce the sorted array of n elements

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22

Efficient Sorting Algorithms: Merge Sort (cont.)

```
mergesort(data[])
  if data have at least two elements
    mergesort(left half of data);
    mergesort(right half of data);
    merge(both halves into a sorted list);
```

- Key operation
 - Merge the sorted halves of the array into a single array
 - these halves must be sorted, which occurs by merging the sorted halves of these halves

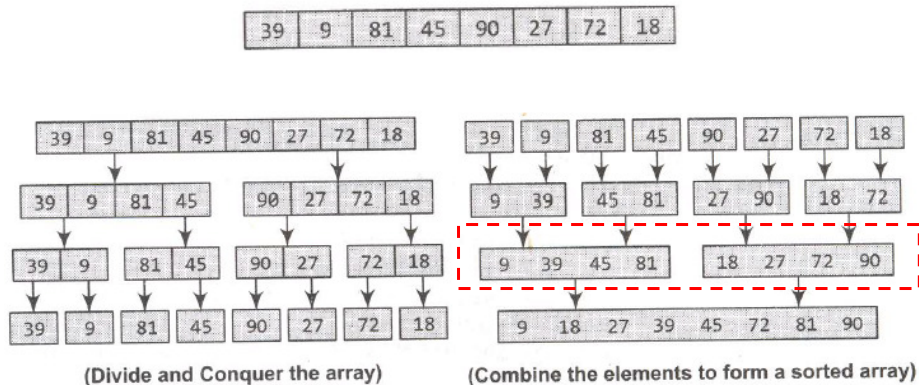
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23

Efficient Sorting Algorithms: Merge Sort (cont.)

- e.g., sort the array using merge sort



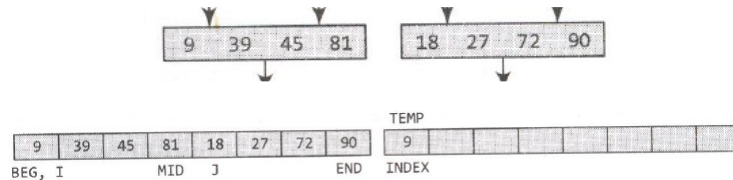
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24

Efficient Sorting Algorithms: Merge Sort (cont.)

- e.g., sort the array using merge sort (cont.)



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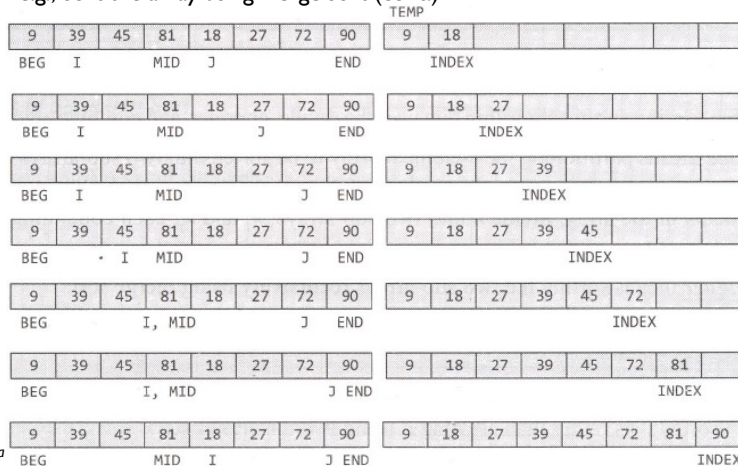


25

Efficient Sorting Algorithms: Merge Sort (cont.)

when $I > MID$ then copy
the remaining elements of
the right sub-array in TEMP

- e.g., sort the array using merge sort (cont.)



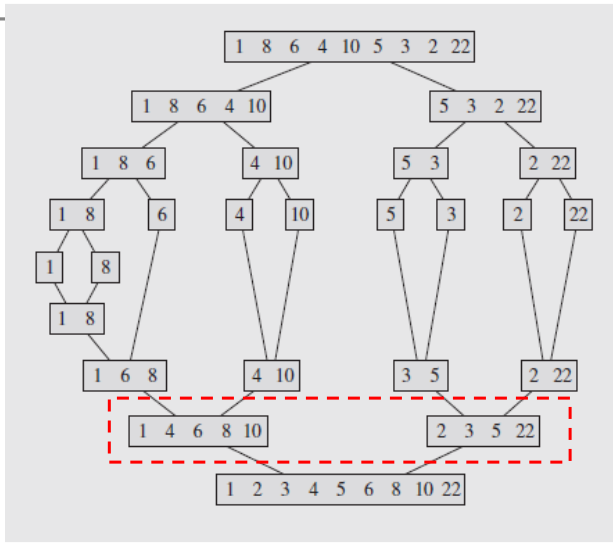
CS2413: Data



26

Efficient Sorting Algorithms: Merge Sort (cont.)

- e.g., the array [1 8 6 4 10 5 3 2 22] sorted by merge sort
- Drawback of merge sort?
 - additional storage for merging array



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27

Efficient Sorting Algorithms: Radix (or Bucket) Sort

- A **non-comparative** sorting algorithm
 - sort data with integer keys by grouping keys by the individual digits which share the same significant **position** and **value**
 - e.g., a list of sorted names
 - 26 radix (or 26 buckets) – 26 alphabet letters
- Frequently used in everyday applications
 - sort library cards by author
 - create a separate pile for each card based on the **first letter** of the name
 - each pile is then further sorted into smaller piles based on the **second letter** of the name

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28

Efficient Sorting Algorithms: Radix (or Bucket) Sort

```
radixsort()
  for d = 1 to the position of the leftmost digit of longest
    number distribute all numbers among piles 0 through 9
    according to the dth digit;
    put all integers on one list;
```

- e.g., sort the list [23 123 234 567 3]
 - [123 23 234 3 567] →
 - [003 023 123 234 567]

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29

Efficient Sorting Algorithms: Radix (or Bucket) Sort (cont.)

- Sort the numbers using radix sort

345, 654, 924, 123, 567, 472, 555, 808, 911

Number	0	1	2	3	4	5	6	7	8	9
345						345				
654					654					
924					924					
123				123						
567								567		
472			472							
555						555				
808									808	
911		911								

sort based on the right-most digit

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30

Efficient Sorting Algorithms: Radix (or Bucket) Sort (cont.)

- Sort the numbers using radix sort (cont.)

345, 654, 924, 123, 567, 472, 555, 808, 911

Number	0	1	2	3	4	5	6	7	8	9
911		911								
472								472		
123			123							
654						654				
924			924							
345				345						
555						555				
567							567			
808	808									

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sort based on the second right-most digit



31

Efficient Sorting Algorithms: Radix (or Bucket) Sort (cont.)

- Sort the numbers using radix sort (cont.)

345, 654, 924, 123, 567, 472, 555, 808, 911

Number	0	1	2	3	4	5	6	7	8	9
808									808	
911										911
123		123								924
924										
345				345						
654							654			
555						555				
567						567				
472					472					


sort based on the third right-most digit

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123, 345, 472, 555, 567, 654, 808, 911, 924.



32



Efficient Sorting Algorithms: Radix (or Bucket) Sort (cont.)

- Another example,


data = [10 1234 9 7234 67 9181 733 197 7 3]

				3	7234			7		
	10	9181		733	1234			197		
piles:	0	1	2	3	4	5	6	7	8	9


data = [10 9181 733 3 1234 7234 67 197 7 9]

				7234						
	9			1234						
	7			733			67		9181	197
piles:	0	1	2	3	4	5	6	7	8	9

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33



Efficient Sorting Algorithms: Radix (or Bucket) Sort (cont.)

data = [3 7 9 10 733 1234 7234 67 9181 197]


	67									
	10									
	9									
	7	197	7234				773			
	3	9181	1234							
piles:	0	1	2	3	4	5	6	7	8	9

data = [3 7 9 10 67 9181 197 1234 7234 733]

	733									
	197									
	67									
	10									
	9									
	7									
	3	1234					7234		9181	
piles:	0	1	2	3	4	5	6	7	8	9

data = [3 7 9 10 67 197 733 1234 7234 9181]

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34