Binary Subtractors

Lecture Overview

- Binary Subtraction
- Sign/Magnitude Binary Numbers
- 2’s Complement Binary Numbers
- Designing a Binary Subtractor
- Implementation in Verilog HDL

Binary Subtraction

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Binary Subtraction

Let’s start by looking at by-hand subtraction.

$$\begin{array}{r} 147 \\ -\,63 \\ \hline \end{array}$$

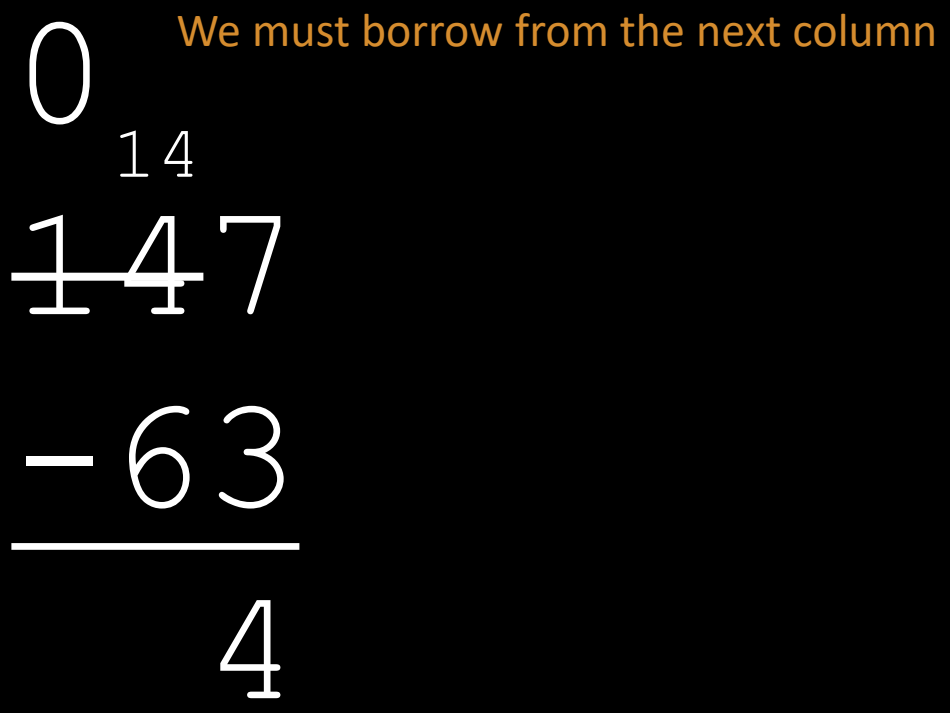
Binary Subtraction

Let’s start by looking at by-hand subtraction.

$$\begin{array}{r} 147 \\ -\,63 \\ \hline 4 \end{array}$$

Binary Subtraction

Let’s start by looking at by-hand subtraction.



Binary Subtraction

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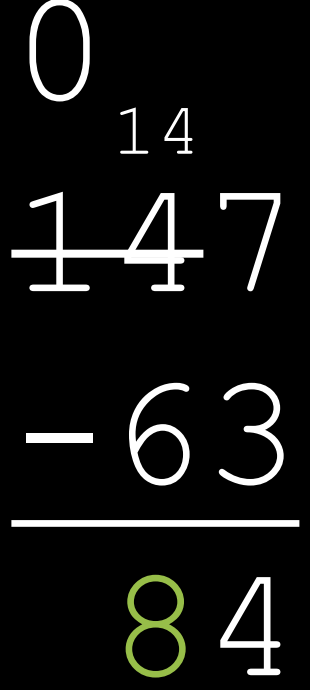

Binary Subtraction

Let’s start by looking at by-hand subtraction.

$$\begin{array}{r} 1010 \\ -0110 \\ \hline \end{array}$$

Binary Subtraction

Let’s start by looking at by-hand subtraction.

$$\begin{array}{r} 1010 \\ -0110 \\ \hline 0 \end{array}$$

Binary Subtraction

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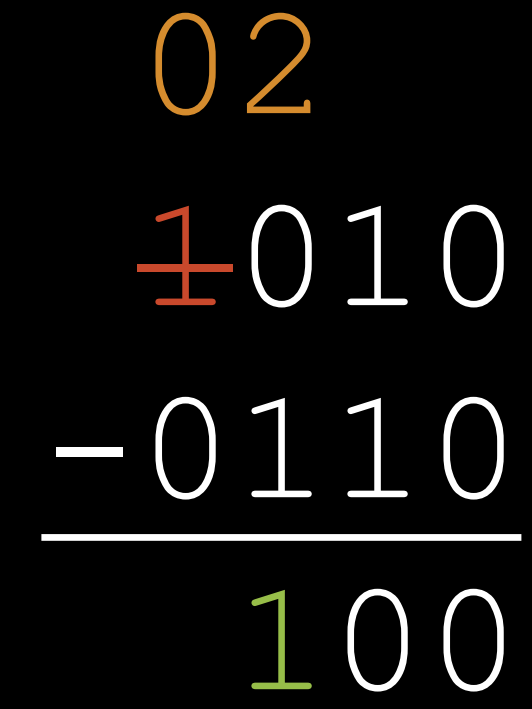
Binary Subtraction

Let’s start by looking at by-hand subtraction.



Binary Subtraction

Let’s start by looking at by-hand subtraction.

$$
\begin{array}{r}
 {}^{0}2 \\
 -1010 \\
\hline
 0100
\end{array}
$$

Binary Subtraction

Let’s start by looking at by-hand subtraction.

```
  1010        ??
- 0110      - ??
------     ------
  0100        ??
```

Binary Subtraction

Let’s start by looking at by-hand subtraction.

```
  1010        10
 -0110       -??
------      ----
  0100        ??
```

Binary Subtraction

Let’s start by looking at by-hand subtraction.

```
  1010        10
- 0110      - 06
------      ----
  0100        ??
```

Binary Subtraction

Let’s start by looking at by-hand subtraction.

```
  1010        10
- 0110      - 06
------     ----
  0100        04
```

What questions do you have?

Exam Problem | Binary Subtraction

What is the result of the given binary subtraction operation?

A. 10110

B. 10010

C. 10011

D. 10110

$$\begin{array}{r} 11101 \\ -1011 \\ \hline \end{array}$$

Exam Problem | Binary Subtraction

What is the result of the given binary subtraction operation?

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B. 10010

~~C. 10011~~

~~D. 10110~~

```
 11101
-1011
------
```

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~~D. 10110~~

```
 11101
-1011
------
     0
```

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~~A. 10110~~

B. 10010

~~C. 10011~~

~~D. 10110~~

```
   02
 11101
-1011
------
    10
```

Exam Problem | Binary Subtraction

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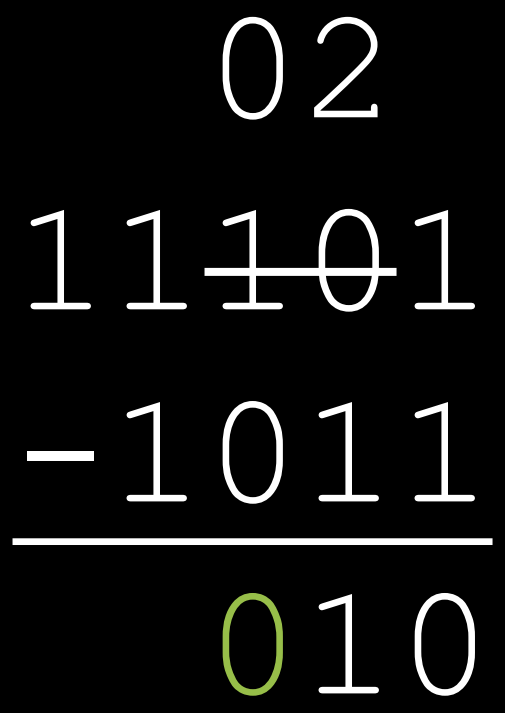
~~D. 10110~~



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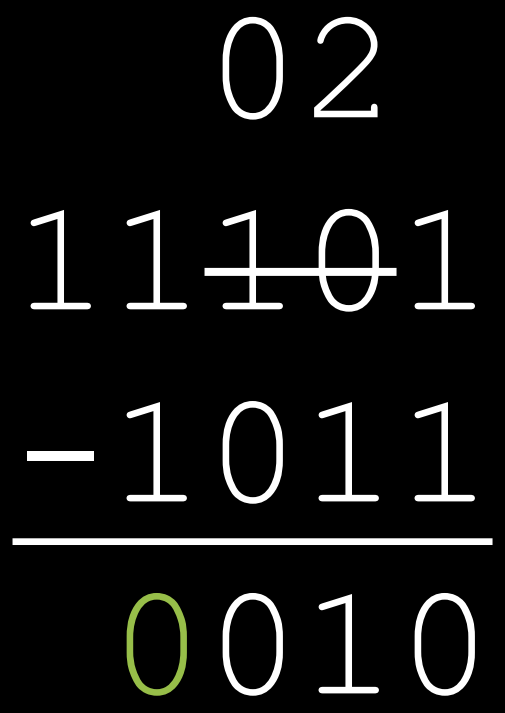

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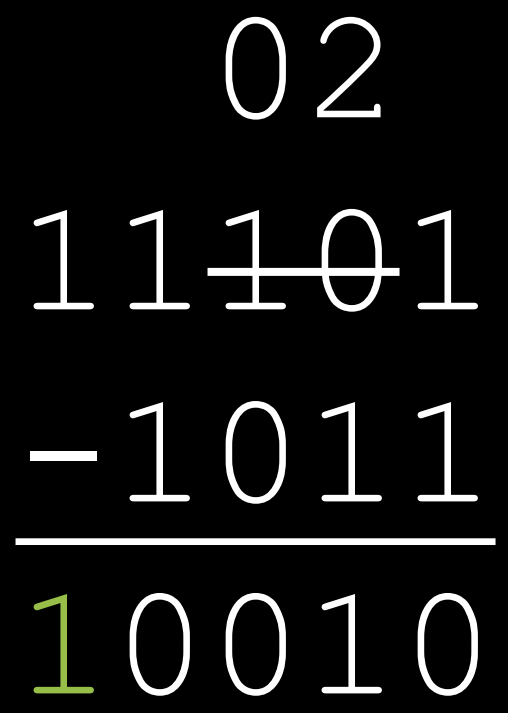

Binary Subtraction

We could design a logic circuit to do this, but there is an easier way.

Instead of performing $A - B$, lets use $A + (-B)$.

But first, we need to represent negative numbers in binary.

Sign/Magnitude Binary Numbers

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Sign/Magnitude Binary Numbers

This is the simplest way to represent negative numbers.

If the sign is **0**, the number is **POSITIVE**

If the sign is **1**, the number is **NEGATIVE**

Sign/Magnitude Binary Numbers

0000 = 0	1000 = -0 ???
0001 = 1	1001 = -1
0010 = 2	1010 = -2
0011 = 3	1011 = -3
0100 = 4	1100 = -4
0101 = 5	1101 = -5
0110 = 6	1110 = -6
0111 = 7	1111 = -7

The **RANGE** of Binary Numbers

Notice that the largest value of our 4-bit binary numbers is reduced when using a signed bit.

```
Unsigned 0000 -> 1111: 0  to 15
```

$0\ to\ 2^n - 1$

```
Signed   0000 -> 1111: -7 to +7
```

$-2^{n-1} + 1\ to\ 2^{n-1} - 1$

What questions do you have?

Exam Problem | Sign/Magnitude Binary

What is the decimal number which is given in sign/magnitude binary?

A. +18
B. -18
C. +22
D. -22

1 1 0 1 1 0

Exam Problem | Sign/Magnitude Binary

What is the decimal number which is given in sign/magnitude binary?

~~A. +18~~

~~B. -18~~

~~C. +22~~

D. -22

110110

Exam Problem | Sign/Magnitude Binary

What is the decimal number which is given in sign/magnitude binary?

~~A. +18~~

~~B. -18~~

~~C. +22~~

D. -22

1 1 0 1 1 0

The sign is 1(negative) The magnitude is 22

Exam Problem | Range of Binary Numbers

What is the range of a 6-bit signed binary number?

$A.\ 0\ to\ 63$

$B.\ -63\ to\ 63$

$C.\ 0\ to\ 31$

$D.\ -31\ to\ 31$

Exam Problem | Range of Binary Numbers

What is the range of a 6-bit signed binary number?

~~*A.* $0\ to\ 63$~~

~~*B.* $-63\ to\ 63$~~

~~*C.* $0\ to\ 31$~~

D. $-31\ to\ 31$

$$N = 6$$

$$-2^{N-1} + 1\ to\ 2^{N-1} - 1$$

$$-2^{5} + 1\ to\ 2^{5} - 1$$

$$-31\ to\ 31$$

2’s Complement Binary Numbers

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2’s Complement Binary Numbers

Sign/magnitude binary numbers are very easy to read, but difficult to work with.

2’s Complement binary offers a much more workable system for representing negative numbers.

Sign/Magnitude	**2’s Complement**
-3 = 1011	-3 = 1101
-4 = 1100	-4 = 1100
-6 = 1110	-6 = 1010
-7 = 1111	-7 = 1001

2’s Complement Binary Numbers

2’s complement conversion involves 2 steps.

1. Invert all the bits
2. Add 1 to the binary number

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+6 to −6

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+6 to −6

0110

2’s Complement Binary Numbers

2’s complement conversion involves 2 steps.

1. Invert all the bits
2. Add 1 to the binary number

+6 to −6

0110

1001

2’s Complement Binary Numbers

2’s complement conversion involves 2 steps.

1. Invert all the bits
2. Add 1 to the binary number

+6 to −6

```
  0110
  1001
+    1
------
```

2’s Complement Binary Numbers

2’s complement conversion involves 2 steps.

1. Invert all the bits
2. Add 1 to the binary number

+6 to −6

```
  0110
  1001
+    1
------
  1010
```

2’s Complement Binary Numbers

2’s complement conversion works in the other direction.

```
-6 to +6
```

2’s Complement Binary Numbers

2’s complement conversion works in the other direction.

−6 to +6

1010

2’s Complement Binary Numbers

2’s complement conversion works in the other direction.

−6 to +6

1010

0101

2’s Complement Binary Numbers

2’s complement conversion works in the other direction.

−6 to +6

```
 1010
 0101
+   1
------
```

2’s Complement Binary Numbers

2’s complement conversion works in the other direction.

−6 to +6

```
  1010
  0101
+    1
------
  0110
```

What questions do you have?

Exam Problem | 2’s Complement Conversion

What is the 2’s complement representation of the number -14?

A. 00010

B. 01101

C. 10010

D. 11101

Exam Problem | 2’s Complement Conversion

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14 = 1110

Exam Problem | 2's Complement Conversion

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14 = 01110

Exam Problem | 2's Complement Conversion

What is the 2's complement representation of the number -14?

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C. 10010

~~D. 11101~~

```
14 = 1110

14 = 01110

     10001
```

Exam Problem | 2's Complement Conversion

What is the 2's complement representation of the number -14?

~~A. 00010~~

~~B. 01101~~

C. 10010

~~D. 11101~~

```
14 = 1110

14 = 01110

     10001
   +     1
   ------
```

Exam Problem | 2's Complement Conversion

What is the 2's complement representation of the number -14?

~~A. 00010~~

~~B. 01101~~

C. 10010

~~D. 11101~~

```
14 = 1110

14 = 01110

     10001
   +     1
   -------
     10010
```

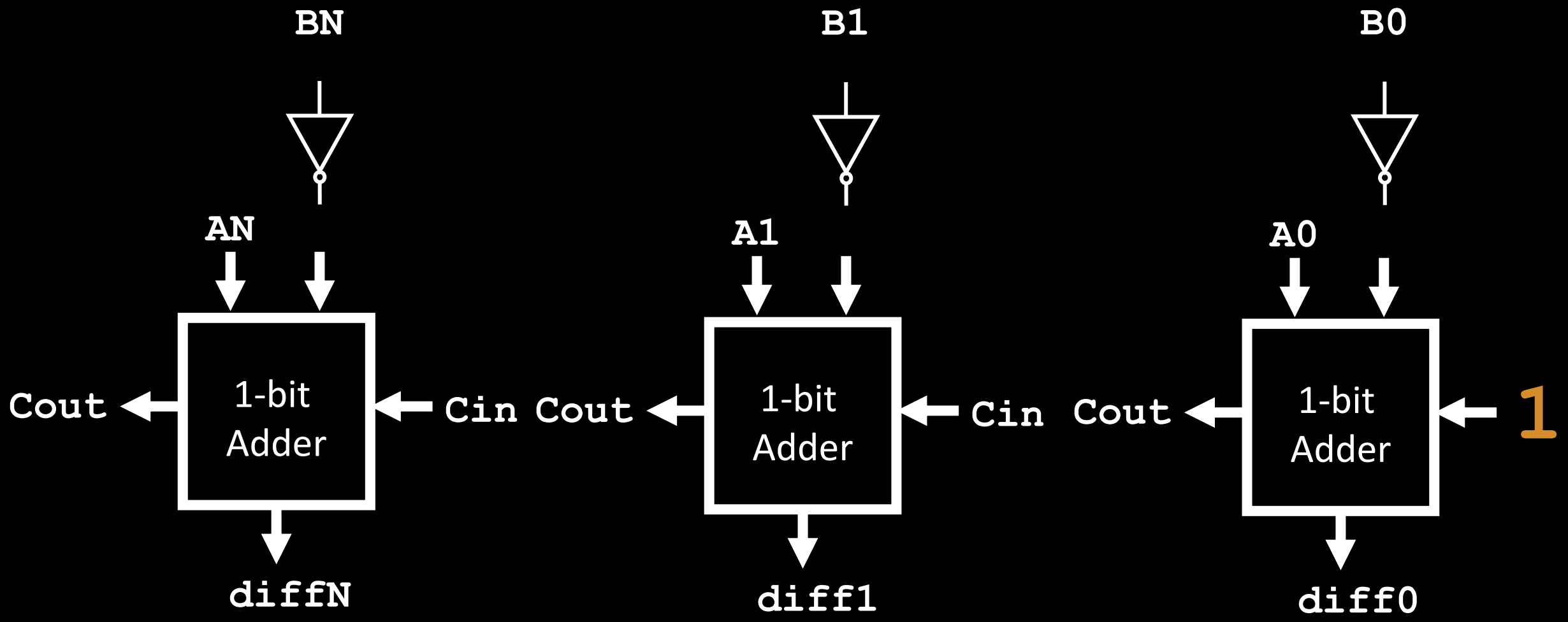
Designing a Binary Subtractor

Designing a Binary Subtractor

Instead of designing the logic circuit from scratch, we'll adapt the Ripple-Carry Adder

$$A - B = A + (-B)$$

Perform a 2's complement conversion on B to get –B and add it to A.


BN
B1
B0
AN
A1
A0
Cout
1-bit Adder
Cin Cout
1-bit Adder
Cin Cout
1-bit Adder
1
diffN
diff1
diff0

What questions do you have?

Implementation in Verilog HDL

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Implementation in Verilog HDL

We only need to make slight modifications to our adder code.

Implementation in Verilog HDL

```
 8  `include "adder.v"
 9
10  module subtractor(A, B, Diff);
11      // ...
12  endmodule
13
```

Implementation in Verilog HDL

```
12    // DATA INPUTS
13    input [7:0] A, B;
14
15    // DATA OUTPUTS
16    output [7:0] Diff;
17
18    // Wires
19    wire [6:0] carry;
```

Implementation in Verilog HDL

```
21   // LOGIC CIRCUIT
22   adder U0(.A(A[0]), .B(~B[0]), .Cin(1'b1),      .Cout(carry[0]), .Sum(Diff[0]));
23   adder U1(.A(A[1]), .B(~B[1]), .Cin(carry[0]), .Cout(carry[1]), .Sum(Diff[1]));
24   adder U2(.A(A[2]), .B(~B[2]), .Cin(carry[1]), .Cout(carry[2]), .Sum(Diff[2]));
25   adder U3(.A(A[3]), .B(~B[3]), .Cin(carry[2]), .Cout(carry[3]), .Sum(Diff[3]));
26   adder U4(.A(A[4]), .B(~B[4]), .Cin(carry[3]), .Cout(carry[4]), .Sum(Diff[4]));
27   adder U5(.A(A[5]), .B(~B[5]), .Cin(carry[4]), .Cout(carry[5]), .Sum(Diff[5]));
28   adder U6(.A(A[6]), .B(~B[6]), .Cin(carry[5]), .Cout(carry[6]), .Sum(Diff[6]));
29   adder U7(.A(A[7]), .B(~B[7]), .Cin(carry[6]), .Cout(       ), .Sum(Diff[7]));
```

What questions do you have?

Lecture Recap

- Binary Subtraction
- Sign/Magnitude Binary Numbers
- 2’s Complement Binary Numbers
- Designing a Binary Subtracter
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Binary Subtracters

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