Note:

The circuit simulation was made on IBM circuit simulator and it was run on 15q-ibmq_16_melbourne

The comparison of the results:

When doing 1 measurement only on the circuit, a state |01> or |10> will result in a probability of 100%, obviously. Both states will appear when we increase the number of measurements as observed in 10, 100 and 1000 measurements. In an ideal environment, increasing the number of measurements of the outcome makes the two available states likely probable to be measured with a probability 50% 50% each; like tossing a coin, you will get 50% 50% chances of heads and tails if you toss the coin very large number of times (using a non-biased system). The past argument was based on the previous knowledge that the states |01> and |10> have 50% 50% chance of appearing, but another argument could be made. Increasing the number of measurements will retrieve the probability of each of the states |01> and |10> that they were produced in, because it is like doing a measurement on 1000 identically prepared systems. But here we have quantum noise in probability due to the higher number of measurements of the outcomes which cause other states to appear. And the appearance of states |00>, |11> was the result of the average phase shift that happened to the superposed wave function of the qubits during the measurement process and during the preforming of the unitary transformation of the qubits.

Bonus question answer:

To produce a state of only |01>+|10> states, you just need to have a qubit in state |0>+|1> and the other qubit to be in state |1> just before using the CNOT gate.