

3b) To Find  $P(FH = \text{true})$

$$P(FH) = \sum_{FS, FM, NDG} P(FH, FS, FM, NDG) \quad (\text{Marginalization})$$

$$= \sum_{FS, FM, NDG} P(FH | FS, FM, NDG) P(FS, FM, NDG) \quad (1) \quad \text{chain rule}$$

on eqn (1)

$$P(FS, FM, NDG) = \sum_{NA} P(FS, FM, NDG, NA) \quad (\text{Marginalization})$$

$$= \sum_{NA} \underbrace{P(FS | FM, NDG, NA)}_{FS \text{ is independent of } FM, NDG, NA} P(FM, NDG, NA)$$

FS is independent of FM, NDG, NA

$$\begin{aligned} \text{So } P(FS, FM, NDG) &= \sum_{NA} P(FS) P(NDG | FM, NA) P(NA, FM) \\ &= P(FS) \sum_{NA} P(NDG | NA, FM) P(NA) P(FM) \quad (2) \end{aligned}$$

Substituting (2) in (1)

$$P(FH) = \sum_{FS, FM, NDG} P(FH | FS, FM, NDG) P(FS) \sum_{NA} P(NDG | NA, FM) P(NA) P(FM)$$

$$P(FH) = \sum_{FS, FM, NDG} \underbrace{P(FH | FS, FM, NDG)}_{f_1} \underbrace{P(FS)}_{f_2} \underbrace{P(FM)}_{f_3} \sum_{NA} \underbrace{P(NDG | NA, FM)}_{f_4}$$



3c) To Find  $P(FS = \text{true} | FM = \text{true}, FH = \text{true})$   
By Bayes Rule,

$$P(FS | FM, FH) = \frac{P(FH | FS, FM) P(FS, FM)}{P(FH, FM)} \quad (1)$$

On eqn (1),

$$P(FH | FS, FM) = \frac{P(FH, FS, FM)}{P(FS, FM)} \quad (2) \text{ Chain rule}$$

Substituting (2) in (1)

$$P(FS | FM, FH) = \frac{P(FH, FS, FM)}{P(FH, FM)} \quad (3)$$

Treating denominator in (3)

as normalized constant

$$P(FS | FM, FH) = \frac{P(FH, FS, FM)}{Z} \quad (4)$$

In (4), numerator becomes

$$P(FH, FS, FM) = \sum_{NDG, NA} P(FH, FS, FM, NDG, NA)$$

$$= \sum_{NDG, NA} P(FH | FS, FM, NDG, NA) P(FS, FM, NDG, NA) \quad (\text{Chain rule})$$

$$= \sum_{NDG, NA} P(FH | FS, FM, NDG) P(NDG | FS, FM, NA) P(FS, FM, NA)$$

$$= \sum_{NDG, NA} P(FH | FS, FM, NDG) P(NDG | FM, NA) P(FS) P(FM) P(NA)$$

$\downarrow$   
 $f_1$

$\downarrow$   
 $f_2$

$\downarrow$   
 $f_3$

$\downarrow$   
 $f_4$

$\downarrow$   
 $f_5$



3d) To Find  $P(FS=true | FB=true, FM=true, FH=true)$   
 Bayes Rule  
 So 
$$P(FS | FB, FM, FH) = \frac{P(FH | FS, FB, FM) P(FS, FB, FM)}{P(FH, FB, FM)} \quad (1)$$

In eqn (1)

$$P(FH | FS, FB, FM) = \frac{P(FH, FS, FB, FM)}{P(FS, FB, FM)} \quad (2) \text{ Chain Rule}$$

Substituting (2) in (1) 
$$P(FS | FB, FM, FH) = \frac{P(FH, FS, FB, FM)}{P(FH, FB, FM)} \quad (3)$$

In eqn (3), treating denominator as  $\alpha$ , normalization constant

$$P(FS | FB, FM, FH) = \frac{P(FH, FS, FB, FM)}{\alpha} \quad (4)$$

In (4), numerator becomes

$$P(FH, FS, FB, FM) = \sum_{NDG, NA} P(FH, FS, FB, FM, NDG, NA)$$

$$= \sum_{NDG, NA} P(FH | FS, FB, FM, NDG, NA) P(FS, FB, FM, NDG, NA)$$

$FH$  is independent of  $FB, NA$

So 
$$P(FH, FS, FB, FM) = \sum_{NDG, NA} P(FH | FS, FM, NDG) P(NDG | FS, FB, FM, NA) P(FS, FB, FM, NA)$$

$$= \sum_{NDG, NA} P(FH | FS, FM, NDG) P(NDG | NA, FM) P(FB | FS, FM, NA) P(FS, FM, NA)$$

$$= \sum_{NDG, NA} P(FH | FS, FM, NDG) P(NDG | NA, FM) P(FB | FS) P(FS) P(FM) P(NA)$$

$\downarrow$   
 $f_1$

$\downarrow$   
 $f_2$

$\downarrow$   
 $f_3$

$\downarrow$   
 $f_4$

$\downarrow$   
 $f_5$

$\downarrow$   
 $f_6$



3e) To Find  $P(FS=true | FB=true, FM=true, FH=true, NA=true)$

By Bayes rule

$$P(FS|FB,FM,FH,NA) = \frac{P(FH|FS,FB,FH,NA) P(FS,FB,FM,NA)}{P(FH,FB,FH,NA)} \quad (1)$$

Ans (1)

$$P(FH|FS,FB,FM,NA) = \frac{P(FH,FS,FB,FM,NA)}{P(FS,FB,FM,NA)} \quad \text{(Chain Rule)} \quad (2)$$

Substituting (2) in (1)

$$P(FS|FB,FM,FH,NA) = \frac{P(FH,FS,FB,FM,NA)}{P(FS,FB,FM,NA)} \quad (3)$$

Taking denominator as  $\propto$ , which will be taken care by normalization

$$P(FS|FB,FM,FH,NA) = \frac{P(FH,FS,FB,FM,NA)}{\propto} \quad (4)$$

Ans (4) the numerator becomes,

$$P(FH,FS,FB,FM,NA) = \sum_{NDG_1} P(FH,FS,FB,FM,NA,NDG_1)$$

$$= \sum_{NDG_1} \underbrace{P(FH|FS,FB,FM,NA,NDG_1)}_{FH \text{ independent of } FB, NA} P(FS,FB,FM,NA,NDG_1)$$

$$= \sum_{NDG_1} P(FH|FS,FM,NDG_1) P(FB|FS,FM,NA,NDG_1) P(FS,FM,NA,NDG_1)$$

$$= \sum_{NDG_1} P(FH|FS,FM,NDG_1) P(FB|FS) P(NDG_1|NA,FS,FM) P(NA,FS,FM)$$

$$= \sum_{NDG_1} P(FH|FS,FM,NDG_1) P(FB|FS) P(NDG_1|NA,FS,FM) P(NA) P(FS) P(FM)$$

↓  
 $f_1$

↓  
 $f_2$

↓  
 $f_3$

↓  
 $f_4$

↓  
 $f_5$

↓  
 $f_6$