3b) To Find $P(FH = \text{true})$

$$P(FH) = \sum_{FS, FM, NDG} P(FH, FS, FM, NDG) \quad \text{(Marginalization)}$$

$$= \sum_{FS, FM, NDG} P(FH \mid FS, FM, NDG)\, P(FS, FM, NDG) \quad -(1)$$

Chain rule

In eqn (1)

$$P(FS, FM, NDG) = \sum_{NA} P(FS, FM, NDG, NA) \quad \text{(Marginalization)}$$

$$= \sum_{NA} \underbrace{P(FS \mid FM, NDG, NA)}_{\text{FS is independent of FM, NDG, NA}}\, P(FM, NDG, NA)$$

So $$P(FS, FM, NDG) = \sum_{NA} P(FS)\, P(NDG \mid FM, NA)\, P(NA, FM)$$

$$= P(FS) \sum_{NA} P(NDG \mid NA, FM)\, P(NA)\, P(FM) \quad -(2)$$

Substituting (2) in (1)

$$P(FH) = \sum_{FS, FM, NDG} P(FH \mid FS, FM, NDG)\, P(FS) \sum_{NA} P(NDG \mid NA, FM)\, P(NA)\, P(FM)$$

$$P(FH) = \sum_{FS, FM, NDG} \underbrace{P(FH \mid FS, FM, NDG)}_{f_1}\, \underbrace{P(FS)}_{f_2}\, \underbrace{P(FM)}_{f_3} \sum_{NA} \underbrace{P(NDG \mid NA, FM)}_{f_4}\, \underbrace{P(NA)}_{f_5}$$

3c) To Find $P(FS=true \mid FM=true, FH=true)$

By Bayes Rule,

$$P(FS \mid FM, FH) = \frac{P(FH \mid FS, FM)\, P(FS, FM)}{P(FH, FM)} \quad -(1)$$

In eqn (1),

$$P(FH \mid FS, FM) = \frac{P(FH, FS, FM)}{P(FS, FM)} \quad -(2) \text{ Chain rule}$$

Substituting (2) in (1)

$$P(FS \mid FM, FH) = \frac{P(FH, FS, FM)}{P(FH, FM)} \quad -(3)$$

Treating denominator in (3) as normalization constant $P(FS \mid FM, FH) = \frac{P(FH, FS, FM)}{\alpha} \quad -(4)$

In (4), numerator becomes

$$P(FH, FS, FM) = \sum_{NDG, NA} P(FH, FS, FM, NDG, NA)$$

$$= \sum_{NDG, NA} P(FH \mid FS, FM, NDG, NA)\, P(FS, FM, NDG, NA) \quad \text{(Chain rule)}$$

$$= \sum_{NDG, NA} P(FH \mid FS, FM, NDG)\, P(NDG \mid FS, FM, NA)\, P(FS, FM, NA)$$

$$= \sum_{NDG, NA} \underbrace{P(FH \mid FS, FM, NDG)}_{f_1}\, \underbrace{P(NDG \mid FM, NA)}_{f_2}\, \underbrace{P(FS)}_{f_3}\, \underbrace{P(FM)}_{f_4}\, \underbrace{P(NA)}_{f_5}$$

3d) To Find $P(FS=true \mid FB=true, FM=true, FH=true)$

Bayes Rule

So $$P(FS \mid FB, FM, FH) = \frac{P(FH \mid FS, FB, FM)\, P(FS, FB, FM)}{P(FH, FB, FM)} \quad -(1)$$

In eqn (1)

$$P(FH \mid FS, FB, FM) = \frac{P(FH, FS, FB, FM)}{P(FS, FB, FM)} \quad -(2) \text{ Chain Rule}$$

Substituting (2) in (1) $$P(FS \mid FB, FM, FH) = \frac{P(FH, FS, FB, FM)}{P(FH, FB, FM)} \quad -(3)$$

In eqn (3), treating denominator as α, normalization constant

$$P(FS \mid FB, FM, FH) = \frac{P(FH, FS, FB, FM)}{\alpha} \quad -(4)$$

In (4), numerator becomes

$$P(FH, FS, FB, FM) = \sum_{NDG, NA} P(FH, FS, FB, FM, NDG, NA)$$

$$= \sum_{NDG, NA} \underbrace{P(FH \mid FS, FB, FM, NDG, NA)}_{FH \text{ is independent of } FB, NA} P(FS, FB, FM, NDG, NA)$$

So $$P(FH, FS, FB, FM) = \sum_{NDG, NA} P(FH \mid FS, FM, NDG)\, P(NDG \mid FS, FB, FM, NA)\, P(FS, FB, FM, NA)$$

$$= \sum_{NDG, NA} P(FH \mid FS, FM, NDG)\, P(NDG \mid NA, FM)\, P(FB \mid FS, FM, NA)\, P(FS, FM, NA)$$

$$= \sum_{NDG, NA} \underbrace{P(FH \mid FS, FM, NDG)}_{f_1}\, \underbrace{P(NDG \mid NA, FM)}_{f_2}\, \underbrace{P(FB \mid FS)}_{f_3}\, \underbrace{P(FS)}_{f_4}\, \underbrace{P(FM)}_{f_5}\, \underbrace{P(NA)}_{f_6}$$

3e) To Find $P(FS=true \mid FB=true, FM=true, FH=true, NA=true)$

By Bayes Rule

$$P(FS \mid FB, FM, FH, NA) = \frac{P(FH \mid FS, FB, FM, NA)\, P(FS, FB, FM, NA)}{P(FH, FB, FM, NA)} \quad -(1)$$

In (1)

$$P(FH \mid FS, FB, FM, NA) = \frac{P(FH, FS, FB, FM, NA)}{P(FS, FB, FM, NA)} \quad \text{(Chain Rule)} \quad -(2)$$

Substituting (2) in (1)

$$P(FS \mid FB, FM, FH, NA) = \frac{P(FH, FS, FB, FM, NA)}{P(FS, FB, FM, NA)} \quad -(3)$$

Taking denominator as α, which will be taken care by normalization

$$P(FS \mid FB, FM, FH, NA) = \frac{P(FH, FS, FB, FM, NA)}{\alpha} \quad -(4)$$

In (4) the numerator becomes,

$$P(FH, FS, FB, FM, NA) = \sum_{NDG} P(FH, FS, FB, FM, NA, NDG)$$

$$= \sum_{NDG} \underbrace{P(FH \mid FS, FB, FM, NA, NDG)}_{\text{FH independent of FB, NA}} P(FS, FB, FM, NA, NDG)$$

$$= \sum_{NDG} P(FH \mid FS, FM, NDG)\, P(FB \mid FS, FM, NA, NDG)\, P(FS, FM, NA, NDG)$$

$$= \sum_{NDG} P(FH \mid FS, FM, NDG)\, P(FB \mid FS)\, P(NDG \mid NA, FS, FM)\, P(NA, FS, FM)$$

$$= \sum_{NDG} \underbrace{P(FH \mid FS, FM, NDG)}_{f_1}\, \underbrace{P(FB \mid FS)}_{f_2}\, \underbrace{P(NDG \mid NA, FS, FM)}_{f_3}\, \underbrace{P(NA)}_{f_4}\, \underbrace{P(FS)}_{f_5}\, \underbrace{P(FM)}_{f_6}$$