RBE501 HW2

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October 9, 2016

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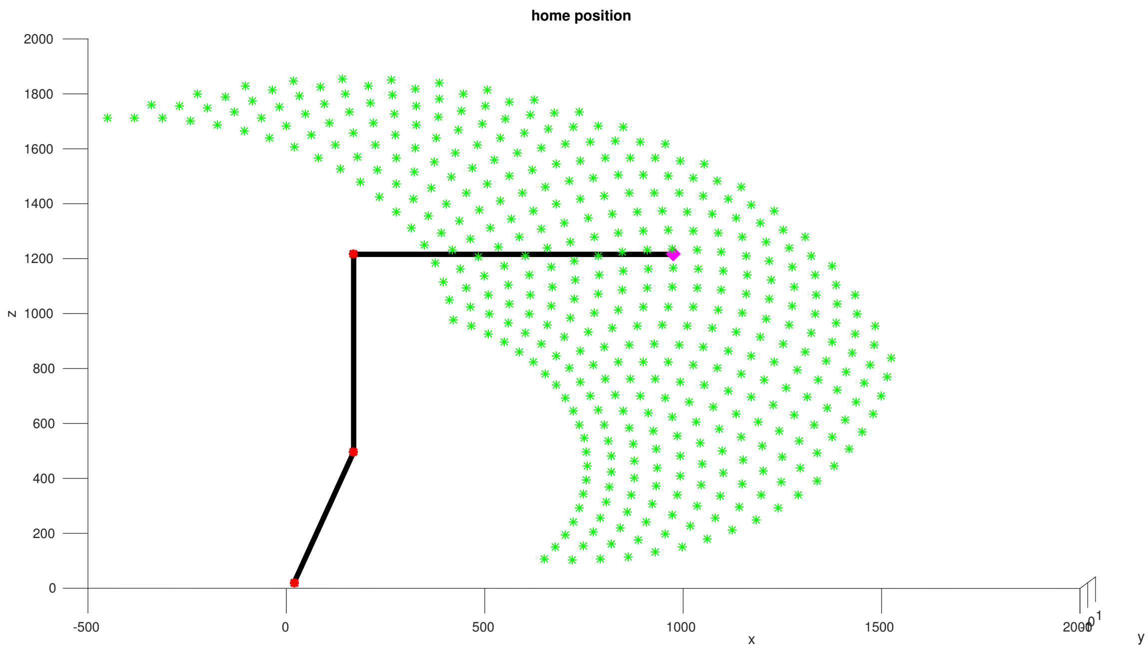
Part A

$$
\begin{aligned}
&x = 150 + 720 * cos(\theta_2) + 805 * cos(\theta_2 + \theta_3) \\
&z = 475 + 720 * sin(\theta_2) + 805 * sin(\theta_2 + \theta_3) \\
&y = 0; \\
&\quad where, -50 < \theta_2 < 50, -45 < \theta_3 < 45,
\end{aligned} \tag{1}
$$



Part B

$$
\begin{aligned}
&\theta_1 = atan2(Py, Px) \\
&\theta_2 = atan2(Pz, Px) - atan2(805 * sin(\theta_3), (720 + 805 * cos(\theta_3))) \\
&\theta_3 = acos\left[\frac{(Px^2 + Pz^2 - (720)^2 - (805)^2)}{(2(720)(805)))}\right] \\
&\theta_4 = \theta_5 = \theta_6 = \infty
\end{aligned} \tag{2}
$$

Part C

$$\begin{aligned}
\theta_1 &= 16.8584 \\
\theta_2 &= 23.7742 \\
\theta_3 &= 90.3410 \\
\theta_4 &= \theta_5 = \theta_6 = \infty
\end{aligned} \tag{3}$$

Part D

See appendix for the exact form of the T matrixs. Matlab was used to generate them. The jacobean of matrix was then found using the equation in the book.

$$J = \begin{bmatrix}
-a_2S_1C_2 - a_3S_1C_{23} & -a_2S_2C_1 - a_3S_{23}C_1 & -a_3S_{23}C_1 \\
a_2C_1C_2 + a_3C_1C_{23} & -a_2S_2S_1 - a_3S_{23}S_1 & -a_3S_1C_{23} \\
0 & a_2C_2 + a_3C_{23} & a_3C_{23} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 1 & 1
\end{bmatrix} \tag{4}$$

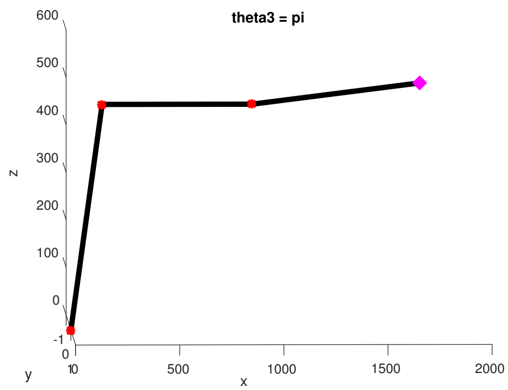
Part E

$$\dot{\vec{\theta}} = J^+\dot{\vec{x}} = [-0.026, 0.0471, -0.0211]^T \tag{5}$$

Part F

Taking the determent of Jacobian of reveals the singularities of the kinematic chain.

$$\begin{aligned} det(J) &= a_2 a_3 S_3 (a_2 C_2 + a_3 C_{23}) = 0 \\ &\Rightarrow sin(\theta_3) = 0 \rightarrow \theta_3 = 0, \pi \\ &\Rightarrow a_2 C_2 + a_3 C_{23} = 0 \end{aligned} \tag{6}$$



Extra Credit

see Appendix $(do_i k.m)$

Appendix

Part A

```
clear all;
close all;
clc;
links = zeros(3,4);
A = ones(4,4,length(links(:,1)));
T = ones(4,4,length(links(:,1)));

%go to home
theta = [ 0 0  0 0 0 0  ];
links(1,:) = [ 150 degtorad(90) 475 degtorad(theta(1))];
        links(2,:) = [ 720 0 0 degtorad(theta(2)+90)];
        links(3,:) = [ 805 0 0 degtorad(theta(3)-90 )];
% links(4,:) = [ 0    0 0   theta(4)];
% links(5,:) = [ 0  -90 0   theta(5)];
% links(6,:) = [ 0   90 0   theta(6)];
%get the A and T matrix
A = getA(links)
```

```
Undefined function 'getA' for input arguments of type 'double'.
```

```
T = getT(A)

%plot
figure(1);
title('home position')
plotArm(T)
%
%
hold on;

    for theta2 = -50:5:50
        for theta3 = -45:5:45
            theta = [0 theta2 theta3 ];
            links(1,:) = [ 150 degtorad(90) 475 degtorad(theta(1))];
            links(2,:) = [ 720 0 0 degtorad(theta(2)+90)];
            links(3,:) = [ 805 0 0 degtorad(theta(3)-90 )];

            %get the A and T matrix
            A = getA(links);
            T = getT(A);
            plot3( T(1,4,end),0,T(3,4,end)-50,'g*');
        end
    end
```

Part E

```
clear all;
close all;
clc;
%links = zeros(3,4);
```

```
%A = ones(4,4,length(links(:,1)));
%T = ones(4,4,length(links(:,1)));

%go to home
syms theta1 theta2 theta3 a1 a2 a3 d1
theta = [ theta1 theta2  theta3 ]
```

theta = $\begin{pmatrix} \theta_1 & \theta_2 & \theta_3 \end{pmatrix}$

```
links(1,:) = [ a1  90 d1 theta1]
```

links = $\begin{pmatrix} a_1 & 90 & d_1 & \theta_1 \end{pmatrix}$

```
links(2,:) = [ a2  0 0   theta2]
```

links =

$$\begin{pmatrix} a_1 & 90 & d_1 & \theta_1 \\ a_2 & 0 & 0 & \theta_2 \end{pmatrix}$$

```
links(3,:) = [ a3  0 0   theta3]
```

links =

$$\begin{pmatrix} a_1 & 90 & d_1 & \theta_1 \\ a_2 & 0 & 0 & \theta_2 \\ a_3 & 0 & 0 & \theta_3 \end{pmatrix}$$

```
A = getA(links)
```

A(:,:,1) =

$$\begin{pmatrix} \cos(\theta_1) & -\cos(90)\sin(\theta_1) & \sin(90)\sin(\theta_1) & a_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(90)\cos(\theta_1) & -\sin(90)\cos(\theta_1) & a_1\sin(\theta_1) \\ 0 & \sin(90) & \cos(90) & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A(:,:,2) =

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A(:,:,3) =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & a_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T = getT(A)
```

```
T(:,:,1) =
```

$$\begin{pmatrix} \cos(\theta_1) & -\cos(90)\sin(\theta_1) & \sin(90)\sin(\theta_1) & a_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(90)\cos(\theta_1) & -\sin(90)\cos(\theta_1) & a_1\sin(\theta_1) \\ 0 & \sin(90) & \cos(90) & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T(:,:,2) =
```

$$\begin{pmatrix} \cos(\theta_1)\cos(\theta_2) - \cos(90)\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1)\sin(\theta_2) - \cos(90)\cos(\theta_2)\sin(\theta_1) & \sin(90 \\ \cos(\theta_2)\sin(\theta_1) + \cos(90)\cos(\theta_1)\sin(\theta_2) & \cos(90)\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) & -\sin(9 \\ \sin(90)\sin(\theta_2) & \sin(90)\cos(\theta_2) & \cos \\ 0 & 0 & \end{pmatrix}$$

```
T(:,:,3) =
```

$$\begin{pmatrix} \cos(\theta_3)\sigma_2 - \sin(\theta_3)\sigma_4 & -\cos(\theta_3)\sigma_4 - \sin(\theta_3)\sigma_2 \\ \cos(\theta_3)\sigma_3 - \sin(\theta_3)\sigma_1 & -\cos(\theta_3)\sigma_1 - \sin(\theta_3)\sigma_3 \\ \sin(90)\cos(\theta_2)\sin(\theta_3) + \sin(90)\cos(\theta_3)\sin(\theta_2) & \sin(90)\cos(\theta_2)\cos(\theta_3) - \sin(90)\sin(\theta_2)\sin \\ 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \sin(\theta_1)\sin(\theta_2) - \cos(90)\cos(\theta_1)\cos(\theta_2)$$

$$\sigma_2 = \cos(\theta_1)\cos(\theta_2) - \cos(90)\sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_2)\sin(\theta_1) + \cos(90)\cos(\theta_1)\sin(\theta_2)$$

$$\sigma_4 = \cos(\theta_1)\sin(\theta_2) + \cos(90)\cos(\theta_2)\sin(\theta_1)$$