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# Problem 1

We can ignore link1 because it does not move

```
clear all;
close all;
clc;
```

variables

```
syms l1 l2 l3 m1 m2 g
      t1(t) t2(t) t3(t) t1d(t) t2d(t) t3d(t) t1dd(t) t2dd(t) t3dd(t)
syms t1_temp t2_temp t3_temp
```

Get the links

```
x1 = [ l2*cos(t2(t))*cos(t1(t));...
        l2*cos(t2(t))*sin(t1(t));...
        l1 + l2*sin(t2(t))];

x2 = [ x1(1) + l3*cos(t2(t) + t3(t));...
        x1(2) + l3*cos(t2(t) + t3(t));...
        x1(3) + l3*sin(t2(t) + t3(t))];
```

get the velocities

```
x1_d = diff(x1,t);
x1_d = subs(x1_d, {diff(t1(t), t) , diff(t2(t), t)}, {t1d(t),t2d(t)});

x2_d = diff(x2,t);
x2_d = subs(x2_d, {diff(t1(t), t) , diff(t2(t), t), diff(t3(t), t) },
            {t1d(t),t2d(t),t3d(t)});
```

get the Kinetic energy

```
K1 = 0.5*m1*(x1_d.'*x1_d);
K2 = 0.5*m1*(x2_d.'*x2_d);
```

get the Potential energy

```
P1 = m1*g*x1(3);
P2 = m1*g*x2(3);
```

get the Lagrange

```
L = (K1+K2) - (P1+P2);
```

get dL/dq

```
temp = subs(L,{t1(t)},{t1_temp});
dL_dt1 = diff( temp , t1_temp);
dL_dt1 = subs(dL_dt1,{t1_temp},{t1(t)});

temp = subs(L,{t2(t)},{t2_temp});
dL_dt2 = diff( temp , t2_temp);
```

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```

dL_dt2 = subs(dL_dt2,{t2_temp},{t2(t)});

temp = subs(L,{t3(t)},{t3_temp});
dL_dt3 = diff( temp , t3_temp);
dL_dt3 = subs(dL_dt3,{t3_temp},{t3(t)});

get dL/dq_d

temp = subs(L,{t1d(t)},{t1_temp});
dL_ddt1 = diff( temp , t1_temp);
dL_ddt1 = subs(dL_ddt1,{t1_temp},{t1d(t)});

temp = subs(L,{t2d(t)},{t2_temp});
dL_ddt2 = diff( temp , t2_temp);
dL_ddt2 = subs(dL_ddt2,{t2_temp},{t2d(t)});

temp = subs(L,{t3d(t)},{t3_temp});
dL_ddt3 = diff( temp , t3_temp);
dL_ddt3 = subs(dL_ddt3,{t3_temp},{t3d(t)});

Get the d/dt(dL/dq_d)

d_dt_1 = diff(dL_ddt1,t);
d_dt_1 = subs( d_dt_1, {diff(t1d(t), t),diff(t2d(t), t),diff(t3d(t),
t)},...
               {t1dd(t), t2dd(t), t3dd(t)});
d_dt_1 = subs(d_dt_1, {diff(t1(t), t) , diff(t2(t), t), diff(t3(t),
t) }, {t1d(t),t2d(t),t3d(t)});
%
d_dt_2 = diff(dL_ddt2,t);
d_dt_2 = subs( d_dt_2, {diff(t1d(t), t),diff(t2d(t), t),diff(t3d(t),
t)},...
               {t1dd(t), t2dd(t), t3dd(t)});
d_dt_2 = subs(d_dt_2, {diff(t1(t), t) , diff(t2(t), t), diff(t3(t),
t) }, {t1d(t),t2d(t),t3d(t)});
%
d_dt_3 = diff(dL_ddt3,t);
d_dt_3 = subs( d_dt_3, {diff(t1d(t), t),diff(t2d(t), t),diff(t3d(t),
t)},...
               {t1dd(t), t2dd(t), t3dd(t)});
d_dt_3 = subs(d_dt_3, {diff(t1(t), t) , diff(t2(t), t), diff(t3(t),
t) }, {t1d(t),t2d(t),t3d(t)});

% get torques

tau_1 = d_dt_1 - dL_dt1;
tau_2 = d_dt_2 - dL_dt2;
tau_3 = d_dt_3 - dL_dt3;

tau = [tau_1; tau_2 ;tau_3]
factored_tau = collect( tau, [t1dd(t) t2dd(t) t3dd(t) ])

M_qdd = subs( tau, {t1d(t) t2d(t) t3d(t) g}, {0 0 0 0});
M = collect( M_qdd, [t1dd(t) t2dd(t) t3dd(t) ])

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```
V = subs( tau, {g}, {0}) - M_qdd
C = collect( V, [t1d(t) t2d(t) t3d(t) ])
```

```
G = tau - V - M_qdd
```

```
tau =
```

```
(m1*(2*l2*cos(t2(t))*sin(t1(t))*(l3*sin(t2(t) + t3(t))*(t2dd(t)
+ t3dd(t)) + l3*cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 +
l2*cos(t2(t))*sin(t1(t))*t1dd(t) + l2*cos(t1(t))*sin(t2(t))*t2dd(t) +
l2*cos(t1(t))*cos(t2(t))*t1d(t)^2 + l2*cos(t1(t))*cos(t2(t))*t2d(t)^2
- 2*l2*sin(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) -
2*l2*cos(t1(t))*cos(t2(t))*(l3*sin(t2(t) + t3(t))*(t2dd(t)
+ t3dd(t)) + l3*cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 -
l2*cos(t1(t))*cos(t2(t))*t1dd(t) + l2*sin(t1(t))*sin(t2(t))*t2dd(t) +
l2*cos(t2(t))*sin(t1(t))*t1d(t)^2 + l2*cos(t2(t))*sin(t1(t))*t2d(t)^2
+ 2*l2*cos(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) +
2*l2*cos(t2(t))*sin(t1(t))*t1d(t)*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t)
+ l2*sin(t1(t))*sin(t2(t))*t2d(t)) +
2*l2*cos(t1(t))*sin(t2(t))*t2d(t)*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t)
+ l2*sin(t1(t))*sin(t2(t))*t2d(t)) -
2*l2*sin(t1(t))*sin(t2(t))*t2d(t)*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) + l2*cos(t2(t))*sin(t1(t))*t1d(t)
+ l2*cos(t1(t))*sin(t2(t))*t2d(t)) +
2*l2*cos(t1(t))*cos(t2(t))*t1d(t)*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) + l2*cos(t2(t))*sin(t1(t))*t1d(t)
+ l2*cos(t1(t))*sin(t2(t))*t2d(t)))/2 -
(m1*(2*(l2*cos(t2(t))*sin(t1(t))*t1d(t) +
l2*cos(t1(t))*sin(t2(t))*t2d(t))*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t) +
l2*sin(t1(t))*sin(t2(t))*t2d(t)) + 2*(l2*cos(t1(t))*cos(t2(t))*t1d(t)
- l2*sin(t1(t))*sin(t2(t))*t2d(t))*(l3*sin(t2(t) +
t3(t))*(t2d(t) + t3d(t)) + l2*cos(t2(t))*sin(t1(t))*t1d(t)
+ l2*cos(t1(t))*sin(t2(t))*t2d(t)))/2 -
(m1*(2*l2*cos(t1(t))*cos(t2(t))*(l2*sin(t1(t))*sin(t2(t))*t2dd(t) -
l2*cos(t1(t))*cos(t2(t))*t1dd(t) + l2*cos(t2(t))*sin(t1(t))*t1d(t)^2
+ l2*cos(t2(t))*sin(t1(t))*t2d(t)^2 +
2*l2*cos(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) -
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$$\begin{aligned}
& 2*l2*cos(t2(t))*sin(t1(t))*(l2*cos(t2(t))*sin(t1(t))*t1dd(t) + \\
& l2*cos(t1(t))*sin(t2(t))*t2dd(t) + l2*cos(t1(t))*cos(t2(t))*t1d(t)^2 \\
& + l2*cos(t1(t))*cos(t2(t))*t2d(t)^2 - \\
& 2*l2*sin(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*l2*cos(t1(t))*cos(t2(t))*t1d(t)*(l2*cos(t2(t))*sin(t1(t))*t1d(t) \\
& + l2*cos(t1(t))*sin(t2(t))*t2d(t)) + \\
& 2*l2*cos(t2(t))*sin(t1(t))*t1d(t)*(l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t2d(t)) + \\
& 2*l2*cos(t1(t))*sin(t2(t))*t2d(t)*(l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t2d(t)) + \\
& 2*l2*sin(t1(t))*sin(t2(t))*t2d(t)*(l2*cos(t2(t))*sin(t1(t))*t1d(t) + \\
& l2*cos(t1(t))*sin(t2(t))*t2d(t)))/2 \\
& (m1*(2*l2^2*cos(t2(t))^2*t2dd(t) - \\
& 4*l2^2*cos(t2(t))*sin(t2(t))*t2d(t)^2 + \\
& 2*l2*cos(t1(t))*sin(t2(t))*(l2*cos(t2(t))*sin(t1(t))*t1dd(t) + \\
& l2*cos(t1(t))*sin(t2(t))*t2dd(t) + l2*cos(t1(t))*cos(t2(t))*t1d(t)^2 \\
& + l2*cos(t1(t))*cos(t2(t))*t2d(t)^2 - \\
& 2*l2*sin(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*l2*sin(t1(t))*sin(t2(t))*(l2*sin(t1(t))*sin(t2(t))*t2dd(t) - \\
& l2*cos(t1(t))*cos(t2(t))*t1dd(t) + l2*cos(t2(t))*sin(t1(t))*t1d(t)^2 \\
& + l2*cos(t2(t))*sin(t1(t))*t2d(t)^2 + \\
& 2*l2*cos(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*l2*cos(t1(t))*cos(t2(t))*t2d(t)*(l2*cos(t2(t))*sin(t1(t))*t1d(t) \\
& + l2*cos(t1(t))*sin(t2(t))*t2d(t)) - \\
& 2*l2*cos(t1(t))*sin(t2(t))*t1d(t)*(l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t2d(t)) - \\
& 2*l2*cos(t2(t))*sin(t1(t))*t2d(t)*(l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t2d(t)) - \\
& 2*l2*sin(t1(t))*sin(t2(t))*t1d(t)*(l2*cos(t2(t))*sin(t1(t))*t1d(t) \\
& + l2*cos(t1(t))*sin(t2(t))*t2d(t)))/2 + \\
& (m1*(2*(l2*cos(t1(t))*sin(t2(t))*t1d(t) + \\
& l2*cos(t2(t))*sin(t1(t))*t2d(t))*(l2*cos(t1(t))*cos(t2(t))*t1d(t) - \\
& l2*sin(t1(t))*sin(t2(t))*t2d(t)) - 2*(l2*cos(t2(t))*sin(t1(t))*t1d(t) \\
& + l2*cos(t1(t))*sin(t2(t))*t2d(t))*(l2*cos(t1(t))*cos(t2(t))*t2d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t1d(t)) + \\
& 2*l2^2*cos(t2(t))*sin(t2(t))*t2d(t)^2))/2 + (m1*(2*(l3*sin(t2(t) + \\
& t3(t)) + l2*sin(t1(t))*sin(t2(t)))*(l3*sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + l3*cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 - \\
& l2*cos(t1(t))*cos(t2(t))*t1dd(t) + l2*sin(t1(t))*sin(t2(t))*t2dd(t) + \\
& l2*cos(t2(t))*sin(t1(t))*t1d(t)^2 + l2*cos(t2(t))*sin(t1(t))*t2d(t)^2 \\
& + 2*l2*cos(t1(t))*sin(t2(t))*t1d(t)*t2d(t)) - 2*(l2*sin(t2(t))*t2d(t) \\
& + l3*sin(t2(t) + t3(t))*(t2d(t) + t3d(t)))*(l3*cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*cos(t2(t))*t2d(t)) - 2*(l2*cos(t2(t)) \\
& + l3*cos(t2(t) + t3(t)))*(l2*sin(t2(t))*t2d(t)^2 - l3*cos(t2(t) \\
& + t3(t))*(t2dd(t) + t3dd(t)) + l3*sin(t2(t) + t3(t))*(t2d(t) \\
& + t3d(t))^2 - l2*cos(t2(t))*t2dd(t)) + 2*(l3*cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*cos(t1(t))*sin(t2(t))*t1d(t) \\
& + l2*cos(t2(t))*sin(t1(t))*t2d(t))*(l3*sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& + l2*sin(t1(t))*sin(t2(t))*t2d(t)) + 2*(l3*cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*cos(t1(t))*cos(t2(t))*t2d(t) \\
& - l2*sin(t1(t))*sin(t2(t))*t1d(t))*(l3*sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*cos(t2(t))*sin(t1(t))*t1d(t) + \\
& l2*cos(t1(t))*sin(t2(t))*t2d(t)) + 2*(l3*sin(t2(t) + t3(t))
\end{aligned}$$


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$$\begin{aligned}
& + l2*\cos(t1(t))*\sin(t2(t))* (l3*\sin(t2(t) + t3(t))* (t2dd(t) \\
& + t3dd(t)) + l3*\cos(t2(t) + t3(t))* (t2d(t) + t3d(t))^2 + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1dd(t) + l2*\cos(t1(t))*\sin(t2(t))*t2dd(t) + \\
& l2*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 + l2*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 \\
& - 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)))/2 \\
& - (m1*(2*(l3*\cos(t2(t) + t3(t))* (t2d(t) + \\
& t3d(t)) + l2*\cos(t1(t))*\sin(t2(t))*t1d(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t2d(t))* (l3*\sin(t2(t) + \\
& t3(t))* (t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*(l2*\sin(t2(t))*t2d(t) \\
& + l3*\sin(t2(t) + t3(t))* (t2d(t) + t3d(t)))* (l3*\cos(t2(t) + \\
& t3(t))* (t2d(t) + t3d(t)) + l2*\cos(t2(t))*t2d(t)) + 2*(l3*\cos(t2(t) \\
& + t3(t))* (t2d(t) + t3d(t)) + l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t))* (l3*\sin(t2(t) + \\
& t3(t))* (t2d(t) + t3d(t)) + l2*\cos(t2(t))*\sin(t1(t))*t1d(t) + \\
& l2*\cos(t1(t))*\sin(t2(t))*t2d(t)))/2 + g*m1*(l2*\cos(t2(t)) + \\
& l3*\cos(t2(t) + t3(t))) + g*l2*m1*\cos(t2(t))
\end{aligned}$$

$$\begin{aligned}
& (m1*(2*l3*\sin(t2(t) + t3(t))* (l3*\sin(t2(t) + t3(t))* (t2dd(t) \\
& + t3dd(t)) + l3*\cos(t2(t) + t3(t))* (t2d(t) + t3d(t))^2 + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1dd(t) + l2*\cos(t1(t))*\sin(t2(t))*t2dd(t) + \\
& l2*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 + l2*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 \\
& - 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*l3*\cos(t2(t) + t3(t))* (l2*\sin(t2(t))*t2d(t)^2 - \\
& l3*\cos(t2(t) + t3(t))* (t2dd(t) + t3dd(t)) + l3*\sin(t2(t) \\
& + t3(t))* (t2d(t) + t3d(t))^2 - l2*\cos(t2(t))*t2dd(t)) + \\
& 2*l3*\sin(t2(t) + t3(t))* (l3*\sin(t2(t) + t3(t))* (t2dd(t) \\
& + t3dd(t)) + l3*\cos(t2(t) + t3(t))* (t2d(t) + t3d(t))^2 - \\
& l2*\cos(t1(t))*\cos(t2(t))*t1dd(t) + l2*\sin(t1(t))*\sin(t2(t))*t2dd(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1d(t)^2 + l2*\cos(t2(t))*\sin(t1(t))*t2d(t)^2
\end{aligned}$$


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$$\begin{aligned}
& + 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) + 2*l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*\sin(t2(t) + t3(t))*(t2d(t) + t3d(t)) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1d(t) + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) \\
& + 2*l3*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*\cos(t2(t) + t3(t))*(t2d(t) + \\
& t3d(t)) + l2*\cos(t2(t))*t2d(t)))/2 - (m1*(2*l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*\sin(t2(t) + t3(t))*(t2d(t) + t3d(t)) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1d(t) + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) \\
& + 2*l3*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t)) + \\
& l2*\cos(t2(t))*t2d(t)))/2 + g*l3*m1*\cos(t2(t) + t3(t))
\end{aligned}$$

factored\_tau =

$$\begin{aligned}
& (m1*(2*l2^2*\cos(t1(t))^2*\cos(t2(t))^2 \\
& + 2*l2^2*\cos(t2(t))^2*\sin(t1(t))^2))*t1dd(t) + (- \\
& (m1*(2*l2*\cos(t1(t))*\cos(t2(t))*(l3*\sin(t2(t) + t3(t)) + \\
& l2*\sin(t1(t))*\sin(t2(t))) - 2*l2*\cos(t2(t))*\sin(t1(t))*(l3*\sin(t2(t) \\
& + t3(t)) + l2*\cos(t1(t))*\sin(t2(t))))/2)*t2dd(t) + (- \\
& (m1*(2*l2*l3*\cos(t1(t))*\cos(t2(t))*\sin(t2(t) + t3(t)) - \\
& 2*l2*l3*\cos(t2(t))*\sin(t1(t))*\sin(t2(t) + t3(t)))/2)*t3dd(t) \\
& + (m1*(2*l2*\cos(t2(t))*\sin(t1(t))*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))^2 + l2*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 \\
& + l2*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 - \\
& 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*l2*\cos(t1(t))*\cos(t2(t))*(l3*\cos(t2(t) + t3(t))*(t2d(t) \\
& + t3d(t))^2 + l2*\cos(t2(t))*\sin(t1(t))*t1d(t)^2 \\
& + l2*\cos(t2(t))*\sin(t1(t))*t2d(t)^2 + \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*l2*\cos(t2(t))*\sin(t1(t))*t1d(t)*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*t2d(t)*(l3*\sin(t2(t) +
\end{aligned}$$

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$$\begin{aligned}
& t3(t)) * (t2d(t) + t3d(t)) - l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& + l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) - \\
& 2 * l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) + l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) \\
& + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)) + \\
& 2 * l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) + l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) \\
& + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t))) / 2 - \\
& (m1 * (2 * (l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) + \\
& l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) - l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) + \\
& l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) + 2 * (l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& - l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) + l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) \\
& + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)))) / 2 - \\
& (m1 * (2 * l2 * \cos(t1(t)) * \cos(t2(t)) * (l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t)^2 \\
& + 2 * l2 * \cos(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t) \\
& + l2 * \cos(t2(t)) * \sin(t1(t)) * t2d(t)^2) - \\
& 2 * l2 * \cos(t2(t)) * \sin(t1(t)) * (l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t)^2 \\
& - 2 * l2 * \sin(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t) \\
& + l2 * \cos(t1(t)) * \cos(t2(t)) * t2d(t)^2) - \\
& 2 * l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) * (l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) \\
& + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)) + \\
& 2 * l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) * (l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& - l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) + \\
& 2 * l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t) * (l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& - l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) + \\
& 2 * l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t) * (l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) + \\
& l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)))) / 2 \\
& (- (m1 * (2 * l2 * \cos(t1(t)) * \cos(t2(t)) * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \sin(t1(t)) * \sin(t2(t))) - 2 * l2 * \cos(t2(t)) * \sin(t1(t)) * (l3 * \sin(t2(t) \\
& + t3(t)) + l2 * \cos(t1(t)) * \sin(t2(t)))) / 2) * t1dd(t) + \\
& ((m1 * (2 * l2^2 * \cos(t1(t))^2 * \sin(t2(t))^2 + 2 * l2^2 * \cos(t2(t))^2 \\
& + 2 * l2^2 * \sin(t1(t))^2 * \sin(t2(t))^2)) / 2 + (m1 * (2 * (l2 * \cos(t2(t)) \\
& + l3 * \cos(t2(t) + t3(t)))^2 + 2 * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \cos(t1(t)) * \sin(t2(t)))^2 + 2 * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \sin(t1(t)) * \sin(t2(t)))^2)) / 2) * t2dd(t) + ((m1 * (2 * l3 * \cos(t2(t) \\
& + t3(t)) * (l2 * \cos(t2(t)) + l3 * \cos(t2(t) + t3(t))) + 2 * l3 * \sin(t2(t) \\
& + t3(t)) * (l3 * \sin(t2(t) + t3(t)) + l2 * \cos(t1(t)) * \sin(t2(t))) \\
& + 2 * l3 * \sin(t2(t) + t3(t)) * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \sin(t1(t)) * \sin(t2(t)))) / 2) * t3dd(t) + (m1 * (2 * (l3 * \sin(t2(t) \\
& + t3(t)) + l2 * \cos(t1(t)) * \sin(t2(t))) * (l3 * \cos(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t))^2 + l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t)^2 \\
& + l2 * \cos(t1(t)) * \cos(t2(t)) * t2d(t)^2 - \\
& 2 * l2 * \sin(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t)) - 2 * (l2 * \sin(t2(t)) * t2d(t) \\
& + l3 * \sin(t2(t) + t3(t)) * (t2d(t) + t3d(t))) * (l3 * \cos(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) + l2 * \cos(t2(t)) * t2d(t)) + 2 * (l3 * \sin(t2(t) \\
& + t3(t)) + l2 * \sin(t1(t)) * \sin(t2(t))) * (l3 * \cos(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t))^2 + l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t)^2 \\
& + l2 * \cos(t2(t)) * \sin(t1(t)) * t2d(t)^2 + \\
& 2 * l2 * \cos(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t)) + 2 * (l3 * \cos(t2(t) \\
& + t3(t)) * (t2d(t) + t3d(t)) + l2 * \cos(t1(t)) * \sin(t2(t)) * t1d(t) \\
& + l2 * \cos(t2(t)) * \sin(t1(t)) * t2d(t)) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) - l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t)
\end{aligned}$$


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$$\begin{aligned}
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) + 2*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) - \\
& 2*(l2*\sin(t2(t))*t2d(t)^2 + l3*\sin(t2(t) + t3(t))*(t2d(t) \\
& + t3d(t))^2)*(l2*\cos(t2(t)) + l3*\cos(t2(t) + t3(t))))/2 \\
& - (m1*(4*l2^2*\cos(t2(t))*\sin(t2(t))*t2d(t)^2 - \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 \\
& - 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t) \\
& + l2*\cos(t1(t))*\cos(t2(t))*t2d(t)^2) - \\
& 2*l2*\sin(t1(t))*\sin(t2(t))*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t)^2 \\
& + 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t) \\
& + l2*\cos(t2(t))*\sin(t1(t))*t2d(t)^2) - \\
& 2*l2*\cos(t1(t))*\cos(t2(t))*t2d(t)*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*l2*\cos(t2(t))*\sin(t1(t))*t2d(t)*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t))))/2 + \\
& (m1*(2*(l2*\cos(t1(t))*\sin(t2(t))*t1d(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t2d(t))*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) - \\
& l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t))*(l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t)) + \\
& 2*l2^2*\cos(t2(t))*\sin(t2(t))*t2d(t)^2))/2 - (m1*(2*(l3*\cos(t2(t) \\
& + t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\sin(t2(t))*t1d(t) \\
& + l2*\cos(t2(t))*\sin(t1(t))*t2d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*(l2*\sin(t2(t))*t2d(t) \\
& + l3*\sin(t2(t) + t3(t))*(t2d(t) + t3d(t)))*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*t2d(t)) + 2*(l3*\cos(t2(t) \\
& + t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*\sin(t1(t))*t1d(t) + \\
& l2*\cos(t1(t))*\sin(t2(t))*t2d(t))))/2 + g*m1*(l2*\cos(t2(t)) + \\
& l3*\cos(t2(t) + t3(t))) + g*l2*m1*\cos(t2(t))
\end{aligned}$$


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$$\begin{aligned}
& (- (m1 * (2 * l2 * l3 * \cos(t1(t)) * \cos(t2(t)) * \sin(t2(t) + t3(t)) \\
& - 2 * l2 * l3 * \cos(t2(t)) * \sin(t1(t)) * \sin(t2(t) + t3(t)))) / 2 * t1dd(t) \\
& + ((m1 * (2 * l3 * \cos(t2(t) + t3(t)) * (l2 * \cos(t2(t)) + l3 * \cos(t2(t) \\
& + t3(t))) + 2 * l3 * \sin(t2(t) + t3(t)) * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \cos(t1(t)) * \sin(t2(t))) + 2 * l3 * \sin(t2(t) + t3(t)) * (l3 * \sin(t2(t) \\
& + t3(t)) + l2 * \sin(t1(t)) * \sin(t2(t)))) / 2 * t2dd(t) + \\
& ((m1 * (2 * l3^2 * \cos(t2(t) + t3(t))^2 + 4 * l3^2 * \sin(t2(t) \\
& + t3(t))^2)) / 2 * t3dd(t) + (m1 * (2 * l3 * \sin(t2(t) + \\
& t3(t)) * (l3 * \cos(t2(t) + t3(t)) * (t2d(t) + t3d(t))^2 + \\
& l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t)^2 + l2 * \cos(t1(t)) * \cos(t2(t)) * t2d(t)^2 \\
& - 2 * l2 * \sin(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t)) - 2 * l3 * \cos(t2(t) \\
& + t3(t)) * (l2 * \sin(t2(t)) * t2d(t)^2 + l3 * \sin(t2(t) \\
& + t3(t)) * (t2d(t) + t3d(t))^2 + 2 * l3 * \sin(t2(t) + \\
& t3(t)) * (l3 * \cos(t2(t) + t3(t)) * (t2d(t) + t3d(t))^2 + \\
& l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t)^2 + l2 * \cos(t2(t)) * \sin(t1(t)) * t2d(t)^2 \\
& + 2 * l2 * \cos(t1(t)) * \sin(t2(t)) * t1d(t) * t2d(t)) + 2 * l3 * \cos(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) * (l3 * \sin(t2(t) + t3(t)) * (t2d(t) + t3d(t)) + \\
& l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)) \\
& + 2 * l3 * \cos(t2(t) + t3(t)) * (t2d(t) + t3d(t)) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) - l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& + l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) - 2 * l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) * (l3 * \cos(t2(t) + t3(t)) * (t2d(t) + \\
& t3d(t)) + l2 * \cos(t2(t)) * t2d(t)))) / 2 - (m1 * (2 * l3 * \cos(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) * (l3 * \sin(t2(t) + t3(t)) * (t2d(t) + t3d(t)) + \\
& l2 * \cos(t2(t)) * \sin(t1(t)) * t1d(t) + l2 * \cos(t1(t)) * \sin(t2(t)) * t2d(t)) \\
& + 2 * l3 * \cos(t2(t) + t3(t)) * (t2d(t) + t3d(t)) * (l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) - l2 * \cos(t1(t)) * \cos(t2(t)) * t1d(t) \\
& + l2 * \sin(t1(t)) * \sin(t2(t)) * t2d(t)) - 2 * l3 * \sin(t2(t) + \\
& t3(t)) * (t2d(t) + t3d(t)) * (l3 * \cos(t2(t) + t3(t)) * (t2d(t) + t3d(t)) + \\
& l2 * \cos(t2(t)) * t2d(t)))) / 2 + g * l3 * m1 * \cos(t2(t) + t3(t))
\end{aligned}$$

$M =$

$$\begin{aligned}
& (m1 * (2 * l2^2 * \cos(t1(t))^2 * \cos(t2(t))^2 \\
& + 2 * l2^2 * \cos(t2(t))^2 * \sin(t1(t))^2)) * t1dd(t) + (- \\
& (m1 * (2 * l2 * \cos(t1(t)) * \cos(t2(t)) * (l3 * \sin(t2(t) + t3(t)) + \\
& l2 * \sin(t1(t)) * \sin(t2(t))) - 2 * l2 * \cos(t2(t)) * \sin(t1(t)) * (l3 * \sin(t2(t) \\
& + t3(t)) + l2 * \cos(t1(t)) * \sin(t2(t)))) / 2 * t2dd(t) + (-
\end{aligned}$$

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$$\begin{aligned}
& (m1*(2*l2*l3*cos(t1(t))*cos(t2(t))*sin(t2(t) + t3(t)) - \\
& 2*l2*l3*cos(t2(t))*sin(t1(t))*sin(t2(t) + t3(t)))/2)*t3dd(t) \\
& (- (m1*(2*l2*cos(t1(t))*cos(t2(t))*(l3*sin(t2(t) + t3(t)) + \\
& l2*sin(t1(t))*sin(t2(t))) - 2*l2*cos(t2(t))*sin(t1(t))*(l3*sin(t2(t) \\
& + t3(t)) + l2*cos(t1(t))*sin(t2(t))))/2)*t1dd(t) + \\
& ((m1*(2*l2^2*cos(t1(t))^2*sin(t2(t))^2 + 2*l2^2*cos(t2(t))^2 \\
& + 2*l2^2*sin(t1(t))^2*sin(t2(t))^2))/2 + (m1*(2*(l2*cos(t2(t)) \\
& + l3*cos(t2(t) + t3(t)))^2 + 2*(l3*sin(t2(t) + t3(t)) + \\
& l2*cos(t1(t))*sin(t2(t)))^2 + 2*(l3*sin(t2(t) + t3(t)) + \\
& l2*sin(t1(t))*sin(t2(t)))^2))/2)*t2dd(t) + ((m1*(2*l3*cos(t2(t) \\
& + t3(t))*(l2*cos(t2(t)) + l3*cos(t2(t) + t3(t))) + 2*l3*sin(t2(t) \\
& + t3(t))*(l3*sin(t2(t) + t3(t)) + l2*cos(t1(t))*sin(t2(t))) \\
& + 2*l3*sin(t2(t) + t3(t))*(l3*sin(t2(t) + t3(t)) + \\
& l2*sin(t1(t))*sin(t2(t))))/2)*t3dd(t)
\end{aligned}$$

$$\begin{aligned}
& (- (m1*(2*l2*l3*cos(t1(t))*cos(t2(t))*sin(t2(t) + t3(t)) \\
& - 2*l2*l3*cos(t2(t))*sin(t1(t))*sin(t2(t) + t3(t)))/2)*t1dd(t) \\
& + ((m1*(2*l3*cos(t2(t) + t3(t))*(l2*cos(t2(t)) + l3*cos(t2(t) \\
& + t3(t))) + 2*l3*sin(t2(t) + t3(t))*(l3*sin(t2(t) + t3(t)) + \\
& l2*cos(t1(t))*sin(t2(t))) + 2*l3*sin(t2(t) + t3(t))*(l3*sin(t2(t) \\
& + t3(t)) + l2*sin(t1(t))*sin(t2(t))))/2)*t2dd(t) + \\
& ((m1*(2*l3^2*cos(t2(t) + t3(t))^2 + 4*l3^2*sin(t2(t) + \\
& t3(t))^2))/2)*t3dd(t)
\end{aligned}$$

V =

$$\begin{aligned}
& (m1*(2*l2*cos(t1(t))*cos(t2(t))*(l3*sin(t2(t) \\
& + t3(t))*(t2dd(t) + t3dd(t)) - l2*cos(t1(t))*cos(t2(t))*t1dd(t) \\
& + l2*sin(t1(t))*sin(t2(t))*t2dd(t)) - \\
& 2*l2*cos(t2(t))*sin(t1(t))*(l3*sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + l2*cos(t2(t))*sin(t1(t))*t1dd(t) \\
& + l2*cos(t1(t))*sin(t2(t))*t2dd(t)))/2 - \\
& (m1*(2*l2*cos(t1(t))*cos(t2(t))*(l2*sin(t1(t))*sin(t2(t))*t2dd(t) - \\
& l2*cos(t1(t))*cos(t2(t))*t1dd(t) + l2*cos(t2(t))*sin(t1(t))*t1d(t)^2 \\
& + l2*cos(t2(t))*sin(t1(t))*t2d(t)^2 +
\end{aligned}$$


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$$\begin{aligned}
& 2*12*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*12*\cos(t2(t))*\sin(t1(t))*(12*\cos(t2(t))*\sin(t1(t))*t1dd(t) + \\
& 12*\cos(t1(t))*\sin(t2(t))*t2dd(t) + 12*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 \\
& + 12*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 - \\
& 2*12*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*12*\cos(t1(t))*\cos(t2(t))*t1d(t)*(12*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*12*\cos(t2(t))*\sin(t1(t))*t1d(t)*(12*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - 12*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*12*\cos(t1(t))*\sin(t2(t))*t2d(t)*(12*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - 12*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*12*\sin(t1(t))*\sin(t2(t))*t2d(t)*(12*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2d(t)))/2 - \\
& (m1*(2*12*\cos(t1(t))*\cos(t2(t))*(12*\cos(t1(t))*\cos(t2(t))*t1dd(t) \\
& - 12*\sin(t1(t))*\sin(t2(t))*t2dd(t)) + \\
& 2*12*\cos(t2(t))*\sin(t1(t))*(12*\cos(t2(t))*\sin(t1(t))*t1dd(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2dd(t)))/2 - \\
& (m1*(2*(12*\cos(t2(t))*\sin(t1(t))*t1d(t) + \\
& 12*\cos(t1(t))*\sin(t2(t))*t2d(t))*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - 12*\cos(t1(t))*\cos(t2(t))*t1d(t) + \\
& 12*\sin(t1(t))*\sin(t2(t))*t2d(t)) + 2*(12*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - 12*\sin(t1(t))*\sin(t2(t))*t2d(t))*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + 12*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2d(t)))/2 + \\
& (m1*(2*12*\cos(t2(t))*\sin(t1(t))*(13*\sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + 13*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 + \\
& 12*\cos(t2(t))*\sin(t1(t))*t1dd(t) + 12*\cos(t1(t))*\sin(t2(t))*t2dd(t) + \\
& 12*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 + 12*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 \\
& - 2*12*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - \\
& 2*12*\cos(t1(t))*\cos(t2(t))*(13*\sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + 13*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 - \\
& 12*\cos(t1(t))*\cos(t2(t))*t1dd(t) + 12*\sin(t1(t))*\sin(t2(t))*t2dd(t) + \\
& 12*\cos(t2(t))*\sin(t1(t))*t1d(t)^2 + 12*\cos(t2(t))*\sin(t1(t))*t2d(t)^2 \\
& + 2*12*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*12*\cos(t2(t))*\sin(t1(t))*t1d(t)*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - 12*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + 12*\sin(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*12*\cos(t1(t))*\sin(t2(t))*t2d(t)*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - 12*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + 12*\sin(t1(t))*\sin(t2(t))*t2d(t)) - \\
& 2*12*\sin(t1(t))*\sin(t2(t))*t2d(t)*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + 12*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2d(t)) + \\
& 2*12*\cos(t1(t))*\cos(t2(t))*t1d(t)*(13*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + 12*\cos(t2(t))*\sin(t1(t))*t1d(t) + \\
& 12*\cos(t1(t))*\sin(t2(t))*t2d(t)))/2 \\
& (m1*(2*12^2*\cos(t2(t))^2*t2dd(t) - \\
& 4*12^2*\cos(t2(t))*\sin(t2(t))*t2d(t)^2 + \\
& 2*12*\cos(t1(t))*\sin(t2(t))*(12*\cos(t2(t))*\sin(t1(t))*t1dd(t) + \\
& 12*\cos(t1(t))*\sin(t2(t))*t2dd(t) + 12*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 \\
& + 12*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 - \\
& 2*12*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*12*\sin(t1(t))*\sin(t2(t))*(12*\sin(t1(t))*\sin(t2(t))*t2dd(t) - \\
& 12*\cos(t1(t))*\cos(t2(t))*t1dd(t) + 12*\cos(t2(t))*\sin(t1(t))*t1d(t)^2
\end{aligned}$$


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$$\begin{aligned}
& + l2*\cos(t2(t))*\sin(t1(t))*t2d(t)^2 + \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) + \\
& 2*l2*\cos(t1(t))*\cos(t2(t))*t2d(t)*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) - \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - \\
& 2*l2*\cos(t2(t))*\sin(t1(t))*t2d(t)*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - \\
& 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t)))/2 \\
& - (m1*(2*l2^2*\cos(t2(t))^2*t2dd(t) + \\
& 2*l2*\cos(t1(t))*\sin(t2(t))*(l2*\cos(t2(t))*\sin(t1(t))*t1dd(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2dd(t)) - \\
& 2*l2*\sin(t1(t))*\sin(t2(t))*(l2*\cos(t1(t))*\cos(t2(t))*t1dd(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t2dd(t)))/2 + \\
& (m1*(2*(l2*\cos(t1(t))*\sin(t2(t))*t1d(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t2d(t))*(l2*\cos(t1(t))*\cos(t2(t))*t1d(t) - \\
& l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*(l2*\cos(t2(t))*\sin(t1(t))*t1d(t) \\
& + l2*\cos(t1(t))*\sin(t2(t))*t2d(t))*(l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t)) + \\
& 2*l2^2*\cos(t2(t))*\sin(t2(t))*t2d(t)^2))/2 + (m1*(2*(l3*\sin(t2(t) + \\
& t3(t)) + l2*\sin(t1(t))*\sin(t2(t)))*(l3*\sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + l3*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 - \\
& l2*\cos(t1(t))*\cos(t2(t))*t1dd(t) + l2*\sin(t1(t))*\sin(t2(t))*t2dd(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1d(t)^2 + l2*\cos(t2(t))*\sin(t1(t))*t2d(t)^2 \\
& + 2*l2*\cos(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)) - 2*(l2*\sin(t2(t))*t2d(t) \\
& + l3*\sin(t2(t) + t3(t))*(t2d(t) + t3d(t)))*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*t2d(t)) - 2*(l2*\cos(t2(t)) \\
& + l3*\cos(t2(t) + t3(t)))*(l2*\sin(t2(t))*t2d(t)^2 - l3*\cos(t2(t) \\
& + t3(t))*(t2dd(t) + t3dd(t)) + l3*\sin(t2(t) + t3(t))*(t2d(t) \\
& + t3d(t))^2 - l2*\cos(t2(t))*t2dd(t)) + 2*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\sin(t2(t))*t1d(t) \\
& + l2*\cos(t2(t))*\sin(t1(t))*t2d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) + 2*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*\sin(t1(t))*t1d(t) + \\
& l2*\cos(t1(t))*\sin(t2(t))*t2d(t)) + 2*(l3*\sin(t2(t) + t3(t)) \\
& + l2*\cos(t1(t))*\sin(t2(t)))*(l3*\sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) + l3*\cos(t2(t) + t3(t))*(t2d(t) + t3d(t))^2 + \\
& l2*\cos(t2(t))*\sin(t1(t))*t1dd(t) + l2*\cos(t1(t))*\sin(t2(t))*t2dd(t) + \\
& l2*\cos(t1(t))*\cos(t2(t))*t1d(t)^2 + l2*\cos(t1(t))*\cos(t2(t))*t2d(t)^2 \\
& - 2*l2*\sin(t1(t))*\sin(t2(t))*t1d(t)*t2d(t)))/2 \\
& - (m1*(2*(l3*\cos(t2(t) + t3(t))*(t2d(t) + \\
& t3d(t)) + l2*\cos(t1(t))*\sin(t2(t))*t1d(t) + \\
& l2*\cos(t2(t))*\sin(t1(t))*t2d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*\cos(t1(t))*\cos(t2(t))*t1d(t) \\
& + l2*\sin(t1(t))*\sin(t2(t))*t2d(t)) - 2*(l2*\sin(t2(t))*t2d(t) \\
& + l3*\sin(t2(t) + t3(t))*(t2d(t) + t3d(t)))*(l3*\cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*t2d(t)) + 2*(l3*\cos(t2(t) \\
& + t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t1(t))*\cos(t2(t))*t2d(t) \\
& - l2*\sin(t1(t))*\sin(t2(t))*t1d(t))*(l3*\sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) + l2*\cos(t2(t))*\sin(t1(t))*t1d(t) +
\end{aligned}$$


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$$\begin{aligned}
& 12*\cos(t1(t))*\sin(t2(t))*t2\ddot{d}(t)))/2 - (m1*(2*(12*\cos(t2(t)) \\
& + 13*\cos(t2(t) + t3(t)))*(13*\cos(t2(t) + t3(t))*(t2\ddot{d}(t) \\
& + t3\ddot{d}(t)) + 12*\cos(t2(t))*t2\ddot{d}(t)) + 2*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t)))*(13*\sin(t2(t) + \\
& t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t)) + 12*\cos(t2(t))*\sin(t1(t))*t1\ddot{d}(t) \\
& + 12*\cos(t1(t))*\sin(t2(t))*t2\ddot{d}(t)) + 2*(13*\sin(t2(t) \\
& + t3(t)) + 12*\sin(t1(t))*\sin(t2(t)))*(13*\sin(t2(t) + \\
& t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t)) - 12*\cos(t1(t))*\cos(t2(t))*t1\ddot{d}(t) + \\
& 12*\sin(t1(t))*\sin(t2(t))*t2\ddot{d}(t)))/2
\end{aligned}$$

$$\begin{aligned}
& (m1*(2*13*\sin(t2(t) + t3(t)))*(13*\sin(t2(t) + t3(t))*(t2\ddot{d}(t) \\
& + t3\ddot{d}(t)) + 13*\cos(t2(t) + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t))^2 + \\
& 12*\cos(t2(t))*\sin(t1(t))*t1\ddot{d}(t) + 12*\cos(t1(t))*\sin(t2(t))*t2\ddot{d}(t) + \\
& 12*\cos(t1(t))*\cos(t2(t))*t1\ddot{d}(t)^2 + 12*\cos(t1(t))*\cos(t2(t))*t2\ddot{d}(t)^2 \\
& - 2*12*\sin(t1(t))*\sin(t2(t))*t1\ddot{d}(t)*t2\ddot{d}(t)) - \\
& 2*13*\cos(t2(t) + t3(t))*(12*\sin(t2(t))*t2\ddot{d}(t)^2 - \\
& 13*\cos(t2(t) + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t)) + 13*\sin(t2(t) \\
& + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t))^2 - 12*\cos(t2(t))*t2\ddot{d}(t)) + \\
& 2*13*\sin(t2(t) + t3(t))*(13*\sin(t2(t) + t3(t))*(t2\ddot{d}(t) \\
& + t3\ddot{d}(t)) + 13*\cos(t2(t) + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t))^2 - \\
& 12*\cos(t1(t))*\cos(t2(t))*t1\ddot{d}(t) + 12*\sin(t1(t))*\sin(t2(t))*t2\ddot{d}(t) + \\
& 12*\cos(t2(t))*\sin(t1(t))*t1\ddot{d}(t)^2 + 12*\cos(t2(t))*\sin(t1(t))*t2\ddot{d}(t)^2 \\
& + 2*12*\cos(t1(t))*\sin(t2(t))*t1\ddot{d}(t)*t2\ddot{d}(t)) + 2*13*\cos(t2(t) + \\
& t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t))*(13*\sin(t2(t) + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t)) + \\
& 12*\cos(t2(t))*\sin(t1(t))*t1\ddot{d}(t) + 12*\cos(t1(t))*\sin(t2(t))*t2\ddot{d}(t)) \\
& + 2*13*\cos(t2(t) + t3(t))*(t2\ddot{d}(t) + t3\ddot{d}(t))*(13*\sin(t2(t) +
\end{aligned}$$

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$$\begin{aligned}
& t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& + l2*sin(t1(t))*sin(t2(t))*t2d(t)) - 2*l3*sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*cos(t2(t) + t3(t))*(t2d(t) + \\
& t3d(t)) + l2*cos(t2(t))*t2d(t)))/2 - (m1*(2*l3*cos(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*sin(t2(t) + t3(t))*(t2d(t) + t3d(t)) + \\
& l2*cos(t2(t))*sin(t1(t))*t1d(t) + l2*cos(t1(t))*sin(t2(t))*t2d(t)) \\
& + 2*l3*cos(t2(t) + t3(t))*(t2d(t) + t3d(t))*(l3*sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t)) - l2*cos(t1(t))*cos(t2(t))*t1d(t) \\
& + l2*sin(t1(t))*sin(t2(t))*t2d(t)) - 2*l3*sin(t2(t) + \\
& t3(t))*(t2d(t) + t3d(t))*(l3*cos(t2(t) + t3(t))*(t2d(t) + \\
& t3d(t)) + l2*cos(t2(t))*t2d(t)))/2 - (m1*(2*l3*sin(t2(t) \\
& + t3(t))*(l3*sin(t2(t) + t3(t))*(t2dd(t) + t3dd(t)) + \\
& l2*cos(t2(t))*sin(t1(t))*t1dd(t) + l2*cos(t1(t))*sin(t2(t))*t2dd(t)) \\
& + 2*l3*sin(t2(t) + t3(t))*(l3*sin(t2(t) + t3(t))*(t2dd(t) \\
& + t3dd(t)) - l2*cos(t1(t))*cos(t2(t))*t1dd(t) + \\
& l2*sin(t1(t))*sin(t2(t))*t2dd(t)) + 2*l3*cos(t2(t) + \\
& t3(t))*(l3*cos(t2(t) + t3(t))*(t2dd(t) + t3dd(t)) + \\
& l2*cos(t2(t))*t2dd(t)))/2
\end{aligned}$$

$C =$

$$\begin{aligned}
& ((m1*(2*l2^2*cos(t1(t))^2*cos(t2(t))*sin(t2(t)) \\
& + 2*l2^2*cos(t2(t))*sin(t1(t))^2*sin(t2(t)) - \\
& 2*l2*cos(t1(t))*cos(t2(t))*(l3*sin(t2(t) + t3(t)) + \\
& l2*cos(t1(t))*sin(t2(t))) - 2*l2*cos(t2(t))*sin(t1(t))*(l3*sin(t2(t) \\
& + t3(t)) + l2*sin(t1(t))*sin(t2(t))))/2 - \\
& (m1*(4*cos(t2(t))*sin(t2(t))*l2^2*cos(t1(t))^2 + \\
& 4*cos(t2(t))*sin(t2(t))*l2^2*sin(t1(t))^2))/2 - \\
& (m1*(6*l2^2*cos(t1(t))^2*cos(t2(t))*sin(t2(t)) \\
& + 6*l2^2*cos(t2(t))*sin(t1(t))^2*sin(t2(t)) - \\
& 2*l2*cos(t1(t))*cos(t2(t))*(l3*sin(t2(t) + t3(t)) +
\end{aligned}$$


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$$\begin{aligned}
& 12*\cos(t1(t))*\sin(t2(t))) - 2*12*\cos(t2(t))*\sin(t1(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\sin(t1(t))*\sin(t2(t))))/2)*t1d(t)*t2d(t) + \\
& (- (m1*(2*12*\cos(t1(t))*\sin(t2(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t))) - 2*12*\sin(t1(t))*\sin(t2(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t)))))/2 - \\
& (m1*(2*12*\cos(t1(t))*\cos(t2(t))*(13*\cos(t2(t) + t3(t)) + \\
& 12*\cos(t2(t))*\sin(t1(t))) - 2*12*\cos(t2(t))*\sin(t1(t))*(13*\cos(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\cos(t2(t))) - \\
& 2*12*\cos(t1(t))*\sin(t2(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t))) + 2*12*\sin(t1(t))*\sin(t2(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t)))))/2)*t2d(t)^2 + (- \\
& (m1*(4*12*13*\cos(t1(t))*\cos(t2(t))*\cos(t2(t) + t3(t)) \\
& - 4*12*13*\cos(t2(t))*\sin(t1(t))*\cos(t2(t) + t3(t)) \\
& - 2*12*13*\cos(t1(t))*\sin(t2(t))*\sin(t2(t) + t3(t)) + \\
& 2*12*13*\sin(t1(t))*\sin(t2(t))*\sin(t2(t) + t3(t))))/2 - \\
& (m1*(2*12*13*\cos(t1(t))*\sin(t2(t))*\sin(t2(t) + t3(t)) - \\
& 2*12*13*\sin(t1(t))*\sin(t2(t))*\sin(t2(t) + t3(t))))/2)*t2d(t)*t3d(t) \\
& + (- (m1*(2*12*13*\cos(t1(t))*\cos(t2(t))*\cos(t2(t) + t3(t)) - \\
& 2*12*13*\cos(t2(t))*\sin(t1(t))*\cos(t2(t) + t3(t))))/2)*t3d(t)^2 \\
& (m1*(2*\cos(t2(t))*\sin(t2(t))*12^2*\cos(t1(t))^2 \\
& + 2*\cos(t2(t))*\sin(t2(t))*12^2*\sin(t1(t))^2) - \\
& (m1*(2*12^2*\cos(t1(t))^2*\cos(t2(t))*\sin(t2(t)) \\
& + 2*12^2*\cos(t2(t))*\sin(t1(t))^2*\sin(t2(t)) - \\
& 2*12*\cos(t1(t))*\cos(t2(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\cos(t1(t))*\sin(t2(t))) - 2*12*\cos(t2(t))*\sin(t1(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\sin(t1(t))*\sin(t2(t)))))/2)*t1d(t)^2 + \\
& ((m1*(2*12*\cos(t1(t))*\cos(t2(t))*(13*\cos(t2(t) + t3(t)) + \\
& 12*\cos(t2(t))*\sin(t1(t))) - 2*12*\cos(t2(t))*\sin(t1(t))*(13*\cos(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\cos(t2(t))) - \\
& 2*12*\cos(t1(t))*\sin(t2(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t))) + 2*12*\sin(t1(t))*\sin(t2(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t)))))/2 - \\
& (m1*(2*12*\cos(t1(t))*\cos(t2(t))*(13*\cos(t2(t) + t3(t)) + \\
& 12*\cos(t2(t))*\sin(t1(t))) - 2*12*\cos(t2(t))*\sin(t1(t))*(13*\cos(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\cos(t2(t))) - \\
& 6*12*\cos(t1(t))*\sin(t2(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t))) + 6*12*\sin(t1(t))*\sin(t2(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t)))))/2)*t1d(t)*t2d(t) \\
& + ((m1*(4*\cos(t2(t))*\sin(t2(t))*12^2*\cos(t1(t))^2 \\
& + 4*\cos(t2(t))*\sin(t2(t))*12^2*\sin(t1(t))^2 \\
& - 4*\cos(t2(t))*\sin(t2(t))*12^2))/2 - \\
& (m1*(2*\cos(t2(t))*\sin(t2(t))*12^2*\cos(t1(t))^2 \\
& + 2*\cos(t2(t))*\sin(t2(t))*12^2*\sin(t1(t))^2 - \\
& 2*\cos(t2(t))*\sin(t2(t))*12^2))/2 - (m1*(2*(13*\cos(t2(t) + \\
& t3(t)) + 12*\cos(t1(t))*\cos(t2(t)))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\cos(t1(t))*\sin(t2(t))) - 2*(12*\cos(t2(t)) + 13*\cos(t2(t) + \\
& t3(t)))*(12*\sin(t2(t)) + 13*\sin(t2(t) + t3(t))) + 2*(13*\cos(t2(t) \\
& + t3(t)) + 12*\cos(t2(t))*\sin(t1(t)))*(13*\sin(t2(t) + t3(t)) \\
& + 12*\sin(t1(t))*\sin(t2(t)))))/2 + (m1*(4*(13*\cos(t2(t) + \\
& t3(t)) + 12*\cos(t1(t))*\cos(t2(t)))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\cos(t1(t))*\sin(t2(t))) - 4*(12*\cos(t2(t)) + 13*\cos(t2(t) + \\
& t3(t)))*(12*\sin(t2(t)) + 13*\sin(t2(t) + t3(t))) + 4*(13*\cos(t2(t) \\
& + t3(t)) + 12*\cos(t2(t))*\sin(t1(t)))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t)))))/2)*t2d(t)^2 + ((m1*(2*13*\sin(t2(t)
\end{aligned}$$


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$$\begin{aligned}
& + t_3(t)) * (l_3 * \cos(t_2(t) + t_3(t)) + l_2 * \cos(t_1(t)) * \cos(t_2(t))) \\
& - 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_2 * \sin(t_2(t)) + l_3 * \sin(t_2(t) + \\
& t_3(t))) - 6 * l_3 * \sin(t_2(t) + t_3(t)) * (l_2 * \cos(t_2(t)) + l_3 * \cos(t_2(t) \\
& + t_3(t))) + 6 * l_3 * \cos(t_2(t) + t_3(t)) * (l_3 * \sin(t_2(t) + t_3(t)) + \\
& l_2 * \cos(t_1(t)) * \sin(t_2(t))) + 2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_3 * \cos(t_2(t) \\
& + t_3(t)) + l_2 * \cos(t_2(t)) * \sin(t_1(t))) + 6 * l_3 * \cos(t_2(t) + \\
& t_3(t)) * (l_3 * \sin(t_2(t) + t_3(t)) + l_2 * \sin(t_1(t)) * \sin(t_2(t)))) / 2 \\
& - (m_1 * (2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_3 * \cos(t_2(t) + t_3(t)) + \\
& l_2 * \cos(t_1(t)) * \cos(t_2(t))) - 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_2 * \sin(t_2(t)) \\
& + l_3 * \sin(t_2(t) + t_3(t))) - 2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_2 * \cos(t_2(t)) \\
& + l_3 * \cos(t_2(t) + t_3(t))) + 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_3 * \sin(t_2(t) \\
& + t_3(t)) + l_2 * \cos(t_1(t)) * \sin(t_2(t))) + 2 * l_3 * \sin(t_2(t) + \\
& t_3(t)) * (l_3 * \cos(t_2(t) + t_3(t)) + l_2 * \cos(t_2(t)) * \sin(t_1(t))) \\
& + 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_3 * \sin(t_2(t) + t_3(t)) + \\
& l_2 * \sin(t_1(t)) * \sin(t_2(t)))) / 2) * t_2 d(t) * t_3 d(t) + ((m_1 * (2 * l_3^2 * \cos(t_2(t) \\
& + t_3(t)) * \sin(t_2(t) + t_3(t)) - 2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_2 * \cos(t_2(t)) \\
& + l_3 * \cos(t_2(t) + t_3(t))) + 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_3 * \sin(t_2(t) \\
& + t_3(t)) + l_2 * \cos(t_1(t)) * \sin(t_2(t))) + 2 * l_3 * \cos(t_2(t) + \\
& t_3(t)) * (l_3 * \sin(t_2(t) + t_3(t)) + l_2 * \sin(t_1(t)) * \sin(t_2(t)))) / 2 - \\
& l_3^2 * m_1 * \cos(t_2(t) + t_3(t)) * \sin(t_2(t) + t_3(t)) * t_3 d(t)^2
\end{aligned}$$

$$\begin{aligned}
& ((m_1 * (2 * l_2 * l_3 * \cos(t_1(t)) * \cos(t_2(t)) * \sin(t_2(t) \\
& + t_3(t)) + 2 * l_2 * l_3 * \cos(t_2(t)) * \sin(t_1(t)) * \sin(t_2(t) + \\
& t_3(t)))) / 2) * t_1 d(t)^2 + ((m_1 * (2 * l_2 * l_3 * \cos(t_1(t)) * \cos(t_2(t)) * \cos(t_2(t) \\
& + t_3(t)) - 2 * l_2 * l_3 * \cos(t_2(t)) * \sin(t_1(t)) * \cos(t_2(t) + \\
& t_3(t)))) / 2 - (m_1 * (2 * l_2 * l_3 * \cos(t_1(t)) * \cos(t_2(t)) * \cos(t_2(t) \\
& + t_3(t)) - 2 * l_2 * l_3 * \cos(t_2(t)) * \sin(t_1(t)) * \cos(t_2(t) + \\
& t_3(t)) - 4 * l_2 * l_3 * \cos(t_1(t)) * \sin(t_2(t)) * \sin(t_2(t) + t_3(t)) + \\
& 4 * l_2 * l_3 * \sin(t_1(t)) * \sin(t_2(t)) * \sin(t_2(t) + t_3(t)))) / 2) * t_1 d(t) * t_2 d(t) \\
& + ((m_1 * (2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_3 * \cos(t_2(t) + t_3(t)) + \\
& l_2 * \cos(t_1(t)) * \cos(t_2(t))) - 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_2 * \sin(t_2(t)) \\
& + l_3 * \sin(t_2(t) + t_3(t))) - 2 * l_3 * \sin(t_2(t) + t_3(t)) * (l_2 * \cos(t_2(t)) \\
& + l_3 * \cos(t_2(t) + t_3(t))) + 2 * l_3 * \cos(t_2(t) + t_3(t)) * (l_3 * \sin(t_2(t) \\
& + t_3(t)) + l_2 * \cos(t_1(t)) * \sin(t_2(t))) + 2 * l_3 * \sin(t_2(t) + \\
& t_3(t)) * (l_3 * \cos(t_2(t) + t_3(t)) + l_2 * \cos(t_2(t)) * \sin(t_1(t))))
\end{aligned}$$


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$$\begin{aligned}
& + 2*13*\cos(t2(t) + t3(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\sin(t1(t))*\sin(t2(t))))/2 - (m1*(2*13*\cos(t2(t) + \\
& t3(t))*(13*\sin(t2(t) + t3(t)) + 12*\cos(t1(t))*\sin(t2(t))) \\
& - 2*13*\sin(t2(t) + t3(t))*(12*\cos(t2(t)) + 13*\cos(t2(t) + \\
& t3(t))) + 2*13*\cos(t2(t) + t3(t))*(13*\sin(t2(t) + \\
& 12*\sin(t1(t))*\sin(t2(t)))))/2)*t2d(t)^2 + ((m1*(6*13^2*\cos(t2(t) + \\
& t3(t))*\sin(t2(t) + t3(t)) - 2*13*\sin(t2(t) + t3(t))*(12*\cos(t2(t)) \\
& + 13*\cos(t2(t) + t3(t))) + 2*13*\cos(t2(t) + t3(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\cos(t1(t))*\sin(t2(t))) + 2*13*\cos(t2(t) + \\
& t3(t))*(13*\sin(t2(t) + t3(t)) + 12*\sin(t1(t))*\sin(t2(t)))))/2 \\
& - (m1*(2*13^2*\cos(t2(t) + t3(t))*\sin(t2(t) + t3(t)) - \\
& 2*13*\sin(t2(t) + t3(t))*(12*\cos(t2(t)) + 13*\cos(t2(t) + \\
& t3(t))) + 2*13*\cos(t2(t) + t3(t))*(13*\sin(t2(t) + t3(t)) + \\
& 12*\cos(t1(t))*\sin(t2(t))) + 2*13*\cos(t2(t) + t3(t))*(13*\sin(t2(t) \\
& + t3(t)) + 12*\sin(t1(t))*\sin(t2(t)))))/2)*t2d(t)*t3d(t) + \\
& (13^2*m1*\cos(t2(t) + t3(t))*\sin(t2(t) + t3(t))*t3d(t)^2
\end{aligned}$$

$G =$

$$\begin{aligned}
& g*m1*(12*\cos(t2(t)) + 13*\cos(t2(t) + t3(t))) + g*12*m1*\cos(t2(t)) \\
& g*13*m1*\cos(t2(t) + t3(t))
\end{aligned}$$

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## Problem 2

```
function [ tau_solve ] = Problem2fnc( M1,M2,L1,L2,theta1,theta2 )

syms l1 l2 m1 m2 g t1(t) t2(t) t1d(t) t2d(t) t1dd(t) t2dd(t) I1 I2
syms t1_temp t2_temp
```

Get the links

Get the link 1 positions and velocity

```
x1 = [ l1*cos(t1(t));...
        l1*sin(t1(t))];

x2 = [ x1(1) + l2*cos(t1(t) + t2(t));...
        x1(2) + l2*cos(t1(t) + t2(t))];
```

Get the link 2 positions and velocity

```
x1_d = diff(x1,t);
x1_d = subs(x1_d, {diff(t1(t), t) }, {t1d(t)});

x2_d = diff(x2,t);
x2_d = subs(x2_d, {diff(t2(t), t) }, {t2d(t)});
```

Solve for the inertia of the links

```
I1 = (1/3)*m1*(l1)^2;
I2 = (1/12)*m2*(L2)^2;
```

*Not enough input arguments.*

```
Error in Problem2fnc (line 30)
I2 = (1/12)*m2*(L2)^2;
```

solve for the kinetic energy including the rotational energy

```
K1 = 0.5*m1*(x1_d.'*x1_d) + 0.5*I1*t1(t)^2;

K2 = 0.5*m1*(x2_d.'*x2_d) + 0.5*I1*(t1(t)+t2(t))^2;
```

solve for the potential energy of the arm

```
P1 = m1*g*(0.5*l1)*sin(t1(t));
P2 = m2*g*( l1*sin(t1(t)) + .5*l2*sin(t1(t)+t2(t)));
```

get the lagrange

```
L = simplify((K1+K2)-(P1+P2));

get dL/dq for theta 1,2

temp = subs(L,{t1(t)},{t1_temp});
```

---

```

dL_dt1 = diff( temp , t1_temp);
dL_dt1 = subs(dL_dt1,{t1_temp},{t1(t)});

temp = subs(L,{t2(t)},{t2_temp});
dL_dt2 = diff( temp , t2_temp);
dL_dt2 = subs(dL_dt2,{t2_temp},{t2(t)});

get dL/dqd for theta 1,2

temp = subs(L,{t1d(t)},{t1_temp});
dL_ddt1 = diff( temp , t1_temp);
dL_ddt1 = subs(dL_ddt1,{t1_temp},{t1d(t)});

temp = subs(L,{t2d(t)},{t2_temp});
dL_ddt2 = diff( temp , t2_temp);
dL_ddt2 = subs(dL_ddt2,{t2_temp},{t2d(t)});

get d(dL/dqd)/dt for theta 1,2

d_dt_1 = diff(dL_ddt1,t);
d_dt_1 = subs( d_dt_1, {diff(t1d(t), t),diff(t2d(t), t)},{t1dd(t),
    t2dd(t)});
d_dt_1 = subs(d_dt_1, {diff(t1(t), t) , diff(t2(t), t) },
    {t1d(t),t2d(t)});
%
d_dt_2 = diff(dL_ddt2,t);
d_dt_2 = subs( d_dt_2, {diff(t1d(t), t),diff(t2d(t), t)},{t1dd(t),
    t2dd(t)});
d_dt_2 = subs(d_dt_2, {diff(t1(t), t) , diff(t2(t), t) },
    {t1d(t),t2d(t)});

get tau 1,2

tau_1 =simplify( d_dt_1 - dL_dt1)
tau_2 =simplify( d_dt_2 - dL_dt2)
tau = [tau_1;tau_2];

plug in values

tau_solve = simplify(subs(tau, {m1,m2,l1,l2,t1(t),t2(t)},
    {M1,M2,L1,L2,theta1,theta2 })))

end

```

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