Documentation of MIP Model for Food Preparation Optimization

1 Problem Statement

You own a restaurant. Restaurant can prepare 20 dishes. Every dish has to be prepared in multiples of 10 for optimal cost and lasts for 3-10 hours depending on the dish. If profit on dish to sell dish i is P(i) and loss on wastage is l(i) and loss on missing demand is w(i). out on hours How much of one dish should the restaurant prepare on a day, on a hour. Prepare(i,h) where i is dish and h is hours lying between 0 to 10. Requirement for each dish is R(i,h) where i are dishes and h lies between 5 to 10.

2 Variables

2.1 Dummy Prepared Food Variables

- **dummy**: These variables represent the amount of food prepared in each hour for each dish. These are integer variables, with a lower bound of 0, and are denoted as:

$$\text{dummy}_{i,j} \text{ for } i \in \text{non-sold hours} + \text{hours}, j \in \text{dishes}$$

2.2 Used Food Variables

- **used**: These variables denote the amount of food prepared in a given hour that is used in subsequent hours for each dish. These are integer variables with a lower bound of 0, represented as:

$$\text{used}_{i,j,k} \text{ for } i \in \text{non-sold hours} + \text{hours}, j \in \text{hours}, k \in \text{dishes}$$

2.3 Unfulfilled Demand Variables

- **unfilled_demand**: These variables denote the amount of unfulfilled demand for each dish in each hour. These are integer variables with a lower bound of 0, represented as:

$$\text{unfilled\_demand}_{i,j} \text{ for } i \in \text{hours}, j \in \text{dishes}$$

2.4 Wasted Food Variables

- **wasted**: These variables represent the amount of food wasted in each hour for each dish. These are integer variables with a lower bound of 0, represented as:

$$\text{wasted}_{i,j} \text{ for } i \in \text{non-sold hours} + \text{hours}, j \in \text{dishes}$$

3 Data

The model uses the following data inputs:

- **requirement**: A matrix representing the required amount of each dish in each hour.
- **metrics**: A matrix containing data for each dish, including shelf life, profit, loss due to wastage, and loss due to unfulfilled demand.
- **hours**: A list of hours during which food is sold.
- **non-sold hours**: A list of hours before food is sold (e.g., preparation hours).
- **dishes**: A list of dishes being considered.

4 Constraints

The model incorporates the following constraints:

4.1 Non-Sold Food Constraint

This constraint ensures that no food is sold during the non-sold hours:

$$\text{used}_{i,k,j} = 0, \forall i \in \text{non-sold hours}, \forall j \in \text{dishes}, \forall k \text{ non-sold hours}$$

4.2 Sold Food Constraint

This constraint ensures that the food used from the current hour and all previous hours up to the expiration time matches the required amount of food minus the unfulfilled demand:

$$\sum_{k=0}^{\min(i,\text{shelf life})} \text{used}_{i-k,i,j} = \text{requirement}_{i,j} - \text{unfilled\_demand}_{i,j}, \forall i \in \text{hours}, \forall j \in \text{dishes}$$

4.3 Wasted Food Constraint

This constraint ensures that the food prepared in each hour is equal to the amount used in the current and subsequent hours up to the expiration time plus the amount wasted:

$$\text{dummy}_{i,j} \times 10 = \sum_{k=0}^{\min(\text{shelf life}, \text{last hour} - i)} \text{used}_{i,i+k,j} + \text{wasted}_{i,j}, \forall i \in \text{non-sold hours} + \text{hours}, \forall j \in \text{dishes}$$

5 Objective Function

The objective function maximizes the total profit, taking into account the profit from sold food, and subtracting the losses due to wastage and unfulfilled demand:

$$\begin{aligned} \text{Maximize: } \sum_{i \in \text{hours}} \sum_{j \in \text{dishes}} \Bigg[& \left(\text{requirement}_{i,j} - \text{unfilled\_demand}_{i,j}\right) \times \text{profit}_j \\ & - \text{unfilled\_demand}_{i,j} \times \text{loss due to unfulfillment}_j \\ & - \text{wasted}_{i,j} \times \text{loss due to wastage}_j \Bigg] \end{aligned}$$

6 Optimization

The model is solved using Gurobi's optimizer, and the optimal objective value is displayed upon successful completion.