Truck Delivery Optimization Model

1 Introduction

This optimization model aims to minimize the cost of delivering goods to various locations while ensuring that each location receives the required weekly weight. The model involves decision variables, constraints, and an objective function to determine optimal delivery patterns using two types of trucks: 15-ton and 10-ton.

2 Decision Variables

- **Trucks[i,h,k]:**
  - **Type:** Integer
  - **Mathematical Representation:** $\text{Trucks}_{i,h,k}$
  - **Description:** The number of trucks of type k (15-ton or 10-ton) delivered to location i on day h.
- **Select[i,h]:**
  - **Type:** Binary
  - **Mathematical Representation:** $\text{Select}_{i,h}$
  - **Description:** Indicates whether location i selects delivery pattern h. It is a binary variable that takes the value 1 if the pattern is selected and 0 otherwise.
- **Min_trucks:**
  - **Type:** Integer
  - **Mathematical Representation:** Min_trucks
  - **Description:** The minimum number of absolute difference between the 15-ton trucks and 10-ton trucks required to satisfy the requirements across all days.

3 Constraints

3.1 Weekly Weight Requirement

$$\sum_{h=1}^{H}\sum_{k=1}^{K}\text{Truck\_types}[k] \times \text{Trucks}_{i,h,k} = W[i], \quad \forall i \in \{1, \ldots, I\} \tag{1}$$

This constraint ensures that the total weight delivered by all trucks to location i over the week equals the exact weekly weight requirement $W[i]$.

3.2 Pattern Selection

$$\sum_{h=1}^{H} \text{Select}_{i,h} = 1, \quad \forall i \in \{1, \ldots, I\} \tag{2}$$

This constraint ensures that each location i selects exactly one delivery pattern from the available options.

3.3 Pattern Requirements

$$\sum_{k=1}^{K} \text{Truck\_types}[k] \times \text{Trucks}_{i,d,k} \geq \frac{W[i] \times \text{Select}_{i,h}}{n}, \quad \forall d \in \text{pattern}[h], \quad \forall i \in \{1, \ldots, I\} \tag{3}$$

For a location i that selects pattern h, the total weight delivered on each day d of the pattern must meet the required weight fraction for that pattern. The fraction n depends on the number of days in the pattern.

3.4 Minimum Truck Requirements

$$\text{Min\_trucks} \geq \sum_{i=1}^{I} \text{Trucks}_{i,j,1} - \text{Trucks}_{i,j,0}, \quad \forall j \in \{1, \ldots, H\} \tag{4}$$

$$\text{Min\_trucks} \geq \sum_{i=1}^{I} \text{Trucks}_{i,j,0} - \text{Trucks}_{i,j,1}, \quad \forall j \in \{1, \ldots, H\} \tag{5}$$

These constraints ensure that the minimum number of 10-ton and 15-ton trucks required to meet the delivery needs across all locations and days are satisfied.

4 Objective Function

The objective function has two main goals:

4.1 Minimize Truck Costs

$$\min \sum_{i=1}^{I} \sum_{h=1}^{H} \sum_{k=1}^{K} \text{Truck\_costs}[k] \times \text{Trucks}_{i,h,k} \tag{6}$$

This part of the objective function aims to minimize the total cost of trucks used, considering the number of trucks and their respective costs.

4.2 Minimize Total Number of Trucks

$$\min(\text{Min\_trucks}) \tag{7}$$

This part of the objective function seeks to minimize the total number of trucks used across all days and locations.

5 Input Data

- **File Path:** The input data is read from an Excel file located at `'delivery system/truckdata.xlsx'`.
- **DataFrame (df):** The first 10 rows represent 10 different locations.
- **Days:** The week is represented by the days `['M', 'T', 'W', 'R', 'F']`.
- **Truck Types:** Two truck types are considered: 15-ton and 10-ton.
- **Truck Costs:** The cost of using a 15-ton truck is $150, while the cost for a 10-ton truck is $120.
- **Weekly Weight Requirement (W):** Represents the weekly weight that needs to be delivered to each location.
- **Patterns:** The available delivery patterns are:
  - **0:** Five times a week (Monday to Friday)
  - **1:** Once a week (Monday)
  - **2:** Twice a week (Monday and Thursday)
  - **3:** Thrice a week (Monday, Wednesday, and Friday)

This model is structured to ensure that each location's weekly weight requirement is met efficiently while minimizing costs and the total number of trucks used.