Shift Scheduling Optimization Model Documentation

1 Model Overview

This model optimizes the shift scheduling of employees over a specified time horizon, ensuring that hourly employee requirements are met while minimizing scheduling costs. The model incorporates constraints for shift timing, breaks, and overnight work.

2 Sets and Indices

2.1 Sets

- E: Set of employees.
- H: Set of hours, where $H = \{0, 1, \ldots, \text{hour*days - work\_hours}\}$
- D: Set of days, where $D = \{0, 1, \ldots, \text{days} - 1\}$

2.2 Indices

- e: Index for employees, $i \in E$
- h: Index for starting hours, $j \in H$
- k: Index for worked hours, $k \in H$
- d: Index for days, $d \in D$

3 Parameters

- **hour**: Total scheduling horizon (24)
- **days**: Total number of days (30)
- **work_hours**: Number of hours an employee works in a shift (9)
- **employee_cost**: Cost per employee per hour (80)

- **overnight_cost**: Cost per employee for working overnight (80)
- **shift_break**: Break between shifts(12)
- **requirement**: A numpy array generated randomly, representing the number of employees needed for each hour.

4 Decision Variables

- **Work Shifts**:
  - $\text{work}_{(e,h,k)}$: Binary variable indicating if employee e starts working at hour h and works until hour k.
  - $\forall e \in E, \forall h \in H, \forall k \in [j, j+1, \ldots, j + work\_hours - 1]$
  - $\text{work}_{(e,h,k)} \in \{0, 1\}$
- **Overnight Work**:
  - $\text{overnight}_{(d,e)}$: Binary variable indicating if employee e works overnight during day d.
  - $\forall d \in D, \forall e \in E$.
  - $\text{overnight}_{(d,e)} \in \{0, 1\}$
- **Work Days**:
  - $\text{work\_day}_{(e,d)}$: Binary variable indicating whether employee e works on day d.
  - $\forall e \in E, \forall d \in D$.
  - $\text{work\_day}_{(e,d)} \in \{0, 1\}$

5 Constraints

- **Shift Gap Constraint**: This constarint ensures that if the employee starts work at hour h then his shift ends at $h + 8$ and he cannot start his shift until the start of $h + 21$

$$\text{work}_{(e,h,h)} + \text{work}_{(e,k,k)} \leq 1, \quad \forall e \in E, \forall h \in H, \forall k \in [h+1, h+work\_hours+shift\_break-1] \tag{1}$$

- **Break and Work Hour Constraints**:

$$\sum_{k=h+3}^{h+5} \text{work}_{(e,h,k)} = 2 \cdot \text{work}_{(e,h,h)}, \quad \forall e \in E, \forall h \in H \tag{2}$$

$$\sum_{k=h}^{h+\text{work\_hours}} \text{work}_{(e,h,k)} = 8 \cdot \text{work}_{(e,h,h)}, \quad \forall e \in E, \forall h \in H \tag{3}$$

- **Overnight Shift Count**: An employee works overnight if they start any shift between 1AM to 6AM.

$$\text{overnight}_{(d,e)} \geq \text{work}_{(e,h,h)}, \quad \forall e \in E, \forall d \in D, \forall h \in range(max(0, 24*d-7), 24*d+6) \tag{4}$$

- **Requirement Satisfaction**: The total number of employees working at any hour must meet or exceed the requirement for that hour.

$$\sum_{e \in E} \sum_{h=\max(0,k-\text{work\_hours}+1)}^{k} \text{work}_{(e,h,k)} \geq \text{requirement}[k], \quad \forall k \in H \tag{5}$$

- **Working days in a month**: Each employee must take at least 1 days and at most 3 holiday days during the scheduling period.

$$\sum_{d \in D} \text{work\_day}_{(e,d)} \geq days - 3, \quad \forall e \in E \tag{6}$$

$$\sum_{d \in D} \text{work\_day}_{(e,d)} \leq days - 1, \quad \forall e \in E \tag{7}$$

- **Workday Definition Based on Shifts**:
  - If an employee works any shift during a day, $\text{work\_day}_{(e,d)}$ is set to 1.
  - If no shifts are assigned during the day, $\text{work\_day}_{(e,d)}$ is set to 0.

$$\sum_{k=24d}^{24(d+1)} \sum_{h=\max(0,k-8)}^{\min(\text{hours},k)} \text{work}_{(e,h,k)} \leq 24 \cdot \text{work\_day}_{(e,d)}, \quad \forall e \in E, \forall d \in D \tag{8}$$

6 Objective Function

The objective function minimizes the total cost associated with employee scheduling:

$$\text{Minimize: } Z = 10 \cdot \sum_{e \in E} \sum_{h \in H} \sum_{k=h}^{h+\text{work\_hours}-1} \text{work}_{(e,h,k)} + 20 \cdot \sum_{d \in D} \sum_{e \in E} \text{overnight}_{(d,e)} \tag{9}$$

Where:

- The first term accounts for the cost of scheduled shifts.
- The second term accounts for the cost of overnight work.

7 Conclusion

This model effectively schedules employee shifts while adhering to various constraints and minimizing costs. The optimal solution, if found, provides an efficient staffing plan based on the generated requirements.