

Python for Capital Asset Pricing Model: CAPM Part 2

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I What is the Multi-factor Model?



- ✓ **CAPM** \doteq **Single Factor Model** estimates the expected return and the risk of individual stocks with a single market portfolio.
- ✓ Therefore, how to overcome the **CAPM anomaly** is one of the most important missions in a market.
- ✓ One of the solutions is “**Arbitrage Price Theory**” by Stephen Ross (University of Pennsylvania).
- ✓ The **CAPM anomaly** is caused by the **small size effect**, **value effect**, and **momentum effect, etc.**
- ✓ By controlling the above three anomalies, it is possible to predict the stock price of individual stocks.

2 Multi-Factor Model



- ✓ **Multi-factor model** explains that multiple common factors determine each individual stock return in the market.
- ✓ Assuming that $E[R_i]$ is the expected return of stock i , f_j is the number of common factor f in the market is j , and $\beta_{i,j}$ is the sensitivity of individual stock i to factor f_j , the **multi-factor model** can be formulated as follows.

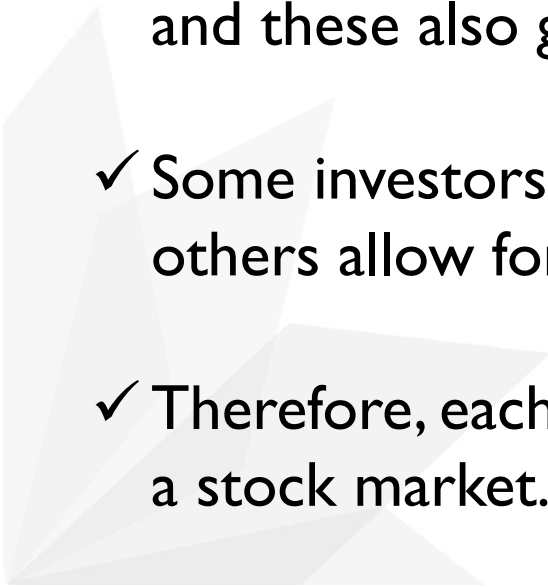
$$E[R_i] - R_f = \sum_{j=1}^N \beta_{i,j} f_j + \mu_i$$



3 Single-factor Model Criticism

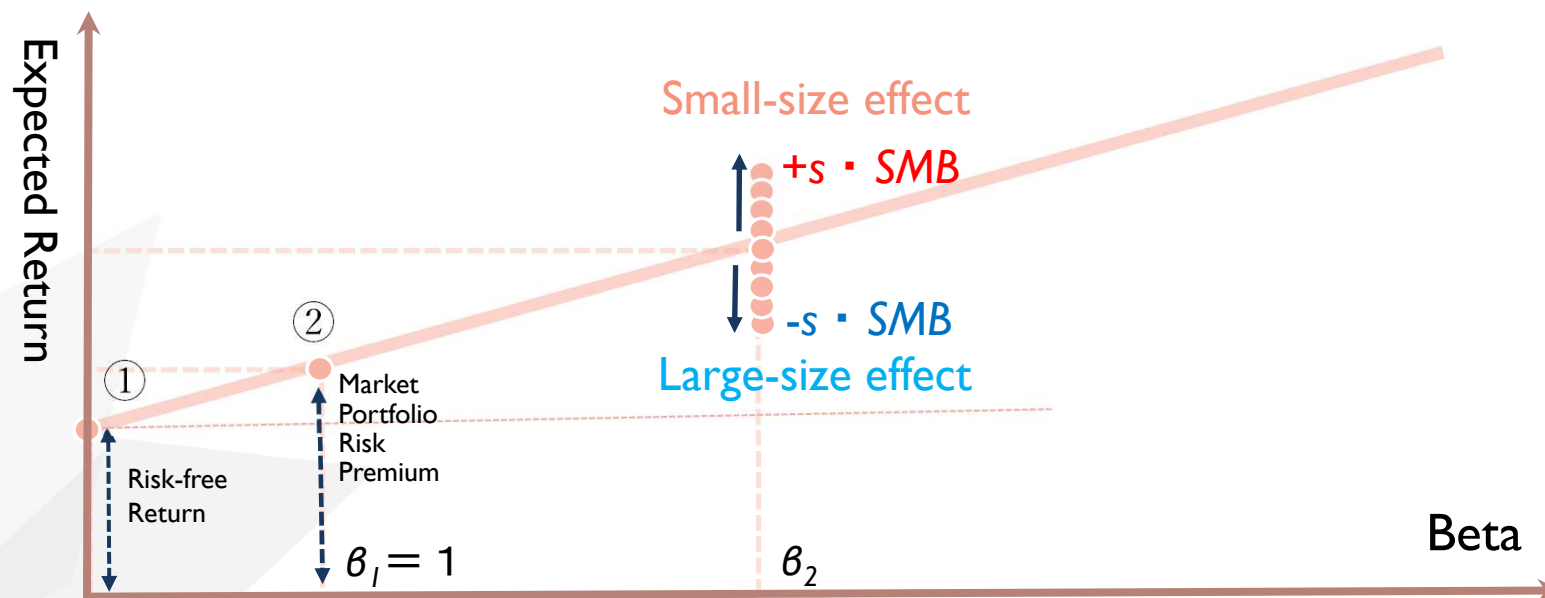


- ✓ Do all market participants really have a risk appetite based on the **mean-variance** approach?
- ✓ Investors own **assets other than financial assets** such as stocks and these also generate returns.
- ✓ Some investors aim to maximize **short-term returns**, while others allow for long-term investments.
- ✓ Therefore, each individual investor's **risk appetite** is diversified in a stock market.



4 CAPM Anomaly: Small Size Effect

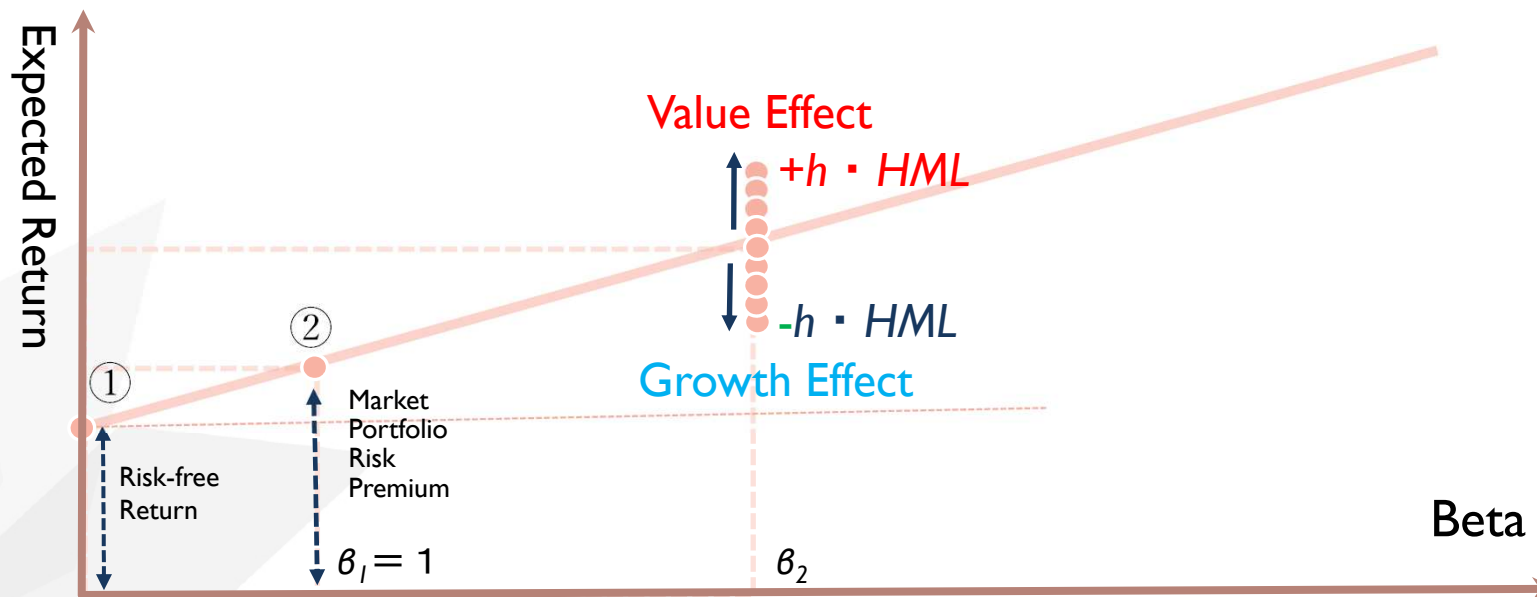
- ✓ The **small size effect** (**large size effect**) is defined as a phenomenon in which the **smaller** (**larger**) the **market capitalization** of a stock, the greater the expected return relative to the risk (**β value**).



5 CAPM Anomaly: Value Effect



- ✓ The **value effect** (**growth effect**) is defined as a phenomenon in which the **smaller** (**larger**) the **market-to-book ratio** is, the higher the expected return relative to the risk (**β value**).



6 Fama-French Three Factor Model

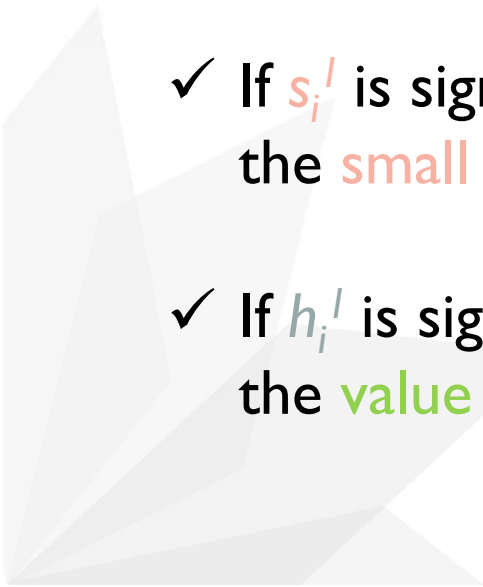


- ✓ Now, R_{it} is the return of an individual stock or portfolio i , R_{ft} is the risk-free asset yield, R_{Mt} is the weighted average market portfolio return, and b_i^1 is its sensitivity.
- ✓ SMB_t is the **small size effect** that is common to the market, HML_t is the **value effect** that similarly affects all stocks, and s_i^1 and h_i^1 are the sensitivity of each anomaly proxy variable to the return of individual stock i .
- ✓ At this time, the three-factor model is expressed by the following equation.

$$R_{it} - R_{ft} = a_i^1 + b_i^1(R_{Mt} - R_{ft}) + s_i^1 SMB_t + h_i^1 HML_t + e_{it}^1$$



- ✓ If b_i' is a positive value, it means that the return of individual stocks after controlling for the **small size effect** and **value effect** has a positive relationship with the market portfolio return.
- ✓ If s_i' is significantly positive, this stock price is affected by the **small size effect**.
- ✓ If h_i' is significantly positive, this stock price is affected by the **value effect**.



7 How to Calculate Two Factors



Top 50% Market
Cap Firms

Bottom 50% Market
Cap Firms

BH	BM	BL
①	②	③
SH	SM	SL
①	②	③

Top 30%
Book to Market
Ratio

Middle 40%
Book to Market
Ratio

Bottom 30%
Book to Market
Ratio



$$\text{SMB} = (\textcircled{\textbf{1}} + \textcircled{\textbf{2}} + \textcircled{\textbf{3}}) / 3 - (\textcircled{1} + \textcircled{2} + \textcircled{3}) / 3$$

$$\text{HML} = (\textcircled{\textbf{1}} + \textcircled{1}) / 2 - (\textcircled{\textbf{3}} + \textcircled{3}) / 2$$

[https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#](https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#International)
International



8 Carhart Four Factor Model



- ✓ The four-factor model is defined as a multifactor model with a **momentum effect (MOM)** factor in addition to the **small size effect (SMB)** and **value effect (HML)**.
- ✓ The **momentum effect** refers that stocks that have had high (or low) return trend in the past have price movements that will continue in the same trend in the future.

$$R_{it} - R_{ft} = a_i^2 + b_i^2(R_{Mt} - R_{Ft}) + s_i^2SMB_t + h_i^2HML_t + m_iMOM_t + e_{it}^2$$

9 Fama-French Five Factor Model



- ✓ The five-factor model does not use proxy variables for momentum effects, but instead employs variables related to firm profitability effect and investment effect in addition to size effect and value effect variables.
- ✓ In the following equation, RMW is the profitability effect, and CMA is the investment effect that similarly affects all stocks.

$$R_{it} - R_{ft} = a_i^3 + b_i^3(R_{Mt} - R_{ft}) + s_i^3SMB_t + h_i^3HML_t + r_i^3RMW_t + c_i^3CMA_t + e_{it}^3$$

10 Three Factor Model : Tesla vs. GM



- ✓ First, install the package to obtain the proxy variables for the small size effect and value effect required for the three-factor model.

```
pip install getFamaFrenchFactors
```

- ✓ Next, import 5 libraries.

```
#[1] Library Imports
```

```
import pandas as pd
```

```
import pandas_datareader as data
```

```
import datetime
```

```
import statsmodels.api as sm
```

```
import getFamaFrenchFactors as gff
```

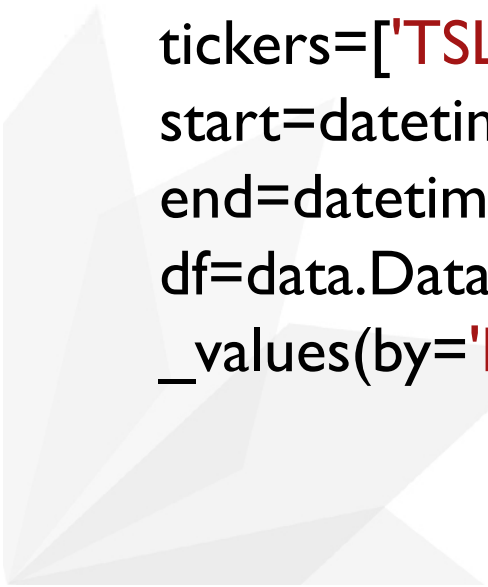
II Three Factor Model : Data Retrieval



✓ Next, we obtain the daily stock prices.

#[2] Stock Price Data Retrieval

```
tickers=['TSLA.US','GM.US']  
start=datetime.date(2012,1,1)  
end=datetime.date(2022,12,3)  
df=data.DataReader(tickers,'stooq',start=start,end=end).sort  
_values(by='Date',ascending=True)
```



12 Three Factor Model : Pre-processing



- ✓ Store only the variable 'Close' in dataframe `df` in a new dataframe.
- ✓ Next, change the column name from `['Close']` to `['Tesla', 'GM']`.
- ✓ Instruct to change the column name (variable name) to `['Tesla', 'GM']` on the third line.

#[3] Pre-processing

```
df = df['Close']  
company_list=['Tesla','GM']  
df.columns = company_list  
df.describe()  
df.head()
```

I3 Three Factor Model : Other Data Retrieval



- ✓ Get the following 3 variables from the library `getFamaFrenchFactors` (gff).
- ✓ *Mkt-RF*: Market Portfolio Return minus Risk Free Asset Return
- SMB*: Proxy Variable for Small Size Effect
- HML*: Proxy Variable for Value Effect

#[3] Small Size and Value Effect Data Retrieval

```
Fama_French_3=gff.famaFrench3Factor(frequency='m')
Fama_French_3.rename(columns={"date_ff_factors":'Date'},inplace=True)
Fama_French_3.set_index('Date',inplace=True)
Returns=df.resample('M').last().pct_change().dropna()
Fama_French_data=Fama_French_3.merge>Returns,on='Date')
```




I 4 Three Factor Model : Tesla

- ✓ Finally, we estimate the β value, **small size effect**, and **value effect**.
- ✓ Independent variables of X are the market portfolio return (**Mkt-RF**), small size effect proxy (**SMB**), and value effect proxy(**HML**)
- ✓ Dependent variable of Y is the market return of Tesla (**TSLA.US**).

#[4]Three Factor Model Estimation: Tesla

```
X=Fama_French_data[['Mkt-RF','SMB','HML']]
y=Fama_French_data['Tesla']-Fama_French_data['RF']
X=sm.add_constant(X)
Fama_French_Model1=sm.OLS(y,X).fit()
print(Fama_French_Model1.summary())
```

15 Three Factor Model : GM



- ✓ Looking at the results, we can see that Tesla has a growth effect, and GM has a small size effect and value effect.
- ✓ A further important finding is that due to these CAPM anomalies, the β value differs significantly between single-factor and multi-factor models.

#[5] Three Factor Model Estimation: GM

```
X=Fama_French_data[['Mkt-RF','SMB','HML']]
z=Fama_French_data['GM']-Fama_French_data['RF']
X=sm.add_constant(X)
Fama_French_Model2=sm.OLS(z,X).fit()
print(Fama_French_Model2.summary())
```



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