Epistemic Graphs for Representing and Reasoning with Positive and Negative Influences of Arguments

Technical Report for Seminar “Formal Argumentation”

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1 Introduction and Motivation

An argument contains a set of rules or premises and one claim or conclusion. A premise provides a reason to believe the claim. Arguments are used in arguing, which is a social activity that requires one or more people, and it occurs when people think that they disagree with each other. People argue for different purposes, such as to make a decision or persuade a person to a point of view or to negotiate an agreement, etc. An abstract argumentation framework (AAF) is a way to deal with such scenarios where the information contained in the arguments is modeled and subsequently, conclusions are drawn. Phan Ming Dung was the first person to lay the foundations for AAF, in which a set of arguments is represented in the form of a directed graph such that nodes are arguments and arrows constitute an attack relation between two arguments.

With Dung's methodology, the acceptance of arguments is precisely defined, and eventually an argument is either accepted or rejected [1]; whereas for a successful argumentation this does not suffice. We also have to deal with complex scenarios, such as acknowledging different perspectives of agents on a given issue, context-sensitivity, handling irrational arguers, etc. To address these challenges, Hunter et al.[2] introduce a new framework - Epistemic Graphs as a generalization of the epistemic approach to probabilistic argumentation. In the following, we elaborate on the challenges that motivated the authors to design the new formalism.

1. **Modelling fine-grained acceptability:** It is not always the case that people either fully believe or disbelieve an argument. Alternatively, a person may believe or disbelieve an argument to some degree. Therefore, there is a need to express belief in terms of a multi-valued scale in order to further accept or reject an argument.

2. **Modelling positive and negative relations between arguments:** An argument may influence other arguments positively or negatively. So, in order to analyze the belief in an argument, we must take into account all the influences it experiences.

3. **Modelling context-sensitivity:** Let A and B be two argumentation graphs that are structural replicas. If the arguments in A and B are provided with an additional description, the same user may interpret the graphs differently depending on the context.

4. **Modelling different perspectives:** In contrast to the above example, let A be presented to two different agents P and Q. For a given argument in A, agent P may have a different degree of belief than Q, or P may believe and Q may disbelieve.

5. **Modelling imperfect agents:** After all, agents are human beings, there is room for error in the way they assess the belief in arguments due to varying domain knowledge and cognitive biases.

6. **Modelling incomplete situations:** This refers to argumentation graphs being incomplete. Incompleteness occurs due to missing arguments or missing information regarding the relations between two arguments. As a result, we incorrectly believe or disbelieve the arguments.

2 Outline

Rest of the paper is organised as follows: Section 3 covers the basics about argument graphs; Section 4 lays the Syntactic and Semantic rules for Epistemic Language; Section 5 introduces concepts that are needed to interpret relations between epistemic formulae; Section 6 introduces the notion of Epistemic Graphs which are used for analysing arguments and relations between them; in Section 7 we draw conclusions on the original work done by the authors.

3 Basics

An argument graph is a directed graph in which nodes represent arguments and arcs represent a relation. The notion of arcs depends on the different frameworks proposed in the past[1, 3, 4]. However, in our case, they represent support, attack, or dependency relations, where a dependency may be neither attacking nor supporting. A labelling function is used to assign a label from label set $\Omega = \{+, -, *\}$ to every arc in the argument graph. However, multiple labels can be assigned to a same arc.

Definition 1. Let $\mathcal{G} = (V, A)$, where $A \subseteq V \times V$, be a directed graph. A **labelled graph** is a tuple $X = (\mathcal{G}, \mathcal{L})$ where $\mathcal{L} : A \to 2^{\Omega}$ is labelling function and Ω is a set of possible labels. X is **fully labelled** iff for every $\alpha \in A, \mathcal{L}(\alpha) \neq \varnothing$. X is **uni-labelled** iff for every $\alpha \in A, |\mathcal{L}(\alpha)| = 1$.

The set of nodes V in graph $\mathcal{G}$ is denoted by $\mathsf{Nodes}(\mathcal{G})$ and set of arcs A in graph $\mathcal{G}$ is denoted by $\mathsf{Arcs}(\mathcal{G})$. For a given node $\mathtt{B} \in \mathsf{Nodes}(\mathcal{G})$ parents of $\mathtt{B}$ are $\mathsf{Parent}(\mathtt{B}) = \{ \mathtt{A} \mid (\mathtt{A},\mathtt{B}) \in \mathsf{Arcs}(\mathcal{G})\}$.

4 Epistemic Language

To communicate effectively with humans, we need a natural language and an associated set of syntactic and semantic rules defined over a set of tokens or words. Similarly, to deal with epistemic graphs, an epistemic language with corresponding rules has been introduced by the authors. Terms act as building blocks of the epistemic language, which are analogous to tokens in a natural language. A term is a propositional formula over arguments. The probabilities of such terms can be added or subtracted together to form an operational formula. Comparing operational formula to a numerical value through equalities and inequalities forms epistemic atoms which can be further combined via negation, disjunction, and conjunction to yield epistemic formulae.

4.1 Syntax and Semantics

Definition 2. The epistemic language based on $\mathcal{G}$ is defined as follows:

- a **term** is a Boolean combination of arguments which is formed through connectives like $\neg, \wedge$ and $\vee$. These connectives derive secondary connectives like $\rightarrow$ as usual. $\mathsf{Terms}(\mathcal{G})$ denotes all the terms that can be formed from the arguments in $\mathcal{G}$.

- an **operational formula** is of the form $p(\alpha_i) \star_1 \ldots \star_{k-1} p(\alpha_k)$ where all $\alpha_i \in \mathsf{Terms}(\mathcal{G})$ and $\star_j \in \{+, -\}$. $\mathsf{OFormulae}(\mathcal{G})$ denotes all possible operational formulae of $\mathcal{G}$ and we read $p(\alpha)$ as "probability of α".

- an **epistemic atom** is of the form $f \# x$ where $\# \in \{=, \neq, \geq, \leq, >, <\}$, $x \in [0, 1]$ and $f \in \mathsf{OFormulae}(\mathcal{G})$.

- an **epistemic formula** is a Boolean combination of epistemic atoms. $\mathsf{EFormulae}(\mathcal{G})$ denotes the set of all possible epistemic formulae of $\mathcal{G}$.

For $\alpha \in \mathsf{Terms}(\mathcal{G})$, $\mathsf{Args}(\alpha)$ is set of all arguments in α. Given an epistemic formula $\psi \in \mathsf{EFormulae}(\mathcal{G})$, $\mathsf{FTerms}(\psi)$ is set of terms in ψ and $\mathsf{FArgs}(\psi)$ is set of arguments in ψ which can be expressed as $\mathsf{Args}(\mathsf{FTerms}(\psi))$.

Example 1. To understand the above definitions clearly, let us consider a graph $\mathcal{G}$ s.t. $\{\texttt{A,B,C}\} \subseteq \mathsf{Nodes}(\mathcal{G})$. If $\psi : p(\texttt{A}) + p(\texttt{B} \vee \texttt{C}) > 0.5$ is an epistemic formula on $\mathcal{G}$ then $\mathsf{FTerms}(\psi) = \{\texttt{A,B} \vee \texttt{C}\}$ and $\mathsf{FArgs}(\psi) = \{\texttt{A,B,C}\}$

The semantics of the epistemic language is defined in the form of belief distributions. A **belief distribution** on arguments is a function $P : 2^{\mathsf{Nodes}(\mathcal{G})} \to [0,1]$ s.t. $\sum_{\Gamma \subseteq \mathsf{Nodes}(\mathcal{G})} P(\Gamma) = 1$. Here $2^{\mathsf{Nodes}(\mathcal{G})}$ corresponds to the power set of $\mathsf{Nodes}(\mathcal{G})$. Function P maps each subset available in power set to a probability such that the sum over all subsets is equal to 1. With $\mathsf{Dist}(\mathcal{G})$ set of all belief distributions on $\mathsf{Nodes}(\mathcal{G})$ is denoted

Definition 3. Let φ be an epistemic atom $p(\alpha_i)\star_1 \ldots \star_{k-1}\ p(\alpha_k)\#\mathrm{b}$. The satisfying distributions of φ are defined as $\mathsf{Sat}(\varphi) = \{P \in \mathsf{Dist}(\mathcal{G}) \mid P(\alpha_i)\star_1 \ldots \star_{k-1}\ P(\alpha_k)\#\mathrm{b}\}$.

Here $\mathsf{Sat}(\varphi)$ is a set of all probability distributions such that the epistemic atom φ holds true.

Example 2. Consider a graph with nodes $\{\texttt{A,B}\}$ and a formula $\psi : p(\texttt{A}) - p(\texttt{B}) < 0.2$. Let P_1 and P_2 be probability distributions as shown in the below table. We can observe that sum of all the probabilities yields 1 in both distributions. Here $P_1 \notin \mathsf{Sat}(\psi)$ since $0.5-0$ is greater than 0.2, whereas $P_2 \in \mathsf{Sat}(\psi)$

	$\varnothing$	$\{\texttt{A}\}$	$\{\texttt{B}\}$	$\{\texttt{A,B}\}$
P_1	0	0.5	0	0.5
P_2	0.1	0.3	0.2	0.4

5 Reasoning with the Epistemic Language

Once the grammar rules are laid out for a language, the next step is to build the sentences satisfying the rules and further understand and interpret them. Similarly, since we already covered the rules for epistemic language; in this section, we will try to understand how to interpret a given epistemic formula. Subsequently, we will learn how two epistemic formulae can be related by considering their satisfying probability distributions.

5.1 Epistemic Entailment

We use an epistemic entailment relation to relate formulae based on their satisfying probability distributions, which is defined as follows :

Definition 4. Let $\{\phi_1, ..., \phi_n\} \subseteq \mathsf{EFormulae}(\mathcal{G})$ be a set of epistemic formulae, and $\psi \in \mathsf{EFormulae}(\mathcal{G})$ be an epistemic formula. The **epistemic entailment relation**, denoted $|\models$, is defined as follows.
$\{\phi_1, ..., \phi_n\} \mid\models \psi$ iff $\mathsf{Sat}(\{\phi_1, ..., \phi_n\}) \subseteq \mathsf{Sat}(\psi)$

In simple terms, if ϕ and ψ are two epistemic formulae, then $\phi \mid\models \psi$ only when a set of all distributions that satisfy ϕ also satisfy ψ. We will make it more clear with the help of an example below.

Example 3. Let $\phi_1 : p(\texttt{A}) < 0.5$, $\phi_2 : p(\texttt{A}) < 0.9$, $\phi_3 : p(\texttt{A}) < 0.4$ be epistemic formulae in $\mathcal{G}$. Then $p(\texttt{A}) < 0.5 \mid\models p(\texttt{A}) < 0.9$. Because any value of $p(\texttt{A})$ that is less than 0.5 is always less than 0.9. Also $p(\texttt{A}) < 0.5 \mid\not\models p(\texttt{A}) < 0.4$ because for $p(\texttt{A}) = 0.45$ it holds for ϕ_1 but violates ϕ_3.

5.2 Closure

Closure is a function that produces a set of all formulae that can be derived from a given epistemic formula. The notion of closure will be more clear when used for the analysis of relation coverage and labelings in Sections [6.1.2] and [6.2].

Definition 5. Let $\Phi \subseteq \mathsf{EFormulae}(\mathcal{G})$. The **epistemic closure function** is defined as follows.

$$\mathsf{Closure}(\Phi) = \{\psi \mid \Phi \mid\models \psi\}$$

Example 4. If $\Phi : p(\mathtt{A}) < 0.3$ then $\mathsf{Closure}(\Phi) = \{ p(\mathtt{A}) < x \mid x \in [0.3, 1]\}$.
Here for example, $p(\mathtt{A}) < 0.4$, $p(\mathtt{A}) < 0.8$, $p(\mathtt{A}) < 1 \in \mathsf{Closure}(\Phi)$

6 Epistemic Graphs

Epistemic Graphs are labelled graphs equipped with a set of epistemic formulae containing the beliefs in the arguments and the interrelationship between them. Epistemic formulae are used to represent epistemic constraints formally.

Definition 6. An **epistemic constraint** is an epistemic formula $\psi \in \mathsf{EFormulae}(\mathcal{G})$ s.t. $\mathsf{FArgs}(\psi) \neq \emptyset$. An **epistemic graph** is a tuple $(\mathcal{G}, \mathcal{L}, \mathcal{C})$ where $(\mathcal{G}, \mathcal{L})$ is a labelled graph, and $\mathcal{C} \in \mathsf{EFormulae}(\mathcal{G})$ is a set of epistemic constraints associated with the graph.

The set of constraints may not always cover all the aspects of the graph. Hence we cannot always construct the entire graph from an available set of constraints. Also, it is not possible to determine the direction of edges in the graph with the help of constraints. For example, if we had two arguments A and B connected by an edge, a constraint of the form $p(\mathtt{A}) < 0.5 \vee p(\mathtt{B}) < 0.5$ does not give any hint about the direction of the edge. We will now look at an example to understand the notion of epistemic graphs and epistemic constraints.

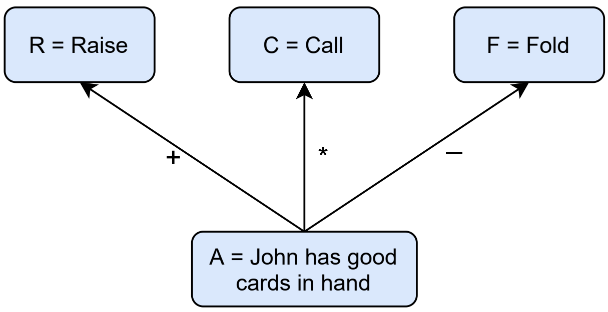

Figure 1: John's choice graph. The $+$ labels denote support, $-$ attack, and $*$ dependency

Example 5. Let us consider an example in which John and Mark are playing poker. Mark has bet 100 euros and now it is John's turn. Figure 1 depicts the choices John can make. Poker is a card game where a player with the best hand wins the pot. A player can either `call` (match the bet amount) or `raise` (increase the bet) or `fold` (give up) for a previous bet. Below is the set of constraints expressing what John can do in his next move.

- If John strongly disbelieves that he has good cards (e.g., just high cards), he will `fold`:

  $\phi_1 : p(\mathtt{A}) \leq 0.2 \rightarrow p(\mathtt{R}) < 0.5 \wedge p(\mathtt{C}) < 0.5 \wedge p(\mathtt{F}) > 0.5$

- If John moderately does not believe that he has good cards (e.g., two pairs), he will either `fold` thinking Mark has a better hand or `call` thinking Mark is bluffing:

  $\phi_2 : p(\mathtt{A}) > 0.2 \wedge p(\mathtt{A}) \leq 0.5 \rightarrow p(\mathtt{R}) < 0.5 \wedge (\ p(\mathtt{C}) > 0.5 \vee p(\mathtt{F}) > 0.5\ )$

- If John moderately believes that he has good cards (e.g., Flush draw), he will either `call` or `raise`:

  $\phi_3 : p(\mathtt{A}) > 0.5 \wedge p(\mathtt{A}) < 0.8 \rightarrow (\ p(\mathtt{R}) > 0.5 \vee p(\mathtt{C}) > 0.5\ ) \wedge p(\mathtt{F}) < 0.5\ )$

- If John strongly believes that he has good cards (e.g., Four of a kind), he will `raise` the bet:

  $\phi_4 : p(\mathtt{A}) \geq 0.8 \rightarrow p(\mathtt{R}) > 0.5 \wedge p(\mathtt{C}) < 0.5 \wedge p(\mathtt{F}) < 0.5$

  If we observe ϕ_1, we can deduce the relation between A and F as conflicting, because when A is disbelieved (to a sufficiently high degree) then F is believed. On the contrary, according to ϕ_4 the relation between A and R is supporting, because when A is believed (to a sufficiently high degree) then R is believed. If we observe ϕ_2 and ϕ_3 we cannot precisely establish the relation between A and C, because C can be believed when A is either moderately disbelieved or moderately believed.

6.1 Coverage

We have already mentioned in the introduction of epistemic graphs that constraints do not necessarily cover all aspects of graphs in terms of arguments and relations between them. However, if we manage to accurately analyze the graph with the available set of constraints covering all scenarios, this is very helpful in cases where we have limited knowledge about the opponent. Hence, to measure the extent to which given constraints take into account arguments and relations between them, we use coverage. This is accomplished by varying the beliefs in certain arguments and observing whether this leads to certain behaviours in the arguments in which we are interested. The varying component is called the constraint combination, which is formally defined below.

Definition 7. Let $F = \{\mathtt{A}_1, ..., \mathtt{A}_m\} \subseteq \mathsf{Nodes}(\mathcal{G})$ be a set of arguments. An **exact constraint combination** for F is a set $\mathcal{CC}^F = \{\ p(\mathtt{A}_1) = x_1,\ p(\mathtt{A}_2) = x_2,...,p(\mathtt{A}_m) = x_m\ \}$, where $x_1,...,x_m \in [0,1]$. A **soft constraint combination** for F is a set $\mathcal{CC}^F = \{\ p(\mathtt{A}_1)\ \#_1\ x_1,\ p(\mathtt{A}_2)\ \#_2\ x_2,...,p(\mathtt{A}_m)\ \#_m\ x_m\ \}$, where $x_1,...,x_m \in [0,1]$ and $\#_1, ..., \#_m \in \{=, \neq, \geq, \leq, >, <\}$. With $\mathcal{CC}^F|_G$ for $G \subseteq \mathsf{Nodes}(\mathcal{G})$ we denote the subset of $\mathcal{CC}^F$ that consists of all and only constraints of $\mathcal{CC}^F$ that are on arguments contained in $F \cap G$.

A constraints combination is formed by taking a subset of nodes in $\mathcal{G}$ and assigning them some probabilities. In the **exact** case, the probabilities are exactly equal to a number in the closed interval of 0 and 1, while in the **soft** case $=$ is replaced by any of the relational operators, so that the probabilities of the arguments fall under larger domain.

6.1.1 Argument Coverage

An argument is said to be covered if there is at least one value that cannot be assigned to its degree of belief. The value may be implicit from the constraints or may be inferred by imposing additional restrictions on the degree of belief of other arguments.

Definition 8. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph. We say that an argument $\mathtt{A} \in \mathsf{Nodes}(\mathcal{G})$ is **default covered** in X if there is a value $x \in [0,1]$ s.t. $\mathcal{C} \Vdash p(\mathtt{A}) \neq x$.

We say that an argument is default covered if the constraint set $\mathcal{C}$ restricts its belief from taking certain value(s).

Example 6. Consider graph depicted in Figure 2 and its set of constraints $\mathcal{C}$ given below:
$\mathcal{C} = \{\ \phi_1, \phi_2, \phi_3, \phi_4\}$ where $\phi_1 : p(\mathtt{A}) > 0.5$ $\quad \phi_2 : p(\mathtt{A}) > 0.5 \rightarrow p(\mathtt{B}) < 0.5$
$\phi_3 : (p(\mathtt{B}) < 0.5 \wedge p(\mathtt{C}) > 0.5) \rightarrow p(\mathtt{D}) \leq 0.5$ $\quad \phi_4 : p(\mathtt{C}) \leq 0.5 \rightarrow p(\mathtt{D}) > 0.5$

According to ϕ_1, belief in A should be always greater than 0.5. To put it formally $\mathcal{C} \Vdash p(\mathtt{A}) > 0.5$. Alternatively, we can say that $\mathcal{C} \Vdash p(\mathtt{A}) \neq v,\ \forall\ v \in [0,5]$. From ϕ_1 and ϕ_2 we can derive the restrictions for B. For example, $\mathcal{C} \Vdash p(\mathtt{B}) \neq 0.6$. Thus A and B are default covered. However, C and D are not default covered. Because their belief can take any value in $[0,1]$.

Although C and D are not default covered, if we restrict the belief of C such that $p(\mathtt{C}) > 0.5$, then $p(\mathtt{D})$ cannot be assigned any value that is greater than 0.5. Therefore, D is somewhat covered under certain additional conditions.

Example 7. Consider graph depicted in Figure 3 and its set of constraints $\mathcal{C}$ given below:
$\{\ \phi_1 : p(\mathtt{B}) > 0.5 \rightarrow p(\mathtt{C}) \leq 0.5,\ \phi_2 : (p(\mathtt{B}) > 0.5 \wedge p(\mathtt{C}) \geq 0.5\ ) \rightarrow p(\mathtt{A}) < 0.5\ \}$

In the context of ϕ_2, the probability of A is constrained if the condition is satisfied. But if ϕ_2 is not satisfied, A can assume any probability. Thus, the coverage is in a sense "partial". To explain this behaviour, we introduce two additional notions of coverage.

Definition 9. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph, $\mathtt{A} \in \mathsf{Nodes}(\mathcal{G})$ an argument and $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{A}\}$ a set of arguments. We say that A is:

- **partially covered** by F in X if there exists a constraint combination $\mathcal{CC}^F$ and a value $x \in [0,1]$ s.t. $\mathcal{CC}^F \cup \mathcal{C} \not\Vdash \perp$ and $\mathcal{CC}^F \cup \mathcal{C} \Vdash p(\mathtt{A}) \neq x$
- **fully covered** by F in X if for every constraint combination $\mathcal{CC}^F$ s.t. $\mathcal{CC}^F \cup \mathcal{C} \not\Vdash \perp$, there exists a value $x \in [0,1]$ s.t. $\mathcal{CC}^F \cup \mathcal{C} \Vdash p(\mathtt{A}) \neq x$

An argument is said to be partially covered by a set of arguments F if we can find a belief assignment to F such that constraints are not violated and result in our argument being unable to take some values, while an argument is fully covered if any suitable belief assignment to F results in our argument being unable to take some values. Here, F is created with the subset of nodes in $\mathcal{G}$ by excluding the argument under analysis. Note that the constraint combination derived from F must be consistent together with the existing set of constraints $\mathcal{G}$.

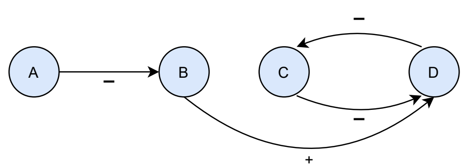

Figure 2: Argument graph for Example 6

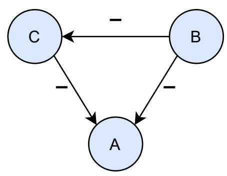

Figure 3: Argument graph for Example 7

Example 8. Let us consider the graph from Figure 2 from Example 6 and analyse the coverage for argument C. The first step is to create F. Let $F = \{\mathtt{A},\mathtt{B}\}$ so that $\mathcal{CC}^F = \{\ p(\mathtt{A}) = x,\ p(\mathtt{B}) = y\}$. We need to assign values to x and y s.t. constraints in $\mathcal{C}$ are not violated. Looking at ϕ_1 and ϕ_2, we can restrict the domain of x and y as follows: $x \in (0.5, 1]$ and $y \in [0, 0.5)$. No matter what values we assign to x and y from the restricted domain, we cannot find a value $z \in [0,1]$ s.t. $\mathcal{CC}^F \cup \mathcal{C} \Vdash p(\mathtt{C}) \neq z$. Therefore, C is not partially covered by F.

Now we will analyse the coverage for D. Let $F = \{\mathtt{C}\}$ so that $\mathcal{CC}^F = \{\ p(\mathtt{C}) = x\ \}$. Similar to the previous case , we will assign a value to x s.t. constraints in $\mathcal{C}$ are not violated. We proved in Example 6 that C is not default covered, so x can take any value in the closed interval [0,1]. Hence, $\mathcal{CC}^F$ is consistent together with $\mathcal{C}$ for any value of x in [0,1]. Let us slice the interval $[0,1]$ into two small intervals s.t $i_1 = [0, 0.5]$ a closed interval and $i_2 = (0.5, 1]$ a half-open interval. Now, if x takes its values from i_1(i.e., $x \in [0, 0.5]$), we can deduce that $\mathcal{C} \cup \{\ p(\mathtt{C}) = x\ \} \Vdash p(\mathtt{D}) > 0.5$. Alternatively, we can say, for example, $\mathcal{C} \cup \{\ p(\mathtt{C}) = x\ \} \Vdash p(\mathtt{D}) \neq 0.4$. If $x \in (0.5, 1]$, we can deduce that $\mathcal{C} \cup \{\ p(\mathtt{C}) = x\ \} \Vdash p(\mathtt{D}) \leq 0.5$. Therefore, for example, $\mathcal{C} \cup \{\ p(\mathtt{C}) = x\ \} \Vdash p(\mathtt{D}) \neq 0.6$. Since for any suitable belief assignment to F it holds that D cannot take some values, we conclude that D is fully covered by F.

Example 9. Let us consider the graph from Figure 3 from Example 7 and analyse the coverage for argument A. Now we will consider the arguments B and C to form F, i.e., $F = \{\mathtt{B},\mathtt{C}\}$. The constraint combination $\mathcal{CC}^F = \{\ p(\mathtt{B}) = 1,\ p(\mathtt{C}) = 0.5\ \}$ is consistent with $\mathcal{C}$ i.e., $\mathcal{CC}^F \cup \mathcal{C} \not\Vdash \perp$. According to ϕ_2, $\mathcal{CC}^F \cup \mathcal{C} \Vdash p(\mathtt{A}) < 0.5$. In other words, we can say, for example $\mathcal{CC}^F \cup \mathcal{C} \Vdash p(\mathtt{A}) \neq 0.6$. So, we have partial coverage here.

Consider another constraint combination $\mathcal{CC}^F = \{ \ p(\mathtt{B}) = 0.5, \ p(\mathtt{C}) = 0.5 \ \}$. With this, ϕ_2 is not satisfied, so we cannot constrain the belief in $\mathtt{A}$. Therefore $\mathtt{A}$ is not fully covered by the arguments set $\{\mathtt{B},\mathtt{C}\}$.

So far we have considered a set F, on the basis of which we have tested the existence of partial or full coverage of arguments. Sometimes we do not need to know what exactly constitutes F as long as there is such a set that an argument is covered. To take this into account, we additionally introduce the notion of **arbitrary coverage**, which is defined as follows:

Definition 10. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph. An argument $\mathtt{A} \in \mathsf{Nodes}(\mathcal{G})$ has **arbitrary full/partial coverage** iff there exists a set of arguments $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{A}\}$ s.t. $\mathtt{A}$ is fully or partially covered w.r.t. F.

The following relationships between the various forms of coverage can be shown straightforwardly:

Proposition 1. *Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph, $\mathtt{A} \in \mathsf{Nodes}(\mathcal{G})$ be an argument and $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{A}\}$ be a set of arguments. The following hold:*

- *If $\mathtt{A}$ is default covered in X, then it is partially and fully covered w.r.t. any set of arguments G $\subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{A}\}$, but not necessarily vice versa.*
- *If $\mathtt{A}$ is fully covered in X w.r.t. F, then it is partially covered in X w.r.t. F, but not necessarily vice* versa

6.1.2 Relation Coverage

Given a relation in the argument graph, it is not necessary that the source argument always have an effect on the target argument. In other words, a change of belief in the source argument may have no effect on the target argument. Therefore, to capture this effectiveness of one argument over the other, we introduce the notion of relation coverage or **effectiveness**.

Definition 11. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph, $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{B}\}$ and $G = F \setminus \{\mathtt{A}\}$ be sets of arguments. The relation represented by $(\mathtt{A},\mathtt{B}) \in \mathsf{Nodes}(\mathcal{G}) \times \mathsf{Nodes}(\mathcal{G})$ is:

- **effective** w.r.t. F if there exists a constraint combination $\mathcal{CC}^F$ and values x,$y \in [0, 1]$ s.t.
  - $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash \bot$, and
  - $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash \bot$, and
  - at least one of the following conditions holds:
    * $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq x$ and $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \Vdash p(\mathtt{B}) \neq x$, or
    * $\mathcal{C} \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \neq x$ and $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash p(\mathtt{B}) \neq x$.
- **strongly effective** w.r.t. F if for every constraint combination $\mathcal{CC}^F$ s.t. $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash \bot$, there exist values x,$y \in [0, 1]$ s.t. $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash \bot$, and at least one of the following conditions holds:
  - $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq x$ and $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \Vdash p(\mathtt{B}) \neq x$, or
  - $\mathcal{C} \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \neq x$ and $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash p(\mathtt{B}) \neq x$.

A relation is said to be effective w.r.t. a set of arguments F with a particular belief assignment if the belief in the target can initially take any value, but is unable to take some values once the belief in the source is altered or vice versa. A relation is strongly effective if we observe the same behaviour for any suitable belief assignments to F.

Example 10. Let us consider a graph $\mathcal{G}$ with set of constraints given below:

$\mathcal{C} = \{\ \phi_1 : p(\mathtt{A}) > 0.5 \rightarrow p(\mathtt{B}) > 0.5,\ \phi_2 : p(\mathtt{C}) > 0.5 \rightarrow p(\mathtt{B}) > 0.9\}$

Consider relation $(\mathtt{A},\mathtt{B})$, where $\mathtt{A}$ is source and $\mathtt{B}$ is the target. We obtain F by removing the target $\mathtt{B}$ from $\mathsf{Nodes}(\mathcal{G})$. Hence $F = \{\mathtt{A},\mathtt{C}\}$, further G is obtained by removing source argument $\mathtt{A}$ from F. Hence $G = \{\mathtt{C}\}$. With F and G we will construct the constraint combinations as below:

$$\mathcal{CC}^F = \{\ p(\mathtt{A}) = 0,\ p(\mathtt{C}) = 0\} \quad \mathcal{CC}^F|_G = \{\ p(\mathtt{C}) = 0\}$$

We can deduce that $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq 0.2$. Because when $p(\mathtt{A}) = 0$, $p(\mathtt{B})$ can take any value in $[0,1]$. Now let us alter the belief we have in $\mathtt{A}$. Let $p(\mathtt{A}) = 0.6$. Then we can further deduce that $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A} = 0.6)\} \Vdash p(\mathtt{B}) \neq 0.2$. Initially the belief in $\mathtt{B}$ could take any value, but the moment we altered the belief in $\mathtt{A}$ to 0.6, it cannot take 0.2. Therefore the relation $(\mathtt{A},\mathtt{B})$ is effective w.r.t. F.

Now let us consider the relation $(\mathtt{C},\mathtt{B})$. Below are the constraints set:

$$\mathcal{CC}^F = \{\ p(\mathtt{A}) = x,\ p(\mathtt{C}) = y\} \quad \mathcal{CC}^F|_G = \{\ p(\mathtt{A}) = x\}$$

$\mathcal{CC}^F$ is consistent for any values of x,$y \in [0,1]$. But for y, let us slice the interval $[0,1]$ into $y_1 = [0,0.5]$ and $y_2 = (0.5,1]$. If we consider x and $y = y_1$, then $\mathcal{C} \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq 0.9$. When we alter the belief in $\mathtt{C}$ as $p(\mathtt{C}) = 1$, then $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{C} = 1)\} \Vdash p(\mathtt{B}) \neq 0.9$. If we consider x and $y = y_2$, then initially $\mathcal{C} \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \neq 0.9$. But when we change the belief in $\mathtt{C}$ to 0 then $\mathcal{C} \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{C} = 0)\} \not\Vdash p(\mathtt{B}) \neq 0.9$. Thus, for every value of x and y it holds that change in belief of $\mathtt{C}$ has an effect on the belief of $\mathtt{B}$. So we conclude that relation $(\mathtt{C},\mathtt{B})$ is strongly effective w.r.t F or $\{\mathtt{A},\mathtt{C}\}$.

In certain scenarios, constraints are defined in such a way that it is not possible to check the effectiveness of relations. For example, if arguments are covered by default, we do not have the flexibility to vary belief in arguments to test the effectiveness of relations. To overcome this, we relax the rules of effectiveness, where effectiveness is determined using derived constraints from $\mathcal{C}$ rather than from $\mathcal{C}$ itself. This form of effectiveness is called semi-effectiveness.

Definition 12. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph, $Z \subseteq \mathsf{Closure}(\mathcal{C})$ be a consistent set of epistemic constraints, $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{B}\}$ and $G = F \setminus \{\mathtt{A}\}$ be sets of arguments. The relation represented by $(\mathtt{A},\mathtt{B}) \in \mathsf{Nodes}(\mathcal{G}) \times \mathsf{Nodes}(\mathcal{G})$ is:

- **semi-effective** w.r.t. (Z,F) if there exists a constraint combination $\mathcal{CC}^F$ and values x,$y \in [0,1]$ s.t.
  - $Z \cup \mathcal{CC}^F \not\Vdash \perp$, and
  - $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash \perp$, and
  - at least one of the following conditions holds:
    - $Z \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq x$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \Vdash p(\mathtt{B}) \neq x$, or
    - $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \neq x$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash p(\mathtt{B}) \neq x$.
- **strongly semi-effective** w.r.t. (Z,F) if for every constraint combination $\mathcal{CC}^F$ s.t. $Z \cup \mathcal{CC}^F \not\Vdash \perp$, there exist values x,$y \in [0,1]$ s.t. $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash \perp$, and at least one of the following conditions holds:
  - $Z \cup \mathcal{CC}^F \not\Vdash p(\mathtt{B}) \neq x$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \Vdash p(\mathtt{B}) \neq x$, or
  - $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \neq x$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) = y\} \not\Vdash p(\mathtt{B}) \neq x$.

The definitions for semi-effectiveness are the same as for effectiveness, except that to test effectiveness we use constraints that are derivable from the original constraints.

6.2 Relation Types

When we instantiate an argument graph with the available information in the knowledge base, the nature of the relationship between any two arguments is determined by the constraints that affect them. The influence we see in such graphs from the parent to its target can sometimes be an illusion. For example, if we have: A supports B, C attacks B, A supports C, if we consider the first constraint, we might think that A supports B. But in a bigger picture, if we analyse all constraints, we find that A supports B's attacker, therefore, from the global point of view, A has a negative influence on B.Similarly, an attacker cannot negatively influence the attackee all the time. In short, to determine the relationship between any two arguments, we must also consider other arguments in the graph. Let us look at two examples covering these scenarios.

Example 11. Consider the graph from Figure 4 and below set of constraints:

$\phi_1 : (p(\mathtt{B}) > 0.5 \wedge p(\mathtt{C}) < 0.5) \rightarrow p(\mathtt{A}) > 0.5$
$\phi_2 : p(\mathtt{B}) > 0.5 \rightarrow p(\mathtt{C}) > 0.5$
$\phi_3 : p(\mathtt{C}) > 0.5 \rightarrow p(\mathtt{A}) < 0.5$

If we want to determine the relation B-A by looking at constraints that concern both of them (ϕ_1), it is evident that B supports A. However, if we consider all the constraints, believing B would result in believing C which in turn leads to disbelieving A.Therefore, B hardly influences A in a positive way.

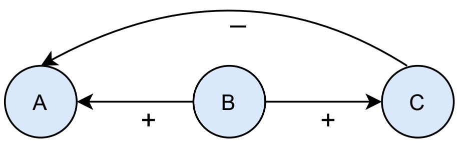

Figure 4: Argument graph for Example 11

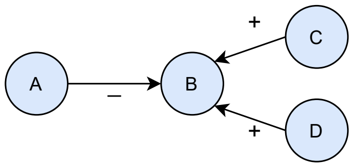

Figure 5: Argument graph for Example 12

Example 12. Consider the graph from Figure 5 and a constraint

$\phi : p(\mathtt{A}) > 0.5 \wedge p(\mathtt{C}) < 0.5 \wedge p(\mathtt{D}) < 0.5 \rightarrow p(\mathtt{B}) < 0.5$

Now we will analyse the relation A-B. If A is believed, then B is disbelieved unless C and D are believed. Therefore the attack of A over B is nullified by the support from C and D. Thus, if we believe A, we do not necessarily disbelieve B because of the presence of C and D. For example, we can construct a constraint like below to reflect this.

$\psi : p(\mathtt{C}) > 0.5 \vee p(\mathtt{D}) > 0.5 \rightarrow p(\mathtt{B}) > 0.5$

Looking at examples 11 and 12, we can see that determining the nature of a given relation depends on the constraints chosen, and the process of choosing such constraints is not always objective. Therefore, we redefine the terms "attack", "support" and "dependence" and introduce two additional terms, viz. "subtle" and "unspecified".

Definition 13. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph. Let $Z \subseteq \mathsf{Closure}(\mathcal{C})$ be a consistent set of epistemic constraints, $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{B}\}$ and $G = F \setminus \{\mathtt{A}\}$ be sets of arguments. The relation represented by $(\mathtt{A},\mathtt{B}) \in \mathsf{Nodes}(\mathcal{G}) \times \mathsf{Nodes}(\mathcal{G})$ is:

- **attacking** w.r.t. (Z,F) if it is semi–effective w.r.t. (Z,F) and for every constraint combination $\mathcal{CC}^F$ s.t. $Z \cup \mathcal{CC}^F \not\Vdash \bot$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) > 0.5\} \not\Vdash \bot$, if $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \leq 0.5$ then $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) > 0.5\} \Vdash p(\mathtt{B}) \leq 0.5$
- **supporting** w.r.t. (Z,F) if it is semi–effective w.r.t. (Z,F) and for every constraint combination $\mathcal{CC}^F$ s.t. $Z \cup \mathcal{CC}^F \not\Vdash \bot$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) > 0.5\} \not\Vdash \bot$, if $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \geq 0.5$ then $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) > 0.5\} \Vdash p(\mathtt{B}) \geq 0.5$

- **dependent** w.r.t. (Z,F) if it is semi–effective but neither attacking nor supporting w.r.t. (Z,F)
- **subtle** w.r.t. (Z,F) if it is semi–effective and both attacking and supporting w.r.t. (Z,F)
- **unspecified** w.r.t. (Z,F) if it is not semi–effective w.r.t. (Z,F)

Here we want to interpret attack as a relation which has no positive attack, by removing the rigidity that it must have a negative effect. An attack, then, is a relation in which a target argument that is not believed persists as such even if the source is believed. Support is defined in a similar way.

Based on the constraints and arguments we use to test, the relation can be considered attacking or supporting due to vacuous truth. (i.e., lack of evidence to prove that a relation is non-attacking or non-supporting). For example, we can never find $\mathcal{CC}^F$ that is consistent with Z. To go further, even if we find $\mathcal{CC}^F$, it may not be suitable in a way that $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \leq 0.5$ to show that the relation is indeed attacking. To avoid this, we enforce additional conditions as below.

Definition 14. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be a consistent epistemic graph. Let $Z \subseteq \mathsf{Closure}(\mathcal{C})$ be a consistent set of epistemic constraints, $F \subseteq \mathsf{Nodes}(\mathcal{G}) \setminus \{\mathtt{B}\}$ and $G = F \setminus \{\mathtt{A}\}$ be sets of arguments. Then a supporting (resp. attacking, dependent, subtle) relation $(\mathtt{A},\mathtt{B})$ is **strong** w.r.t. (Z,F) if:

- for every constraint combination $\mathcal{CC}^F$ it holds that $Z \cup \mathcal{CC}^F \not\Vdash \bot$ and $Z \cup \mathcal{CC}^F|_G \cup \{p(\mathtt{A}) > 0.5\} \not\Vdash \bot$
- and there is at least one constraint combination $\mathcal{CC}^F$ s.t. $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \geq 0.5$ (resp. $Z \cup \mathcal{CC}^F \Vdash p(\mathtt{B}) \leq 0.5$ or both)

Example 13. We will consider below set of constraints and try to analyse the relation $(\mathtt{B},\mathtt{A})$:

$\phi_1 : p(\mathtt{A}) > 0.5 \rightarrow p(\mathtt{B}) > 0.5 \vee p(\mathtt{C}) > 0.5$
$\phi_2 : (p(\mathtt{D}) < 0.5 \wedge (p(\mathtt{B}) > 0.5 \vee p(\mathtt{C}) > 0.5)) \rightarrow p(\mathtt{A}) > 0.5$
$\phi_3 : p(\mathtt{B}) > 0.5 \rightarrow p(\mathtt{D}) > 0.5$
$\phi_4 : p(\mathtt{D}) > 0.5 \rightarrow p(\mathtt{A}) < 0.5$

Let $Z = \mathcal{C}$ and $F = \{\mathtt{B},\mathtt{C}\}$ so that we have a constraint combination $W = \{p(\mathtt{B}) = x, p(\mathtt{C}) = y\}$. For every $x,y \in [0,1]$ W is consistent with Z. If we take $W_1 = \{p(\mathtt{B}) = 1, p(\mathtt{C}) = 1\}$ and $W_2 = \{p(\mathtt{B}) = 0, p(\mathtt{C}) = 1\}$ then $Z \cup W_1 \Vdash p(\mathtt{A}) \neq 1$ and $Z \cup W_2 \not\Vdash p(\mathtt{A}) \neq 1$. Therefore relation $(\mathtt{B},\mathtt{A})$ is semi–effective w.r.t. (Z,F). Furthermore, $Z \cup \{p(\mathtt{C}) = y\} \cup \{p(\mathtt{B}) > 0.5\} \not\Vdash \bot$. For $x > 0.5$, $Z \cup W \Vdash p(\mathtt{A}) < 0.5$, for $x \leq 0.5$ and $y \leq 0.5$, $Z \cup W \Vdash p(\mathtt{A}) < 0.5$, and for $x \leq 0.5$ and $y > 0.5$, $\mathtt{A}$ can take any probability. Hence we conclude that $(\mathtt{B},\mathtt{A})$ is strongly attacking w.r.t. (Z,F).

6.3 Epistemic Semantics

The task of epistemic semantics is to assign probabilities to arguments by analysing the epistemic graph. We define epistemic semantics formally as follows:

Definition 16. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be an epistemic graph. An **epistemic semantics** associates X with a set $\mathcal{R} \subseteq \mathsf{Dist}(\mathcal{G})$, where $\mathsf{Dist}(\mathcal{G})$ is the set of all belief distributions over $\mathcal{G}$.

Despite the availability of such distributions, it is reasonable to accept only those that respect the constraints defined over the graph. Therefore, we further introduce the notion of satisfaction semantics.

Definition 17. For an epistemic graph $(\mathcal{G}, \mathcal{L}, \mathcal{C})$, a distribution $P \in \mathsf{Dist}(\mathcal{G})$ meets the **satisfaction semantics** iff $P \in \mathsf{Sat}(\mathcal{C})$

Once we have a set of satisfying distributions, we want to further select them based on a preference. To do this, we use the following means of comparison between any two distributions.

Definition 18. Let $X = (\mathcal{G}, \mathcal{L}, \mathcal{C})$ be an epistemic graph and $P, P' \in \mathsf{Dist}(\mathcal{G})$ be probability distributions. We say that:

- $P \lesssim_A P'$ iff $\{ \mathtt{A} \mid P(\mathtt{A}) > 0.5 \} \subseteq \{ \mathtt{A} \mid P'(\mathtt{A}) > 0.5 \}$
- $P \lesssim_R P'$ iff $\{ \mathtt{A} \mid P(\mathtt{A}) < 0.5 \} \subseteq \{ \mathtt{A} \mid P'(\mathtt{A}) < 0.5 \}$
- $P \lesssim_U P'$ iff $\{ \mathtt{A} \mid P(\mathtt{A}) = 0.5 \} \subseteq \{ \mathtt{A} \mid P'(\mathtt{A}) = 0.5 \}$
- $P \lesssim_I P'$ iff $\{ \mathtt{A} \mid P(\mathtt{A}) > 0.5 \} \subseteq \{ \mathtt{A} \mid P'(\mathtt{A}) > 0.5 \}$ and $\{ \mathtt{A} \mid p(\mathtt{A}) < 0.5 \} \subseteq \{ \mathtt{A} \mid P'(\mathtt{A}) < 0.5 \}$

We refer to these orderings as acceptance, rejection, undecided and information orderings. A distribution P' is preferred over P if P' leads to believing more or the same number of arguments as those of P. Other orderings can be understood in a similar fashion. These approaches can be further refined to take the actual degrees (instead $> 0.5, < 0.5$ or $= 0.5$) by considering belief maximization and minimization approaches such as entropy.

Definition 19. For a probability distribution P, the **entropy** $H(P)$ of P is defined as

$$H(P) = - \sum_{\Gamma \subseteq \mathsf{Nodes}(\mathcal{G})} P(\Gamma) \log P(\Gamma)$$

with $0 \log 0 = 0$.

Entropy as a general term refers to randomness or uncertainty. The entropy of a probability distribution measures its indeterminacy. If a probability distribution is absolutely certain as to the belief and disbelief of all the arguments in the graph, then the entropy is minimal, i.e., if $P_1(\Gamma) = 1$ for some $\Gamma \subseteq \mathsf{Nodes}(\mathcal{G})$ and $P_1(\Gamma') = 0$ for every other $\Gamma' \subseteq \mathsf{Nodes}(\mathcal{G})$ then $H(P_1) = 0$. If the distribution is uniform (absolutely no idea about the arguments), i.e., $P_0 : 2^{\mathsf{Nodes}(\mathcal{G})} \rightarrow x$ where x is a constant and $x = \frac{1}{2^{\mathsf{Nodes}(\mathcal{G})}}$ then $H(P_0) = - \log \frac{1}{2^{\mathsf{Nodes}(\mathcal{G})}}$ (maximal entropy). With this approach we prefer a distribution that yields minimal entropy. This is because the observed arguments are most likely to occur under this distribution.

7 Conclusion

In this paper, we have looked at a new formalism for representing arguments that overcomes the challenges in existing frameworks. We will consider each of these previously mentioned challenges and see how the proposed framework - Epistemic Graphs - effectively addresses them.

Epistemic graphs allow us to assign probabilities to arguments reflecting the belief we have in them, rather than just saying they are either accepted or rejected. Thus, there is **fine-grained acceptance** for arguments. Between any two arguments, a local support cannot be an obvious support, or a local attack cannot be an obvious attack. Therefore, any relationship between arguments can be modelled as **positive, negative, or dependent** using epistemic graphs by analysing the relation w.r.t. all available constraints.

Two different scenarios for a given argument graph can be handled by creating two different sets of constraints to deal with **context sensitivity**. For the same argument graph, two people may have different beliefs about the arguments in it. We can model such **different perspectives** by creating constraints so that certain arguments are accepted or rejected depending on the perspective. Epistemic graphs offer us flexibility in defining constraints and beliefs in arguments. With this agents can put their own version of analysis for the graphs, even though it may seem irrational. Thus, we can account for **imperfect agents**. **Incomplete graphs** are those graphs that do not cover all the information that an agent possesses. We can handle such graphs by safely omitting constraints concerning certain arguments and relations that we are not sure how the agent interprets.

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