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A hybrid artificial neural network-GJR modeling approach to forecasting currency exchange rate volatility

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ABSTRACT

The study examines the integration of an asymmetric Glosten, Jagannathan, and Runkle (GJR) model into an artificial neural network (ANN) comprising of a NARX (Nonlinear Autoregressive model with eXogenous inputs) augmented by a NAR network. The empirical results obtained from the study of five (5) major currency pairings reveal that, compared to the benchmark generalized autoregressive conditional heteroskedasticity (GARCH), GJR and Asymmetric Power Generalised Autoregressive Conditional Heteroskedasticity (APGARCH) models, the proposed hybrid ANN-GJR model provides an overall substantial improvement in exchange rate volatility forecasting precision by effectively capturing asymmetric volatility as well as volatility clusters. In terms of the MSE, the MAD and the MAPE measures, a significant improvement of the measured forecast accuracy was found when using the ANN hybrid model compared to the benchmark models. Applying measurements from a heteroscedasticity-adjusted mean squared error (HMSE) model, the hybrid ANN-GJR model can significantly reduce the error associated with using only the GJR model by 90%. The study also investigated the effect of incorporating commodity prices series of oil and gold as additional input variables. It was found that depending on the specific currency pair understudy, the inclusion of commodity price series potentially improves model performance over different forecast horizons.

1. Introduction

A major phenomenon that has rapidly transformed and improved the socio-economic status of individual corporations, institutions and our world in its entirety is globalization. In fact, financial globalization is in particular widely regarded as inevitably central to the survival of financial institutions and national economies. With its key drivers including the liberalization of national financial and capital markets as well as rapid improvements in information technology, it has become a mainstay of the world economy while its reach and associated impact is expected to keep increasing [1,2]. In recent decades, as a direct consequence of the aforementioned drivers, there has been a rapid spurt in the growth of foreign exchange markets through the increased cross-border capital. Thus unsurprisingly, the foreign exchange market is the largest and most liquid of the current financial markets. A number of economic indices exist for this market but perhaps the most crucial is exchange rates [3]. Primarily, the foreign exchange market comprises of three inter-related parts; spot transactions, forward transactions, and derivative contracts. As traders react to new information, currency markets can be volatile and exhibit periods of volatility clustering [4]. When compared to the volatility of inflation rates and relative price levels, exchange rates and their rates of change with time are more volatile [5,6]. In the context of international trade, the impact of this volatility on national monetary policies especially for countries whose economic growth is largely dependent on export growth has been identified by national governments as worthy of consideration [1,7,8]. Accordingly, central banks often monitor exchange rate movements and periodical fluctuations for macroeconomic analysis and market surveillance purposes. The subject of effectively monitoring and managing exchange rate fluctuations and movements has attracted not only the interests of national policy makers but also financial economists, investors and corporate managers who given the growth of the number of international portfolios have embraced the need to address exchange rate volatility related risks [9,10].

In academia, contributions to this all-important subject can be categorized into two collective empirical modeling approaches. The first and relatively more modern approach involves time series modeling. This approach focuses on time series analysis of financial returns such as leverage effects, and volatility clustering

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and persistence. In this regard, the univariate Autoregressive Conditional Heteroscedasticity (ARCH) / Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, the "work-horse" of financial and economic time series modeling have been successfully employed in modeling volatilities of foreign exchange rates. The works of [11] and [12] are two earlier examples of the applications of such models. More recent applications of these univariate models can be found in other studies such as that of [13] who utilized data from 30/06/2003 to 31/03/2008 in evaluating volatility forecasts of the United States Dollar (USD) to Mauritius Rupee (MUR) i.e. USD/MUR exchange rate using GARCH (1, 1) models with GED and Student's error distributions. Their findings indicated that the GARCH (1, 1) model with GED errors slightly outperformed the other models in their study. Also using daily data from a different time period i.e., 03/07/2006 to 30/04/2012, [14] analyzed the Bangladesh Taka ((BDT) to USD (BDT/USD) exchange rate volatility using GARCH-type models and discovered on the basis of their empirical findings that current volatility is significantly affected by past volatilities. In yet another recent study, the in-sample and out-of-sample forecast performance of Log-Linear Realized GARCH was found to be superior to other models as reported by Xu et al. [15].

Prior to the deployment of time series models, earlier attempts often focused on the premise that a number of macroeconomic variables such as inflation, gross domestic product (GDP), and interest rates can in general help in the explanation of exchange rates movements and fluctuations. For instance, while employing a multivariate approach, Meese, and Rogoff [16] identified macroeconomic variables including trade nominal income, trade balance, and money supply as determinants of exchange rate changes. Their findings are popularly referred to as exchange rate disconnect puzzle in literature as their proposed macroeconomic variables proved to be weaker predictors while higher predictive power was instead realized for a random walk model. Cheung et al. [17] analyzed the forecasting ability of four models namely Behavioral Equilibrium Exchange Rate (BEER), Stick-price monetary model, Balassa-Samuelson and Uncovered Interest Rate Parity adopting the criterion of mean squared error. They used exchange rates data for CAD/USD, GBP/USD, DM/USD, CHF/USD, JPY/USD, spanning the second quarter of 1973 to fourth of 2004. Cheung et al. [17] however found that whiles none of the tested models outperforms the random walk on forecasting, over longer horizons, structural models however presented, on mean, a higher forecasting power than the random walk hypothesis.

Lately, there has been growing interest in the adoption of artificial neural networks (ANNs) to analyze historical data and provide predictions on future movements in the foreign exchange market. Researchers have revealed increasingly improved performance using artificial neural network models which have proven to outperform other models in time series forecasts [1,18–20]. Multiple distinguishing features of ANNs make them valuable and attractive in forecasting. One of such is the fact that they are non-linear. In addition, ANNs are data-driven and can generalize. Previous studies to forecast exchange rate volatility with Neural Networks include the work of Podding [21] who investigated the problem of predicting the trend of the United States Dollar to German Deutsche Mark (USD/DEM) in comparison with results from the regression analysis. On the other hand, considerable research effort has gone into ANNs for forecasting exchange rates directly as against predicting volatility, Kuan and Liu [22] use both feed-forward (FFNN) and recurrent neural networks (RNN) to forecast British Pound to United States Dollar (GBP/USD), Canada Dollar to United States Dollar (CAD/USD), German Deutsche Mark to United States Dollar (DEM/ USD), Japanese Yen to United States Dollar (JPY/ USD), Swiss Franc to United States Dollar (CHF/ USD). Wu [23] also compares neural networks with Autoregressive integrated moving average (ARIMA) models in forecasting Taiwan New Dollar to United States Dollar (TWD/USD) exchange rates. Hann, and Steurer[24] make comparisons between the neural network and linear model in USD/DEM forecasting.

In spite of the numerous research conducted in the area of foreign exchange rate volatility forecast, to the best of our knowledge, previous studies on the use of neural network-based hybrid approaches is scanty. Other areas of finance are replete with successful implementations of such hybrid models which generally improve the forecasting ability of traditional models. For instance, Tseng et al. [25] used an asymmetric hybrid volatility forecast model together with a Neural Network to improve the forecasting power of the financial derivatives prices in the Taiwanese stock market. Based on their findings, they concluded that the Grey-Exponential generalized autoregressive conditional heteroskedastic (Grey -EGARCH) model has the highest forecasting power compared to the other models used in their study. In a recent application, Kristjanpoller and Minutolo [26] successfully employed a hybrid artificial neural network – GARCH model to forecast the volatility of oil price. They found that their proposed hybrid model improves the forecasting precision of the traditional models by up to 30% in terms of their adopted performance criterion. Various GARCH models have been applied in the modeling of the volatility of exchange rates in different countries. However, asymmetric GARCH models by addressing the challenge of heavy tails in financial data have gained significant attention. Brownlees et al. [27] introduced the Glosten Jagannathan and Runkle (GJR) model as a threshold autoregressive conditional heteroskedasticity (TARCH) model and reported that it was the best forecaster among asymmetric models and GARCH for one step or multi-step ahead forecasting. In more recent studies this observation is further confirmed [28–31]. Therefore the GJR model has promising prospect for incorporation in hybrid systems/models for the task of volatility forecasting.

In this paper, we seek to improve the foreign exchange rates volatility forecast precision of traditional models by the use of a hybrid ANN-GJR model. Exchange rates data for five (5) major currencies i.e., British Pound to United States Dollar (GBP/USD), Euro to United States Dollar (EUR/USD), Canada Dollar to United States Dollar (CAD/USD), Swiss Franc to United States Dollar (CHF/ USD), and Japanese Yen to United States Dollar (JPY/ USD) are utilized for the purpose of the study. Using heteroskedastically modified mean standard (HMSE) and absolute errors (HMAE) we assess the relative performance of the proposed hybrid model vis a vis a GARCH (1,1) model and a GJR (1,1) model. The study employs the GARCH (1,1) owing to its popularity and use in several previous works as a benchmark model. Since improving the asymmetric GARCH variant i.e., the GJR model is a direct target of the study it is also employed as a benchmark. We adopt a moving window approach for which the calculations are based on some suggested innovations in previous volatility related forecasting studies [26,32]. The moving window calculations are applied in generating both forecasts from the benchmark models as well as the neural network. A base model with GJR estimates and squared returns is firstly built. Followed by assessing the impact of two major commodity prices i.e., Oil and Gold as external input variables on the forecasting performance of the hybrid model. We finally investigate the best architecture for forecasting using the hybrid model by varying number of hidden layer sizes/nodes of the neural network. To determine the superiority of models generated from these combinations we also apply the Model Confidence Set (MCS) test procedure [33–35].

The need to improve exchange rate volatility forecast is very crucial thus reflecting the significance of the current work. Our study provides an extension to the field of expert systems, exchange rate volatility modeling and forecasting by applying an artificial neural network to an asymmetric GJR model in developing

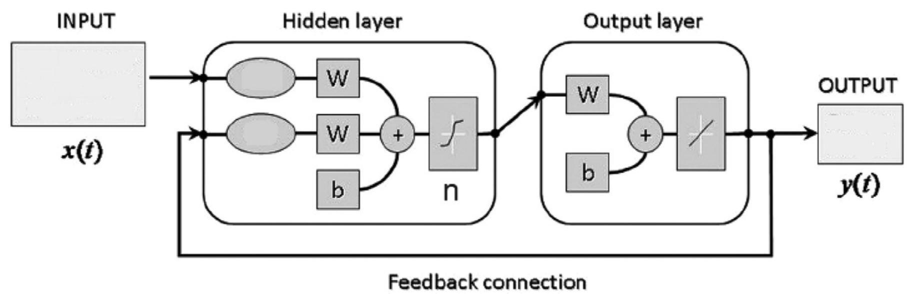

Fig. 1. An illustration of the NARX network architecture.

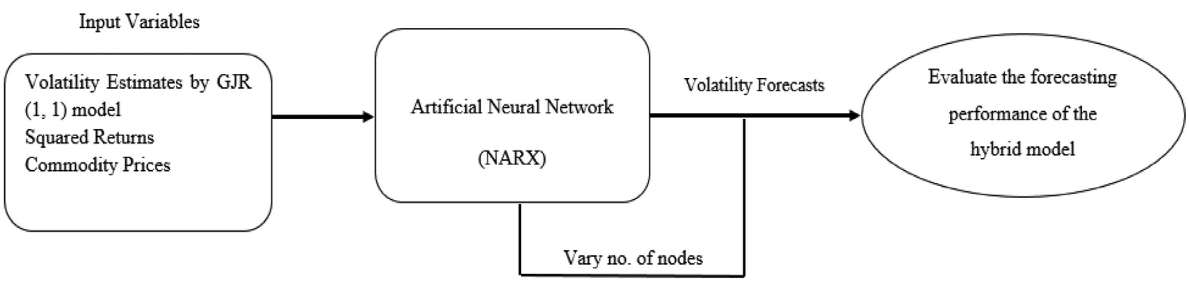

Fig. 2. Schematic diagram of the proposed hybrid model process.

a hybrid ANN-GJR model. In general, understanding and estimating exchange rate volatility is important for asset pricing, portfolio allocation, and risk management. More specifically, improved forecast from the proposed ANN-GJR model will be useful to multinational firms, financial institutions as well as traders who wish to hedge currency risks. Most traders of foreign currency options often attempt to make profits from either buying options on the basis of expectations that exchange rate volatility will rise or be writing options based on expectations that it will drop relative to that currently implied in currency option premiums [4]. Moreover, knowledge of exchange rate volatility is also important because exchange-rate risk may increase transaction costs and reduce gains to international trade.

The organization of this paper is as follows: section one provides an introduction which includes illustrations of the state of the art in this subject. Section two gives the methodology and provides details on the models utilized in the study. We also define the methods of comparison for measuring the accuracy of the models. In the following section, we discuss the characteristics of the employed data sets. The fourth section contains presentation and analysis of the obtained results and we finally follow that up with a summary of our key findings and their implications in the last section.

2. Methodology

Modeling and forecasting volatility is undoubtedly one of the most essential developments in empirical finance. The rising popularity of this field is albeit a direct consequence of the rapid growth in financial derivatives, quantitative trading and risk modeling. It is worth noting however that volatility modeling is quite demanding with associated difficulties derived from the latent nature of financial market volatility. An effective volatility study will require a prior determination of forecast targets and scope. Hence in this section, we define our objectives and discuss the competing models used in this study to forecast exchange rate volatility while illustrating our modeling approach.

For a given series comprising of successive daily currency exchange rates with assigned index t, let P_t denote the corresponding exchange rate while r_t denotes the computed exchange rates return which is the first difference of the natural logarithm of the exchange rate and is expressed as:

$$r_t = \log P_t - \log P_{t-1} \tag{1}$$

The forecasting objective of this study can thus be expressed in terms of P_t and r_t as;

$$\mathrm{RV}_t = \frac{1}{n}\sum_{i=t+1}^{t+21}(r_i - \bar{r}_t)^2 \tag{2}$$

Where RV_t is the realized volatility in the rates which as seen in Eq. (2) is calculated as sample variance log returns in an n, number of days window into the future [36]. Because realized volatility RV_t is observable, we can model and forecast it using standard and relatively straightforward time-series techniques. Thus, for this study, realized volatility is the forecasted parameter through a GARCH (1,1) model, a GJR (1,1) model and a hybrid artificial neural network model. The study firstly considered a 21-day window representing approximately one month of transactions and subsequently varied n according to $n = 14$, $n = 28$.

2.1. GARCH models

Forecasting exchange rate volatility has been fairly popular in finance and economic journals particularly in the last decade. A wide range of volatility models have been proposed in various studies conducted over different time periods and using different data frequency as well as exchange rate pairs. In spite of the numerous proposed models, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model and its related types have conventionally been employed more frequently. The GARCH family of models constitute a category of conditional variance models and can be said to primarily use optimal exponential weighting of historical returns to obtain a volatility forecast. These models have the ability to catch stylized facts, as not predictability of returns, presence of heavy tails in asset returns and volatility clustering. The autoregressive conditional heteroscedasticity (ARCH) model was the first such model suggested in 1982 by Engle. However, its successor, the generalized ARCH model (GARCH) proposed independently by Bollerslev [37] and Taylor [38] has gained more popularity owing to the fact that it is more parsimonious and avoids

Table 1
Descriptive statistics of the logarithmic returns of daily exchange rates.

	Mean (%)	Standard deviation (%)	Min (%)	Max (%)	Skewness	Kurtosis	Normality Test	ADF
AUD/USD	0.01451	0.88923	−7.61111	8.23363	−0.46	11.56	18,798	−14.89
CAD/USD	0.01023	0.59285	−3.24419	3.87902	−0.14	3.21	1446.6	−15.97
CHF/USD	0.01706	0.70366	−9.08968	4.35815	−0.54	10.35	15,168	−15.80
EUR/USD	0.01091	0.64406	−2.78099	3.73332	0.02	1.48	308.8	−14.35
GBP/USD	0.00267	0.59809	−3.96444	3.46029	−0.24	3.45	1694.1	−14.66

Note: The Normality Test is the Jarque–Bera test, which has a $\chi 2$ (q) distribution with 2 degrees of freedom and $p < 0.01$. The Augmented Dickey Fuller test tests up to the fourteenth lag and the $p < 0.01$.

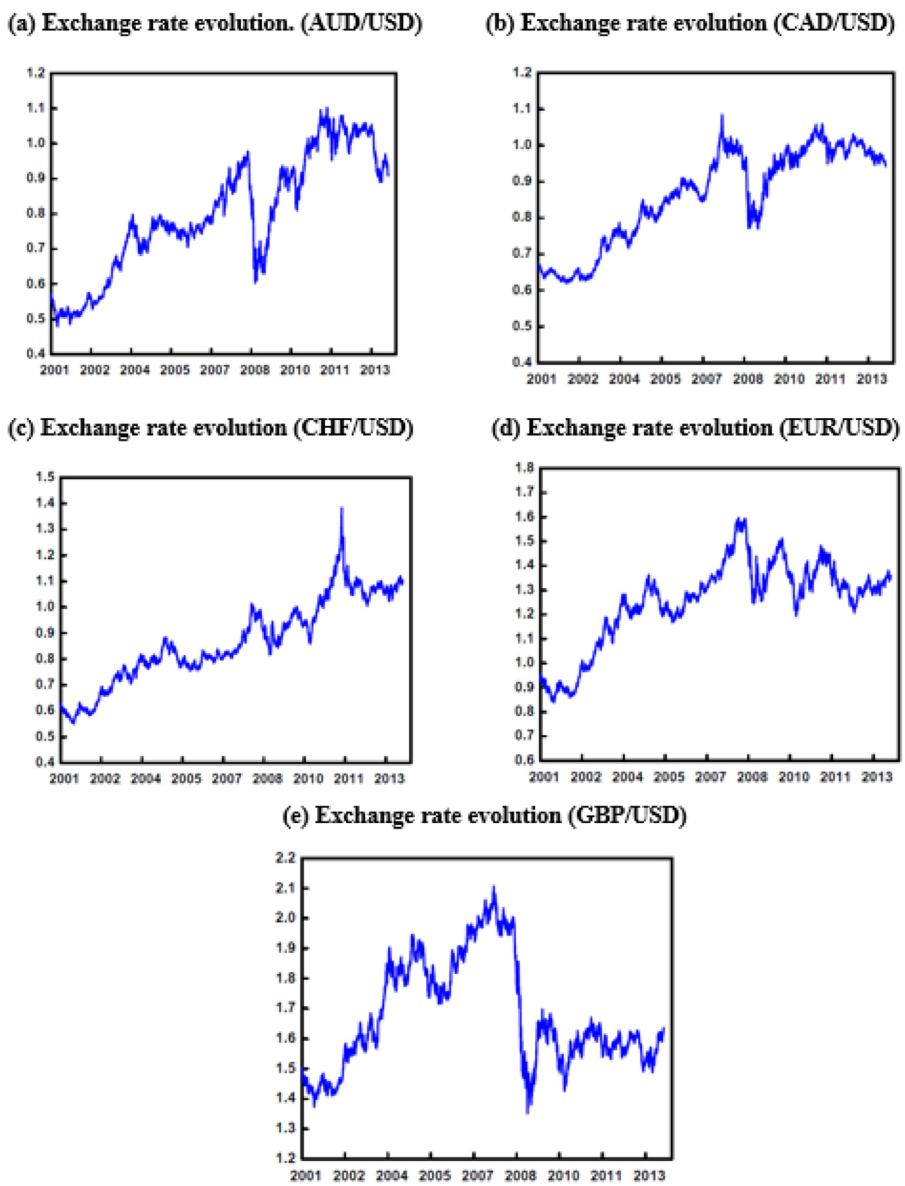

Fig. 3. Plots of the exchange rate evolution of the five major currencies against the USD.

overfitting thus reducing the likelihood to breach the non-negativity constraint. The principal restriction of the GARCH model is that all the explanatory variables in a GARCH and therefore ARCH model must be positive, this is referred to as the non-negativity constraint, clearly it is impossible to have a negative variance, as it consists of squared variables. The GARCH model is very often the basis for most volatility models. In this study, we deploy both symmetric and asymmetric GARCH models. The symmetric model is the GARCH (1,1) while the asymmetric model is the GJR- GARCH variant of Glosten et al. [39]. The GARCH model is symmetric because of the sign of the disturbance being ignored. On the other hand, the GJR-GARCH is an extension of GARCH with an additional leverage term added to capture possible asymmetries. As a result, large negative changes are more likely to be followed by large negative changes than positive changes in the GJR-GARCH model.

In this study, the GARCH (1,1) model and the GJR (1,1) model are used to forecast the volatility attributed to heteroscedasticity, using a moving window length of 252 days back (one year of

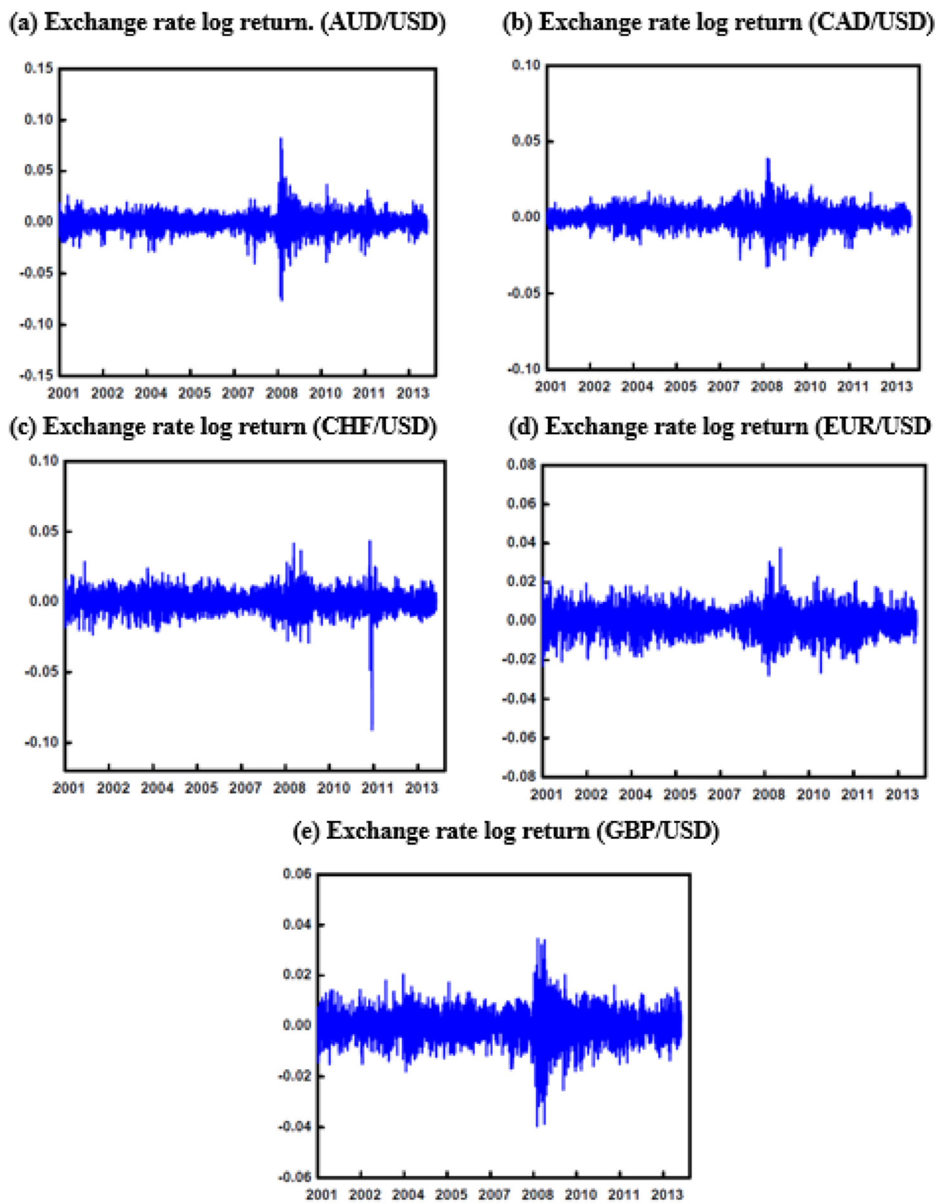

Fig. 4. Plots of the exchange rate log returns of the five major currencies against the USD.

Table 2
Performance results of benchmark models based on various loss functions and AIC/BIC criteria for forecasting AUD/USD exchange rate volatility.

Models	AUD/USD volatility (14d)			AUD/USD volatility (21d)			AUD/USD volatility (28d)			AUD/USD	
	MSE	MAD	MAPE	MSE	MAD	MAPE	MSE	MAD	MAPE	AIC	BIC
GARCH	0.2712	5.1478	68.85	0.3117	5.4870	63.81	0.3423	5.8739	63.34	−6.9537	−6.8808
GJR	0.2667	5.0187	66.72	0.3060	5.2992	61.55	0.3350	5.6237	60.63	−6.9602	−6.8630
APGARCH	0.3804	5.1943	51.43	0.3054	4.9063	46.92	0.2680	4.7962	44.83	−7.3556	−7.2856
	HMSE	HMAE		HMSE	HMAE		HMSE	HMAE			
GARCH	1.0727	0.6885		0.8242	0.6381		0.7989	0.6334			
GJR	0.9830	0.6672		0.7461	0.6155		0.7136	0.6063			
APGARCH	0.4546	0.5143		0.3429	0.4692		0.2987	0.4483			

MSE and MAD are multiplied by 10^7 and 10^5 respectively.

transactions). The general formulas for the GARCH (1,1) and GJR (1,1) models employed in this study are provided in Eqs. (3)–(5).

$$y_t = \mu + \varepsilon_t \tag{3}$$

Where $\varepsilon_t = \sigma_t z_t$ and for the GARCH (1,1)

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \tag{4}$$

Whereas for the GJR (1,1)

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \xi_1 I[\varepsilon_{t-1} < 0]\varepsilon_{t-1}^2 \tag{5}$$

Alongside the classic GARCH (1,1) and GJR (1,1) models we also apply the more advanced Asymmetric Power Generalised Autoregressive Conditional Heteroskedasticity (APGARCH) structure of [40] as benchmark. APGARCH incorporates all of the modifications and derivatives of the original GARCH model and innovates by in-

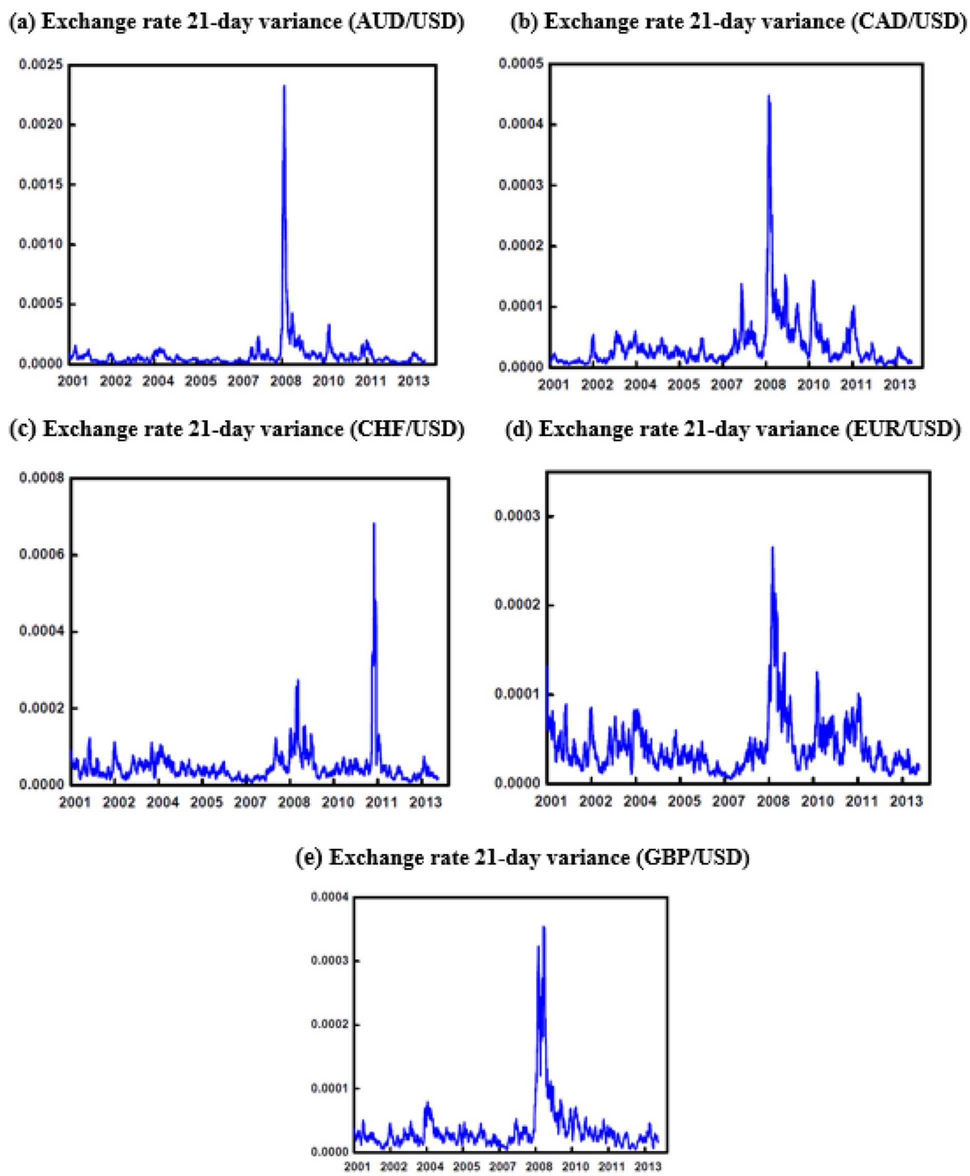

Fig. 5. Plots of the exchange rate 21-day variance of the five major currencies against the USD.

cluding a power term. It has interesting features in volatility modeling with the flexibility to be a linear relationship of standard deviation, $d = 1$, establishing itself as a model of stochastic volatility. The general formula of this model is provided in Eq. (6).

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|\varepsilon_{t-1}|\gamma_1|\varepsilon_{t-1}|)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{6}$$

where $\alpha > 0$, $\delta \geq 0$, $\beta_{\mathrm{j}} \geq 0$, $\alpha_{\mathrm{i}} \geq 0$, $-1 < \gamma_{\mathrm{i}} < 1$ and γ_{i} reflects the leverage effect.

2.2. Neural network model

Inspired by the quest to simulate the connections of input and output signals in the human brain, the introduction of artificial neural networks largely revolutionized the ability to solve a variety of problems in diverse fields. Classification, pattern recognition, signal filtering and interpolation are amongst the host of applications where artificial neural networks have being successfully deployed.

In specific applications such as time series prediction, a variety of ANNs have proved very useful even in instances where the related series display chaotic behavior. One of such ANNs is the Nonlinear AutoRegressive model with eXogenous inputs (NARX). It is a popular ANN which comprises of a multilayered perceptron, a simple recurrent network, and a feedforward backpropagation network. It uses the nonlinear mapping ability of neural networks and introduces the time-series concept of the linear autoregressive (AR) model, rendering the NARX model with a strong dynamic performance and a high immunity to noise. The underlying equation of the proposed NARX neural network can be stated as

$$y(t) = f(y(t-1), y(t-2), \ldots, y(t-d), x(t-1), x(t-2), \ldots, x(t-d)) \tag{7}$$

From the above Eq. (7) the output (target) y at time t ($y(t)$) is regressed on previous values (d) of the output and previous values of an exogenous (i.e., independent) input series ($x(t)$). To construct this neural network, we begin with the structure of a feedforward perceptron network in order to learn the behavior of the output (target) y_t by using inputs x_t, and modeled as a nonlinear

Table 3
Performance results based on various loss functions and AIC/BIC criteria for AUD /USD exchange rate volatility forecasting using the ANN-GJR hybrid model (1 hidden layer with 10 nodes).

Models	AUD/USD volatility (14d)						AUD/USD volatility (21d)						AUD/USD volatility (28d)					
	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)
ANN-GJR 2 VARS	0.0832	−68.82	1.5899	−68.32	**14.40**	**−78.42**	0.0427	−86.05	1.1723	−77.88	**9.37**	**−84.78**	0.0215	−93.58	**0.8246**	**−85.34**	**6.83**	**−88.73**
ANN-GJR 3 VARS (oil)	0.0461	−82.71	**1.3926**	**−72.25**	14.53	−78.22	0.0363	−88.14	**1.0405**	**−80.36**	9.60	−84.40	0.0162	−95.16	0.8313	−85.22	7.20	−88.13
ANN-GJR 3 VARS (gold)	**0.0431**	**−83.84**	1.4999	−70.11	15.86	−76.23	0.0385	−87.43	1.1269	−78.73	9.54	−84.50	**0.0146**	**−95.64**	0.8436	−85.00	7.37	−87.84
ANN-GJR 4 VARS	0.0526	−80.29	1.5593	−68.93	15.73	−76.43	**0.0254**	**−91.70**	1.1099	−79.06	10.09	−83.61	0.0154	−95.39	0.8523	−84.84	7.64	−87.39
	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC
ANN-GJR 2 VARS	**0.0484**	**−95.08**	**0.144**	**−78.42**	−9.1139	−8.0985	**0.0210**	−97.19	**0.0937**	**−84.78**	−9.4290	−8.4136	**0.0116**	**−98.37**	**0.0683**	**−88.73**	−9.6205	−8.6051
ANN-GJR 3 VARS (oil)	0.0493	−94.98	0.1453	−78.22	−8.0275	−6.7614	0.0213	−97.15	0.0960	−84.40	−8.5408	−7.2746	0.0117	−98.36	0.0720	−88.12	−8.7967	−7.5305
ANN-GJR 3 VARS (gold)	0.0663	−93.26	0.1586	−76.23	−7.5741	−6.3079	0.0197	**−97.36**	0.0955	−84.48	−8.2739	−7.0078	0.0121	−98.30	0.0737	−87.84	−8.8655	−7.5994
ANN-GJR 4 VARS	0.0575	−94.15	0.1573	−76.42	−8.2084	−6.6915	0.0218	−97.08	0.1009	−83.61	−8.4496	−6.9328	0.0138	−98.07	0.0764	−87.40	−7.9935	−6.4767

The number of variables means the number of inputs supplied to the ANN. For each model we use 2840 forecasts in computing the loss functions. The variation is calculated relative to the performance of the best benchmark model. The best performances are indicated in bold letters. MSE and MAD are multiplied by 10^7 and 10^5 respectively.

functional form of a regression model for y (output layer):

$$y_t = \Phi\left[\beta_0 + \sum_{i=1}^{q} \beta_i h_{it}\right] \tag{8}$$

Where Φ is the activation function for the output, which in this case is $\Phi(x) = x$, the linear function; β_0 is the output bias; β_i are the output layer weights. The hidden layer (h_{it}) can be mathematically represented by:

$$h_{it} = \Psi\left[\gamma_{i0} + \sum_{j=1}^{n} \gamma_{ij} x_{jt}\right] \tag{9}$$

With Ψ representing the activation function for the hidden neurons; γ_{i0} as the input bias; and γ_{ij} as the weights of the input layer. i is the subindex of the q neurons, and j is the subindex of the n inputs. Substituting Eq. (9) into Eq. (8) gives:

$$y_t = \Phi\left\{\beta_0 + \sum_{i=1}^{q} \beta_i \Psi\left[\gamma_{i0} + \sum_{j=1}^{n} \gamma_{ij} x_{jt}\right]\right\} \tag{10}$$

We now add the dynamic term, an autoregression on the output in order to describe a recurrent network where the hidden layers are described by Eq. (11):

$$h_{it} = \Psi\left[\gamma_{i0} + \sum_{j=1}^{n} \gamma_{ij} x_{jt} + \sum_{r=1}^{q} \delta_{ir} h_{r,t-1}\right] \tag{11}$$

where $h_{r,t-1}$ is the delayed feedback term and δ_{ir} its weight. By replacing Eq. (11) into Eq. (8), we finally have:

$$y_t = \Phi\left\{\beta_0 + \sum_{i=1}^{q} \beta_i \Psi\left[\gamma_{i0} + \sum_{j=1}^{n} \gamma_{ij} x_{jt} + \sum_{r=1}^{q} \delta_{ir} h_{r,t-1}\right]\right\} \tag{12}$$

An illustration of a typical NARX network structure is provided in Fig. 1.

2.3. Exchange rates and commodity prices

While factors such as interest rates and inflation rates have been shown to impact exchange rate volatility. Previous related studies are replete with empirical evidences of relationships between commodity prices and the behavior of exchange rates. Since economic growth and exports are directly related to a country's domestic industry, it is natural for some currencies to be correlated with commodity prices. For instance Chan et al. [41] reported on the basis of their findings that commodity/currency relationships exist contemporaneously. They further suggested that commodity returns information is rapidly incorporated into currency returns on a daily level. Arezki et al. [42] also reported that gold price volatility plays a key role in explaining the excessive exchange rate volatility of the Rand. With regards to the impact of oil price on exchange rates movements, Bodenstein et al. [43] illustrated that theoretically, an oil price shock may be transmitted to currency exchange rate through two main channels. These channels are the terms of trade channel and the wealth effects channel. A negative terms of trade shock (e.g., a drop-in oil prices for an oil exporter) drives down the price of non-traded goods in the domestic economy which in turn reduces real exchange rate. Nominal exchange rate may be required to depreciate too following adjustment of the real exchange rate if prices of non-traded goods tend to be sticky. Regarding the wealth effects channel, Beckmann and Czudaj [44] stated that when oil prices rise, wealth is transferred to oil exporting countries (in United States dollar terms) and is reflected as an improvement in exports and the current account balance in domestic currency terms. Hence, currencies of oil-exporting countries tend to appreciate while currencies of oil-importers tend to

Table 4
Performance results for ANN-GJR forecasts using different number of hidden layer nodes (AUD /USD).

Models	AUD/USD volatility (14d)				AUD/USD volatility (21d)				AUD/USD volatility (28d)			
	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC
2n	**0.0425**	**0.1319***	−9.6413	−9.4281	**0.0145**	**0.0792***	−10.0948	−9.8817	**0.0101**	**0.0609***	−10.3203	−10.1072
5n	**0.0425***	0.1352	−9.4997	−8.9857	0.0151*	0.0810	−9.5191	−9.0051	**0.0101***	0.0648	−10.1080	−9.5941
10n	0.0484	0.1440	−9.1139	−8.0985	0.0210	0.0937	−9.429	−8.4136	0.0116	0.0683	−9.6205	−8.6051
15n	0.0641	0.1496	−8.7995	−7.2826	0.0232	0.0975	−8.7191	−7.2022	0.0151	0.0737	−8.8133	−7.2965
20n	0.0610	0.1548	−8.2396	−6.2213	0.0279	0.1032	−8.3933	−6.375	0.0172	0.0783	−8.6036	−6.5853

The best performances are shown in bold letters and * means models that belong to the Model Confidence Set using a 10% confidence level.
The models with the best performing optimal number of variables were used in each case.

Table 5
Performance results of benchmark models based on various loss functions and AIC/BIC criteria for forecasting CAD/USD exchange rate volatility.

Models	CAD/USD volatility (14d)			CAD/USD volatility (21d)			CAD/USD volatility (28d)			CAD/USD	
	MSE	MAD	MAPE	MSE	MAD	MAPE	MSE	MAD	MAPE	AIC	BIC
GARCH	0.0118	1.8245	61.97	0.0125	1.8039	55.82	0.0129	1.8144	54.38	−7.5203	−7.4474
GJR	0.0120	1.8282	61.04	0.0126	1.7873	54.60	0.0130	1.8116	53.13	−7.5213	−7.4241
APGARCH	0.0136	1.8839	60.07	0.0151	1.9032	54.24	0.0167	1.9635	53.44	−7.5291	−7.4591
	HMSE	HMAE		HMSE	HMAE		HMSE	HMAE			
GARCH	0.9538	0.6197		0.6939	0.5582		0.6239	0.5438			
GJR	0.9145	0.6104		0.6588	0.5460		0.5861	0.5313			
APGARCH	0.8899	0.6006		0.6662	0.5423		0.6316	0.5344			

MSE and MAD are multiplied by 10^7 and 10^5 respectively.

depreciate in effective terms after a rise in oil prices. Also, due to the wealth effect leading to potential reinvestment of revenues in US dollar assets by oil-exporting countries, the US dollar risks appreciating in the short-run. Furthermore, the sharp increase in oil prices and oil price volatility in recent decades is believed to coincide with a closer link between oil prices and asset prices, including exchange rates. Of particular interest is the well-known example of a strong negative correlation between spot oil prices and USD exchange rates from the early 2000s. Undoubtedly, these studies point to the potential significance of commodity prices (in this case oil and gold) on the behavior of currency exchange rates.

2.4. The hybrid model

In this study, to develop and estimate our hybrid model we generally follow the procedure of [32]. However, our study distinguishes itself by focusing on improving forecasts obtained from an asymmetric GJR model. Additionally, while their employed neural network is the multilayer perceptron (MLP) network, we employ a Nonlinear AutoRegressive model with eXogenous inputs (NARX). The difference in the two neural networks lies in the fact that the latter is a combination of a simple recurrent network, a feedforward backpropagation network, and a multilayered perceptron.

For the initial forecast horizon (i.e., $h = 21$), the target realized volatility to forecast using the hybrid model is $t + 22$, which implies the standard deviation calculated 22 days from t, taking the last 21 data to calculate it [26]. The rolling window length is 252 days, which is roughly one trading year.

The independent input variables of the artificial neural network model are:

- GJR (1, 1) estimate: this is the result of the GJR model, forecasting 22 days in the future of t.
- Squared return: this is the squared exchange rate log return which can be a good predictor of the realized volatility.
- Commodity price: this is the price of selected commodities.

The neural network used is a NARX network implemented in MATLAB environment. The parameters are 1 hidden layer, 10 nodes and 2 input delays as well as feedback delays. In general, no rule of thumb exists for selecting the number of hidden layer nodes/sizes and delays. The best configuration depends on the nature of problem to be solved by the network. In this study we investigate the robustness of the proposed hybrid model by varying the number of nodes in the hidden layer. Hence, we conduct further analysis using 2, 5, 15 and 20 nodes. The NARX network typically divides data into three subsets. These subsets are the training set, validation set, and the testing set. The configured percentage for the sets that was used in this study is 70% training, 15% validation and 15% testing. For the training phase, the Levenberg–Marquardt backpropagation algorithm was employed. To ensure effective training, the dependent input variable was normalized within the range (0, 1). Also, we apply a filter to the outputs of the neural network such that if a negative variance is forecasted the last positive variance is instead used. Furthermore, for the purpose of our study, an initial base model (i.e., ANN-GJR 2 Var) was constructed using the GJR estimates and squared returns. After running and ensuring stability of the base model, we add oil price data to construct a different model (i.e., ANN-GJR 3 Var (oil)). A third hybrid model was also built by addition of gold price data to the base model (i.e., ANN-GJR 3 Var (gold)). A final model was subsequently constructed by adding both oil and gold price data to the base model (i.e., ANN-GJR 4 Var). A summary of the hybrid model process is provided in Fig. 2.

2.5. Performance measurement criteria

The above overall methodology was used in this study to in each case forecast the volatility of five different exchange rate pairings of the Australian dollar (AUD), Canadian dollar (CAD), Swiss Franc (CHF), the Euro (EUR) and the Great Britain pound (GBP) against the United States dollar (USD). Model performance was evaluated by use of selected loss functions. Mean Square Error (MSE), Mean Absolute Deviation (MAD) and Mean Absolute Percentage Error (MAPE) are amongst the measures by which model performance is assessed. These measures are relevant in order to analyze the errors. Their respective formulas are provided below:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(RV_i - \hat{\sigma}_i^2\right)^2 \tag{13}$$

Table 6
Performance results based on various loss functions and AIC/BIC criteria for CAD /USD exchange rate volatility forecasting using the ANN-GJR hybrid model (1 hidden layer with 10 nodes).

Models	CAD/USD volatility (14d)						CAD/USD volatility (21d)						CAD/USD volatility (28d)					
	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)
ANN-GJR 2 VARS	0.00301	−74.46	0.6252	−65.74	**16.51**	**−72.94**	0.00145	−88.40	0.4269	−76.11	10.75	−80.32	0.00079	−93.92	0.3219	−82.23	**7.84**	**−85.25**
ANN-GJR 3 VARS (oil)	**0.00184**	**−84.38**	**0.5956**	**−67.36**	16.87	−72.35	**0.00083**	**−93.39**	**0.4040**	**−77.39**	**10.63**	**−80.54**	**0.00054**	**−95.78**	**0.3096**	**−82.91**	7.89	−85.16
ANN-GJR 3 VARS (gold)	0.00241	−79.53	0.6336	−65.28	17.47	−71.37	0.00206	−83.50	0.4489	−74.88	11.13	−79.62	0.00058	−95.54	0.3201	−82.33	8.22	−84.52
ANN-GJR 4 VARS	0.00242	−79.42	0.6438	−64.72	18.03	−70.46	0.00144	−88.45	0.4380	−75.49	11.34	−79.23	0.00055	−95.77	0.3250	−82.06	8.43	−84.13
	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC
ANN-GJR 2 VARS	**0.0659**	**−92.79**	**0.1652**	**−72.94**	−6.9993	−5.9839	0.0282	−95.72	0.1075	−80.31	−6.7298	−5.7143	0.0146	−97.51	**0.0784**	**−85.24**	−8.1729	−7.1575
ANN-GJR 3 VARS (oil)	0.0685	−92.51	0.1687	−72.36	−6.6636	−5.3975	**0.0232**	**−96.48**	**0.1063**	**−80.53**	−7.9074	−6.6412	**0.0132**	**−97.75**	0.0789	−85.15	−8.1809	−6.9148
ANN-GJR 3 VARS(gold)	0.0726	−92.06	0.1748	−71.36	−7.2647	−5.9986	0.0262	−96.02	0.1113	−79.62	−7.7589	−6.4928	0.0151	−97.42	0.0822	−84.53	−7.8305	−6.5643
ANN-GJR 4 VARS	0.0788	−91.38	0.1803	−70.46	−7.4874	−5.9706	0.0267	−95.95	0.1134	−79.23	−7.9436	−6.4268	0.0157	−97.32	0.0843	−84.13	−8.0879	−6.5710

The number of variables means the number of inputs supplied to the ANN. For each model we use 2840 forecasts in computing the loss functions. The variation is calculated relative to the performance of the best benchmark model. The best performances are indicated in bold letters. MSE and MAD are multiplied by 10^7 and 10^5 respectively.

$$MAD = \frac{1}{n}\sum_{i=1}^{n}\left|RV_i - \hat{\sigma}_i^2\right| \quad (14)$$

$$MAPE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{RV_i - \hat{\sigma}_i^2}{RV_i}\right)\right] * 100 \quad (15)$$

However, for this study two other measures were identified as the most relevant. They are the Heteroscedasticity-adjusted Mean Square Error (HMSE) and the Heteroscedasticity-adjusted Mean Absolute Error (HMAE). HMSE typically corresponds to the mean of the squared percentage error and is described in Eq. (16). Congruently, HMAE corresponds to the mean of the absolute percentage error and is computed by use of Eq. (17). From the two Eqs. (16) and (17) it can be seen that both HMSE and HMAE demonstrate the relative extent of the error with respect to the objective value through the inter-day volatility.

$$HMSE = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\hat{\sigma}_i^2}{RV_i}\right)^2 \quad (16)$$

$$HMAE = \frac{1}{n}\sum_{i=1}^{n}\left|1 - \frac{\hat{\sigma}_i^2}{RV_i}\right| \quad (17)$$

HMSE and HMAE are thus considered the most relevant on the basis of suitability to the non-linear nature of the model studied herein. As noted in previous studies, assessment of non-linear models requires non-linear loss measurements. Since, for volatility series, it is possible to use their Heteroscedasticity in measuring loss functions, the two selected loss functions i.e. HMSE and HMAE were accordingly deemed more appropriate.

For the purpose of investigating which network configuration (using a different number of nodes) provides optimum performance we further apply only the HMSE and HMAE in evaluating different configurations of the base model using the model confidence set (MCS) proposed by Hansen et al. [33]. The MCS estimates a set of superior models from an initially chosen set, where superiority is defined by a user-specified loss function (i.e., HMSE and HMAE in our case). Hansen et al. [33] device a stepwise procedure in which they repeatedly test the null hypothesis that all currently selected models are identical with respect to expected loss. Upon rejection, they remove a model from the current set and repeat until rejection fails. Their approach provides an important advantage over standard practice by allowing for an asymptotic control of the family-wise error rate within the process of model selection [35,45,46]. Thus the general theory and formulations adopted in the study to conduct the test follow from previous work by Hansen et al. [33].

In addition to the above error measurements we also apply the Akaike Information Criterion (AIC) and the Bayesian information criterion (BIC). These two criteria have been commonly used in model selection for decades. The Akaike criterion is a measure of goodness-of-fit (likelihood) of an estimated statistical model. It can be expressed mathematically as:

$$AIC = (-2lnL)/n + (2k/n) \quad (18)$$

Where L is the maximized value of the likelihood function, k is the number of free parameters used in the model and n is the number of observations. This measure essentially comprises of two parts for which there is a compromise between the log maximized likelihood (i.e., log L, the lack of fit component) and k (the penalty component) which increases with the number of parameters and it is used to prevent overfitting. Penalizing the likelihood in the model is an attempt to select a more parsimonious model. It was designed to be an asymptotically unbiased estimate of the Kullback–Leibler index of the fitted model relative to the true model.

The BIC is a modification of the AIC and it provides a consistent criterion for models defined in terms of their posterior probability

Table 7
Performance results for ANN-GJR forecasts using a different number of hidden layer nodes (CAD /USD).

Models	CAD/USD volatility (14d)				CAD/USD volatility (21d)				CAD/USD volatility (28d)			
	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC
2n	**0.0504***	**0.1447***	−8.6477	−8.4346	**0.0173***	**0.0908***	−8.9307	−8.7176	**0.0117***	**0.0689***	−8.0325	−7.8194
5n	0.0602	0.1545	−8.0963	−7.5824	0.0224	0.1011	−7.5568	−7.0428	0.0122	0.0743	−9.0006	−8.4866
10n	0.0659	0.1652	−6.9993	−5.9839	0.0232	0.1063	−6.7298	−5.7143	0.0146	0.0789	−8.1729	−7.1575
15n	0.0755	0.1714	−7.1427	−5.6258	0.0254	0.1127	−7.4093	−5.8925	0.0159	0.0844	−7.6911	−6.1742
20n	0.0793	0.1794	−6.4729	−4.4546	0.0335	0.1235	−7.1337	−5.1154	0.0193	0.0919	−7.6378	−5.6195

The best performances are shown in bold letters and * means models that belong to the Model Confidence Set using a 10% confidence level.
The models with the best performing optimal number of variables were used in each case.

Table 8
Performance results of benchmark models based on various loss functions and AIC/BIC criteria for forecasting CHF/USD exchange rate volatility.

Models	CHF/USD volatility (14d)			CHF/USD volatility (21d)			CHF/USD volatility (28d)			CHF/USD	
	MSE	MAD	MAPE	MSE	MAD	MAPE	MSE	MAD	MAPE	AIC	BIC
GARCH	0.0531	2.8855	67.21	0.0448	2.8067	62.15	0.0452	2.8771	62.72	−7.1858	−7.1129
GJR	0.0358	2.5688	62.04	0.0284	2.5114	57.90	0.0275	2.5601	58.40	−7.1912	−7.0940
APGARCH	0.0338	2.4251	56.38	0.0251	2.2854	49.98	0.0237	2.2581	48.07	−7.2061	−7.1360
	HMSE	HMAE		HMSE	HMAE		HMSE	HMAE			
GARCH	1.4975	0.6721		1.2541	0.6215		1.2306	0.6272			
GJR	1.0254	0.6204		0.9472	0.5790		0.9929	0.5840			
APGARCH	0.7501	0.5637		0.5669	0.4998		0.5200	0.4807			

MSE and MAD are multiplied by 10^7 and 10^5 respectively.

(Bayesian approach) which is given as:

$$BIC = (-2\,ln\,L)/n + (k\,ln\,n)/n \tag{19}$$

Where the definitions of L, k and n are the same as stated above. The difference between the BIC and the AIC is the greater penalty imposed for the number of parameters by the former than the latter. In general information criteria provides a measure of information that strikes a balance between the measure of goodness of fit and parsimonious specification of the model. Hence the chosen or preferred model is the one which minimizes the value of the information criterion, so that a model with larger number of lags would only be selected if the minimized value of log-likelihood outweighed the value of the penalty term.

3. Data

The analysis was conducted using data sets for foreign exchange rates of five (5) major currencies against the US Dollar. The currency pairings studied are the AUD/USD, CAD/USD, CHF/USD, EUR/USD and GBP/USD. Each of the data sets obtained from investing.com covered a sample period starting from January 2001 to November 2013. Descriptive statistics of the returns for each exchange rate pairing are provided in Table 1. The daily mean return for AUD, CAD, CHF and EUR against the USD are all above 0.01% whiles that of the GBP/USD is around 0.0026%. Their standard deviations were all above 0.5% but less than 0.0.9%. Also, for all the five pairings the skewness of their corresponding daily returns indicates that each series is fairly symmetrical. The Kurtosis for AUD/USD and CHF/USD was much higher than 3, which represents the value for which the series is usually Mesokurtic. Hence in order to correctly describe the conditional distribution of the returns of these series, fat-tailed distributions are necessary as their series present a high degree of concentration around the central values of the variable. Those of CAD/USD and GBP/USD was also above 3 but just slightly. On the other hand, the Kurtosis for EUR/USD is less than 3. On the basis of the test results obtained from the Jarque–Bera Normality Test, the empirical distribution of the daily returns of the exchange rates studied herein exhibit significantly heavier tails than in a normal distribution since the model errors are not normally distributed. To test for stationarity, we conduct the augmented Dickey–Fuller (ADF) test for each exchange rate pairing, the results indicate that the individual series can be modeled as they are all stationary.

In order to examine the behavior of the exchange rate pairings, we provide plots of their respective rates series, returns as well as their 21-day variance evolutions in Figs. 3–5. For all the currency pairings, a spike in the volatility appears to emerge starting in 2008. This can be attributed to the credit crunch which began in August 2007. Volatility clusters can be seen for each pairing at different periods of each exchange rate pair.

4. Results

In this section, we report the findings of our empirical analysis by firstly examining the results for each exchange rate pairing and subsequently discuss existing general trends and probable deviations that may aid in explaining the strengths or weakness of the models developed herein. To assess the performance of the hybrid model we deploy the traditional GARCH (1,1) model as well as the GJR (1,1) model as benchmarks. The more advanced APGARCH model is also adopted as an additional benchmark. The initial forecasts were thus performed using these two models. For each exchange rate currency understudy, the performance of the hybrid neural network model over the benchmark models is determined by calculating the relative error variation. The primary forecasting horizon used in each case is the 21-day window. After which the horizon is changed to 14 days and 28 days to further analyze the performance of the models at relatively shorter and longer horizons respectively.

Table 2 presents the results for the initial forecasts obtained for the benchmark models in forecasting the volatility of the AUD/USD exchange rate pairing. The results in Table 2 indicate that at all the forecasting horizons except 14-day horizon, the APGARCH model outperforms the other benchmarks in terms of all applied loss functions. The superiority of the APGARCH model is further confirmed by its AIC and BIC values which are lesser than those of the other benchmark models. Interestingly, while both the GARCH and GJR models show big MAPE greater than 60%, the APGARCH model's MAPE is significantly lesser. Also from Table 2, clearly on the basis of both HMSE and HMAE loss functions, the volatility forecasting ability of the APGARCH model is superior on average

Table 9
Performance results based on various loss functions and AIC/BIC criteria for CHF /USD exchange rate volatility forecasting using the ANN-GJR hybrid model (1 hidden layer with 10 nodes).

Models	CHF/USD volatility (14d)						CHF/USD volatility (21d)						CHF/USD volatility (28d)					
	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)
ANN-GJR 2 VARS	0.00681	−81.00	**0.8956**	**−65.14**	**18.73**	**−69.80**	0.00706	−75.12	0.6533	−73.99	**12.61**	**−78.22**	0.00893	−67.56	**0.4916**	**−80.80**	**9.06**	**−84.48**
ANN-GJR 3 VARS (oil)	0.00830	−76.84	0.9321	−63.72	19.84	−68.01	**0.00265**	**−90.64**	**0.6104**	**−75.69**	13.05	−77.46	**0.00325**	**−88.22**	0.5015	−80.41	9.87	−83.11
ANN-GJR 3 VARS (gold)	0.00965	−73.07	0.9690	−62.28	20.97	−66.19	0.00341	−87.99	0.6586	−73.77	13.78	−76.19	0.00405	−85.31	0.5252	−79.49	9.83	−83.17
ANN-GJR 4 VARS	**0.00573**	**−84.01**	0.9427	−63.30	21.67	−65.07	0.00403	−85.80	0.6540	−73.96	13.75	−76.25	0.00395	−85.65	0.5303	−79.29	9.89	−83.07
	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC
ANN-GJR 2 VARS	**0.0659**	**−93.57**	**0.1652**	**−73.37**	−7.8942	−6.8788	0.0282	−97.02	0.1075	−81.43	−8.3245	−7.3091	0.0146	−98.53	**0.0784**	**−86.58**	−8.3656	−7.3502
ANN-GJR 3 VARS (oil)	0.0685	−93.32	0.1687	−72.81	−7.2477	−5.9815	**0.0232**	**−97.55**	**0.1063**	**−81.64**	−7.7361	−6.4699	**0.0132**	**−98.67**	0.0789	−86.49	−7.7195	−6.4534
ANN-GJR 3 VARS (gold)	0.0726	−92.92	0.1748	−71.82	−7.3671	−6.101	0.0262	−97.23	0.1113	−80.78	−7.4146	−6.1485	0.0151	−98.48	0.0822	−85.92	−7.913	−6.6469
ANN-GJR 4 VARS	0.0788	−92.32	0.1803	−70.94	−7.0268	−5.5099	0.0267	−97.18	0.1134	−80.41	−7.0744	−5.5576	0.0157	−98.42	0.0843	−85.57	−7.9735	−6.4566

The number of variables means the number of inputs supplied to the ANN. For each model we use 2840 forecasts in computing the loss functions. The variation is calculated relative to the performance of the best benchmark model. The best performances are indicated in bold letters. MSE and MAD are multiplied by 10^7 and 10^5 respectively.

while its difference in forecasting precision relative to that of the GARCH model is lower in terms of HMAE. In comparison with the benchmark models, the forecasting performance of the base hybrid model (i.e., built with GJR estimates and the squared returns) is significantly superior in terms of MSE, MAD, MAPE, AIC and BIC. Of particular notice is the relatively lower MAPE values. When gold prices are added to the base model, the performance appears to slightly improve for both MSE and MAD but not MAPE. This observation is also true when only oil prices are added to the base model. Interestingly, the best model differs depending on which loss function is used.

From Table 3, it can also be observed that forecasting with the base model, the HMSE of the best performing benchmark model i.e., the GJR model reduces by 97% indicating substantially improved forecasting performance and greater superiority of the base hybrid model. When gold prices are added, the performance can be observed to further improve in terms of HMSE. In fact, this hybrid model provides the best 21-day volatility forecasts on the basis of HMSE. However, with oil prices as an additional input variable, it is observed that the difference in reduction of both HMSE and HMAE slightly reduces. The difference is further reduced when forecasting with the 4 independent input variables model (i.e., adding Oil and Gold, in that order to the base model of 2 inputs). Nonetheless their respective performances still constitute substantial improvements on the 21-day volatility forecasts provided by the GJR model. When the forecasting horizon is changed to a shorter horizon i.e., 14 days, the forecasting performance of the benchmark models as well as the hybrid models is generally observed to reduce. The best 14-day volatility forecasts considering all loss functions, were however provided by the base hybrid model. Based on the results presented in Table 3, at a longer forecast horizon of 28 days, the base model provides marginally better forecasts in terms of MAD and MAPE. For MSE, the best forecasts were provided by the gold prices 3 variables model. The base model also averagely provides the best in terms of both loss functions while the forecasting precision of the 3-input model with oil prices as the third input is almost akin to that of the 2-input model judging by their computed HMSEs. The observed reduction for all the hybrid models at all the horizons is smaller for the HMAE loss function relative to the HMSE loss function. This observation suggests that the improvements associated with the use of the hybrid model are to a greater extent in the high volatility clusters. In this study, we also investigate the best architectural combination in terms of number of hidden layer nodes. Thus, for the five variations considered herein, we report their forecast results in Table 4. At all horizons and on the basis of the MCS test, the 5n ANN-GJR model is the only model that belongs to the 10% model confidence set using HMSE while the 2n ANN-GJR model is the only one for HMAE. The other models were eliminated. However, when comparing both HMSE and HMAE values, the 2n ANN-GJR model recorded the smallest at each horizon. On the other hand, the highest and essentially weakest forecasts were recorded by the 20n ANN-GJR model. Furthermore using the BIC and AIC model selection criteria the superior model is further confirmed to be the 2n ANN-GJR model with the 5n model following closely. These results suggest that for the volatility forecasting of the AUD/USD exchange rate, optimum forecasting performance of our proposed ANN-GJR model, requires the use of either 2 or 5n.

The volatility forecast results of the CAD/USD pairing are also shown in Tables 5–7. The performance of the benchmark models were almost similar in terms of MSE, MAD and MAPE (See Table 5). The GJR model notwithstanding provides better overall forecasts. In terms of their AIC and BIC it is also the model of choice with least values of −7.5291 and −7.4591, respectively. However, the results presented in Table 6 indicate that in comparison with the hybrid models, the GJR model is substantially inferior. In fact, at

Table 10
Performance results for ANN-GJR forecasts using a different number of hidden layer nodes (CHF/USD).

Models	CHF/USD volatility (14d)				CHF/USD volatility (21d)				CHF/USD volatility (28d)			
	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC
2n	**0.1347***	**0.1777***	−7.8847	−7.6716	**0.0506***	**0.1181***	−8.8087	−8.5956	**0.0218***	**0.0815***	−8.9601	−8.7469
5n	0.1677	0.1849	−8.0368	−7.5228	0.0605	0.1258	−8.5698	−8.0558	0.0374	0.0896	−8.8763	−8.3623
10n	0.0659	0.1652	−7.8942	−6.8788	0.0232	0.1063	−8.3245	−7.3091	0.0132	0.0789	−8.3656	−7.3502
15n	0.2088	0.2019	−7.5611	−6.0443	0.0945	0.1407	−7.7416	−6.2247	0.0351	0.1006	−8.1167	−6.5998
20n	0.1832	0.2133	−7.1672	−5.1489	0.1605	0.1535	−7.4171	−5.3988	0.0945	0.1137	−7.664	−5.6457

The best performances are shown in bold letters and * means models that belong to the Model Confidence Set using a 10% confidence level.
The models with the best performing optimal number of variables were used in each case.

Table 11
Performance results of benchmark models based on various loss functions and AIC/BIC criteria for forecasting EUR/USD exchange rate volatility.

Models	EUR/USD volatility (14d)			EUR /USD volatility (21d)			EUR /USD volatility (28d)			EUR /USD	
	MSE	MAD	MAPE	MSE	MAD	MAPE	MSE	MAD	MAPE	AIC	BIC
GARCH	0.0078	1.8212	57.73	0.0070	1.6981	50.98	0.0069	1.6786	49.30	−7.3541	−7.2812
GJR	0.0080	1.8286	57.25	0.0072	1.7004	50.29	0.0071	1.6879	48.70	−7.3497	−7.2526
APGARCH	0.0067	1.7571	55.43	0.0057	1.6072	48.18	0.0055	1.5605	45.99	−7.3556	−7.2856
	HMSE	HMAE		HMSE	HMAE		HMSE	HMAE			
GARCH	0.7072	0.5773		0.5395	0.5098		0.5089	0.4930			
GJR	0.6994	0.5725		0.5257	0.5029		0.4954	0.4870			
APGARCH	0.6282	0.5542		0.4575	0.4819		0.4078	0.4600			

MSE and MAD are multiplied by 10^7 and 10^5 respectively.

all forecasting horizons, the best forecasts in terms of MSE, MAD and MAPE were provided when the hybrid model with oil prices is employed. The only exceptions are for the 14 day and 28-day forecasts for which the base model recorded the least error according to MAPE. From Table 5, the 28-day volatility forecasting accuracies of both benchmark models appear to improve while contrastingly, the forecast at the shorter 14d horizon is observed to be less accurate with the HMSE and HMAE of the GJR model increasing by a difference of 39% and 12% respectively compared to 37% and 11% for the forecasts of the GARCH model. However, compared to the hybrid ANN-GJR model, their performance is significantly poorer at all horizons considering both HMSE and HMAE. The forecasts of the 3-input hybrid model with oil prices as the third input was found to have the least HMSE as well as HMAE at the forecasting horizon of 21 suggesting better forecasting performance (See Table 6). The 3-input model with gold prices as third input, followed closely with the second-best forecasts. It's worth noting that for the 21-day volatility forecasting of CAD/USD exchange rate, the forecasting accuracy of the 4 input variables model is observed to be better than that of the 2-input variable albeit slightly.

The 28-day volatility forecasts of this hybrid model were also found to be the best using the HMSE loss function while for HMAE the 2-input variable model recorded the least. For the 14-day forecast window. The 2-input model provided the best precision for both loss functions. In addition, the results shown in Table 7 also indicate that for best performance of the ANN-GJR at different horizons, the 2n must be applied according to the HMSE and HMAE values at all horizons. Perhaps more importantly, using all comparisons and at all forecasting horizons, the 2n model is the only model included in the 10% model confidence set. Interestingly, based on both the AIC and BIC the superior model or the model of choice is instead 5n model.

In the case of CHF against the US dollar, at all horizons and for each error function the APGARCH model outperforms the traditional GARCH (1,1) and GJR models (See Table 8). This observation is further confirmed by the fact that it is the best model based on the obtained AIC and BIC values. But as shown in Table 14, better forecasts are obtained when the hybrid model is used. For both MSE and MAD, the best 21-day forecasts were provided by the oil prices added hybrid model. This is however not the case for MAPE as the base model recorded the least MAPE at this forecast horizon. The base model also provides the overall best accuracy when the forecast horizon is increased to 28 or decreased to 14. For the 14-day forecasts hybrid model which incorporates both oil and gold prices presented the lowest MSE of 0.00573. For the 21-day exchange rate volatility forecast, the APGARCH had an HMSE of 0.5669 compared to 0.9472 recorded by the GJR model. When the forecasting window is decreased the original values increase thereby indicating reduced forecasting precision at the shorter horizon.

However, when the window size is increased the performance improves. These two later observations are also true for the forecast results obtained for the ANN-GJR model. However, the ANN-GJR models significantly outperform the benchmark models at all the horizons and in terms of each loss function. The improvement in the forecasting accuracy of the GJR by use of the ANN-GJR models is exemplified by the substantial reduction by 93%, 97% and 98% for 14d, 21d and 28d respectively in terms of HMSE when the base ANN-GJR model is compared to the forecast for the GJR (1,1) model. Additionally, both the 21d and 28d horizons, when comparing average performance in terms of both HMSE and HMAE, the 3 input ANN-GJR model with oil prices included was found to provide the best forecasting precision. However, based on the results provided in Table 9, for 14d, the 2 input ANN-GJR model outperforms the other models in regards to both loss functions and also in terms of both applied information criteria model selection methods. Comparing the performances of the 3-input model including gold as input to that of the 4-input model, it is observed that the 14, 21 and 28-day volatility forecasts of the former are more precise with lower HMSE and HMAE values. This observation is corroborated by the AIC and BIC results provided in the Table 9. The 2n ANN-GJR model is the best model at all horizons using the information criteria selection methods of AIC and BIC and also considering that from the MCS test it was the only model included in the set thereby statistically suggesting its superiority over the other models including the default 10n ANN-GJR model (See Table 10).

The results obtained for the EUR/USD pairing are reported in Tables 11–13. The GJR and GARCH (1,1) models once again provide almost similar forecast accuracy at all horizons. However,

Table 12
Performance results based on various loss functions and AIC/BIC criteria for EUR /USD exchange rate volatility forecasting using the ANN-GJR hybrid model (1 hidden layer with 10 nodes).

Models	EUR/USD volatility (14d)						EUR/USD volatility (21d)						EUR/USD volatility (28d)					
	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)
ANN-GJR 2 VARS	**0.00117**	**−84.99**	**0.5810**	**−68.10**	**15.87**	**−72.28**	**0.00059**	**−91.47**	**0.3895**	**−77.07**	**10.14**	**−79.84**	0.00037	−94.68	**0.3060**	**−81.77**	**7.62**	**−84.36**
ANN-GJR 3 VARS (oil)	0.00121	−84.46	0.6054	−66.76	16.20	−71.70	0.00060	−91.34	0.4179	−75.39	10.89	−78.35	0.00039	−94.39	0.3066	−81.73	7.83	−83.93
ANN-GJR 3 VARS (gold)	0.00126	−83.94	0.6199	−65.96	17.12	−70.09	0.00068	−90.19	0.4203	−75.25	11.05	−78.03	0.00036	−94.80	0.3263	−80.56	8.20	−83.16
ANN-GJR 4 VARS	0.00130	−83.42	0.6372	−65.01	17.24	−69.88	0.00065	−90.70	0.4346	−74.41	11.41	−77.31	**0.00033**	**−95.20**	0.3196	−80.96	8.18	−83.21
	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC
ANN-GJR 2 VARS	0.0601	−91.41	**0.1587**	**−72.28**	−9.1139	−8.0985	**0.0227**	**−95.68**	**0.1014**	**−79.84**	−9.429	−8.4136	0.0131	−97.36	**0.0762**	**−84.35**	−9.6205	−8.6051
ANN-GJR 3 VARS (oil)	**0.0567**	**−91.89**	0.1620	−71.70	−8.0275	−6.7614	0.0251	−95.23	0.1089	−78.35	−8.5408	−7.2746	**0.0129**	**−97.40**	0.0783	−83.92	−8.7967	−7.5305
ANN-GJR 3 VARS (gold)	0.0680	−90.28	0.1713	−70.08	−7.5741	−6.3079	0.0271	−94.84	0.1104	−78.05	−8.2739	−7.0078	0.0136	−97.25	0.0821	−83.14	−8.8655	−7.5994
ANN-GJR 4 VARS	0.0657	−90.61	0.1724	−69.89	−8.2084	−6.6915	0.027	−94.86	0.1141	−77.31	−8.4496	−6.9328	0.0133	−97.32	0.0818	−83.20	−7.9935	−6.4767

The number of variables means the number of inputs supplied to the ANN. For each model we use 2840 forecasts in computing the loss functions. The variation is calculated relative to the performance of the best benchmark model. The best performances are indicated in bold letters.

Table 13
Performance results for ANN-GJR forecasts using a different number of hidden layer nodes (EUR /USD).

Models	EUR/USD volatility (14d)				EUR/USD volatility (21d)				EUR/USD volatility (28d)			
	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC
2n	**0.0492***	**0.1454***	−9.6413	−9.4281	**0.0169***	**0.0874***	−10.0948	−9.8817	**0.0095***	**0.0675***	−10.3203	−10.1072
5n	0.0524	0.1550	−9.4997	−8.9857	0.0198	0.0960	−9.5191	−9.0051	0.0113	0.0740	−10.1080	−9.5941
10n	0.0567	0.1620	−9.1139	−8.0985	0.0227	0.1014	−9.429	−8.4136	0.0129	0.0783	−9.6205	−8.6051
15n	0.0687	0.1742	−8.7995	−7.2826	0.0276	0.1091	−8.7191	−7.2022	0.0152	0.0834	−8.8133	−7.2965
20n	0.0673	0.1774	−8.2396	−6.2213	0.0280	0.1101	−8.3933	−6.375	0.0167	0.0867	−8.6036	−6.5853

The best performances are shown in bold letters and * means models that belong to the Model Confidence Set using a 10% confidence level.
The models with the best performing optimal number of variables were used in each case.

Table 14
Performance results of benchmark models based on various loss functions and AIC/BIC criteria for forecasting GBP /USD exchange rate volatility.

Models	GBP/USD volatility (14d)			GBP /USD volatility (21d)			GBP /USD volatility (28d)			GBP /USD	
	MSE	MAD	MAPE	MSE	MAD	MAPE	MSE	MAD	MAPE	AIC	BIC
GARCH	0.0117	1.7223	56.14	0.0118	1.6911	49.01	0.0120	1.7241	46.10	−7.5445	−7.4716
GJR	0.0109	1.6944	55.59	0.0110	1.6596	48.49	0.0112	1.6813	45.37	−7.5428	−7.4456
APGARCH	0.0105	1.6849	56.92	0.0099	1.6133	49.23	0.0095	1.5741	45.49	−7.5446	−7.4745
	HMSE	HMAE		HMSE	HMAE		HMSE	HMAE			
GARCH	0.6914	0.5614		0.4843	0.4901		0.4051	0.4610			
GJR	0.6747	0.5559		0.4698	0.4849		0.3931	0.4537			
APGARCH	0.7523	0.5692		0.5093	0.4923		0.4051	0.4549			

MSE and MAD are multiplied by 10^7 and 10^5 respectively.

Table 15
Performance results based on various loss functions and AIC/BIC criteria for GBP /USD exchange rate volatility forecasting using the ANN-GJR hybrid model (1 hidden layer with 10 nodes).

Models	GBP/USD volatility (14d)						GBP/USD volatility (21d)						GBP/USD volatility (28d)					
	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)	MSE	Var(%)	MAD	Var(%)	MAPE	Var(%)
ANN-GJR 2 VARS	0.00214	−80.37	**0.5877**	**−65.32**	**15.77**	**−71.63**	0.00124	−88.78	0.4067	−75.49	**10.18**	**−79.01**	0.00069	−93.87	0.3123	−81.42	**7.67**	**−83.10**
ANN-GJR 3 VARS (oil)	0.00263	−75.84	0.6332	−62.63	16.70	−69.96	0.00085	−92.31	**0.3873**	**−76.66**	10.49	−78.36	0.00057	−94.94	**0.3088**	**−81.63**	7.89	−82.60
ANN-GJR 3 VARS (gold)	**0.00206**	**−81.10**	0.6128	−63.83	16.96	−69.48	0.00086	−92.17	0.3955	−76.17	10.62	−78.10	0.00070	−93.81	0.3285	−80.46	8.16	−82.01
ANN-GJR 4 VARS	0.00216	−80.14	0.6121	−63.87	17.04	−69.35	**0.00076**	**−93.12**	0.3956	−76.16	10.91	−77.50	**0.00056**	**−95.00**	0.3115	−81.47	8.08	−82.19
	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC	HMSE	Var(%)	HMAE	Var(%)	AIC	BIC
ANN-GJR 2 VARS	**0.0583**	**−91.36**	**0.1577**	**−71.63**	−9.1139	−8.0985	**0.0222**	**−95.27**	**0.1018**	**−79.01**	−9.429	−8.4136	**0.0127**	**−96.77**	**0.0766**	**−83.12**	−9.6205	−8.6051
ANN-GJR 3 VARS (oil)	0.0633	−90.62	0.1670	−69.96	−8.0275	−6.7614	0.0230	−95.10	0.1049	−78.37	−8.5408	−7.2746	0.0134	−96.59	0.0789	−82.61	−8.7967	−7.5305
ANN-GJR 3 VARS (gold)	0.0628	−90.69	0.1696	−69.49	−7.5741	−6.3079	0.0232	−95.06	0.1062	−78.10	−8.2739	−7.0078	0.0143	−96.36	0.0816	−82.01	−8.8655	−7.5994
ANN-GJR 4 VARS	0.0623	−90.77	0.1704	−69.35	−8.2084	−6.6915	0.0235	−95.00	0.1091	−77.50	−8.4496	−6.9328	0.0145	−96.31	0.0808	−82.19	−7.9935	−6.4767

The number of variables means the number of inputs supplied to the ANN. For each model we use 2840 forecasts in computing the loss functions. The variation is calculated relative to the performance of the best benchmark model. The best performances are indicated in bold letters.

Table 16
Performance results for ANN-GJR forecasts using a different number of hidden layer nodes (GBP/USD).

Models	GBP/USD volatility (14d)				GBP/USD volatility (21d)				GBP/USD volatility (28d)			
	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC	HMSE	HMAE	AIC	BIC
2n	**0.0492***	**0.1402***	−9.6413	−9.4281	**0.0183***	**0.0866***	−10.0948	−9.8817	**0.0094***	**0.0656***	−10.3203	−10.1072
5n	0.0508	0.1458	−9.4997	−8.9857	0.0191	0.0940	−9.5191	−9.0051	0.0105	0.0707	−10.1080	−9.5941
10n	0.0583	0.1577	−9.1139	−8.0985	0.0222	0.1018	−9.429	−8.4136	0.0127	0.0766	−9.6205	−8.6051
15n	0.0627	0.1624	−8.7995	−7.2826	0.0249	0.1067	−8.7191	−7.2022	0.0145	0.0803	−8.8133	−7.2965
20n	0.0643	0.1673	−8.2396	−6.2213	0.0289	0.1118	−8.3933	−6.3750	0.0165	0.0856	−8.6036	−6.5853

The best performances are shown in bold letters and * means models that belong to the Model Confidence Set using a 10% confidence level.
The models with the best performing optimal number of variables were used in each case.

compared to their performances for the previous currencies they appear to forecast the volatility of EUR/USD better. Nonetheless, the APGARCH provides comparatively superior forecasts at each horizon. According to the obtained AIC and BIC values of the benchmark models, the preferred model is the APGARCH model followed by the GARCH model. Also, for the EUR/USD using MSE, MAD, and MAPE, the hybrid ANN-GJR model significantly improves forecasting accuracy by up to 94%. MSE, MAD and MAPE values are particularly minimized when the base hybrid model is applied. This is with the exception of the 28-day horizon for which the MSE of the gold and oil prices added hybrid model slightly outperforms the other hybrid types (Table 12).

We further analyze model performance based on HMSE and HMAE. It can be seen that within the benchmark models, the GJR model slightly underperforms against the APGARCH model in terms of both loss functions at all the horizons (Table 11). However, at the initial horizon of 21d, the use of the ANN-GJR base model largely improves the forecast of the GJR by reducing the HMSE by 95% and HMAE 79%. At this horizon, the best ANN-GJR model was found to be the 2 input variables model with a value of 0.0227 using the HMSE loss function. Assessing the impact of adding different variables, the 21d volatility forecast accuracy appears to decrease when either gold/oil is added or both are. This observation is true regarding both loss functions. At the other two forecast horizons, the addition of oil prices tends to reduce the HMSE of the base model thereby improving forecast precision. The HMAE values of the oil included 3 input variables model is however still higher than the base 2 input model. Additionally, for the EUR/USD exchange rate volatility forecast, adding both oil and gold in the 4-input variable provides better forecasts relative to the gold added 3 input hybrid models. Regarding the best architectural combination of hidden sizes, a similar observation as ones reported for the volatility forecasting of the two earlier major currency pairs i.e. CAD, CHF against the USD was realized. The results in Table 13 indicate that the AIC and BIC values of the 2n model were more favorable suggesting that it is the best amongst the competing models. This is further corroborated by the MCS test as the 2n model once again appears to be superior to the other models while the 15n and 20n are the weakest at all the horizons.

Tables 14–16 shows the empirical results obtained for the volatility forecast of the GBP/USD exchange rate. With regards to the initial forecasts using the benchmark models, the observed results followed a similar trend as the one already established from the earlier analysis of the results obtained for the other currency exchange rate pairings studied herein. Thus the GJR model was superior to the GARCH model for each of the three horizons and in terms of all applied loss functions by recording the least errors in each case (See Table 14). However, it is interesting to note that for this currency pairing the GJR model appears to outperform the APGARCH model at the 14 and 21 day forecast horizons. The AIC and BIC on the other hand selected APGARCH as the optimal model with slightly higher level of performance. From the results obtained for the proposed hybrid ANN-GJR model in Table 15 it can be seen that in general the errors (MSE, MAD and MAPE) associated with the benchmark models can be effectively reduced if any of the hybrid model types are instead used to forecast volatility of GBP/USD. Here the AIC and BIC select the 2-var ANN-GJR model as the best model regardless of the horizon being considered. The incorporation of oil and gold prices on the bases of AIC and BIC results is detrimental as on the basis of both the AIC and BIC of the competing models, the 4-var ANN-GJR model is the worst performing model In addition to all forecasting horizons, very little differences exist between the forecast ability of the various hybrid types in terms of MSE, MAD and MAPE (Table 15).

Significantly reduced HMSE value of 0.0222 and HMAE of 0.1018 were however recorded for the 2-var ANN-GJR model at the same horizon. This implies an impressive reduction by 95% and further highlights the apparent superiority of the ANN-GJR model in forecasting currency exchange rate volatility. The ANN-GJR model with either oil or gold prices included, on the basis of both loss functions, also performed slightly better than the 4-var input variable with both oil and gold prices as additional inputs. In similar fashion to earlier observed trends, the hybrid models with various inputs appear to perform better at a relatively higher forecasting horizon while their performance was weaker at the lower horizon. For the GBP/USD exchange rate volatility forecasting, the base ANN-GJR model outperforms all other models at all the horizons. The 14d horizon forecast results suggest that relative to the benchmark GJR model, the addition of gold prices improves forecasting accuracy better than the inclusion of oil prices. This is true in terms of both applied loss functions. In fact, the 4-input model also performs better than the oil prices included 3 input ANN-GJR model. For optimum forecasting performance, the results in Table 16 also suggest the use of the 2n ANN-GJR model.

Based on the findings reported for each of the currency pairing. The combination of the inherent characteristics of the individual parts of the hybrid asymmetric volatility method is beneficial. As it reduces the stochastic and nonlinearity of the error term sequence while simultaneously capturing the asymmetric volatility. The ANN–GJR approach allows for the possibility to incorporate other input variables into the ANN. Hence another significant merit of the ANN–GJR approach is that it provides the ability to determine the influences that variables exert by either incremental improvements in the accuracy of forecasts or poorer performance relative to the classical form of their relationship in the fit of the model. Consequently, financial variables that are significant for the volatility of exchange rates in the classical models can be incorporated. This fact is very important, since the focus is not on the behavior in-sample, but the influence of the variables is measured according to its contribution in the forecast out-of-sample. Notwithstanding these advantages, a key limitation of the hybrid model is its complexity. Other limitations stem from the fact that the performance depends on the quality of data used as input since the results depend on historic data and also relatively longer period of data is required to obtain forecasts over a short horizon. For future studies it is recommended to apply a feature selection algorithm to select most relevant features instead of performing several model trials. There is also room to further improve the accuracy of the proposed model by further reducing noise level of the data.

5. Conclusion

This paper examines the implementation of a hybrid neural network model in forecasting currency exchange rate volatility. The currency pairings utilized are the AUD/USD, CAD/USD, CHF/USD, EUR/USD and GBP/USD. The empirical evidence firstly suggests that the hybrid ANN-GJR, developed in this study is remarkably superior to all applied benchmark models. In terms of the MSE, the MAD and the MAPE measures, a significant improvement of the measured forecast accuracy was found when using the ANN hybrid model compared to the benchmark models. This error reduction is consistent with every window size and every number of variables included in the model. However, within the different hybrid model types, the results suggest that the performances based on a different number of inputs are not significant in most cases. The base ANN model with two variables generally provides a slightly better forecast for AUD/USD and EUR/USD. But for the other currencies forecasting accuracy may benefit from the inclusion of either oil or gold prices. This is particularly the case for CAD/USD as the best volatility forecasts are obtained when oil prices are added.

For all the currencies and in terms of employed heteroskedastic loss functions, the use of the hybrid model can significantly improve forecasting precision by over 90% using a forecast horizon of 21 days. While the forecasting accuracy of the hybrid model decreases at a shorter horizon, its performance is nonetheless better than the benchmark models. At a longer forecasting horizon of 28 days, the results from all five currency pairings also suggest that the forecasting accuracy of the hybrid model slightly improves. In general, the improvement experienced for HMAE is smaller compared to that obtained for HMSE hence it can be inferred that the improvements associated with the hybrid model are to a greater extent in the high volatility clusters.

Furthermore, we investigate the effect of incorporating commodity prices as additional inputs in our proposed multivariate approach using the ANN-GJR model. We specifically consider the impact of oil and gold prices. In this regard, our findings demonstrate that the inclusion of oil prices as input can potentially improve forecasting precision of the hybrid model. This is particularly the case for AUD, CAD, CHF and EUR against the USD pairings where at different horizons the 3 VAR ANN-GJR model provides the best forecasts. In contrast, for three currency pairings, the addition of gold price series appears to reduce predictive ability relative to the base ANN-GJR model. This assertion is exemplified in the higher error values obtained for these currencies at different horizons. Interestingly, the empirical results for the 21d volatility forecasting of both the AUD and CAD versus the USD show however that on the basis of observed HMSE values, the inclusion of gold price series can provide improved forecasts relative to the initial 2 input base model. In general, the addition of both price series in a 4-input model fails to improve forecast accuracy of the initial hybrid model with the exception of the 21d volatility forecast of CAD/USD exchange rate. The study additionally analyzed the impact of different NN architectures by varying the number of nodes in the hidden layer of the NARX network. Using the heteroskedastic loss functions as well as the MCS test at 10% confidence level, the results to reveal the 2 nodes and the 5 nodes configurations as superior models in obtaining optimum performance of the hybrid model.

To this end, the study conducted herein attains several important functions. It firstly extends the current literature of artificial neural networks in terms of broadening their potential applications in finance and modeling. It also solves the problem of high error associated with the use of traditional models in currency exchange rate volatility forecasting beyond a model's historical data by providing a substantially viable and superior alternative. In particular the results suggest that volatility clustering, asymmetry and nonlinearity characteristics are modeled more efficiently and provide better forecast accuracy with the use of the neural network based hybrid model. The paper applies a method to determine the impact of incorporating other external financial variables i.e. major commodity series in the forecasting procedure and verifies the possibility of improved forecast precision. Future studies will seek to evaluate the extension of the hybrid ANN-GJR model in volatility modeling and forecasting in other major financial markets.

Declaration of Competing Interest

None.

CRediT authorship contribution statement

Alexander Amo Baffour: Conceptualization, Data curation, Writing - original draft, Writing - review & editing, Formal analysis. **Jingchun Feng:** Conceptualization, Data curation, Writing - original draft, Writing - review & editing, Formal analysis. **Evans Kwesi Taylor:** Conceptualization, Data curation, Writing - original draft, Writing - review & editing, Formal analysis.

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