Question 7.2

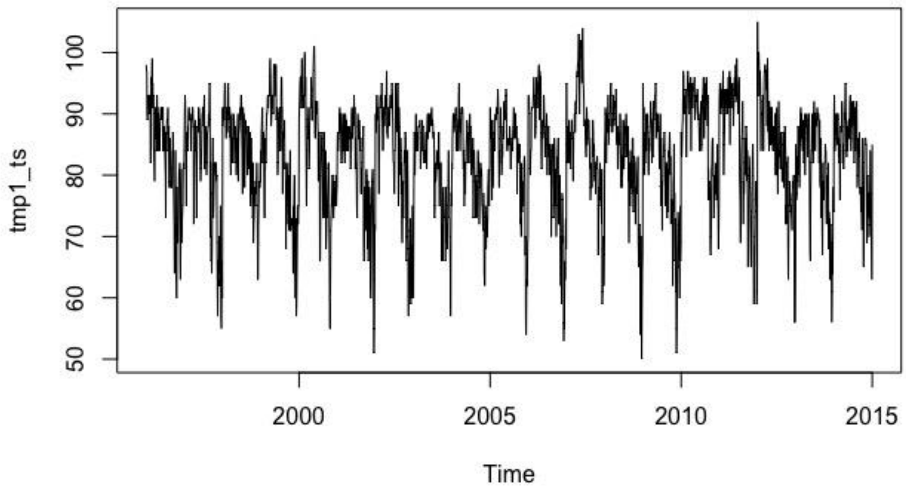
Using the 20 years of daily high temperature data for Atlanta (July through October) from Question 6.2 (file `temps.txt`), build and use an exponential smoothing model to help make a judgment of whether the unofficial end of summer has gotten later over the 20 years. (Part of the point of this assignment is for you to think about how you might use exponential smoothing to answer this question. Feel free to combine it with other models if you'd like to. There's certainly more than one reasonable approach.)

Note: in R, you can use either `HoltWinters` (simpler to use) or the `smooth` package's `es` function (harder to use, but more general). If you use `es`, the Holt-Winters model uses `model="AAM"` in the function call (the first and second constants are used "A"dditively, and the third (seasonality) is used "M"ultiplicatively; the documentation doesn't make that clear).

```
# Single Exponential Smoothness(ES) Values:
                    # alpha: 0.8396301
                       # sum of squared error(SSE):53704.15

# Double ES Values
                    # alpha: 0.8455303
                   # beta : 0.003777803
                     # SSE: 54071.22


# Triple ES Values:
                   # alpha: 0.6677614
                   # beta : 0
                # gamma: 0.6297674
                  # SSE: 63025.97
```



#Single ES has smallest SSE when compared to other two and it is closed to 1. It means there is less randomness in the system and recent temperature reading has more weight in predicting the current temperature.

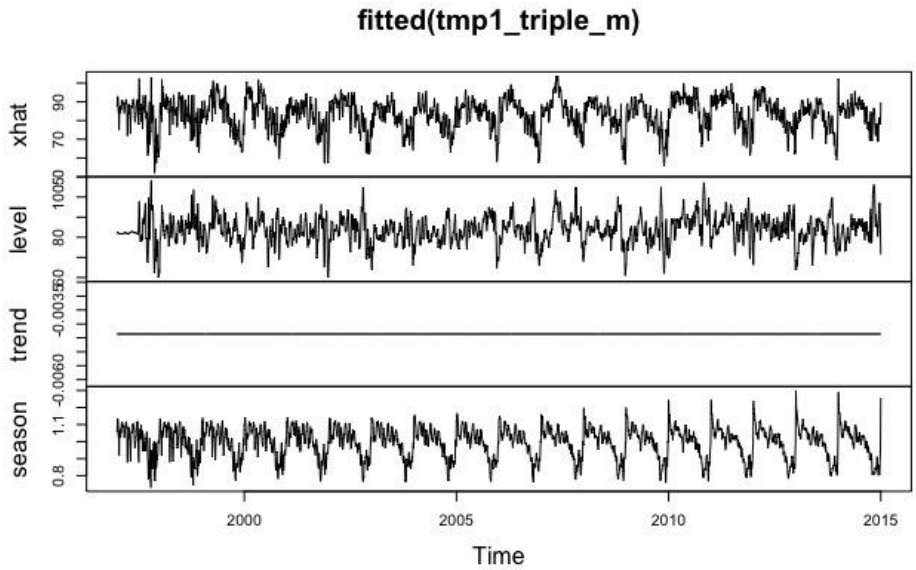
#Seasonality uses Model AAM:

#1. A - additive: seasonal variation is independent

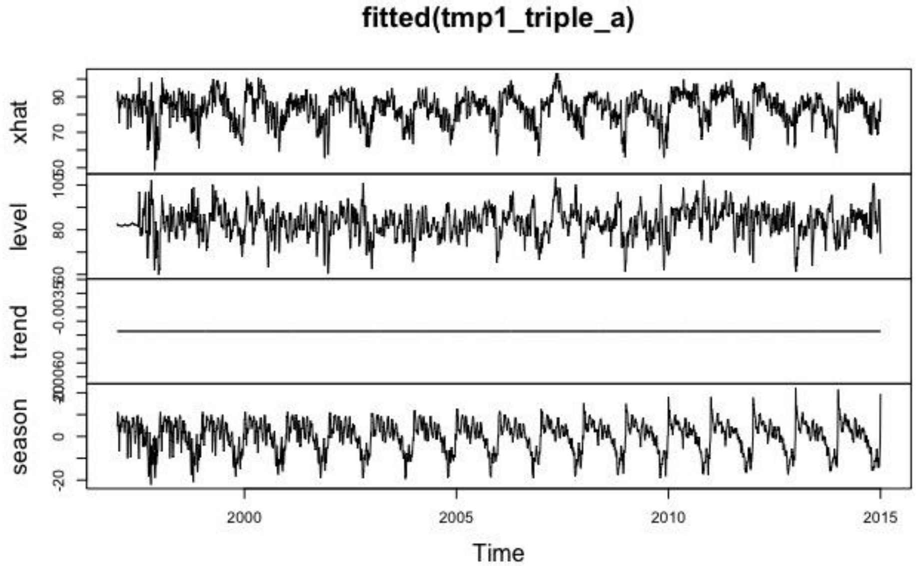
#2. M - multiplicative: seasonal variation is connected.

#Triple Exponential – using multiplicative

#SSE:65648.65



#Triple Exponential - using additive
#SSE:63025.97



Triple ES with additive seasonal factor has better SSE.

From the above chart, Trend line is straight which means no trend.

The season subchart shows that the duration of each season has been constant throughout all 20 years.

Use Chkum formula from previous lesson St = max(0, St-1 + (Xmean - T - C)) as a running equation across each daily temperature observation there wasn't much of a change.

I. R Script

```
## Clear Work Area ##
rm(list=ls()) #clear environment
cat("\014") #clear console

#Load dara from temperature table
getwd()

##Input data
tmp1_data <- read.table("temps.txt", header = TRUE) #read txt data into a table


#Exploring the data
tail(tmp_data)[1:5,]
str(tmp1_data)
print(summary(tmp1_data))

#Marix of temperature data
tmp1_mat <- as.vector(unlist(tmp1_data[,2:21]))
str(tmp1_mat)
tmp1_mat

# Time Series Values
tmp1_ts <- ts(tmp1_mat, start=1996, end = 2015, frequency=123)
tmp1_ts

class(tmp1_ts)
plot(tmp1_ts)

# Define beta to generate trend value to forecast a future trend
tmp1_b_ts <- HoltWinters(tmp1_ts , beta=.5)
plot(tmp1_b_ts)

# Exponential Smoothing
#Simple Exponential #
tmp1_single <- HoltWinters(tmp1_ts,beta=FALSE, gamma=FALSE)

#Double Exponential - model trend #
tmp1_double <- HoltWinters(tmp1_ts,gamma=FALSE)

#Triple Exponential - model trend and seasonality#
```

```
tmp1_triple <- HoltWinters(tmp1_ts, seasonal = "additive")

#Compare 3 kinds of Exp Smoothing#
tmp1_single
tmp1_single$SSE # Sum of Squared Error

tmp1_double
tmp1_double$SSE

tmp1_triple
tmp1_triple$SSE

# Triple Exponential using multiplicative ( Seasonal varioation is connected)
tmp1_triple_m <- HoltWinters(tmp1_ts, seasonal = "multiplicative")
tmp1_triple_m$SSE


# Triple Exponential using multiplicative ( Seasonal varioation is independent of the level)
tmp1_triple_a <- HoltWinters(tmp1_ts, seasonal = "additive")
tmp1_triple_a$SSE

tmp1_triple_a$fitted

plot(fitted(tmp1_triple_a))

plot(fitted(tmp1_triple_m))


#Forecast
library(forecast)
tmp1_forecast=predict( tmp1_b_ts, n.ahead = 100, prediction.interval = TRUE )
plot(forecast( tmp1_b_ts, h = 180 ))
```

R-Script With Answers

```
>
> #Load dara from temperature table
> getwd()
[1] "/Users/nagarajanmurugan/Documents/MS/Data"
>
> ##Input data
> tmp1_data <- read.table("temps.txt", header = TRUE) #read txt data into a table
>
>
> #Exploring the data
> str(tmp1_data)
'data.frame':	123 obs. of  21 variables:
$ DAY  : chr  "1-Jul" "2-Jul" "3-Jul" "4-Jul" ...
$ X1996: int  98 97 97 90 89 93 93 91 93 93 ...
$ X1997: int  86 90 93 91 84 84 75 87 84 87 ...
$ X1998: int  91 88 91 91 91 89 93 95 95 91 ...
$ X1999: int  84 82 87 88 90 91 82 86 87 87 ...
$ X2000: int  89 91 93 95 96 96 96 91 96 99 ...
$ X2001: int  84 87 87 84 86 87 87 89 91 87 ...
$ X2002: int  90 90 87 89 93 93 89 89 90 91 ...
$ X2003: int  73 81 87 86 80 84 87 90 89 84 ...
$ X2004: int  82 81 86 88 90 90 89 87 88 89 ...
$ X2005: int  91 89 86 86 89 82 76 88 89 78 ...
$ X2006: int  93 93 93 91 90 81 80 82 84 84 ...
$ X2007: int  95 85 82 86 88 87 82 82 89 86 ...
$ X2008: int  85 87 91 90 88 82 88 90 89 87 ...
$ X2009: int  95 90 89 91 80 87 86 82 84 84 ...
$ X2010: int  87 84 83 85 88 89 94 97 96 90 ...
$ X2011: int  92 94 95 92 90 90 94 94 91 92 ...
$ X2012: int  105 93 99 98 100 98 93 95 97 95 ...
$ X2013: int  82 85 76 77 83 83 79 88 88 87 ...
$ X2014: int  90 93 87 84 86 87 89 90 90 87 ...
$ X2015: int  85 87 79 85 84 84 90 90 91 93 ...
> print(summary(tmp1_data))
    DAY               X1996          X1997          X1998          X1999
Length:123         Min.   :60.00   Min.   :55.00   Min.   :63.00   Min.   :57.00
Class :character   1st Qu.:79.00   1st Qu.:78.50   1st Qu.:79.50   1st Qu.:75.00
Mode  :character   Median :84.00   Median :84.00   Median :86.00   Median :86.00
                   Mean   :83.72   Mean   :81.67   Mean   :84.26   Mean   :83.36
                   3rd Qu.:90.00   3rd Qu.:88.50   3rd Qu.:89.00   3rd Qu.:91.00
                   Max.   :99.00   Max.   :95.00   Max.   :95.00   Max.   :99.00
    X2000          X2001          X2002          X2003          X2004          X2005
Min.   : 55.00   Min.   :51.00   Min.   :57.00   Min.   :57.00   Min.   :62.00   Min.   :54.00
```

```
1st Qu.: 77.00   1st Qu.:78.00   1st Qu.:78.00   1st Qu.:78.00   1st Qu.:78.00   1st Qu.:81.50
Median : 86.00   Median :84.00   Median :87.00   Median :84.00   Median :82.00   Median :85.00
Mean   : 84.03   Mean   :81.55   Mean   :83.59   Mean   :81.48   Mean   :81.76   Mean   :83.36
3rd Qu.: 91.00   3rd Qu.:87.00   3rd Qu.:91.00   3rd Qu.:87.00   3rd Qu.:87.00   3rd Qu.:88.00
Max.   :101.00   Max.   :93.00   Max.   :97.00   Max.   :91.00   Max.   :95.00   Max.   :94.00
    X2006          X2007          X2008          X2009          X2010          X2011
Min.   :53.00   Min.   : 59.0   Min.   :50.00   Min.   :51.00   Min.   :67.00   Min.   :59.00
1st Qu.:79.00   1st Qu.: 81.0   1st Qu.:79.50   1st Qu.:75.00   1st Qu.:82.00   1st Qu.:79.00
Median :85.00   Median : 86.0   Median :85.00   Median :83.00   Median :90.00   Median :89.00
Mean   :83.05   Mean   : 85.4   Mean   :82.51   Mean   :80.99   Mean   :87.21   Mean   :85.28
3rd Qu.:91.00   3rd Qu.: 89.5   3rd Qu.:88.50   3rd Qu.:88.00   3rd Qu.:93.00   3rd Qu.:94.00
Max.   :98.00   Max.   :104.0   Max.   :95.00   Max.   :95.00   Max.   :97.00   Max.   :99.00
    X2012           X2013          X2014          X2015
Min.   : 56.00   Min.   :56.00   Min.   :63.00   Min.   :56.0
1st Qu.: 79.50   1st Qu.:77.00   1st Qu.:81.50   1st Qu.:77.0
Median : 85.00   Median :84.00   Median :86.00   Median :85.0
Mean   : 84.65   Mean   :81.67   Mean   :83.94   Mean   :83.3
3rd Qu.: 90.50   3rd Qu.:88.00   3rd Qu.:89.00   3rd Qu.:90.0
Max.   :105.00   Max.   :92.00   Max.   :95.00   Max.   :97.0
>
> #Marix of temperature data
> tmp1_mat <- as.vector(unlist(tmp1_data[,2:21]))
> str(tmp1_mat)
 int [1:2460] 98 97 97 90 89 93 93 91 93 93 ...
> tmp1_mat
   [1] 98 97 97 90 89 93 93 91 93 93 90 91 93 93 82 91 96 95 96 99 91 95 91
  [24] 93 84 84 82 79 90 91 87 86 90 84 91 93 88 91 84 90 89 88 86 84 86 89
  [47] 90 91 91 90 89 90 91 91 91 84 88 84 86 88 84 82 80 73 87 84 87 89 89
  [70] 89 91 84 86 88 78 79 86 82 82 78 79 79 78 81 84 84 87 84 79 75 72 64
  [93] 66 72 84 70 66 64 60 78 70 72 69 69 73 79 81 80 82 66 63 68 79 81 69
 [116] 73 73 75 75 81 82 82 81 86 90 93 91 84 84 75 87 84 87 84 88 86 90 91
 [139] 91 89 89 89 90 89 84 87 88 89 89 91 91 89 88 72 80 84 88 89 88 84 84
 [162] 80 73 80 86 88 88 87 88 91 91 89 89 88 82 79 81 82 84 87 90 90 91 91
 [185] 88 88 91 93 81 81 82 86 88 84 80 82 86 87 87 88 88 90 88 91 95 89 70
 [208] 80 82 66 70 64 68 77 86 75 73 75 78 81 82 82 82 80 82 82 79 80 68 63
 [231] 57 66 64 69 70 70 62 63 62 75 71 57 55 64 66 60 91 88 91 91 91 89 93
 [254] 95 95 91 91 86 88 87 91 87 90 91 95 91 91 89 91 91 86 88 80 88 89 90
 [277] 86 86 82 84 86 90 89 89 86 82 87 88 84 86 80 82 86 84 87 90 79 84 87
 [300] 87 88 90 91 89 90 93 93 91 87 84 77 90 91 89 90 89 79 78 81 84 89 87
 [323] 87 88 87 82 80 82 82 88 84 81 82 84 87 80 75 75 86 78 77 82 82 73 82
 [346] 69 72 73 78 78 78 75 79 78 77 78 82 75 73 63 63 72 75 79 79 79 78 82
 [369] 79 84 82 87 88 90 91 82 86 87 87 82 77 73 81 81 86 82 87 88 90 90 91
 [392] 93 93 91 93 93 93 93 97 99 96 93 88 89 91 93 93 93 91 90 96 98 97 98
 [415] 93 93 96 98 98 89 91 91 90 80 82 89 88 90 91 91 84 88 91 84 93 96 96
 [438] 91 91 77 87 87 87 86 87 89 81 81 82 79 68 79 72 75 78 81 82 78 80 77
 [461] 71 73 75 84 71 73 71 73 73 72 72 73 70 64 75 73 77 80 71 66 60 64 73
 [484] 57 59 64 69 75 73 72 75 75 89 91 93 95 96 96 96 91 96 99 96 93 91 93
```

```
[507]  93  93  91  97 100  99  93  96  87  82  75  82  88  91  89  87  86  86  81  84  88  91  91
[530]  91  91  96  95  89  89  89  89  94  97  99 101 101  97  87  86  88  92  92  90  90  92  92
[553]  88  87  79  81  82  87  81  66  66  75  80  82  84  86  87  86  80  75  73  73  84  87  77
[576]  73  81  84  82  68  71  75  73  75  77  79  82  81  82  73  66  55  55  64  71  73  75  75
[599]  77  80  80  80  73  73  75  79  75  75  78  75  78  80  75  77  78  84  87  87  84  86  87
[622]  87  89  91  87  90  90  86  82  82  84  87  88  90  87  84  87  90  84  82  88  90  84  89
[645]  89  87  84  84  84  86  88  84  86  88  87  88  86  86  81  87  84  90  91  91  87  86  88
[668]  90  88  93  90  91  91  81  86  81  82  80  75  73  81  90  88  87  86  86  89  87  84  84
[691]  86  77  77  81  81  82  84  86  87  88  69  66  72  75  78  71  71  75  80  81  80  79  70
[714]  68  79  66  73  75  78  78  75  75  62  60  64  71  75  79  80  81  79  73  64  51  55  63
[737]  72  71  90  90  87  89  93  93  89  89  90  91  84  77  82  88  91  93  93  93  93  91  95
[760]  91  89  87  84  86  89  91  91  88  90  93  91  91  91  93  97  87  87  86  88  89  91  91
[783]  89  88  90  91  93  91  93  93  91  95  93  91  88  84  82  82  78  77  84  84  89  95  93
[806]  91  88  87  91  95  95  90  75  78  91  88  86  81  80  86  84  77  82  73  69  75  75  79
[829]  73  79  82  84  84  82  87  86  80  71  66  70  78  84  79  68  57  66  64  68  71  73  71
[852]  64  59  68  60  68  69  75  75  68  60  73  81  87  86  80  84  87  90  89  84  84  86  87
[875]  84  86  88  88  88  88  88  89  86  81  82  84  87  87  89  88  84  88  84  84  84  82  84
[898]  82  84  84  86  87  84  81  87  89  90  86  89  90  90  87  88  88  90  89  88  89  90  91
[921]  89  88  89  88  86  87  87  84  73  75  81  82  79  80  81  84  82  82  81  81  81  84  87
[944]  82  75  81  80  82  82  82  73  66  71  72  68  66  77  78  75  73  73  73  73  66  78  78
[967]  78  69  72  68  70  75  78  84  78  78  73  73  68  64  57  70  77  75  82  81  86  88  90
[990]  90  89  87  88  89  90  89  91  91  84  84
[ reached getOption("max.print") -- omitted 1460 entries ]
>
> # Time Series Values
> tmp1_ts <- ts(tmp1_mat, start=1996, end = 2015, frequency=123)
> tmp1_ts
Time Series:
Start = c(1996, 1)
End = c(2015, 1)
Frequency = 123
  [1]  98  97  97  90  89  93  93  91  93  93  90  91  93  93  82  91  96  95  96  99  91  95  91
 [24]  93  84  84  82  79  90  91  87  86  90  84  91  93  88  91  84  90  89  88  86  84  86  89
 [47]  90  91  91  90  89  90  91  91  91  84  88  84  86  88  84  82  80  73  87  84  87  89  89
 [70]  89  91  84  86  88  78  79  86  82  82  78  79  79  78  81  84  84  87  84  79  75  72  64
 [93]  66  72  84  70  66  64  60  78  70  72  69  69  73  79  81  80  82  66  63  68  79  81  69
[116]  73  73  75  75  81  82  82  81  86  90  93  91  84  84  75  87  84  87  84  88  86  90  91
[139]  91  89  89  89  90  89  84  87  88  89  89  91  91  89  88  72  80  84  88  89  88  84  84
[162]  80  73  80  86  88  88  87  88  91  91  89  89  88  82  79  81  82  84  87  90  90  91  91
[185]  88  88  91  93  81  81  82  86  88  84  80  82  86  87  87  88  88  90  88  91  95  89  70
[208]  80  82  66  70  64  68  77  86  75  73  75  78  81  82  82  82  80  82  82  79  80  68  63
[231]  57  66  64  69  70  70  62  63  62  75  71  57  55  64  66  60  91  88  91  91  91  89  93
[254]  95  95  91  91  86  88  87  91  87  90  91  95  91  91  89  91  91  86  88  80  88  89  90
[277]  86  86  82  84  86  90  89  89  86  82  87  88  84  86  80  82  86  84  87  90  79  84  87
[300]  87  88  90  91  89  90  93  93  91  87  84  77  90  91  89  90  89  79  78  81  84  89  87
[323]  87  88  87  82  80  82  82  88  84  81  82  84  87  80  75  75  86  78  77  82  82  73  82
[346]  69  72  73  78  78  78  75  79  78  77  78  82  75  73  63  63  72  75  79  79  79  78  82
```

```
[369]  79  84  82  87  88  90  91  82  86  87  87  82  77  73  81  81  86  82  87  88  90  90  91
[392]  93  93  91  93  93  93  93  97  99  96  93  88  89  91  93  93  93  91  90  96  98  97  98
[415]  93  93  96  98  98  89  91  91  90  80  82  89  88  90  91  91  84  88  91  84  93  96  96
[438]  91  91  77  87  87  87  86  87  89  81  81  82  79  68  79  72  75  78  81  82  78  80  77
[461]  71  73  75  84  71  73  71  73  73  72  72  73  70  64  75  73  77  80  71  66  60  64  73
[484]  57  59  64  69  75  73  72  75  75  89  91  93  95  96  96  96  91  96  99  96  93  91  93
[507]  93  93  91  97 100  99  93  96  87  82  75  82  88  91  89  87  86  86  81  84  88  91  91
[530]  91  91  96  95  89  89  89  89  94  97  99 101 101  97  87  86  88  92  92  90  90  92  92
[553]  88  87  79  81  82  87  81  66  66  75  80  82  84  86  87  86  80  75  73  73  84  87  77
[576]  73  81  84  82  68  71  75  73  75  77  79  82  81  82  73  66  55  55  64  71  73  75  75
[599]  77  80  80  80  73  73  75  79  75  75  78  75  78  80  75  77  78  84  87  87  84  86  87
[622]  87  89  91  87  90  90  86  82  82  84  87  88  90  87  84  87  90  84  82  88  90  84  89
[645]  89  87  84  84  84  86  88  84  86  88  87  88  86  86  81  87  84  90  91  91  87  86  88
[668]  90  88  93  90  91  91  81  86  81  82  80  75  73  81  90  88  87  86  86  89  87  84  84
[691]  86  77  77  81  81  82  84  86  87  88  69  66  72  75  78  71  71  75  80  81  80  79  70
[714]  68  79  66  73  75  78  78  75  75  62  60  64  71  75  79  80  81  79  73  64  51  55  63
[737]  72  71  90  90  87  89  93  93  89  89  90  91  84  77  82  88  91  93  93  93  93  91  95
[760]  91  89  87  84  86  89  91  91  88  90  93  91  91  91  93  97  87  87  86  88  89  91  91
[783]  89  88  90  91  93  91  93  93  91  95  93  91  88  84  82  82  78  77  84  84  89  95  93
[806]  91  88  87  91  95  95  90  75  78  91  88  86  81  80  86  84  77  82  73  69  75  75  79
[829]  73  79  82  84  84  82  87  86  80  71  66  70  78  84  79  68  57  66  64  68  71  73  71
[852]  64  59  68  60  68  69  75  75  68  60  73  81  87  86  80  84  87  90  89  84  84  86  87
[875]  84  86  88  88  88  88  88  89  86  81  82  84  87  87  89  88  84  88  84  84  84  82  84
[898]  82  84  84  86  87  84  81  87  89  90  86  89  90  90  87  88  88  90  89  88  89  90  91
[921]  89  88  89  88  86  87  87  84  73  75  81  82  79  80  81  84  82  82  81  81  81  84  87
[944]  82  75  81  80  82  82  82  73  66  71  72  68  66  77  78  75  73  73  73  73  66  78  78
[967]  78  69  72  68  70  75  78  84  78  78  73  73  68  64  57  70  77  75  82  81  86  88  90
[990]  90  89  87  88  89  90  89  91  91  84  84
[ reached getOption("max.print") -- omitted 1338 entries ]
>
> class(tmp1_ts)
[1] "ts"
> plot(tmp1_ts)
>
> # Define beta to generate trend value to forecast a future trend
> tmp1_b_ts <- HoltWinters(tmp1_ts , beta=.5)
> plot(tmp1_b_ts)
>
> # Exponential Smoothing
> #Simple Exponential #
> tmp1_single <- HoltWinters(tmp1_ts,beta=FALSE, gamma=FALSE)
>
> #Double Exponential - model trend #
> tmp1_double <- HoltWinters(tmp1_ts,gamma=FALSE)
>
> #Triple Exponential - model trend and seasonality#
> tmp1_triple <- HoltWinters(tmp1_ts, seasonal = "additive")
```

```
>
> #Compare 3 kinds of Exp Smoothing#
> tmp1_single
Holt-Winters exponential smoothing without trend and without seasonal component.

Call:
HoltWinters(x = tmp1_ts, beta = FALSE, gamma = FALSE)

Smoothing parameters:
alpha: 0.8396301
beta : FALSE
gamma: FALSE

Coefficients:
     [,1]
a 81.62444
> tmp1_single$SSE # Sum of Squared Error
[1] 53704.15
>
> tmp1_double
Holt-Winters exponential smoothing with trend and without seasonal component.

Call:
HoltWinters(x = tmp1_ts, gamma = FALSE)

Smoothing parameters:
alpha: 0.8455303
beta : 0.003777803
gamma: FALSE

Coefficients:
        [,1]
a 81.729657393
b -0.004838906
> tmp1_double$SSE
[1] 54071.22
>
> tmp1_triple
Holt-Winters exponential smoothing with trend and additive seasonal component.

Call:
HoltWinters(x = tmp1_ts, seasonal = "additive")

Smoothing parameters:
alpha: 0.6677614
beta : 0
gamma: 0.6297674
```

Coefficients:

	[,1]
a	66.739214602
b	-0.004362918
s1	17.167113056
s2	12.692593452
s3	11.926233267
s4	12.862822489
s5	11.026083880
s6	8.860499089
s7	9.547553333
s8	7.755384526
s9	4.419013466
s10	2.272689626
s11	4.628251667
s12	2.396834852
s13	3.512957136
s14	1.734948091
s15	3.035023890
s16	6.257944053
s17	5.086362292
s18	8.599153274
s19	5.507486014
s20	10.404819396
s21	10.115801978
s22	9.628840064
s23	7.658623118
s24	7.150473636
s25	6.306599371
s26	5.850691115
s27	5.770487458
s28	4.280481134
s29	7.229771199
s30	4.632381095
s31	6.006248308
s32	6.443645890
s33	5.701166527
s34	3.546887269
s35	3.879569716
s36	3.517339384
s37	2.828550977
s38	2.122971410
s39	2.627923984
s40	1.658896597
s41	0.165866282
s42	-0.001574460

s43	-1.557500303
s44	-2.159601227
s45	-2.260609558
s46	0.474052766
s47	2.501631056
s48	6.552191593
s49	7.240238719
s50	8.395899120
s51	8.633263084
s52	7.504540260
s53	4.804135812
s54	0.449902809
s55	-1.045831475
s56	1.562077049
s57	1.632745190
s58	0.857309158
s59	2.909614779
s60	0.626594899
s61	4.491805650
s62	4.567058619
s63	3.065433531
s64	3.787652805
s65	-2.147135463
s66	1.759895146
s67	1.541155061
s68	1.278521842
s69	0.895959617
s70	2.009912430
s71	3.695537344
s72	4.675235988
s73	4.535880359
s74	1.710420810
s75	0.822675780
s76	2.363162195
s77	1.925012161
s78	-1.656914701
s79	-1.809929506
s80	-0.427021203
s81	0.056812125
s82	-1.137248149
s83	-1.037423821
s84	-2.817503990
s85	-4.578240308
s86	-3.080091372
s87	-2.710719111
s88	-2.255335538
s89	-4.518502545

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s90   -5.159556421
s91   -4.440834373
s92   -5.790113744
s93   -7.461163074
s94   -8.882612687
s95   -8.619859733
s96   -6.200719796
s97   -6.055889182
s98  -11.167287691
s99  -13.489975101
s100 -13.615536188
s101 -14.373453486
s102 -15.142110213
s103 -14.419874185
s104 -14.023613348
s105 -16.187082843
s106 -15.999259045
s107 -12.074075053
s108  -9.199729415
s109 -10.403127076
s110 -12.075113349
s111  -9.722863134
s112  -5.846856763
s113  -8.047801338
s114  -9.636669876
s115 -10.510269852
s116 -12.876648138
s117  -8.657362442
s118  -9.828539578
s119 -14.522204766
s120 -11.852457644
s121  -8.714763993
s122  -4.711332904
s123  18.737998957
> tmp1_triple$SSE
[1] 63025.97
>
> # Triple Exponential using multiplicative ( Seasonal varioation is connected)
> tmp1_triple_m <- HoltWinters(tmp1_ts, seasonal = "multiplicative")
> tmp1_triple_m$SSE
[1] 65648.65
>
>
> # Triple Exponential using multiplicative ( Seasonal varioation is independent of the level)
> tmp1_triple_a <- HoltWinters(tmp1_ts, seasonal = "additive")
> tmp1_triple_a$SSE
[1] 63025.97
```

```
>
> tmp1_triple_a$fitted
Time Series:
Start = c(1997, 1)
End = c(2015, 1)
Frequency = 123
            xhat    level        trend       season
1997.000  87.17619  82.87739 -0.004362918   4.303159495
1997.008  90.32137  82.08762 -0.004362918   8.238118845
1997.016  92.95607  81.86865 -0.004362918  11.091777381
1997.024  90.93226  81.89363 -0.004362918   9.042996893
1997.033  83.99752  81.93450 -0.004362918   2.067387137
1997.041  84.04359  81.93179 -0.004362918   2.116167625
1997.049  75.06703  81.89832 -0.004362918  -6.826921806
1997.057  87.04230  81.84919 -0.004362918   5.197468438
1997.065  84.01782  81.81658 -0.004362918   2.205598519
1997.073  87.05847  81.80032 -0.004362918   5.262509089
1997.081  84.04758  81.75692 -0.004362918   2.295029414
1997.089  88.04397  81.72078 -0.004362918   6.327549739
1997.098  86.02650  81.68706 -0.004362918   4.343809902
1997.106  89.93127  81.66500 -0.004362918   8.270639170
1997.114  90.90776  81.70653 -0.004362918   9.205598519
1997.122  90.94873  81.76376 -0.004362918   9.189338357
1997.130  88.92982  81.79363 -0.004362918   7.140557869
1997.138  88.90728  81.83613 -0.004362918   7.075517219
1997.146  88.88353  81.89368 -0.004362918   6.994216406
1997.154  89.85938  81.96709 -0.004362918   7.896655430
1997.163  88.81884  82.05663 -0.004362918   6.766574129
1997.171  83.84602  82.17324 -0.004362918   1.677143235
1997.179  87.03391  82.27170 -0.004362918   4.766574129
1997.187  88.03942  82.24469 -0.004362918   5.799094454
1997.195  89.02500  82.21400 -0.004362918   6.815354617
1997.203  89.17467  82.19295 -0.004362918   6.986086324
1997.211  91.16749  82.07195 -0.004362918   9.099907462
1997.220  91.17324  81.95574 -0.004362918   9.221858682
1997.228  89.11010  81.83570 -0.004362918   7.278769251
1997.236  87.99157  81.75781 -0.004362918   6.238118845
1997.244  71.81397  81.75908 -0.004362918  -9.940742944
1997.252  79.86066  81.87894 -0.004362918  -2.013913676
1997.260  83.94121  81.96762 -0.004362918   1.977956243
1997.268  88.04928  82.00251 -0.004362918   6.051126975
1997.276  88.94697  81.96524 -0.004362918   6.986086324
1997.285  87.85607  81.99629 -0.004362918   5.864135105
1997.293  83.80148  82.08804 -0.004362918   1.717793641
1997.301  83.75082  82.21625 -0.004362918   1.538931853
1997.309  79.88033  82.37828 -0.004362918  -2.493588472
1997.317  72.87458  82.45383 -0.004362918  -9.574889285
```

1997.325	79.87267	82.53322	-0.004362918	-2.656190098
1997.333	85.84764	82.61388	-0.004362918	3.238118845
1997.341	87.86372	82.71126	-0.004362918	5.156818032
1997.350	87.89345	82.79790	-0.004362918	5.099907462
1997.358	87.04967	82.86469	-0.004362918	4.189338357
1997.366	88.15848	82.82716	-0.004362918	5.335679820
1997.374	91.23528	82.71697	-0.004362918	8.522671690
1997.382	91.20389	82.55550	-0.004362918	8.652752991
1997.390	89.07964	82.41499	-0.004362918	6.669013154
1997.398	88.97332	82.35745	-0.004362918	6.620232666
1997.407	87.97051	82.37090	-0.004362918	5.603972503
1997.415	82.05901	82.38623	-0.004362918	-0.322856765
1997.423	79.16971	82.34246	-0.004362918	-3.168385220
1997.431	81.10080	82.22477	-0.004362918	-1.119604733
1997.439	82.11856	82.15310	-0.004362918	-0.030173838
1997.447	84.01877	82.06956	-0.004362918	1.953565999
1997.455	87.03439	82.05267	-0.004362918	4.986086324
1997.463	90.15341	82.02534	-0.004362918	8.132427788
1997.472	90.25799	81.91854	-0.004362918	8.343809902
1997.480	91.22769	81.74190	-0.004362918	9.490151365
1997.488	91.20137	81.58550	-0.004362918	9.620232666
1997.496	84.21295	81.44667	-0.004362918	2.770639170
1997.504	80.81467	83.97116	-0.004362918	-3.152125058
1997.512	78.66530	88.76488	-0.004362918	-10.095214489
1997.520	100.93009	96.99715	-0.004362918	3.937305836
1997.528	92.62219	91.69738	-0.004362918	0.929175755
1997.537	87.89763	83.93216	-0.004362918	3.969826162
1997.545	85.36047	79.32183	-0.004362918	6.042996893
1997.553	83.25846	77.07348	-0.004362918	6.189338357
1997.561	85.11731	78.89981	-0.004362918	6.221858682
1997.569	89.11107	80.82040	-0.004362918	8.295029414
1997.577	78.74251	77.40306	-0.004362918	1.343809902
1997.585	81.62663	78.23840	-0.004362918	3.392590389
1997.593	83.89598	78.48336	-0.004362918	5.416980633
1997.602	75.35351	79.88398	-0.004362918	-4.526108798
1997.610	84.15061	87.65670	-0.004362918	-3.501718554
1997.618	92.97579	89.55504	-0.004362918	3.425110715
1997.626	85.64879	86.22804	-0.004362918	-0.574889285
1997.634	87.27138	87.79373	-0.004362918	-0.517978716
1997.642	85.13787	89.61143	-0.004362918	-4.469198229
1997.650	88.10164	91.51829	-0.004362918	-3.412287659
1997.659	90.10586	93.44934	-0.004362918	-3.339116928
1997.667	92.38587	96.71309	-0.004362918	-4.322856765
1997.675	93.20999	94.44778	-0.004362918	-1.233425871
1997.683	80.73941	78.94468	-0.004362918	1.799094454
1997.691	80.28195	78.44657	-0.004362918	1.839744861
1997.699	84.38418	79.58945	-0.004362918	4.799094454

1997.707	69.06292	67.30884	-0.004362918	1.758444048
1997.715	64.61113	67.93022	-0.004362918	-3.314726684
1997.724	60.10112	67.51777	-0.004362918	-7.412287659
1997.732	62.37945	72.78797	-0.004362918	-10.404157578
1997.740	64.16252	82.54665	-0.004362918	-18.379767334
1997.748	80.86233	97.12451	-0.004362918	-16.257816115
1997.756	82.99211	93.20551	-0.004362918	-10.209035627
1997.764	88.36418	86.52880	-0.004362918	1.839744861
1997.772	65.40321	77.60035	-0.004362918	-12.192775464
1997.780	69.82676	86.00764	-0.004362918	-16.176515302
1997.789	75.32411	93.46433	-0.004362918	-18.135864895
1997.797	75.81017	97.91788	-0.004362918	-22.103344570
1997.805	97.99605	102.04684	-0.004362918	-4.046434001
1997.813	79.34266	91.36094	-0.004362918	-12.013913676
1997.821	81.91546	91.79552	-0.004362918	-9.875702294
1997.829	79.04072	91.84761	-0.004362918	-12.802531562
1997.837	81.02871	93.81934	-0.004362918	-12.786271399
1997.846	83.65339	92.46029	-0.004362918	-8.802531562
1997.854	87.17691	90.01633	-0.004362918	-2.835051887
1997.862	76.35882	77.20636	-0.004362918	-0.843181968
1997.870	66.44208	68.28150	-0.004362918	-1.835051887
1997.878	62.12453	61.97208	-0.004362918	0.156818032
1997.886	48.70806	64.55560	-0.004362918	-15.843181968
1997.894	55.93132	74.76261	-0.004362918	-18.826921806
1997.902	69.66185	83.48500	-0.004362918	-13.818791725
1997.911	80.89142	83.70644	-0.004362918	-2.810661643
1997.919	75.67923	76.42921	-0.004362918	-0.745620993
1997.927	54.63797	67.29039	-0.004362918	-12.648060017
1997.935	64.29875	72.86987	-0.004362918	-8.566759204
1997.943	62.83254	71.33049	-0.004362918	-8.493588472
1997.951	72.95314	79.45109	-0.004362918	-6.493588472
1997.959	71.65267	78.14250	-0.004362918	-6.485458391
1997.967	67.81504	68.35364	-0.004362918	-0.534238879
1997.976	60.22077	59.79189	-0.004362918	0.433240796
1997.984	62.71564	62.31115	-0.004362918	0.408850552
1997.992	63.84754	64.49996	-0.004362918	-0.648060017
1998.000	65.97906	61.92636	-0.004362918	4.057062406
1998.008	86.79653	78.63002	-0.004362918	8.170877414
1998.016	90.52589	79.42928	-0.004362918	11.100969520
1998.024	88.79432	79.74151	-0.004362918	9.057170233
1998.033	83.27356	81.21002	-0.004362918	2.067905689
1998.041	88.46776	86.36507	-0.004362918	2.107046284
1998.049	79.87081	86.71612	-0.004362918	-6.840946973
1998.057	100.66318	95.47892	-0.004362918	5.188618358
1998.065	93.89042	91.69291	-0.004362918	2.201869966
1998.073	97.67540	92.42948	-0.004362918	5.250275614
1998.081	90.24826	87.96755	-0.004362918	2.285073521

1998.089	94.77916	88.46517	-0.004362918	6.318350547
1998.098	86.93233	82.59842	-0.004362918	4.338264278
1998.106	91.58767	83.30701	-0.004362918	8.285019302
1998.114	89.45972	80.23918	-0.004362918	9.224897674
1998.122	90.45906	81.26336	-0.004362918	9.200065311
1998.130	86.10005	78.94917	-0.004362918	7.155241134
1998.138	88.63960	81.54904	-0.004362918	7.094917086
1998.146	90.13509	83.12087	-0.004362918	7.018585474
1998.154	94.28682	86.36510	-0.004362918	7.926077622
1998.163	90.96605	84.16593	-0.004362918	6.804479491
1998.171	85.88924	84.18424	-0.004362918	1.709361624
1998.179	91.01224	86.25712	-0.004362918	4.759479289
1998.187	92.03107	86.24459	-0.004362918	5.790845815
1998.195	92.35748	85.55172	-0.004362918	6.810124804
1998.203	88.24725	81.30207	-0.004362918	6.949538929
1998.211	90.19311	81.13261	-0.004362918	9.064862730
1998.220	83.50293	74.32168	-0.004362918	9.185612110
1998.228	84.57166	77.32029	-0.004362918	7.255732036
1998.236	86.50852	80.27300	-0.004362918	6.239883268
1998.244	72.69393	82.60011	-0.004362918	-9.901820071
1998.252	89.49191	91.48103	-0.004362918	-1.984759393
1998.260	91.13080	89.14491	-0.004362918	1.990256369
1998.268	89.07980	83.04335	-0.004362918	6.040816683
1998.276	86.63971	79.64689	-0.004362918	6.997182367
1998.285	85.10524	79.21535	-0.004362918	5.894250731
1998.293	84.23449	82.47952	-0.004362918	1.759331552
1998.301	87.24409	85.65738	-0.004362918	1.591069014
1998.309	84.35264	86.82555	-0.004362918	-2.468549302
1998.317	78.36822	87.92123	-0.004362918	-9.548646623
1998.325	87.70812	90.34203	-0.004362918	-2.629547590
1998.333	93.13045	89.86481	-0.004362918	3.269997443
1998.341	91.61551	86.43453	-0.004362918	5.185333016
1998.350	86.46267	81.34483	-0.004362918	5.122201679
1998.358	85.20610	81.03152	-0.004362918	4.178946324
1998.366	82.84888	77.55072	-0.004362918	5.302520582
1998.374	85.44859	76.97951	-0.004362918	8.473442899
1998.382	85.94909	77.34336	-0.004362918	8.610092846
1998.390	82.68546	76.03747	-0.004362918	6.652350510
1998.398	85.53564	78.91419	-0.004362918	6.625816049
1998.407	87.49673	81.89095	-0.004362918	5.610142549
1998.415	75.87323	76.21280	-0.004362918	-0.335203707
1998.423	78.42692	81.63518	-0.004362918	-3.203895037
1998.431	86.21053	87.35559	-0.004362918	-1.140696088
1998.439	87.81906	87.87840	-0.004362918	-0.054980233
1998.447	89.94014	87.99486	-0.004362918	1.949639387
1998.455	93.00500	88.03047	-0.004362918	4.978890292
1998.463	94.78322	86.68725	-0.004362918	8.100330384

1998.472	91.10654	82.82108	-0.004362918	8.289830648
1998.480	91.51595	82.07781	-0.004362918	9.442510769
1998.488	92.63817	83.06443	-0.004362918	9.578100181
1998.496	86.86034	83.30169	-0.004362918	3.563016184
1998.504	84.40855	86.06163	-0.004362918	-1.648715946
1998.512	80.26899	87.78774	-0.004362918	-7.514388171
1998.520	92.54851	90.27480	-0.004362918	2.278067894
1998.528	78.38081	79.88775	-0.004362918	-1.502570622
1998.537	90.16448	87.64223	-0.004362918	2.526614473
1998.545	93.53131	88.19579	-0.004362918	5.339876352
1998.553	91.92420	85.16560	-0.004362918	6.762960169
1998.561	90.69698	83.87633	-0.004362918	6.825012625
1998.569	89.96005	82.73879	-0.004362918	7.225625268
1998.577	77.01828	75.41573	-0.004362918	1.606918095
1998.585	79.53327	76.06692	-0.004362918	3.470711462
1998.593	82.89483	77.04198	-0.004362918	5.857211189
1998.602	75.68197	77.77561	-0.004362918	-2.089278380
1998.610	83.75462	86.66451	-0.004362918	-2.905532868
1998.618	91.20694	88.82729	-0.004362918	2.384010915
1998.626	85.92640	86.01370	-0.004362918	-0.082937756
1998.634	87.44258	87.39401	-0.004362918	0.052937961
1998.642	83.21940	87.09410	-0.004362918	-3.870345199
1998.650	83.46526	86.27548	-0.004362918	-2.805854177
1998.659	81.63768	83.95715	-0.004362918	-2.315100992
1998.667	79.15907	84.19473	-0.004362918	-5.031293693
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1998.683	93.06962	91.42960	-0.004362918	1.644385312
1998.691	87.56375	85.36889	-0.004362918	2.199218069
1998.699	81.92967	80.98151	-0.004362918	0.952515668
1998.707	82.97427	81.02412	-0.004362918	1.954511737
1998.715	78.25774	81.70470	-0.004362918	-3.442595878
1998.724	81.77414	87.53808	-0.004362918	-5.759580500
1998.732	78.99960	86.34902	-0.004362918	-7.345056098
1998.740	69.85887	83.67388	-0.004362918	-13.810644299
1998.748	69.61379	87.10256	-0.004362918	-17.484410097
1998.756	85.73620	98.04028	-0.004362918	-12.299715505
1998.764	91.90913	92.86998	-0.004362918	-0.956483800
1998.772	73.34840	82.90987	-0.004362918	-9.557111515
1998.780	74.83965	88.68272	-0.004362918	-13.838703967
1998.789	76.71635	93.45976	-0.004362918	-16.739047137
1998.797	70.16117	90.97376	-0.004362918	-20.808227064
1998.805	91.47721	98.87491	-0.004362918	-7.393335572
1998.813	71.98039	83.86113	-0.004362918	-11.876376715
1998.821	74.00749	83.86986	-0.004362918	-9.858012730
1998.829	71.00502	83.19274	-0.004362918	-12.183352127
1998.837	74.64424	87.85935	-0.004362918	-13.210744539
1998.846	80.52453	90.09583	-0.004362918	-9.566942039

```
1998.854  81.55383  88.40569 -0.004362918  -6.847496242
1998.862  80.38228  84.02493 -0.004362918  -3.638288239
1998.870  79.28252  83.09753 -0.004362918  -3.810647528
1998.878  83.20008  82.23675 -0.004362918   0.967694205
1998.886  65.44425  78.09221 -0.004362918 -12.643602052
1998.894  70.37521  86.47210 -0.004362918 -16.092522774
1998.902  80.47792  94.23032 -0.004362918 -13.748039245
1998.911  85.47415  90.56801 -0.004362918  -5.089506695
1998.919  78.62177  82.23390 -0.004362918  -3.607767550
1998.927  60.89511  71.79792 -0.004362918 -10.898446041
1998.935  64.14703  73.19912 -0.004362918  -9.047733112
1998.943  72.48656  78.43867 -0.004362918  -5.947753797
1998.951  73.20608  80.11269 -0.004362918  -6.902249366
1998.959  74.42164  83.97728 -0.004362918  -9.551282217
1998.967  83.81024  87.03017 -0.004362918  -3.215568915
1998.976  85.03334  83.81372 -0.004362918   1.223981060
1998.984  80.20445  79.11276 -0.004362918   1.096047196
1998.992  78.84995  80.30740 -0.004362918  -1.453091577
1999.000  89.69115  80.40324 -0.004362918   9.292270132
1999.008  85.01687  76.59855 -0.004362918   8.422683133
1999.016  85.77544  74.57964 -0.004362918  11.200169006
1999.024  84.90729  75.39299 -0.004362918   9.518671355
[ reached getOption("max.print") -- omitted 1965 rows ]
>
> plot(fitted(tmp1_triple_a))
>
> plot(fitted(tmp1_triple_m))
>
>
> #Forecast
> library(forecast)
> tmp1_forecast=predict( tmp1_b_ts, n.ahead = 100, prediction.interval = TRUE )
> plot(forecast( tmp1_b_ts, h = 180 ))
```