Optimal Resource Allocation for wireless networks: a review

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***Abstract*—This report is a review of [1], [2] and [3]. have almost the same subject but with different objectives. All of them are studying and proposing optimization methods for wireless networks. [1] deals with a NFP/SC allocation while [2] and [3] deals with UE and D2D allocation. In terms of optimization problem, they are very similar, since all of them deals with allocation. [1] and [2] are minimizing interference and [2] proposes two different problems: FARA/RARA. [3] maximizing sum rate respecting quality constraints. [1] is able to achieve a very low gap compared to optimality (MIP programming) while [2] and [3] overperform against other methods from literature.**

***Index Terms*—Wireless Networks, Optimization, Mixed Integer Programming.**

I. Introduction

This work will be a review on [1], [2] and [3]. In a worldwide connected world wireless services such as LTE, 5G and 5G+ become more and more important to end users. According to [1] significant effort is still needed to establish the base of 5G+. Works [1], [2] and [3] envision to achieve a network where the target data rate is respected minimizing total system interference.

Article [1] studies the problem of associating SCs with NFPs. SC stands for ultra-dense small cells. An ultra-dense small cell network is an approach to serve 5G systems requirements including higher data rate, energy efficiency and spectrum utilization [1]. NFP stands for network flying platforms. Examples of NFP are unmanned balloons, drones, unmanned aerial vehicles (UAVs). Each of these act as hubs between the core network and SCs. The goal of [1] is to study the association problem of SCs with NFPs in order to achieve a minimized total interference while taking into consideration the following constraints: (1) number of NFP links, (2) NFPs maximum bandwidth and (3) whether a target data rate is maintained.

Works [2] and [3] focus on the same problem but with different solution methods. Before defining the problems, the concept of Device-to-Device (D2D) needs to be defined. It is a mode of communication that two near cellular user equipment’s (UEs) can communicate directly rather than communicating via eNB (Base Station in LTE). Examples can be device applications and services like sharing documents in a conference, downloading media contents in a concert etc. [2]. The problem here is decide how to share resources between D2D and UEs maximizing sum rate respecting other quality of service requirements.

This report will be organized as following: in section II we’ll explain all the methodology used in each article, in section III experimental results will be exposed while conclusion about them will be in section IV.

II. Methodology

In this section we’ll present the methodology used by all articles, it will be split into three subsections to make understanding easier.

A. *ARTICLE [1]*

Work [1] solves the problem of association of NFPs to SCs with two variants: (1) minimizing total interference while satisfying SC data target rate (MIETDR) and (2) minimizing total interference while maintaining system total sum rate target (MITTSR).

One mathematical model for each problem was proposed. The main variable is:

A_{ij} = 1 if i is connected to j. 0 otherwise

Objective function is minimizing interference I_{ij} as below:

$$Minimize \sum_i \sum_j I_{ij} A_{ij}$$

How to calculate I_{ij} is well explained in [1]

Constraints for MIETDR are:

$$\sum_i r_{ij} A_{ij} \geq r_{SCj} \quad \forall\ j \tag{1}$$

$$\sum_j b_{ij} A_{ij} \geq B_i \quad \forall\ i \tag{2}$$

$$\sum_j A_{ij} \leq Nl_i \quad \forall\ i \tag{3}$$

$$\sum_j A_{ij} \leq 1 \quad \forall\ j \tag{4}$$

$$A_{ij} \in \{0,1\} \quad \forall\ i,j \tag{5}$$

(1) ensures that minimum data rate target r_{SCj} for SCs is respected, *(2)* no more than maximum bandwidth B_i is allocated to NFP i *(3)* ensures no more than Nl_i (maximum number of

links) are allocated to NFP_i while *(4)* states that no more than 1 NFP is allocated to SC_j. *(5)* are binary constraints.

For MITTSR, we want to respect the total sum rate target, so we replace *(1)* to:

$$\sum_i \sum_j r_{ij} A_{ij} \geq r_m \tag{6}$$

where r_m is the total sum rate target.

The author proved that both MITTSR and MIETDR are NP-hard in its optimization formats so mathematical models won't be feasible for larger instances. [1] proposes heuristics to solve these problems as below:

1) MIETDR

The algorithm is split into three different parts: (a) check feasibility of total number of SCs associated with NFPs, called NOAS (b) minimize total interference with a method called HBIMTI and then a local search to improve results called MTI (c).

The NOAS method follows the following workflow:

- Sort NFPs based on maximum bandwidth and edges by the division of data rate and bandwidth
- Associate SCs with NFPs depending on the sorted lists. It applies two sorting methods: either the sorting starts from minimum to maximum SC bandwidth or the opposite from maximum to minimum SC bandwidth.
- Selects the sorting method that maximizes the total number of associated SCs, then checks whether all constraints are satisfied.

The NOAS method will then find the number of associated SC. The retrieved total number of associated SCs is compared with the number of SCs j. If the retrieved value from NOAS is equal to J, then the solution for MIETDR is feasible and can be found. Otherwise, if the retrieved value from NOAS is less than j, we find the maximum total number of associated SCs by the same MIP as before, but this time to maximize the number of SCs. If the retrieved value from ILP is less than J, then the solution is infeasible, and we don't need to proceed with next step.

The next step of the algorithm is called HBIMTI and it's based on the well-known Hungarian Algorithm. This method is used to get the optimal assignment between NFPs and SCs and it's repeated until there's no unassigned NFP/SC. Since number of SCs is much higher than NFPs some dummy NFPs needs to be added depending on the input.

The last step called MTI is a local search algorithm. Local search algorithms move from solution to solution in the space of candidate solutions (the search space) by applying local changes, until a solution deemed optimal is found or a time bound is elapsed. [1] used swap and move operators in the local search. Swap chooses two different associations and swap it. Move associates SC_i with another NFP_j and keeps SC_j association with NFP j or vice versa.

Complexity of NOAS is $O(\mathrm{ij})$ and HBIMTI is $O(\mathrm{ij}^3)$, where i is the number of SC and j number of NFPs.

2) MITTSR

The algorithm used for MITTSR has the same structure as the previous one, it starts from NOAS to find total sum rate to them apply the local called MTIBLS, this local search uses three different operators: drop, swap and change. Drop operator removes an existing association. Swap chooses two different SC and swap their association. Move associates SC_i with another NFP_j and keeps SC_j association with NFP_j or vice versa.

Complexity of NOAS is $O(\mathrm{ij})$ and MTIBLS is $O(\mathrm{wi}^2\mathrm{j}^2)$, where i is the number of SC and j number of NFPs and w as the sum of interference matrix.

B. ARTICLE [2]

Paper [2] deals with an assignment problem, where each D2D pair j is assigned to a cellular UE_i in the system. An assignment implies that a D2D pair is sharing the RBs of its assigned cellular UE. The goal is to find a set of assignments which incur minimum interference due to resource sharing while maintaining a target system sum rate. The paper proposes a mathematical model (MIP), where the main set of variables is x_{ij} is a binary variable that indicates whether a D2D pair j shares the RBs of a cellular UE_i or not.

The objective function is minimizing total interference:

$$Minimize \sum_i \sum_j x_{ij} I_{cidj}$$

where $I_{ci,dj}$ represents the interference between UE_{ci} and $D2D_{dj}$.
Constraints:

$$Z = \sum_i (1 - \sum_j x_{ij}) S_{ci,0}\, N_{ci} + \sum_i \sum_j x_{ij} S_{cidj} N_{ci} \tag{7}$$

$$Z \geq T \tag{8}$$

$$\sum_j x_{ij} \leq 1 \quad \forall\ i \tag{9}$$

$$\sum_j x_{ij} = 1 \quad \forall\ j \tag{10}$$

$$\sum_j x_{ij} \leq 1 \quad \forall\ j \tag{11}$$

$$x_{ij} \in \{0,1\} \quad \forall\ i,j \tag{12}$$

(7) expresses the total the total system sum rate. N_{ci} implies the number of RBs allocated to a cellular and $S_{ci,0}$ the sum rate contribution of cellular UE_{ci}. The paper explains how these parameters are calculated. Constraint *(9)* implies that a cellular UE can share RBs with the maximum of one D2D pair. Constraint *(10)* implies that a D2D pair must share RBs with only one cellular UE and constraint *(11)* implies that, a D2D pair can share RBs with maximum one cellular UE or it can remain unassigned.

Based on constraints 12 two different problems are defined:

- **Fair Assignment Resource Allocation (FARA)** scheme where each D2D pair of the system must share RBs of exactly one cellular UE. For FARA problem constraint *(10)* is valid and *(11)* is not
- **Restricted Assignment Resource Allocation (RARA)** scheme where a cellular UE does not share RBs with a D2D, which concludes that either a D2D pair shares RBs of the maximum of one cellular UE or it remains unassigned with any of the cellular UEs. In this case, we replace constraint *(10)* by constraint *(11)* to define the restricted assignment property of this resource allocation problem

Resource allocation problem in D2D communication can be converted into a weighted bipartite matching problem. We propose a resource allocation algorithm for both the fair (FARA) and restricted (RARA) assignment by using weighted bipartite matching approach. So, the final algorithm relies on that property and first part of it is to transform the instance of FARA/RARA into a weighted bipartite matching instance. To do that, the bipartite graph is constituted of two disjoint sets: set of existing cellular UEs and set of D2D pairs. After that, the weighted of the edges must be defined. First of all, the adjacency matrix needs to be square, so a few dummy D2D edges need to be added. In case of FARA, weight for first *m* columns is the system sum rate contributed by c_i and d_j as calculated in the paper [2] for remaining (n-m) dummy D2D pairs, the weight is calculated by sum rate formula defined in the paper as if no D2D pair reuses the RBs of cellular UE, which ensures that dummy D2D pairs do not affect the selection of actual D2D pairs. For RARA, if sharing RBs of c_i with d_j decreases the system sum rate, it's assigned the value of the sum rate obtained for c_i by using by sum rate formula defined in the paper as if no D2D pair reuses the RBs of cellular UE to that edge as we do not want them to be matched in the final solution too.

The next step is the assignment phase, which finds an initial but feasible solution to both problems (FARA and RARA), this phase takes as input the weighted bipartite graph generated by the last step. The only difference between RARA and FARA are the weights (generated previously). Here the Hungarian algorithm is used, to minimize interference and finds optimal allocation. The Hungarian algorithm assigns D2D pairs to cellular UEs and returns the corresponding system sum rate *Z*. If the system sum rate meets the target sum rate *T*, then we consider this allocation as the optimal solution of the resource allocation problem. However, if the result of this minimization bipartite matching fails to surpass the target sum rate then we use Hungarian maximization algorithm (maximum weighted bipartite matching algorithm) to find the possible maximum system sum rate for the same instance of the problem to check whether there exists any solution or not [2]. The complexity of this phase is $O(n^3)$ where n is the total number of UEs.

Finally, a local search (called improvement phase) is used to find local optimum for the solution. In case of FARA, a swap is evaluated and whether it can minimize the interference by maintaining the target system sum rate after this swapping. If an improvement is found with this swapping while maintaining the target sum rate, then we assign RBs of c_i to d_j and c_j to d_i and continue these steps until we find no such pair that can improve the solution. For RARA, it is considered all swapping (change allocations of UE), dropping (eliminate one association) and moving (UE can move its D2D pair to the other one and shares its RBs with no other D2D pairs) operators. The complexity of this phase is $O(\max(n^3, n*m*\mathrm{w}))$, where n is total number of UEs, m is total number of D2D and w is total interference.

C. ARTICLE [3]

Paper [3] deals with an assignment problem, optimization problem where the objective is to maximize the total sum rate of the system while sharing RBs among cellular UEs and D2D pairs and maintaining some quality of service (QoS) requirements.

First of all, it is proposed a integer linear programming to the model. The main variable is x_c^d, that indicates whether the D2D pair d shares RBs with the cellular UE_c or not (binary variable). The objective is to maximize the sum rate while satisfying minimum SINR (ratio between the received signal power and interference with noise power). Model is defined below:

$$Maximize \sum_{c\ in\ n} R_c^{DL} N_c + \sum_{c\ in\ n} \sum_{d\ in\ m} x_c^d R_d^{DL} N_c \tag{13}$$

subject to:

$$\frac{P^{eNB} G^{eNBc}}{T + \sum_d x_c^d\, P^{dt} G^{dtc}} \geq SINR_{c,target}^{DL} \quad \forall\ c\ in\ C \tag{14}$$

$$\frac{\sum_d x_c^d\, P^{dt} G^{dtdr}}{T + P^{eNB} G^{eNBdt}} \geq SINR_{d,target}^{DL} \quad \forall\ d\ in\ D \tag{15}$$

$$\sum_d x_c^d \leq 1 \quad \forall\ c\ in\ C \tag{16}$$

$$\sum_c x_c^d \leq 1 \quad \forall\ d\ in\ D \tag{17}$$

$$x_c^d = \{0,1\} \quad \forall\ c\ in\ C, \forall\ d\ in\ D \tag{18}$$

where Nc implies the number of RBs allocated to a cellular UE c. R_c^{DL} and R_d^{DL} are the sum rate contributed by an individual cellular UE and a D2D, respectively. $G^{dt,dr}$ is the channel gain between transmitter and receiver end of the D2D pair d. T is thermal noise which is also known as the energy of Additive White Gaussian Noise (AWGN) introduced at the receiver end. The calculation of all these parameters are explained in [3].

(14) and *(15)* guarantee that minimum SINR for both UE and D2D are satisfied. *(16)* and *(17)* ensures that a UE is assigned to at most one D2D, and vice versa. *(18)* is binary variable definition

As this problem is NP-hard in its optimization form, an algorithm based on Hungarian method was proposed. Like in previous model the resource allocation problem is translated into a maximum weighted bipartite matching problem where each D2D pair needs to be assigned to a cellular UE. We consider an assignment feasible only if it respects the SINR constraints. Suppose R_{c0} is the system sum rate of a cellular UE if it does not share RBs with anyone and $R_c + R_d$ is the system sum rate contribution if cellular UE c and D2D pair d share RBs. So, a cellular UE c and a D2D pair d can be shared only if $R_c + R_d$ is less than equal to R_{c0}. After that we are able to solve the allocation problem using Hungarian method. Dummy nodes need to be added in order to make matrix square. The complexity of this algorithm is $O(n^3)$ where n is the total number of UEs.

III. EXPERIMENTAL RESULTS

In this section, we investigate the performance of the proposed algorithms, it will again be split into three different subsections, one for each article.

A. ARTICLE[1]

Here the test instances are generated and the algorithm (for both MIETDR and MITTSR) is compared against the MIP presented in the last section.

The parameters for the experimental results instance are defined in Table 1:

Table 1
SIMULATION PARAMETERS [1]

Parameter	**Value**	**Parameter**	**Value**
B_i	200 – 500 MHz	h_d	300 m
J	(30, 40, 50, 60)	I	(20)
J	(60, 70, 80,90,100)	I	(50)
P_t	5 W	Nl	J/5, J/3

The results section in this specific article is very limited since no set of instances were tested and no gap from optimal solution were formalized. Also, the parameters to generate the input data are not big, so it's not possible to see how the algorithms scale with huge datasets.

It can be seen from all tests that interference increases as number of SCs increases, this is expected as the bandwidth will also increase, and more associations will be needed. Comparing algorithms to mathematical model was also done and independently on number of SCs the gap keeps almost constant (less than 10%). The run time is missing in the analysis, as well as MIP Gap returned from solver (Gurobi, in this case).

It was also observed that the total interference in the MIETDR results is higher than the one returned from MITTSR problem. This is because in the MIETDR all SCs should be associated into the NFPs. On the other hand, not all SCs should be associated to the NFPs for MITTSR problem, where the number of associated SCs to the NFPs depends on the total sum rate target.

B. ARTICLE [2]

The simulation parameters are defined in Table 2. In the results presented are an average of 20 different runs

Table 2
SIMULATION PARAMETERS [2]

Parameter	**Value**
Cell Radius	1000 meters
Cellular Users	250
D2D pairs	10 to 250 (increments of 10)
Maximum D2D pair distance	15 meters
Cellular user transmit power	20 dBm
D2D transmit power	20 dBm
Base Station transmit power	46 dBm
Noise power (AWGN)	-174 dBm
Carrier Frequency	1.7 Ghz for LTE
Bandwidth, B	180 kHz [9]
T	Sum rate without any sharing, optimal achievable sum rate

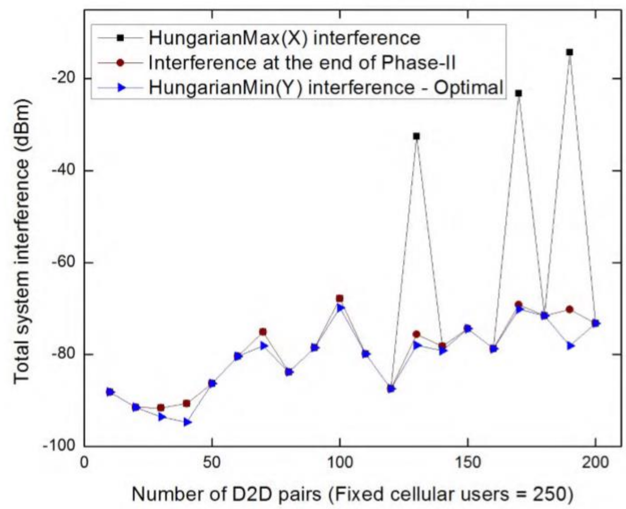

Fig. 1. Total system interference for different algorithms in different phases of FARA and RARA.

The first set of results presented concerns the comparison of different steps of the algorithm. It's is known that in the assignment phase a minimization Hungarian method is run and if no feasible solution considering sum rate is found, the maximization version is run. Fig. 1 shows the results of these phases (red is the final output of the local search phase; blue is Hungarian minimization and black is the Hungarian maximization.

From this graph one is able to observe that optimal solution is always between the black and blue curve, meaning that phase I can introduce more interference with Hungarian Maximization method (if sum rate is not respected), but phase II is able to almost achieve the lower bound (Min Hungarian).

The next set of results aims to compare the existing algorithm with two well-known algorithms in literature: TAFIRA and MIKIRA, these algorithms can provide infeasible solutions to the same problem. FARA is compared to TAFIRA and RARA is compared to MIKIRA, the results are also compared to the mathematical programming model run by Gurobi. Results from FARA are shown in Fig. 2.

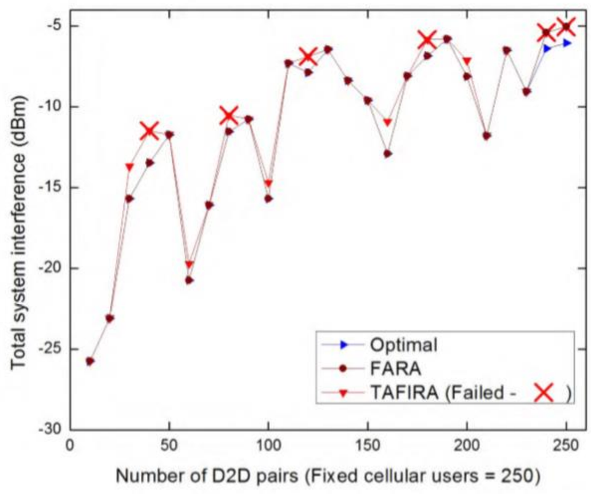

Fig. 2. Comparison of FARA, TAFIRA and Optimal solution.

It can be easily seen that FARA is better for almost all tests interference-wise and also always finds feasible solution. The comparison against optimal solution also shows FARA is very close to optimal. The same effect can be seen in RARA against MIKIRA, as shown in Fig. 3.

It is also found that FARA algorithm gives higher interference than RARA algorithm by giving a fair chance of sharing to all the D2D pairs and our proposed algorithm for both FARA and RARA outperforms TAFIRA and MIKIRA in all the cases [2].

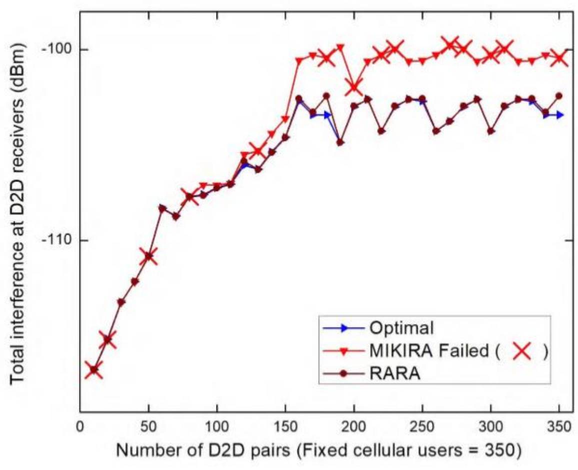

Fig. 3. Comparison of RARA, MIKIRA and Optimal solution.

C. *ARTICLE [3]*

The simulation parameters are defined in **Error! Reference source not found.**. In the results presented are an average of 20 different runs

Table 3

SIMULATION PARAMETERS [3]

Parameter	**Value**
Cell Radius	1000 meters
Cellular Users	250
D2D pairs	10 to 250 (increments of 10)
Maximum D2D pair distance	15 meters
Cellular user transmit power	20 dBm
D2D transmit power	20 dBm
Base Station transmit power	46 dBm
Noise power (AWGN)	-174 dBm
Carrier Frequency	1.7 GHz for LTE
$\mathrm{SINR}^{\mathrm{DL}}_{\mathrm{c,target}}$	Random
$\mathrm{SINR}^{\mathrm{DL}}_{\mathrm{d,target}}$	Random

Four different algorithms were used to compare against the proposed method:

- Greedy Algorithm: cellular UEs are sorted in decreasing order depending on the Channel Quality Identifier (CQI). A D2D device with the lowest channel gain which is not yet assigned is selected for a cellular UE which has higher CQI if QoS constraints are maintained. This method is not optimal (it's a preference rule) and it does not guarantee feasibility.
- Deferred Acceptance Based Algorithm for Resource Allocation (DARA): in this case the assignment is done through preference based on distance. But this is also not optimal, since lower distances are preferable but can also improve interference.
- Local Search Based Resource Allocation Algorithm (LORA): this uses the greedy solution as initial and uses some swapping in it until a local optimum is found.
- Weighted Bipartite Matching Algorithm: others weighted bipartite based from literature

The results can be seen in Fig. 4 and Fig. 5

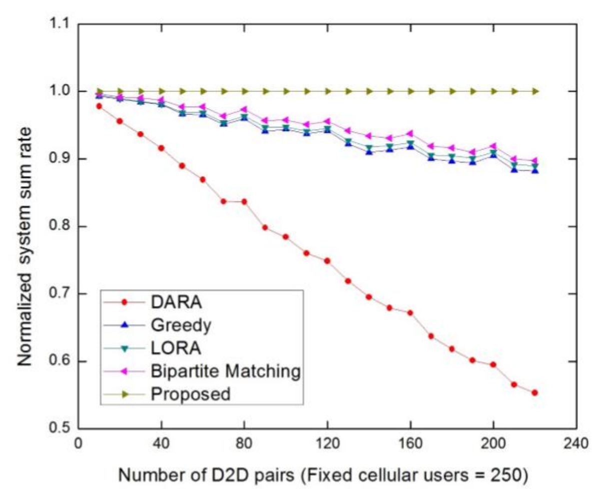

Fig. 4. Comparison of DARA, Greedy, LORA, Bipartite and the proposed method (normalized sum rate)

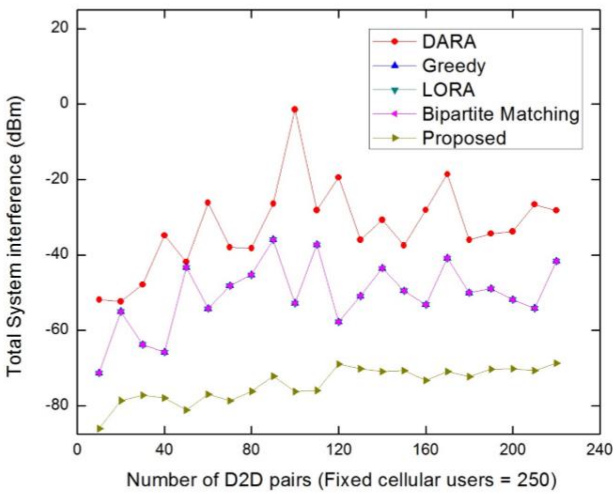

Fig. 5. Comparison of DARA, Greedy, LORA, Bipartite and the proposed method (total system interference)

Simulation results show that our algorithm outperforms other algorithms in terms of the system sum rate and differences get significant with the increase of D2D pairs for a fixed number of cellular UEs [3].

IV. Conclusion

The papers [1], [2] and [3] have almost the same subject but with different objectives. All of them are studying and proposing optimization methods for wireless networks. [1] deals with a NFP/SC allocation while [2] and [3] deals with UE and D2D allocation. In terms of optimization problem, they are very similar, since all of them deals with allocation. [1] and [2] are minimizing interference and [2] proposes two different problems: FARA/RARA. [3] maximizing sum rate respecting quality constraints.

All three problems are known to be NP-hard in its optimization variant so mathematical formulation was proposed but all three were solved by a tailored made algorithm based on the Hungarian method for weighted bipartite graphs. Results presented in [1] are very limited and does not mean a lot since it was tested for only one type of network with not a lot of replications. It was shown that the difference between MIP model solved by Gurobi and the model is not much, however run time is lacking on the analysis. [2] compares its algorithms against TAFIRA/MIKIRA and proposed methods overperform in all studied instances, in feasibility and optimality. [3] compares its results with a lot of methods from literature and overperform them all.

In terms of experimental results, it's missing some more robust analysis on algorithms, such as consider more networks with different structures and replications.

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