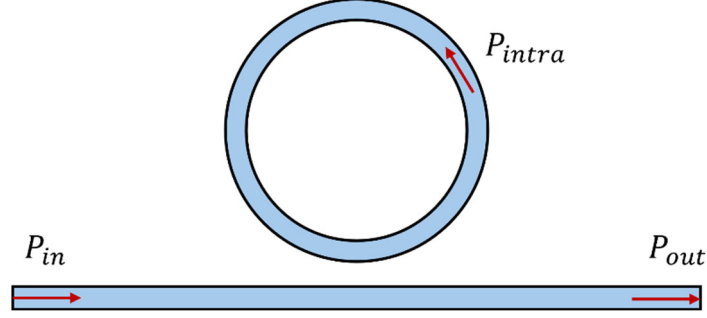


Derivation of ring enhancement factor

Author: Di Yu

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This Note serves to derive the ring enhancement factor, or the ratio of intra-cavity power to the input power on the bus waveguide, for an all-pass ring resonator.



Problem description

Given input power P_{in} , intrinsic and loaded Q factors of the resonator Q_i and Q , calculate the intra-cavity power P_{intra} .

Solution

The power transmission of the ring at a resonant frequency is given by

$$T_{res} = \frac{(a - r)^2}{(1 - ra)^2},$$

where a is roundtrip amplitude transmission coefficient, and r is self-coupling coefficient.

The loaded, intrinsic, and coupling Q factors of the all-pass ring are given by:

$$Q = \frac{\pi n_g L \sqrt{ra}}{\lambda_{res}(1 - ra)} \approx \frac{\pi n_g L}{\lambda_{res}(1 - ra)},$$

$$Q_i = \frac{\pi n_g L \sqrt{a}}{\lambda_{res}(1 - a)} \approx \frac{\pi n_g L}{\lambda_{res}(1 - a)},$$

$$Q_c = \frac{\pi n_g L \sqrt{r}}{\lambda_{res}(1 - r)} \approx \frac{\pi n_g L}{\lambda_{res}(1 - r)},$$

where we assume the roundtrip attenuation and coupling are weak ($r, a \approx 1$).

Now the power transmission can be recast into

$$T_{res} = \frac{[(1 - r) - (1 - a)]^2}{(1 - ra)^2} = \left(\frac{Q}{Q_c} - \frac{Q}{Q_i} \right)^2$$

Next, we express P_{intra} as a function of T_{res} . By the conservation of energy, we have:

$$P_{in}(1 - T_{res}) = P_{diss},$$

Note that

$$\frac{P_{diss}}{f_{res}} = 2\pi \times \frac{\text{intracavity energy}}{Q_i} = 2\pi \times \frac{P_{intra}}{Q_i} \times \frac{n_g L}{c}$$

Combine these equations, we have

$$P_{diss} = 2\pi \frac{f_{res} P_{intra} n_g L}{Q_i c} = P_{in} (1 - T_{res}) = P_{in} \left[1 - Q^2 \left(\frac{1}{Q_c} - \frac{1}{Q_i} \right)^2 \right]$$

Which can be simplified into

$$\frac{P_{intra}}{P_{in}} = \frac{Q_i c}{2\pi n_g L f_{res}} \left[1 - \left(\frac{Q}{Q_c} - \frac{Q}{Q_i} \right)^2 \right]$$

Example usage

Let $Q_c, Q_i = 50 \text{ M}$, $Q = 25 \text{ M}$, $L = 6 \text{ mm}$, $n_g = 1.6$, $f_{res} = 193 \text{ THz}$, $P_{in} = 1 \text{ mW}$, we have

$$P_{intra} = 1,289 \text{ mW}$$

This indicates a ring enhancement factor of $P_{intra}/P_{in} = 1289$.

Reference

[1] https://en.wikipedia.org/wiki/Q_factor

[2] Bogaerts, Wim, et al. "Silicon microring resonators." *Laser & photonics reviews* 6.1 (2012): 47-73.