Abstract

The rise of modern transportation technology has engendered a revolution of driving; indeed, new car-sharing business models have emerged to augment the driving experience. In fact, according to the Wall Street Journal, car-sharing has experienced a three fold rise since 2008 [15]. The newfound popularity begs the mathematical modeling we completed today.

We first focus on categorizing American drivers based on their driving demographics. We employed two independent variables: time spent driving and distance driven to conclude the percentage of drivers in that specific strata. The model was achieved though statistical graphing in an x-y plane of time and distance. The area bounded by specific equations calculated provided relative driving rates in certain strata. These percentages were subsequently extrapolated to the general American population. Furthermore, a sensitivity analysis was conducted to lend validity to our findings.

The second part required further consideration, as we endeavored to produce a mathematical model that would take location and business option and output relative participation rates. We first extrapolated probabilities for individual people, and summed over the people in a city to glean the total participation rates. We applied our model to four representative U.S. cities: Poughkeepsie, NY, Richmond, VA, Riverside, CA, and Knoxville, TN.

The dynamic automobile industry reveals new technology day-by-day, and our third section serves to incorporate two recent technologies into our mathematical model: self-driving cars and renewable energy. We recycled the model from part 2 and further incorporated what we call the Renewable Energy Multiplier (REM) that models the additional participation in car-sharing services caused by renewable energy fueling locations.

While our models accuately incorporate real-world data to produce city rankings, some car-sharing companies may still remain reluctant to use our models.

Share and (Car) Share Alike

Modeling new approaches to mobility

Team 7508

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Contents

1 Introduction

1.1 Background

Ever since the heyday of the automobile during the Roaring Twenties, car usage has exploded to the point where cars are the undisputed, premier form of transportation in America. Not only do 95 percent of American households use cars today [4], but it has become an essential part of our lives, being used to get groceries, go to work, and move from place to place. Yet recently, a revolution in the car industry has been gaining momentum that has the potential to upend modern transportation as we know it: car-sharing. Car-sharing, which is the shared use of a vehicle at the same time, has the potential to save emissions, reduce traffic, and reduce individual costs. This method of transportation has appeals to different types of drivers, depending on their backgrounds.

In addition to current types of car-sharing services such as Lyft, Zipcar, and Uber, there is also new and upcoming technology involving clean energy and completely driverless cars. However, many are unsure of whether this relatively new technology is safe or feasible. In order to be successful, it's best to focus initial efforts of this upstart technology on cities susceptible to it. There are multitude of different car-sharing techniques, the mainstream of which being: round trip car sharing, one-way car sharing floating, one-way car sharing station, and fractional ownership. Mathematical modeling can help us find cities with high likely proportions of people that would be interested in certain technology, and help bring in the start of a new transportation era. In this report, we will explain methodology to understand possible market segments to penetrate and then explore four case study examples and their susceptibility for different types of car-sharing.

1.2 Restatement of the Problem

Given the recent popularity of car-sharing services, we have developed mathematical models providing insight into the emerging car-sharing market by answering the following questions:

1. What is the relationship between distance traveled by car and time spent in the car? How can we categorize American drivers by their amount of time spent using the car and the miles they drive per day?

2. What differentiates one car-sharing business model from another? Given a certain car-sharing business model, can we optimize the location (and subsequent demographics) to best-fit the business model?

3. With technology more dynamic now than ever, how will the emergence of new driving technologies including renewable energy and self-driving cars affect the car-sharing industry? Will new automotive technology augment the car-sharing experience or detract from it?

2 Who's Driving?

The decision to employ car-sharing services is fundamentally based on two key variables related to usage of cars themselves: time spent using the car and miles driven per day. The categorization of these key variables depends on determinants of what should be considered "high" time and distance and what is "low" time and distance as they are comparable. In our model for Part 1, we build a model based largely on geographic location of the car user in relation to urban, suburban, and rural areas. The mathematical model factors in the average distances and times that the driver spends. For distance, there is a 'work cycle' of driving to work in a potentially different region and a 'life cycle' of customary non-work driving. For time, the 'work cycle' and 'life cycle' are also used. This ultimately incorporates the wide spectrum of driving levels that Americans fall into.

2.1 Assumptions and Simplifications

1. **Assumption**: We assume average traffic times to travel from one of the urban, suburban, and rural subsections to the other constant no matter where you are in the US.

   Justification: The time it takes to travel from one section to the other changes with many variables. Every road has a different average speed. It is also unpredictable because accidents will greatly affect the time it takes to travel. So we will not consider the difference between regions in the US in this report.

2. **Assumption**: We assume that the geographic subsections reflect other variables effectively.

   Justification: Where people live is a result of their socioeconomic, physical, and other conditions. The model's estimation of living location is important for these other factors. Furthermore, considering every single determinant would break down the prompt of only nine percentages.

3. **Simplification**: We can divide people's vehicle usage as for a 'work cycle', where they may drive from one region of the city to the other (i.e. suburb to central city), and 'life cycle', where people usually do within their town. According to assumption 1, 'life cycles' are mainly activities requiring little driving. Driving to work, however, is different because many people drive very far to work.

4. **Assumption**: The 'life cycle' starts and ends at the same region.

   Justification: According to data from one of the studies from the National Household Travel Survey [1], people use their car mainly for home, work, recreational activities and shopping. All activities other than work usually require driving to a destination.

5. **Assumption**: 'High', 'Medium', and 'Low' will be qualitatively relative between different combinations of 'work cycle' and 'life cycle'. **Justification**: In order to avoid the basic percentages of making every percentage 1/9, we will interpret the parameters within our model of different distance/time combinations for all people.

6. **Assumption**: The population in question is all US citizens who drive, regardless of public transportation.

   Justification: Car sharing companies are looking to attract consumers, and even people who don't drive on a regular basis can become potential customers.

7. **Assumption**: The time it takes to complete a life cycle (see next section) is the same for people regardless of their residence location.

Justification: One thing people consider about choosing where they live is the size of their life cycle. If people's life cycle in a region is too long, there may be too little competition for businesses in that area; there will be a net inflow of businesses, a subsequent increase in employment, and thus a decrease in the average life cycle.

2.2 Work Cycle and Life Cycle

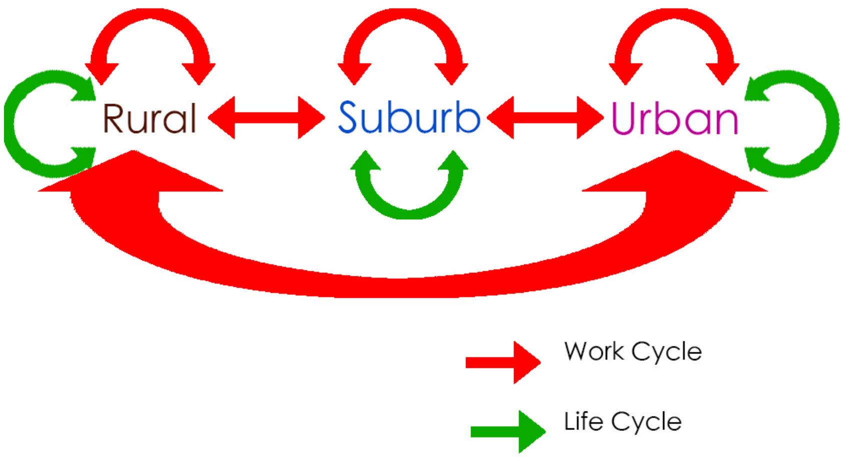

Figure 1: A visual description of our work cycle and life cycle simplification.

As shown in figure 1, an individual's life cycle begins and ends at the same place, while the individual's work cycle can connect two different regions. Using this simplified model, we can define "low," "medium," and "high" travel distances and times for the context of the problem. We describe each individual by two letters showing where they live and where they work. For example, RS means a person that lives in rural area and works in suburbs. For time, since life cycles all consume the same amount of time according to assumption 7, people's time driving depends solely on their work cycle. Apparently RU will have the longest working cycle, and UR, by assumption 2 has the exact same work cycle as RU. SU will have a longer work cycle than RS because although the distance is the same, SU takes longer because the traffic to the cities is worse. The next is RR, SS and UU. Thus the ranking of time from longest to shortest is RU=UR>SU=US>RS=SR>RR>SS>UU. So we'll define the first two ranks, RU,UR,SU and US as high, the next two ranks, RS,SR and RR as medium, and the last two ranks SS and UU as low.

For distance, we have to consider life cycle. Since time is the same for each life cycle, the rural life cycle will be the longest in distance because the traffic in rural area is the best thus businesses can spread a little farther. With that in mind, our ranking for distance from the farthest to the shortest will be RU>UR>SU>US>RS>SR>RR>SS>UU. Similarly, the first three will be characterized as long distance, the next three will be mid distance and the last three will be short distance.

Commuter Statistics	Data (Millions)
Total number of commuters in the U.S.	128.3
Suburb to Suberb Commuters	40.8
Within City Commuters	27.4
Rural to Rural Commuters	20.4
Suburb to City Commuters	18.2
Central City to Suburb Commuters	8
Outer Suburb to Inner Suburb Commuters	3.6
Outer Central City to Inner Suburb	2.6
Central City to Rural "Extreme Commuters"	0.4

Table 1: Commuting categories and counts [2]

Travel Type	Percentage of Entries
Home	34.2
Work	10.2
Shopping/Errands	19.6
Social/Recreational	12.1
Meals	7.4

Table 2: Percentages of entries in the NHTS data set for the 5 most frequent travel types [1]

Distance Categorization	Our two-letter codes
High	RU, UR, SU
Medium	US, RS, SR,
Low	RR, SS, UU

Table 3: Distance Categorizations based on the work/life cycles model

Time Categorization	Our two-letter codes
High	(RU+UR), (SU+US)
Medium	(RS+SR), RR
Low	SS, UU

Table 4: Time Categorizations based on the work/life cycles model

2.3 Analyzing the Data

We found the relationship between travel time and travel distance for the survey respondents. Travel time and travel distance are not entirely independent, since the travel time for some people will depend on the distance. However they are not entirely dependent because the travel time for a given distance can vary based on which route the individual takes. First, we filtered the entries in the dataset to only ones that were from individuals 18 or older, and also only trips to home, work, shopping/errands, social/recreational activities, and meals. Table 2 lists the percentages of the these trip categories from the NHTS dataset.

We then used a linear regression and constrained it to pass through the origin to obtain a slope value relating distance traveled and time taken. Since the travel distance and travel time variables are somewhere between completely independent (with x and y axis perpendicular) and completely dependent (with x and y axis parallel), we used a graph with the Y-axis slanted. A slanted Y axis is between fully parallel axes (dependent) and fully perpendicular axes (fully independent), so by calculating the areas in regions bounded by our low, medium, and high intervals over the total area, we can find the percentages of US drivers in each of the nine specified categories.

2.4 Plugging the numbers in

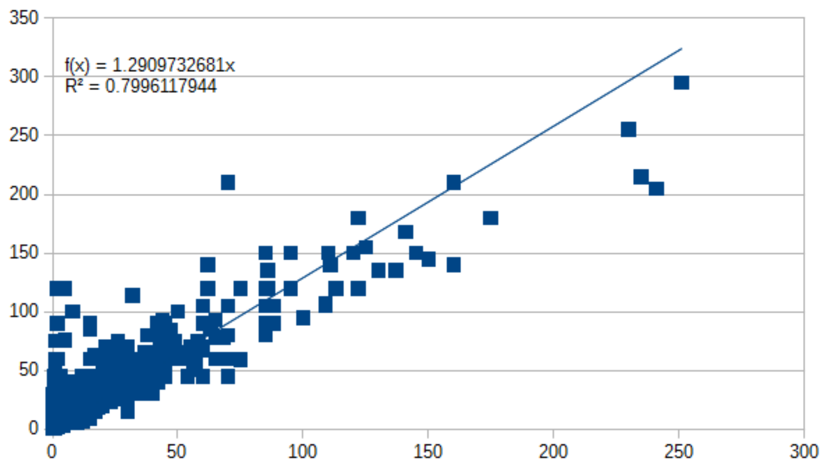

Figure 2: A scatter plot of time vs distance .

We choose 3000 random samples from the data in one of the studies from the National Household Travel Survey [1]. Their scatter plot shows a linear regression with slope 1.29. This tells us that our slant axis should have a slope of 1.29.

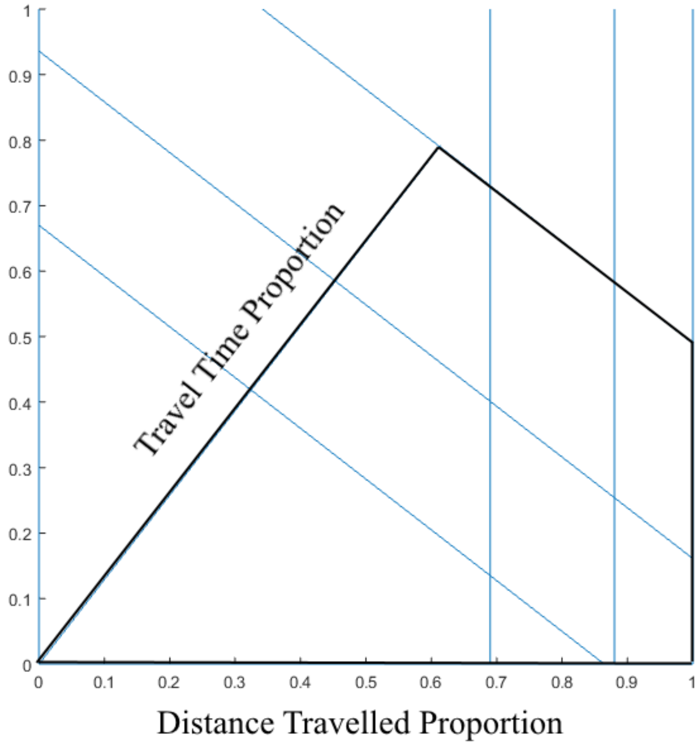

Figure 3: Slanted axis that we used.

Pulling data from Table 2, we draw lines perpendicular to their axis at

$$x = .69, .88, 1.0$$

and at

$$y = .53, .74, 1.0$$

. This will divide the graph into parts, with each representing a unique combination of travel time and travel distance. Calculating the areas of each section gives us

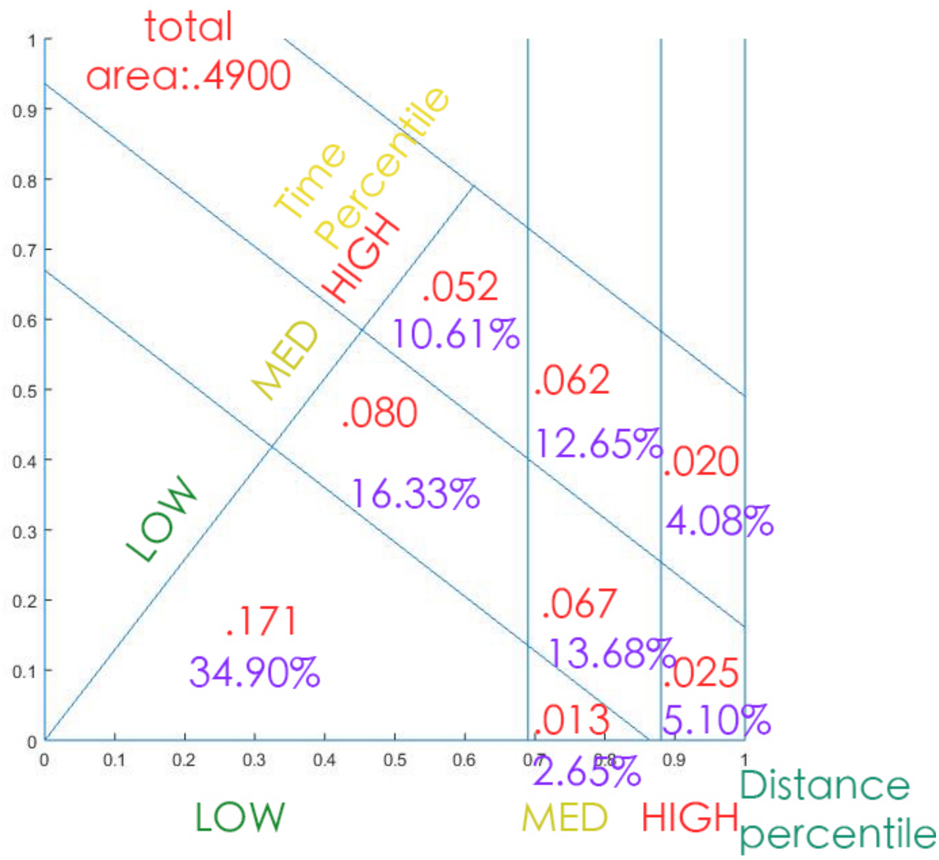

Figure 4: Final Calculation.

Our evaluation of percent of US drivers in each category is below

Distance / Time	Low	Medium	High
Low	34.90%	16.33%	10.61%
Medium	2.65%	13.68%	12.65%
High	0%	5.10%	4.08%

Table 5: Percentages for Low, Medium, and High Distances and Times

2.5 Sensitivity Analysis

If due to demographic changes (say a lot of people moving to suburb from urban areas), boundary changes and it will cause our result to shift. Recently there is a trend that people are moving to cities from suburbs [3]. This will cause an increase in the population characterized as UR,US and UU and a decrease in the population characterized as SR,SS, and SU. This will result in no change to the x boundaries, while resulting in a net increase in the high time section and a net decrease in the medium time section, thus shifting the boundary of medium and high on the time axis down. If the value on the time axis increase by x, then the change in percentage will be

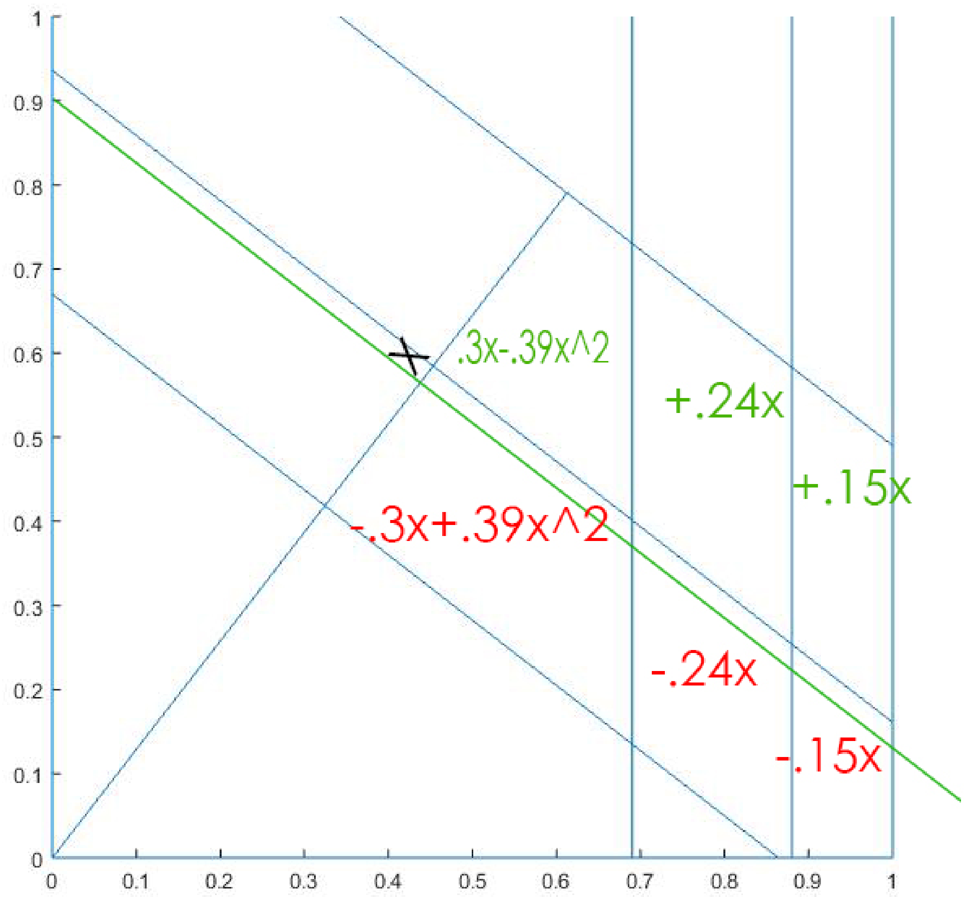

Figure 5: Change in area if the second bar of the time axis decreases by x.

Distance / Time	Low	Medium	High
Low	34.90%	(16.33+(.39x^2-.3x)/.49)%	(10.61+(.3x-.39x^2)/.49)%
Medium	2.65%	(13.68-(.24x/.49))%	(12.65+(.24x/.49))%
High	0%	(5.10-(.15x/.49))%	(4.08+(.15x/.49))%

Table 6: New Percentages for Low, Medium, and High Distances and Times

Let's say, for example, that X shifted by 4 percentage points, that will lead to the new distribution as

Distance / Time	Low	Medium	High
Low	34.90%	16.30%	10.64%
Medium	2.65%	13.66%	12.67%
High	0%	5.09%	4.09%

Table 7: New Percentages for Low, Medium, and High Distances and Times for $\triangle x = .04$

Now let's look a more complicated hypothetical situation. The emergence of new high-tech companies in the cities cause a job flowing into the city and out of the suburb.

In the short run, people are not going to move to adjust to this change, which limit the effect of this change to an increase in RU,SU and UU and an decrease in RS,SS and US. Then by Table 3 and 4 the medium and high boundary of both time and distance will decrease, which will result in the following area change

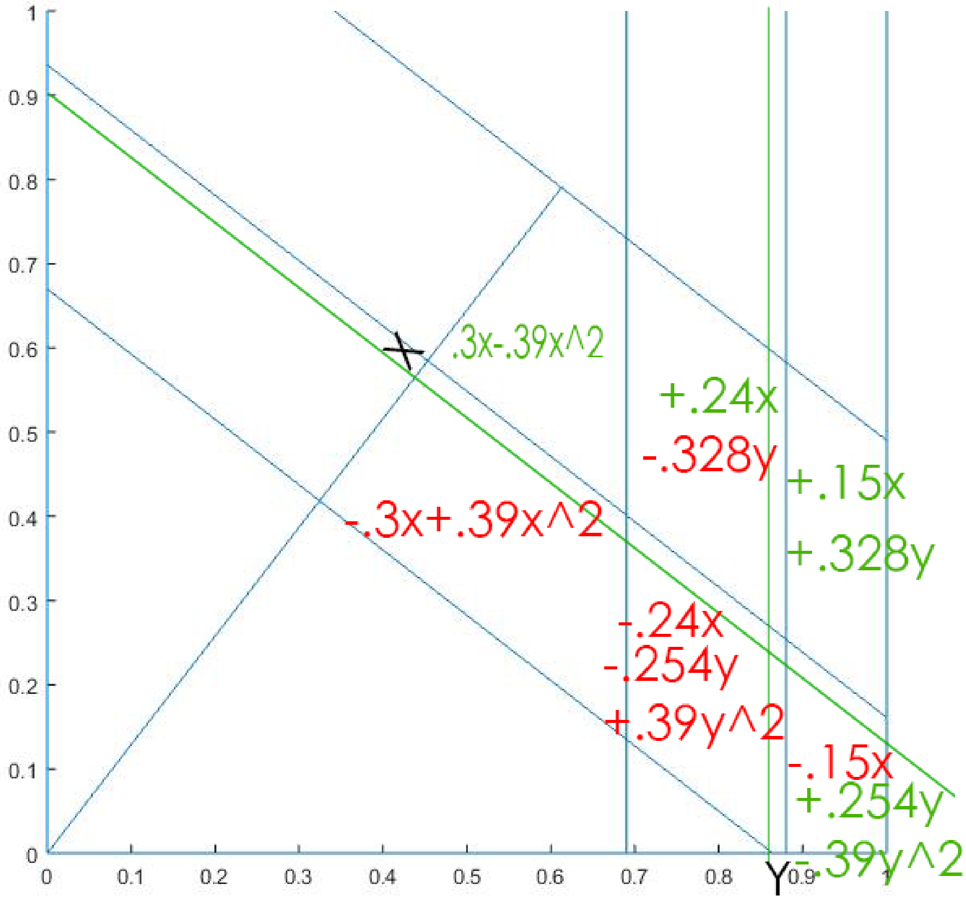

Figure 6: Change in area if the second bar of the time axis decreases by x and the second bar of the distance axis decreases by y.

Overall, population movement will likely to result in a change in driving time distribution only, while a shift in job structure will change both the distance and time distribution. The change, however, is not very big. Table 7 has shown that a 4% change in x will result in a .3% change in low distance medium and high time, a .2% change in medium distance medium and high time, and a .1% change in high distance medium and high time. This is a net change of 1.2% versus a $\triangle x = 4\%$. Shifting in one axis will not cause a overall big shift. The changes in the boundary between low and medium for time and distance will likely cause a more significant overall change, but it is less likely to happen as the definition of rural and suburban areas are becoming more and more vague nowadays.

3 Zippity Do or Don't?

While Zipcar certainly pioneered the pay-by-the-hour car-sharing service, other profitable business models have emerged as well. In this section, we evaluate the feasibility of four specific car-sharing business options: the round trip car sharing model, one-way sharing floating model, the one-way car sharing station model, and the fractional ownership model. We first extrapolated the probability that a single individual would use a certain car-sharing model, and summed up all individual probabilities to calculate the expected value of participation. We then further incorporated Bayesian probabilities to augment the calculation of individual probabilities. Finally, we implemented our mathematical to rank four cities- Poughkeepsie, NY, Richmond, VA, Riverside, VA, and Knoxville, TN- based upon their success with different business options.

3.1 Assumptions and Simplifications

1. **Assumption**: The demographic variability of age and ownership of vehicle is not statistically significant

   Justification: We reason that regardless of the car-sharing option available to individuals, young adults are always going to be more likely to rent cars than adults. Similarly, regardless of the business options, an individual without a car is far more likely to rent a car than an individual with 1 or 2 cars. Thus, we determine that age and ownership of vehicle do not change significantly enough relative to the four models to be included in our mathematical model.

2. **Simplification**: The geographical location of a city is characterized as either urban or rural.

   Justification: The vast majority of geographical locations fall under these two stratifications. While suburban municipalities have come to dominate America, the binary system of urban and rural simplifies the mathematical model further. The other possible geographical location types are not statistically significant.

3. **Assumption**: We assume that the variables included: geographical location, income, and disposable time are independent of each other.

   Justification: It is impossible to assume that we can account for the possible variation between different variables and isolate the effects of each one without conducting a detailed case study. Thus, for simplification, we assume that the variables are independent of each other, contributing towards our mathematical model.

4. **Assumption**: We assume that income and disposable time are both variables that can be quantified in step graphical notation.

   Justification: The data that is accessible does not take into account variables as an increment of 1 unit, rather, it divides units proportionally. Thus, we reason that step graphs most accurately represent the data available from inducing parallelism.

3.2 Defining Variables

In calculating the participation level among residents of a city, we present three defining independent variables for consideration with each car-sharing business option:

1. **Geographical Location**
   We reason that the geographical location of a resident affects his/her propensity to choose one car-sharing business option over another. Our model categorizes three levels of location: urban, rural, and suburban. Indeed, the prevalence of taxis in larger cities suggests the timely, on-the-spot culture of driving one-way to destinations; those living in urban locations are subsequently more inclined to use the one-way car sharing floating model than the round trip car sharing model, for example.

2. **Net Individual Income**
   The varying prices and car amenities of these business options compelled us to take into account net individual income. According to Cervero, Golub, and Nee of Berkeley University, those using car-sharing in general generally identify as upper and middle class. The four car sharing options, however, attract a specific strata as well. Connie Loizos of Tech Crunch finds that "luxury cars within the reach of many more people," [12] while Jane Gottlieb of Governing declares Zipcar, a round-trip sharing service, a service "that poor people can afford." [11] Income levels thus factor into the popularity of car-sharing models.

3. **Average Household Size**
   A consumer's household size affects his/her propensity for convenience. While the one-way car sharing floating model provides great flexibility for drop-off locations as cars are returned merely to defined areas and not to a specific location, the one-way car sharing station model requires drop-off at a specific location, providing less convenience. The round-trip car sharing service, on the other hand, requires drop-off at the pick-up location and is thus the least flexible option. Indeed, those with larger families would gravitate toward the one-way car sharing floating model, and vice versa.

3.3 Expected Value

To calculate the participation rate among residents of a city, we first calculated the probability that an individual resident would use a specific car sharing service. When choosing car sharing services, residents live in one of three geographical locations: urban, suburban, or rural. Indeed, probability that resident j living in the geographical location G will use a given service S is $P_j(S|G)$. This reasoning was extended to include net individual income(I) and household size (H):

$$Probability\ of\ Resident\ J = P_j(S|G, I, H) \tag{1}$$

When G I and T are independent of each other as assumed, we have to take into account certain variables that may affect whether the resident uses a car sharing service but don't affect the probability that the resident will use a specific business model. This is captured in the coefficient .

$$P_j(S|G, I, T) = \alpha * P_j(S|G) * P_j(S|I) * P_j(S|H) \tag{2}$$

This equation provides the probability that a single resident with given parameters G, I and T uses car-sharing business option S. To find total expected value of residents using car-sharing business option S, we simply sum over all residents in the city:

$$E.V.\ of\ Participants = \sum_{j=0}^{CityPopulation} P_j = \sum_{j=0}^{CityPopulation} \alpha * P_j(S|G) * P_j(S|I) * P_j(S|H) \tag{3}$$

3.4 Bayes' Rule

Since little data exists to quantify our model, estimating specific probability values such as $P_j(S|G)$, or the probability that person j will buy the service S given his geographic location G is relatively difficult. Instead, we used Bayes' Rule to change our input values:

$$P_j(S|G) = P_j(G|S) * P_j(S)P_j(G) \tag{4}$$

We can now easily calculate $P_j(G|S)$, or the probability that a person lives in geographic location G given that he/she uses service S by simply calculating $\frac{\#\ of\ people\ currently\ living\ in\ G\ using\ S}{\#\ of\ total\ people\ using\ S}$. We can also calculate $P_j(S)$ by calculating $\frac{\#\ of\ people\ currently\ using\ S}{Current\ Population\ Size}$. Finally, we can calculate $P_j(G)$ by calculating $\frac{\#\ of\ people\ currently\ living\ in\ G}{Current\ Population\ Size}$. The incorporation of Bayes' rule helps augment the real-world application of the model.

3.5 Calculating City/Service Rankings

Unfortunately, we do not have access the data we would need to be able to use this model. To rank cities instead, we compare means of several variables in each service and city. Tables of means for each variable for every service and city are below.

Location	Geography	Income	Average Household Size	
NY	Rural	$39481	2.40	
VA	Urban	$40496	2.33	[13, 14]
CA	Urban	$56529	3.21	
TN	Urban	$33595	2.05	

Table 8: Averages for Each of the Four Analyzed Locations

Service	Geography	Income	Average Household Size	
Round Trip	Rural	$60000	2.02	
1-way Floating	Urban	$55000	2.8	[8, 9, 10]
1-way Station	Urban	$51000	2.6	
Partial Ownership	Rural	$67742	1.5	

Table 9: Averages for Each Type of Car Sharing Service

Location	Round Trip	1-way Floating	1-way Station	Partial Ownership
Poughkeepsie, NY	0.152	0.544	0.501	0.534
Richmond, VA	4.13	0.098	0.053	4.468
Riverside, CA	4.350	0.022	0.067	5.327
Knoxville, TN	4.194	0.223	0.161	4.389

Table 10: Scores for Each Service in Each Location

Service	City
Round Trip	Poughkeepsie, NY; Richmond, VA; Knoxville, TN; Riverside, CA
1-way Floating	Riverside, CA; Richmond, VA; Knoxville, TN; Poughkeepsie, NY
1-way Station	Richmond, VA; Riverside, CA; Knoxville, TN; Poughkeepsie, NY
Partial Ownership	Poughkeepsie, NY; Knoxville, TN; Richmond, VA; Riverside, CA

Table 11: City Rankings for Each Type of Service Based on Scores; Best to Worst Score

To calculate scores that compare each city and each service to see how well they fit, we summed the squares of the percent differences between each variable mean. For the categories of Urban vs Rural, we assumed a 50% difference between urban and rural locations. The following equation was used.

$$Score_{City} = \sum_{variable} \left(\frac{variable_{City} - variable_{Service}}{variable_{Service}}\right)^2 \tag{5}$$

Results of the equation are in table 10. A lower score means a better fit. Table 11 contains the rankings for the cities in order of best to worst score for each service.

4 Road Map to the Future

Now that we have a qualitative model to understand which car-sharing options would be the best in each of the four cities, we must modify it for the ever changing conditions of the car ride word. Specifically, self-driving cars and environmentally friendly cars will adjust the model by rendering older components unnecessary and bringing in new variables to increased importance. We will need to adapt the model for these conditions.

4.1 Assumptions and Simplifications

1. **Assumption**: Self-driving cars will be able to drive themselves to the users and self-park themselves immediately, allowing users to go wherever the need to go without worry of where it is left.

   Justification: Technology is already approaching the perfection of park-assist and other driverless settings. KPMG predicts that by 2030, the portion of completely driverless cars will be 25 percent and only increase from there. [16]

2. **Assumption**: Alternative fuel cars will be reliant on fuel charging stations dispersed throughout the area.

   Justification: Currently, the only feasible method of charging would be those public locations rather than a futuristic source. Also, this is more quantifiable and realistic.

3. **Assumption**: All American cities and towns will be equally receptive in mentality to these new car technologies, meaning that variables will be physical and tangible in nature, rather than psychological or mental.

   Justification: Only the physical factors are truly quantifiable and measurable in studies, whereas the intangible ones would not make sense to use in mathematical modeling. Furthermore, the American culture and society regarding new car technology is generally close enough throughout the nation in mentality to make this assumption.

4. **Assumption**: The ratio of alternative fueling centers in each state for its area nears constant.

   Justification: This simplification helps utilize data on fueling centers necessary for altering our model. Generally, each state's density ratio should be consistent.

5. **Assumption**: The difference of geography between rural and urban is minimized because driverless cars won't mind the distance.

   Justification: The preferences of the passengers and car users in geography are reduced since there is no human driver anymore. This helps quantify the impact of driverless cars through the removal of driver preference.

6. **Assumption**: There will be enough car-sharing storage stations in both rural and urban areas to provide an excess capacity, thus not hindering our model.

   Justification: There are significant car-sharing stations throughout the country and since the distribution is too randomized to include in the model, it is best to assume that there is excess storage. According to TransportationResearchProcedia, rural areas are just as open to electric car-sharing development and usage as urban areas. Especially with the introduction of driverless cars, parking storage efficiency at the stations can be maximized since the cars will be programmed to a tee.

4.2 Adjustments to Previous Model

Because the popularity and participation of a city's populace in the new car technology is founded on whether or not they have easy access to fuel charging stations for the alternative energy, fuel charging station density needs to be added into the model to help rank them. This is the most important added factor because this is where they refuel and can actually utilize their cars in the way that they want. This factor will be added based off of population modeling functions. This is because in the very beginning, participation rates will increase slowly since it will be inconvenient to buy into a program with very few charging stations. As the stations increase, however, it will reach a maximum carrying capacity because there will simply be a few stanch refusers of participating in the program, no matter how dense and easily accessible the stations are. This multiplier will reflect the likely increased market penetration of new car technology in each city. In addition, we will also need to take out the factor of geography. Before this technology, it was a definite factor in ranking cities. However, the new charging station density will reflect the technological advancement of the area. Furthermore, the driverless cars make the difference between rural and urban communities minimal since no human is driving, making driver factors such as time reduced significantly.

4.3 Mathematical Calculations

From the Alternative Fuels Data Center, using the most recently updated fueling center counts in February 2016, we can find the overall fueling center counts for each state. Then, we can divide that number by the total square miles of the state to get the density of fueling centers per each square mile.

$$density = \frac{\#\ of\ alternative\ fueling\ centers\ in\ state}{total\ square\ miles\ of\ state} \tag{6}$$

We made the simplification that this density ratio will remain consistent for the city inside the state to help measure the rankings. For the four case study examples, we found fueling center densities of the following:

State	# of fueling centers	total sq. miles	Density ratio
New York	738	54,555	0.0135
Virginia	359	42,775	0.0084
California	3679	163,695	0.0225
Tennessee	518	42,144	0.0123

Table 12: Number of fueling centers in each state.

By density ratio alone, the states would rank in proportion adopted (highest to lowest) from California, New York, Tennessee, and Virginia. However, we will input this density ratio value into the earlier stated equation to find a realistic growth function of this proportion.

4.4 Revisiting Part 2

We found in part 2 that the expected value of the participants in self-driving services is the sum of the individual probabilities of every person in the city. The emergence of self-driving car technology, however, seemingly eliminates the urban-rural geographical factor, as those living in both urban and rural areas now have similar access to self-driving technology (see Assumptions and Justifications). Our equation can thus be adjusted:

$$Expected\ Value\ of\ Participants = \sum_{j=0}^{City\ Population} P_j = \sum_{j=0}^{City\ Population} \alpha * P_j(S|I) * P_j(S|H) \tag{7}$$

4.5 Incorporating the Renewable Energy Density Ratio

The renewable energy density ratio measures the number of renewable energy stations per square mile. In adjusting part 2 to account for the emergence of renewable energy transportation, we need to incorporate the renewable energy density ratio.

Let's think logically about the economics of renewable energy stations. Originally, when very few renewable energy stations are erected in a city, garnering participation will be difficult because of the relative recency of the technology. Eventually, as more energy stations emerge, the technology will rise to prominence and consumption will increase. Indeed, as greater amounts of renewable energy stations are erected, more people will switch to renewable car-sharing services. However, the population using renewable energy car-sharing services is limited by the population that can afford this novel technology. This model equates to logistic growth. Indeed, as explained by Kucharavy and Guio from The Triz Journal, logistic growth is an effective "quantitative model for predicting the diffusion of new technologies."

We can thus produce a multiplier that incorporates renewable energy. If Y is the number of people using renewable fuel technology, P is the percentage of the population wealthy enough to buy renewable fuel technology, and x is the renewable energy density ratio, we have the following equation:

$$\frac{dY}{dx} = \beta * Y(1 - Y) \tag{8}$$

which has the solution

$$Y = \frac{1}{(1 + ke^{-\beta x})} \tag{9}$$

where β and k are constants, and x is the renewable energy density ratio incremented over time. We can now use this logistic equation to calculate the renewable energy density multiplier, or the value that we can multiply our expected value by. Note that the logistic growth equation shows the percentage of the population that would gravitate toward renewable energy; to find the renewable energy multiplier (REM), we compute:

$$Renewable\ Energy\ Multiplier\ (REM) = 1 + Y = 1 + \frac{1}{(1 + ke^{-\beta x})} \tag{10}$$

We now have:

$$\begin{aligned} Adjusted\ Car\ Sharing\ Participants &= REM * Expected\ Value\ of\ Participants \\ &= (1 + \frac{1}{(1 + ke^{-\beta x})}) * \sum_{j=0}^{City\ Population} \alpha * Pj(S|I) * Pj(S|H) \end{aligned} \tag{11}$$

4.6 Real World Application

We incorporated real world data from Table 12 in Section 4.3, as well as data in Tables 8, 9, 10, and 11 from Section 3.5, and used MATLAB to account for the new multipliers The new rankings for the cities are:

Round Trip: Poughkeepsie, NY

1-way Floating: Richmond, VA

1-way Station: Knoxville, TN

Partial Ownership: Poughkeepsie NY

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